Supplementary Material: Online Appendix

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Estimating the Gains from Trade in Frictional Local Labor Markets

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A Theoretical Framework

This section contains details of the model and of derivations that were omitted in the main text. The presentation is not necessarily self-contained but rather complementary with Section 2 of the paper.

The demand structure, introduced in subsection A.1, is common to all the market structures considered in the paper. Subsections A.2 to A.8 focus on the case of monopolistic competition with free entry and heterogeneous firms (MC-FE-HET). Sections A.9 and A.10 consider the special cases of homogeneous firms (MC-FE-HOM) and restricted entry (MC-RE-HET), respectively. Finally, Section A.11 analyzes a perfectly competitive multi-industry Armington model with frictional labor markets (PC).

A.1 Demand

The preferences of the normative representative consumer in location n are described by a time-separable and stationary two-tier utility function

$$U_n = \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i}, \quad \sum_{i=1}^{I} \alpha_i = 1, \tag{A.1}$$

where the consumption of good i in period t is a CES aggregate

$$Y_{int} = \left[\int_{\omega \in \Omega_{int}} q_{int}(\omega)^{\frac{\sigma_i - 1}{\sigma_i}} d\omega\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1.$$

$q_{int}(\omega)$ denotes the consumption of variety ω of good i and Ω_{int} is the set of varieties available to the consumer. The latter is endogenous under MC-FE-HET and MC-FE-HOM, and exogenous under PC and MC-RE. The price index dual to Y_{int} is

$$P_{int} = \left[\int_{\omega \in \Omega_{int}} p_{int}(\omega)^{1-\sigma_i} d\omega\right]^{\frac{1}{1-\sigma_i}},$$

where $p_{int}(\omega)$ denotes the price of variety ω.

In each location, there is a sequence of markets in one-period-ahead claims to consumption of each good i. We assume that these assets are not tradable across locations. Let a_{int+1} denote the claims to time $t+1$ consumption of good i and Q_{int} denote the price of 1 unit of this asset at time t. Note that both quantity and price of this asset are state-independent in the absence of aggregate uncertainty, a property that holds in equilibrium. The consumer then faces a sequence of budget constraints

$$\sum_i P_{int} Y_{int} + a_{int+1} Q_{int} \leq \sum_i a_{int} P_{int} + W_{nt}, \quad t \geq 1,$$

where W_{nt} denotes aggregate income (labor income and aggregate profits, if any) in location n. We rule out Ponzi schemes by implicitly imposing a natural debt limit.

The first-order conditions with respect to Y_{mnt} for good $m \in \{1, ..., I\}$, and the Langrange multiplier η_{nt} for the time t budget constraint, can be expressed as

$$\alpha_m \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i} (Y_{mnt})^{-1} = \eta_{nt} P_{mnt}, \tag{A.2}$$

$$\sum_i P_{int} Y_{int} + a_{int+1} Q_{int} = \sum_i a_{int} P_{int} + W_{nt}. \tag{A.3}$$

In a stationary equilibruim, $a_{int+1} = 0$ for all i and t, and $W_{nt} = W_n$ for all t. Imposing these conditions in (A.3) and using (A.2) yields

$$\sum_{m=1}^{I} \alpha_m \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i} (\eta_{nt})^{-1} = W_n. \tag{A.4}$$

Let $\tilde{V}_{nt} = \prod_{i=1}^{I} (Y_{int})^{\alpha_i}$. Under stationarity, $\tilde{V}_{nt} = \tilde{V}_n$ and $Y_{mnt} = Y_{mn}$ for all t. Equation (A.4) then becomes

$$\left(\frac{1}{1+\rho}\right)^t = \frac{W_n}{\tilde{V}_n} \eta_{nt}. \tag{A.5}$$

Plugging (A.5) into (A.2) with $Y_{mnt} = Y_{mn}$ for all t, we obtain

$$\tilde{V}_n = \prod_{i=1}^{I} (\alpha_i)^{\alpha_i} \frac{W_n}{\prod_{i=1}^{I} (P_{in})^{\alpha_i}}. \tag{A.6}$$

In turn, plugging (A.6) into (A.1) yields V_n, the indirect utility function in the stationary equilibrium,

$$V_n = (\rho)^{-1} \prod_{i=1}^{I} (\alpha_i)^{\alpha_i} \frac{W_n}{\prod_{i=1}^{I} (P_{in})^{\alpha_i}}. \tag{A.7}$$

A.2 The Firm's Problem

Throughout this section, we consider a firm with poductivity φ in industry i located in city c.

A.2.1 The (Conditional) Revenue Function

Suppose that the firm is employing l production workers and serving a given set of destinations at some point in time. Let $I_{icn}(\varphi)$ denote an export decision indicator for an arbitrary destination n. In this section, we take l and $I_{icn}(\varphi)$ as given and characterize the optimal allocation of workers across destinations served by the firm. This will allow us to derive the firm's revenue function conditional on l and $I_{icn}(\varphi)$.

Let $l_{icn}(\varphi)$ denote the mass of production workers allocated by the firm to serve market n. Then $l = \sum_n I_{icn}(\varphi) l_{icn}(\varphi)$. At a given point in time, the firm's revenue, output and demand in any destination n can be written, respectively, as

$$r_{icn}(\varphi) \equiv p_{icn}(\varphi) q_{icn}(\varphi), \tag{A.8}$$

$$y_{icn}(\varphi) = l_{icn}(\varphi) \varphi, \tag{A.9}$$

$$q_{icn}(\varphi) = X_{in} \frac{(p_{icn}(\varphi))^{-\sigma_i}}{(P_{in})^{1-\sigma_i}}, \tag{A.10}$$

where $l_{icn}(\varphi)$ is the mass of production workers hired by the firm to serve market n.[1] Moreover, due to transportation costs,

$$q_{icn}(\varphi) = y_{icn}(\varphi) (\tau_{icn})^{-1}. \tag{A.11}$$

[1] Note that these expressions apply at any given point in time t, not just in stationary equilibrium. Because in this section we focus on a static problem, however, we simplify notation by omitting the time index.

Using (A.9), (A.10) and (A.11)

$$q_{icn}(\varphi) = l_{icn}(\varphi)\varphi(\tau_{icn})^{-1}, \tag{A.12}$$

$$p_{icn}(\varphi) = \left(\frac{l_{icn}(\varphi)\varphi}{\tau_{icn}A_{in}}\right)^{-\frac{1}{\sigma_i}}, \tag{A.13}$$

where $A_{in} = X_{in}\left(P_{in}\right)^{\sigma_i - 1}$ is the industry-specific demand shifter in destination n.

Equations (A.12) and (A.13), imply that revenue from sales in n can be written as a function of $l_{icn}(\varphi)$,

$$r_{icn}(\varphi) = (A_{in})^{\frac{1}{\sigma_i}}\left(\frac{l_{icn}(\varphi)\varphi}{\tau_{icn}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}. \tag{A.14}$$

Using (A.13), we can then express the marginal revenue of allocating an additional production worker to serve market n as

$$\frac{\partial r_{icn}(\varphi)}{\partial l_{icn}(\varphi)} = p_{icn}(\varphi)\left(\frac{\varphi}{\tau_{icn}}\right)\left(\frac{\sigma_i - 1}{\sigma_i}\right).$$

An efficient allocation of workers requires equating marginal revenue across all destinations. This implies

$$p_{icn}(\varphi) = \tau_{icn}p_{icc}(\varphi), \tag{A.15}$$

for all n. Using (A.13) and (A.15), relative employment across any two destinations n and $n^{'}$ served by the firm can be written as

$$\frac{l_{icn}(\varphi)}{l_{icn'}(\varphi)} = \frac{A_{icn}}{A_{icn'}}\left(\frac{\tau_{icn}}{\tau_{icn'}}\right)^{1-\sigma_i}.$$

For $n^{'} = c$, $\tau_{icc} = 1$ implies

$$l_{icn}(\varphi) = l_{icc}(\varphi)\left(\tau_{icn}\right)^{1-\sigma_i}\left(\frac{A_{in}}{A_{ic}}\right). \tag{A.16}$$

Using $l = \sum_n I_{icn}(\varphi)l_{icn}(\varphi)$,

$$l_{icc}(\varphi) = \frac{A_{ic}}{\sum_{n'} I_{icn'}(\varphi)A_{in'}\left(\tau_{icn'}\right)^{1-\sigma_i}}l. \tag{A.17}$$

Moreover, substituting (A.17) into (A.16) yields

$$l_{icn}(\varphi) = \frac{\left(\tau_{icn}\right)^{1-\sigma_i}A_{in}}{\sum_{n'} I_{icn'}(\varphi)A_{in'}\left(\tau_{icn'}\right)^{1-\sigma_i}}l. \tag{A.18}$$

The firm's total revenue conditional on l is $r_{ic}(l;\varphi) = \sum_n r_{icn}(\varphi)I_{icn}(\varphi)$. Using (A.14) and (A.18), we can express it as

$$r_{ic}(l;\varphi) = \left[\sum_n I_{icn}(\varphi)A_{in}\left(\tau_{icn}\right)^{1-\sigma_i}\right]^{\frac{1}{\sigma_i}}(l\varphi)^{\frac{\sigma_i - 1}{\sigma_i}}. \tag{A.19}$$

A.2.2 Optimal Vacancy Posting

We now study the dynamic behavior of the firm, taking all export decisions as given and constant over time. In a stationary equilibrium, the firm faces a time-invariant revenue function given by (A.19). The firm determines its optimal employment growth by posting vacancies, denoted v, with the goal of maximizing the present value of expected profits. We show that employment in a firm that starts with no workers reaches its optimal long-run level in the following period.

Suppose that the firm is currently employing l production workers. Then it solves

$$\Pi_{ic}(l;\varphi) = \max_{v} \frac{1}{1+\rho}\left\{ r_{ic}(l;\varphi) - w_{ic}(l;\varphi)l - w_{ic}\sum_{n} I_{icn}(\varphi)f_{icn} - k_{ic}v + (1-\delta_c)\,\Pi_{ic}(l';\varphi)\right\},$$
$$s.t. \quad l' = l + m_c(\theta_c)v,$$

where $l^{'}$ is the level of employment next period. For tractability, the firm takes the wage of non-production workers w_{ic} as given when maximizing profits; the main text provides a rationale for this assumption. Note also that the vacancy posting cost is in units of a numeraire whose price is normalized to one in every city and industry.[2]

The first order condition for vacancy posting can be written as:

$$(1-\delta_c)\frac{\partial \Pi_{ic}(l';\varphi)}{\partial l'} = \frac{k_{ic}}{m_c(\theta_c)}, \tag{A.20}$$

Note that optimal employment size is independent of current employment l and constant over time as long as the firm is not forced to exit the market. Hence the firm converges to its optimal employment size in one period. From this point on, $l = l^{'}$. Using this condition and the envelope theorem yields

$$\frac{\partial \Pi_{ic}(l;\varphi)}{\partial l} = \frac{1}{\rho+\delta_c}\left[\frac{\partial r_{ic}(l;\varphi)}{\partial l} - w_{ic}(l;\varphi) - \frac{\partial w_{ic}(l;\varphi)}{\partial l}l\right]. \tag{A.21}$$

Combining (A.20) and (A.21) with $l = l^{'}$, we can obtain the implicit optimal pricing rule of the firm,

$$\frac{\partial r_{ic}(l;\varphi)}{\partial l} = \frac{\partial w_{ic}(l;\varphi)}{\partial l}l + w_{ic}(l;\varphi) + \frac{k_{ic}}{m_c(\theta_c)}\left(\frac{\rho+\delta_c}{1-\delta_c}\right). \tag{A.22}$$

A.2.3 Bargaining

This section follows the analysis in Felbermayr et al. (2011). As in Stole & Zwiebel (1996), we assume that the bargaining outcome over the division of the total surplus from a match satisfies the following surplus-splitting rule:

$$(1-\beta_i)\left[E_{ic}(l;\varphi) - U_c\right] = \beta_i \frac{\partial \Pi_{ic}(l;\varphi)}{\partial l}, \tag{A.23}$$

where U_c is the worker's outside option (i.e. the value of unemployment) and $E_{ic}(l;\varphi)$ is the value of employment in a firm with productivity φ and l production workers. The Bellman equation for workers can be written as:

$$E_{ic}(l;\varphi) - U_c = \frac{w_{ic}(l;\varphi) - \rho U_c}{(\rho+\delta_c)}. \tag{A.24}$$

[2] We can motivate this property by following a standard practice in the trade literature that assumes the existence of a freely traded 'outside' good to pin down a single price of the numeraire across countries. For our purposes, we can assume the existence of an outside city or country that has a comparative advantage in the production of a freely traded homogeneous service used exclusively as an input towards posting vacancies. We choose this service as the numeraire. Parameter k_{ic} can then be interpreted as a fixed number of units of this service required to post one vacancy in industry i of city c. We ignore the analysis of the (uninteresting) outside city elsewhere in the paper.

Inserting (A.21) and (A.24) into (A.23) yields

$$w_{ic}(l;\varphi) = (1-\beta_i)\rho U_c + \beta_i \frac{\partial r_{ic}(l;\varphi)}{\partial l} - \beta_i \frac{\partial w_{ic}(l;\varphi)}{\partial l} l. \tag{A.25}$$

Using the revenue function (A.19), one can verify by direct substitution that

$$w_{ic}(l;\varphi) = (1-\beta_i)\rho U_c + \left(\frac{\sigma_i}{\sigma_i - \beta_i}\right) \frac{\partial r_{ic}(l;\varphi)}{\partial l} \beta_i, \tag{A.26}$$

solves (A.25). Differentiating this equation with respect to l, we obtain:

$$\frac{\partial w_{ic}(l;\varphi)}{\partial l} = \left(-\frac{\beta_i}{\sigma_i - \beta_i}\right) \frac{\frac{\partial r_{ic}(l,\varphi)}{\partial l}}{l}.$$

Substituting this expression in (A.22) yields

$$w_{ic}(l;\varphi) = \left(\frac{\sigma_i}{\sigma_i - \beta_i}\right) \frac{\partial r_{ic}(l;\varphi)}{\partial l} - \frac{k_{ic}}{m_c(\theta_c)} \left(\frac{\rho + \delta_c}{1-\delta_c}\right). \tag{A.27}$$

From (A.26) and (A.27), we can express the Wage Curve as a function of θ:

$$w_{ic} = \rho U_c + \frac{\beta_i}{(1-\beta_i)} \left(\frac{\rho+\delta_c}{1-\delta_c}\right) \frac{k_{ic}}{m_c(\theta_c)}. \tag{A.28}$$

A.2.4 Firm-level Outcomes

Upon entry, the firm starts with zero workers but immediately recruits workers to achieve its optimal size in the following period.

Let $l^T_{ic}(\varphi) = l_{ic}(\varphi) + l^F_{ic}(\varphi)$ denote the firm's optimal employment size, where $l_{ic}(\varphi)$ is the optimal employment of production workers and $l^F_{ic}(\varphi) = f_{icc} + \sum_n I_{icn}(\varphi) f_{icn}$ is the mass of non-production workers, given a set of export decisions $I_{icn}(\varphi)$ for all n. The expected profits of the firm upon entry can then be written as:

$$\Pi_{ic}(0;\varphi) = \frac{1}{1+\rho}\left[-\frac{k_{ic}}{m_c(\theta_c)} l^T_{ic}(\vartheta) + (1-\delta_c)\Pi_{ic}(\varphi)\right], \tag{A.29}$$

where

$$\Pi_{ic}(\varphi) = \frac{1}{1+\rho}\left[r_{ic}(\varphi) - w_{ic} l^T_{ic}(\varphi) + (1-\delta_c)\Pi_{ic}(\varphi)\right] \tag{A.30}$$

is the value function of the vacancy posting problem evaluated at the firm's (constant) optimal employment size. That is, $\Pi_{ic}(\varphi) = \Pi_{ic}(l_{ic}(\varphi);\varphi)$ and $r_{ic}(\varphi) = r_{ic}(l_{ic}(\varphi);\varphi)$, after a slight abuse of notation. Using (A.30) to rewrite (A.29) yields

$$\Pi_{ic}(0;\varphi) = \frac{1}{1+\rho}\left[-\frac{k_{ic}}{m_c(\theta_c)} l^T_{ic}(\varphi) + \left(\frac{1-\delta_c}{\rho+\delta_c}\right)\left(r_{ic}(\varphi) - w_{ic} l^T_{ic}(\varphi)\right)\right]. \tag{A.31}$$

We can now define the cost of labor in industry i of city c, denoted μ_{ic}, as

$$\mu_{ic} = w_{ic} + \left(\frac{\rho+\delta_c}{1-\delta_c}\right) \frac{k_{ic}}{m_c(\theta_c)}. \tag{A.32}$$

μ_{ic} can be interpreted as the per-period cost of hiring an additional worker in industry i of city c. To see this, use (A.32) to rewrite (A.31) as

$$\Pi_{ic}(0;\varphi) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}\left[r_{ic}(\varphi) - \mu_{ic} l^T_{ic}(\varphi)\right]. \tag{A.33}$$

Using the definition of $l_{ic}^{T}(\varphi)$, we can now define the per-period profits of the firm (gross of the entry cost) as

$$\pi_{ic}(\varphi) = r_{ic}(\varphi) - \mu_{ic} l_{ic}(\varphi) - \mu_{ic} \sum_{n} I_{icn}(\varphi) f_{icn}. \tag{A.34}$$

Note that (A.32), (A.27) and the revenue equation (A.19) imply

$$\mu_{ic} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i} \right) \frac{r_{ic}(\varphi)}{l_{ic}(\varphi)}. \tag{A.35}$$

Substituting this into (A.34), we can rewrite the per-period profit function as in the main text,

$$\pi_{ic}(\varphi) = \left(\frac{1 - \beta_i}{\sigma_i - \beta_i} \right) r_{ic}(\varphi) - \mu_{ic} \sum_{n} I_{icn}(\varphi) f_{icn}. \tag{A.36}$$

Equations (A.19) and (A.35) allow us to compute the firm's optimal employment of production workers in terms of μ_{ic}

$$l_{ic}(\varphi) = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i} \right)^{\sigma_i} \frac{(\varphi)^{\sigma_i - 1}}{(\mu_{ic})^{\sigma_i}} \sum_{n} I_{icn}(\varphi) A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.37}$$

Using (A.35) and (A.37), yields the firm's per-period revenue

$$r_{ic}(\varphi) = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i} \right)^{\sigma_i - 1} \left(\frac{\varphi}{\mu_{ic}} \right)^{\sigma_i - 1} \sum_{n} I_{icn}(\varphi) A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.38}$$

Next, use (A.8) and (A.12) to obtain

$$p_{icn}(\varphi) = \frac{r_{icn}(\varphi)}{l_{icn}(\varphi)} \frac{\tau_{icn}}{\varphi}.$$

Combining this with (A.35) yields the profit maximizing price in terms of μ_{ic}

$$p_{icn}(\varphi) = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \right) \left(\frac{\mu_{ic}}{\varphi} \right) \tau_{icn}. \tag{A.39}$$

A.3 Entry

A.3.1 The Cost of Entry

In order to discover its productivity, the firm commits to an investment that requires hiring f_e workers upon entry and in each subsequent period with probability $1 - \delta_c$. This setting ensures that the per-period cost of an entry worker is equal to the per-period cost of hiring production workers, a standard property in frictionless trade models.

The present value of the entry cost can be written as

$$\frac{1}{1+\rho} \left[\frac{k_{ic}}{m_c(\theta_c)} f_e + \left(\frac{1-\delta_c}{1+\rho} \right) f_e w_{ic} + \left(\frac{1-\delta_c}{1+\rho} \right)^2 f_e w_{ic} + \left(\frac{1-\delta_c}{1+\rho} \right)^3 f_e w_{ic} + \ldots\ldots \right].$$

Rearranging terms yields

$$\frac{f_e}{1+\delta_c} \left(\frac{1-\delta_c}{\rho+\delta_c} \right) \left[w_{ic} + \frac{k_{ic}}{m_c(\theta_c)} \left(\frac{\rho+\delta_c}{1-\delta_c} \right) \right].$$

Using (A.32), the present value of the entry cost can be written as a function of the local cost of labor, μ_{ic},

$$\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)} f_e \mu_{ic}.$$

A.3.2 The Free Entry Condition

Under free entry, the expected profits are equal to the present value of the entry cost. Using (A.33) and (A.34), the free entry condition can be written as

$$\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}\int_0^\infty \pi_{ic}(\varphi)dG_{ic}(\varphi)=\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}f_e\mu_{ic},$$

where $\pi_{ic}(\varphi)$ is per-period profit. Substituting the revenue function (A.38) into the per-period profit function (A.36) and imposing $I_{icn}(\varphi)=1$ if $\varphi \geq \varphi^*_{icn}$, the free entry condition becomes

$$f^e_{ic}\mu_{ic}=\sum_n\int_{\varphi^*_{icn}}^\infty\left[\left(\frac{(\sigma_i-\beta_i)\tau_{icn}\mu_{ic}}{(\sigma_i-1)}\right)^{1-\sigma_i}A_{in}\left(\varphi\right)^{\sigma_i-1}\left(\frac{1-\beta_i}{\sigma_i-\beta_i}\right)-f_{icn}\mu_{ic}\right]dG_{ic}(\varphi).$$

Using the cutoff condition in destination n (equation (12) in the main text), we obtain

$$f^e_{ic}=\sum_n\int_{\varphi^*_{icn}}^\infty f_{icn}\left[\left(\frac{\varphi}{\varphi^*_{icn}}\right)^{\sigma_i-1}-1\right]dG_{ic}(\varphi).$$

Assume that $G_{ic}(\varphi)$ is a Pareto distribution, with shape parameter κ_i and lower bound $\varphi_{min,ic}$. If $\kappa_i>\sigma_i-1$, then the integral has a closed-form solution. In this case, the free entry condition simplifies to

$$f^e_{ic}=\frac{(\sigma_i-1)}{(\kappa_i-\sigma_i+1)}\sum_n f_{icn}\left(\frac{\varphi_{min,ic}}{\varphi^*_{icn}}\right)^{\kappa_i}. \tag{A.40}$$

A.4 Gravity

Bilateral exports from city c to destination n in industry i can be decomposed into the mass of exporting firms times average firm exports:

$$X_{icn}=\left(\frac{1-G_{ic}(\varphi^*_{icn})}{1-G_{ic}(\varphi^*_{icc})}\right)M_{ic}\int_{\varphi^*_{icn}}^\infty\left(\frac{\sigma_i-1}{\sigma_i-\beta_i}\right)^{\sigma_i-1}\left(\frac{\varphi}{\mu_{ic}}\right)^{\sigma_i-1}\frac{X_{in}}{(P_{in})^{1-\sigma_i}}\left(\tau_{icn}\right)^{1-\sigma_i}\frac{dG_{ic}(\varphi)}{1-G(\varphi^*_{icn})},$$

using (A.38) to compute export revenue in n.

Under Pareto productivity, we obtain

$$X_{icn}=M_{ic}\left(\frac{\varphi^*_{icc}}{\varphi_{min,ic}}\right)^{\kappa_i}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\tau_{icn}\mu_{ic}\right)^{1-\sigma_i}\frac{X_{in}}{(P_{in})^{1-\sigma_i}}\left(\frac{\kappa_i}{\kappa_i-\sigma_i+1}\right)(\varphi_{min,ic})^{\kappa_i}(\varphi^*_{icn})^{\sigma_i-\kappa_i-1}. \tag{A.41}$$

We can further simplify (A.41) using the export cutoff condition (12) and the aggregate stability condition; i.e. $\delta_c M_{ic}=[1-G_{ic}(\varphi^*_{icc})]M^e_{ic}$. This yields the standard decomposition of bilateral exports into the extensive and intensive margins of trade,

$$X_{icn}=\frac{M^e_{ic}}{\delta_c}\left(\frac{\varphi_{min,ic}}{\varphi^*_{icn}}\right)^{\kappa_i}\mu_{ic}f_{icn}\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)\left(\frac{\kappa_i}{\kappa_i-\sigma_i+1}\right). \tag{A.42}$$

For estimation purposes, it is convenient to work with the share of exports in sectoral revenue, X_{icF}/R_{ic}, where

$$R_{ic}\equiv\sum_v X_{icv}=\frac{M^e_{ic}}{\delta_c}\mu_{ic}\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)\left(\frac{\kappa_i}{\kappa_i-\sigma_i+1}\right)\sum_v\left(\frac{\varphi_{min,ic}}{\varphi^*_{icv}}\right)^{\kappa_i}f_{icv}.$$

Therefore,

$$\frac{X_{icn}}{R_{ic}}=\frac{\left(\frac{\varphi_{min,ic}}{\varphi^*_{icn}}\right)^{\kappa_i}f_{icn}}{\sum_v\left(\frac{\varphi_{min,ic}}{\varphi^*_{icv}}\right)^{\kappa_i}f_{icv}}.$$

Using free entry condition (A.40), the export share simplifies to

$$\frac{X_{icn}}{R_{ic}} = \left(\frac{\sigma_i - 1}{\kappa_i - \sigma_i + 1}\right) \frac{f_{icn}}{f_{ic}^e} \left(\frac{\varphi_{min,ic}}{\varphi^*_{icn}}\right)^{\kappa_i}.$$

Finally, imposing the export cutoff condition (12), we obtain the local gravity equation (14) in the main text,

$$\frac{X_{icn}}{R_{ic}} = \left(\frac{\sigma_i - 1}{\kappa_i - \sigma_i + 1}\right) \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right)^{\frac{\kappa_i \sigma_i}{\sigma_i - 1}} (\varphi_{min,ic})^{\kappa_i} \frac{(f_{icn})^{\frac{\sigma_i - 1 - \kappa_i}{\sigma_i - 1}}}{f_{ic}^e} (A_{in})^{\frac{\kappa_i}{\sigma_i - 1}} (\tau_{icn})^{-\kappa_i} (\mu_{ic})^{\frac{-\kappa_i \sigma_i}{\sigma_i - 1}}.$$

A.5 Labor Demand and Supply

The stationary demand for production workers in industry i of city c can be computed as the sum of destination-specific labor demands for producers serving destination n:

$$\sum_n M_{ic} \frac{1 - G_{ic}(\varphi^*_{icn})}{1 - G_{ic}(\varphi^*_{icc})} \left[\int_{\varphi^*_{icn}}^{\infty} l_{icn}(\varphi) \frac{dG_{ic}(\varphi)}{1 - G_{ic}(\varphi^*_{icn})}\right], \tag{A.43}$$

where the demand for workers producing output for n in firm φ, $l_{icn}(\varphi)$, is obtained from (A.37) by setting $I_{icn}(\varphi) = 1$ and $I_{icv}(\varphi) = 0$ for $v \neq n$. The term in brackets in (A.43) is then the average destination-specific demand for production workers across firms serving n.

Under Pareto productivity, we can evaluate the integral in (A.43) and obtain

$$M_{ic} \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i} \kappa_i \left[\sum_n \frac{X_{in}}{(P_{in})^{1-\sigma_i}} \frac{(\tau_{icn})^{1-\sigma_i}}{(\mu_{ic})^{\sigma_i}} (\varphi^*_{icc})^{\kappa_i} \frac{(\varphi^*_{icn})^{\sigma_i - \kappa_i - 1}}{\kappa_i - \sigma_i + 1}\right].$$

Using the export cutoff conditions yields a simplified expression for the demand for production workers

$$M_{ic} \frac{(\sigma_i - 1)\kappa_i}{(\kappa_i - \sigma_i + 1)(1 - \beta_i)} \sum_n \left(\frac{\varphi^*_{icc}}{\varphi^*_{icn}}\right)^{\kappa_i} f_{icn}.$$

In turn, the industry's stationary labor demand due to fixed and entry costs is:[3]

$$\frac{M_{ic}}{1 - G_{ic}(\varphi^*_{icc})} f_{ic}^e + \sum_n M_{ic} \left[\frac{1 - G_{ic}(\varphi^*_{icn})}{1 - G_{ic}(\varphi^*_{icc})}\right] f_{icn}.$$

Next, we show that M^e_{ic}, the mass of entrants, is proportional to L_{ic}, the mass of workers that are succesfully matched in the industry. Equating L_{ic} to the aggregate labor demand in industry i of city c under Pareto productivity yields

$$L_{ic} = M_{ic} \left[\sum_n \left(\frac{\varphi^*_{icc}}{\varphi^*_{icn}}\right)^{\kappa_i} f_{icn} \left(\frac{(\sigma_i - 1)\kappa_i}{(\kappa_i - \sigma_i + 1)(1 - \beta_i)} + 1\right) + \frac{f^e_{ic}}{\left(\frac{\varphi_{min,ic}}{\varphi^*_{icc}}\right)^{\kappa_i}}\right]. \tag{A.44}$$

Imposing the free entry condition (A.40) and the aggregate stability condition $\delta_c M_{ic} = [1 - G_{ic}(\varphi^*_{icc})] M^e_{ic}$, we obtain:

$$M^e_{ic} = \frac{\delta_c}{\kappa_i f^e_{ic}} \left[\frac{1}{1 - \beta_i} + \frac{1}{\sigma_i - 1}\right]^{-1} L_{ic}. \tag{A.45}$$

[3]It is straightforward to verify that, under the entry protocol described in section A.3.1, the industry's demand for entry workers is equal to $M^e_{ic} f^e_{ic} / \delta_c$ in the stationary equilibrium.

A.6 Price Index

The price index in industry i of city c can be expressed as follows:

$$P_{in}^{1-\sigma_i} = \sum_v M_{iv} \left[\frac{1 - G_{iv}(\varphi_{inv}^*)}{1 - G_{iv}(\varphi_{ivv}^*)} \right] \int_{\varphi_{ivn}^*}^{\infty} p_{ivn}(\varphi)^{1-\sigma_i} dG_{iv}(\varphi | \varphi \geq \varphi_{inv}^*).$$

Substituting optimal firm prices (A.39) and imposing Pareto productivity yields:

$$P_{in}^{1-\sigma_i} = \frac{\kappa_i}{\kappa_i - \sigma_i + 1} \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \right)^{1-\sigma_i} \sum_v M_{iv} (\varphi_{ivv}^*)^{\kappa_i} (\varphi_{ivn}^*)^{\sigma_i - \kappa_i - 1} (\mu_{iv} \tau_{ivn})^{1-\sigma_i}.$$

Using the export cutoff condition, the price index becomes

$$P_{in}^{-\kappa_i} = \frac{\kappa_i}{\kappa_i - \sigma_i + 1} \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \right)^{-\kappa_i} \left(\frac{\sigma_i - \beta_i}{1 - \beta_i} \right)^{1 - \frac{\kappa_i}{\sigma_i - 1}} \sum_v M_{iv} (\varphi_{ivv}^*)^{\kappa_i} (\mu_{iv} \tau_{ivn})^{-\kappa_i} \left(\frac{\mu_{iv} f_{ivn}}{X_{in}} \right)^{1 - \frac{\kappa_i}{\sigma_i - 1}}. \tag{A.46}$$

A.7 Trade Share

From (A.42), the share of total income of location n spent on goods from city c in industry i can be expressed as:

$$\lambda_{icn} = \frac{X_{icn}}{\sum_v X_{ivn}} = \frac{\delta_c^{-1} M_{ic}^e f_{icn} \mu_{ic} \left(\frac{\varphi_{min,ic}}{\varphi_{icn}^*} \right)^{\kappa_i}}{\sum_v \delta_{iv}^{-1} M_{iv}^e f_{ivn} \mu_{iv} \left(\frac{\varphi_{min,iv}}{\varphi_{ivn}^*} \right)^{\kappa_i}}.$$

Using (A.45),

$$\lambda_{icn} = \frac{L_{ic} \left(\frac{f_{icn}}{f_{ic}^e} \right) \mu_{ic} \left(\varphi_{icn}^* \right)^{-\kappa_i} \left(\varphi_{min,ic} \right)^{\kappa_i}}{\sum_v L_{iv} \left(\frac{f_{ivn}}{f_{iv}^e} \right) \mu_{iv} \left(\varphi_{ivn}^* \right)^{-\kappa_i} \left(\varphi_{min,iv} \right)^{\kappa_i}}.$$

Using export cutoff conditions, city c's trade share in location n's expenditure on good i can be written as:

$$\lambda_{icn} = \frac{\left(\frac{L_{ic}}{f_{ic}^e} \right) (f_{icn})^{1 - \frac{\kappa_i}{\sigma_i - 1}} (\mu_{ic})^{1 - \frac{\kappa_i \sigma_i}{\sigma_i - 1}} (\varphi_{min,ic})^{\kappa_i} (\tau_{icn})^{-\kappa_i}}{\sum_v \left(\frac{L_{iv}}{f_{iv}^e} \right) (f_{ivn})^{1 - \frac{\kappa_i}{\sigma_i - 1}} (\mu_{iv})^{1 - \frac{\kappa_i \sigma_i}{\sigma_i - 1}} (\varphi_{min,iv})^{\kappa_i} (\tau_{ivn})^{-\kappa_i}}.$$

A.8 Welfare

From (A.7), the welfare of the normative representative consumer in any location n can be written as

$$V_n = (\rho)^{-1} \prod_{i=1}^{I} (\alpha_i)^{\alpha_i} \frac{\sum_{i=1}^{I} L_{in} w_{in}}{\prod_{i=1}^{I} (P_{in})^{\alpha_i}},$$

where $W_n = \sum_{i=1}^{I} L_{in} w_{in}$ since, net of entry costs, aggregate profits are zero.

Next, we rewrite the price index (A.46) using (i) the stability condition and (A.45) to express the mass of firms as a function of the labor allocation,[4] and (ii) $X_{in} = \alpha_i W_n$, an implication of the Cobb-Douglas assumption:

$$P_{in}^{-\kappa_i} = \frac{\left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \right)^{-\kappa_i} \left(\frac{\sigma_i - \beta_i}{1 - \beta_i} \right)^{1 - \frac{\kappa_i}{\sigma_i - 1}} \sum_v \left[\left(\frac{L_{iv}}{f_{iv}^e} \right) (\varphi_{min,vi}^*)^{\kappa_i} (\tau_{ivn})^{-\kappa_i} (\mu_{iv})^{1 - \frac{\kappa_i \sigma_i}{\sigma_i - 1}} (f_{ivn})^{1 - \frac{\kappa_i}{\sigma_i - 1}} \right]}{(\kappa_i - \sigma_i + 1) \left[\frac{1}{1 - \beta_i} + \frac{1}{\sigma_i - 1} \right] (\alpha_i W_n)^{1 - \frac{\kappa_i}{\sigma_i - 1}}}.$$

[4] $M_{iv} = \left(\frac{\varphi_{min,iv}}{\varphi_{ivv}} \right)^{\kappa_i} \frac{L_{iv}}{\kappa_i f_{iv}^e} \left[\frac{1}{1 - \beta_i} + \frac{1}{\sigma_i - 1} \right]^{-1}.$

In turn, the *domestic* trade share of industry i in location n can be expressed as:

$$\lambda_{inn} = \frac{\left(\frac{L_{in}}{f_{in}^e}\right)(f_{inn})^{1-\frac{\kappa_i}{\sigma_i-1}}(\mu_{in})^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}(\varphi_{min,in})^{\kappa_i}}{\sum_v \left[\left(\frac{L_{iv}}{f_{iv}^e}\right)(f_{inv})^{1-\frac{\kappa_i}{\sigma_i-1}}(\mu_{iv})^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}(\varphi_{min,iv})^{\kappa_i}(\tau_{ivn})^{-\kappa_i}\right]}.$$

We can now express the price index as a function of λ_{inn}:

$$P_{in} = \left[\frac{\lambda_{inn}(\alpha_i W_n)^{1-\frac{\kappa_i}{\sigma_i-1}}(\kappa_i-\sigma_i+1)\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right]}{\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\left(\frac{L_{in}}{f_{in}^e}\right)(f_{inn})^{1-\frac{\kappa_i}{\sigma_i-1}}}\right]^{\frac{1}{\kappa_i}} \frac{(\mu_{in})^{\frac{\sigma_i}{\sigma_i-1}-\frac{1}{\kappa_i}}}{(\varphi_{min,in})}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right).$$

For analytical tractability, suppose that k_{ic}, the cost of posting vacancies in city c, is small (but positive) in all industries. Equations (A.28) and (A.32) imply that cross-industry variaton in both wages and cost of labor becomes arbitrarily small; i.e. $\mu_{ic} \approx w_{ic} \approx w_c$ for all i. The price index can then be approximated as follows:

$$P_{in} \approx \left[\frac{\lambda_{inn}\left(\alpha_i \sum_j L_{jn}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}(\kappa_i-\sigma_i+1)\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right]}{\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\left(\frac{L_{in}}{f_{in}^e}\right)(f_{inn})^{1-\frac{\kappa_i}{\sigma_i-1}}}\right]^{\frac{1}{\kappa_i}} \frac{w_n}{\varphi_{min,in}}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right). \tag{A.47}$$

Consider the effects of an arbitrary shock to the vector of variable trade costs, $\{\tau_{ivn}\}$ for any industry i and any two different locations n and v, on the welfare of city c. For any endogenous variable x, let $\dot{x}$ denote the ratio of x after the shock to x before the shock; i.e. the proportional change in the stationary equilibrium value of x.

Aggregate income satisfies $W_c \approx w_c\left(\sum_{i=1}^I L_{ic}\right) = w_c e_c \bar{L}_c$, where e_c is the employment rate and $\bar{L}_c$ is the endowment of labor in city c. Therefore, $\dot{W}_c \approx \dot{e}_c \dot{w}_c$. The proportional change in welfare is

$$\dot{V}_c \approx \frac{\dot{e}_c \dot{w}_c}{\prod_{i=1}^I \left(\dot{P}_{ic}\right)^{\alpha_i}}. \tag{A.48}$$

Let η_{ic} denote the employment share of industry i in city c. Then, $\dot{L}_{ic} = \dot{\eta}_{ic}\dot{e}_c$. From (A.47), we can thus approximate the proportional change in the price index as

$$\dot{P}_{ic} \approx \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{\frac{1}{\varepsilon_i}}(\dot{e}_n)^{-\Upsilon_i}\dot{w}_c,$$

where $\Upsilon_i \equiv \frac{1}{\sigma_i-1}$ and $\varepsilon_i \equiv \kappa_i$ is the trade elasticity. Substituting this expression into (A.48), we obtain:

$$\dot{V}_c \approx (\dot{e}_c)^{1+\sum_{i=1}^I -\alpha_i\Upsilon_i} \prod_{i=1}^I \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{-\frac{\alpha_i}{\varepsilon_i}}.$$

A.9 Special Case: Monopolistic Competition, Free Entry and Homogenous Firms (MC-FE-HOM)

In this section, we impose a degenerate productivity distribution. In particular, we assume that the labor productivity of *all* firms in any industry i of any location n is equal to φ_{in}. Moreover, we assume $f_{ivn} = 0 = f_{iv}^e$. Instead, there is a fixed startup cost f_{in} that depends on the industry and location of the producer. Note that, in this setting, every firm in any cell in exports to every destination.

A.9.1 Firm Level Outcomes and Zero-Profit Condition

From (A.39), the profit maximizing price that firms in industry i of city c set in destination n is

$$p_{ivn} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right) \frac{\mu_{iv}}{\varphi_{iv}} \tau_{ivn}. \tag{A.49}$$

From (A.38), destination-specific firm revenue can be written as:

$$r_{icn} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\frac{\varphi_{ic}}{\mu_{ic}}\right)^{\sigma_i - 1} A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.50}$$

Let $r_{ic} \equiv \sum_n r_{icn}$ denote total firm revenue. From (A.33) and (A.35), expected profits upon entry can be expressed as:

$$\Pi_{ic}(0; \varphi_{ic}) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)} \left[r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right) - \mu_{ic} f_{ic} \right].$$

Due to free entry in any industry i of city c, the zero-profit condition thus requires:

$$r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right) = \mu_{ic} f_{ic}. \tag{A.51}$$

A.9.2 Gravity

Computing bilateral exports $X_{icn} \equiv M_{ic} r_{icn}$ and sectoral revenue R_{ic}, we obtain the export share:

$$\frac{X_{icn}}{R_{ic}} = \frac{A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}}{\sum_n A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}}.$$

Using the zero-profit condition to rewrite the denominator yields

$$\frac{X_{icn}}{R_{ic}} = A_{in} \left(\frac{\tau_{icn}}{\varphi_{ic}}\right)^{1-\sigma_i} \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right) \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\mu_{ic}\right)^{-\sigma_i} \left(f_{ic}\right)^{-1}.$$

Note that $\sigma_i - 1$ is the trade elasticity.

A.9.3 Labor Demand and Supply

The demand for production workers in cell ic is $M_{ic} l_{ic}$, where firm employment follows from (A.37). Labor demand from fixed costs is simply $M_{ic} f_{ic}$. Hence we obtain:

$$L_{ic} = M_{ic} \left[\left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i} \frac{\left(\varphi_{ic}\right)^{\sigma_i - 1}}{\left(\mu_{ic}\right)^{\sigma_i}} \sum_n A_{in} \left(\tau_{icn}\right)^{1-\sigma_i} + f_{ic} \right]. \tag{A.52}$$

Using the zero-profit condition (A.51), yields a proportional link between the mass of producers and the mass of workers in the industry:

$$L_{ic} = M_{ic} f_{ic} \left[\frac{\sigma_i - \beta_i}{1 - \beta_i}\right]. \tag{A.53}$$

A.9.4 Price Index

The price index is:

$$\left(P_{in}\right)^{1-\sigma_i} = \sum_v M_{iv} \left(p_{ivn}\right)^{1-\sigma_i}.$$

Using (A.49), the price index can be expressed as

$$\left(P_{in}\right)^{1-\sigma_i} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right)^{1-\sigma_i} \sum_v M_{iv} \left(\frac{\mu_{iv}}{\varphi_{iv}}\right)^{1-\sigma_i} \left(\tau_{ivn}\right)^{1-\sigma_i}. \tag{A.54}$$

A.9.5 Trade Share

The trade share is

$$\lambda_{icn} \equiv \frac{X_{icn}}{X_{in}} = \frac{M_{ic} r_{icn}}{\sum_v X_{ivn}}.$$

From revenue (A.50), the domestic trade share can be expressed as

$$\lambda_{inn} = \frac{M_{in} \left(\frac{\varphi_{in}}{\mu_{in}}\right)^{\sigma_i - 1}}{\sum_v M_{iv} \left(\frac{\varphi_{iv}}{\mu_{iv}}\right)^{\sigma_i - 1} (\tau_{ivn})^{1-\sigma_i}}. \tag{A.55}$$

A.9.6 Welfare

Using (A.53) and (A.55), the price index (A.54) in industry i of city c can be written as a function of the domestic trade share:

$$P_{ic} = (\lambda_{icc})^{\frac{1}{\sigma_i - 1}} \left(\frac{L_{ic}}{f_{ic}}\right)^{\frac{1}{1-\sigma_i}} \left[\left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right)\left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} (\varphi_{ic})^{\sigma_i - 1}\right]^{-1} \mu_{ic}. \tag{A.56}$$

If the cost of posting vacancies in city c, k_{ic} is small in all industries, then $\dot{w}_c \approx \dot{\mu_{ic}}$ in all industries. From (A.56),

$$\left(\frac{\dot{w}_c}{\dot{P}_{ic}}\right) \approx \left(\frac{\dot{\lambda}_{icc}}{\dot{L}_{ic}}\right)^{\frac{1}{1-\sigma_i}}. \tag{A.57}$$

From (A.48) and (A.57),

$$\dot{V}_c = \dot{e}_c \prod_{i=1}^{I} \left(\frac{\dot{\lambda}_{icc}}{\dot{L}_{ic}}\right)^{\frac{\alpha_i}{1-\sigma_i}}.$$

Let $\Upsilon_i \equiv \frac{1}{\varepsilon_i}$, where $\varepsilon_i \equiv \sigma_i - 1$ is the trade elasticity. Then, using $\dot{L}_{ic} = \dot{\eta}_{ic}\dot{e}_c$, the welfare gains from trade can be approximated as follows:

$$\dot{V}_c \approx (\dot{e}_c)^{1+\sum_{i=1}^{I} \alpha_i \Upsilon_i} \prod_{i=1}^{I} \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{-\frac{\alpha_i}{\varepsilon_i}}.$$

A.10 Special Case: Monopolistic Competition and Restricted Entry (MC-RE)

In the context of the model of the previous section, here we abandon the free entry condition. In particular, we follow the setup in Arkolakis et al. (2012), where the mass of producers, M_{in}, is fixed and $f_{in} = 0$ for all i and n. A distinct feature of this market structure is that aggregate profits are positive and thus need to be accounted for in the welfare analysis. We assume that profits are distributed back to the representative consumer. Although we focus on the case of homogeneous firms, the derivations below also hold for heterogenous firms with only minor changes.

Aggregate income in city c can be written as:

$$W_c = \sum_{i=1}^{I} \left[L_{ic} w_{ic} + M_{ic} \Pi_{ic}(0, \varphi_{ic})\right], \tag{A.58}$$

where,

$$\Pi_{ic}(0, \varphi_{ic}) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)} r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right),$$

and r_{ic} is defined as in section A.9.1. From (A.52), since now we have $f_{ic} = 0$ for all i and c,

$$L_{ic} = M_{ic} \sum_n l_{icn}, \tag{A.59}$$

where,

$$l_{icn} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i} A_{in} \left(\tau_{icn}\right)^{1-\sigma_i} \frac{\left(\varphi_{ic}\right)^{\sigma_i - 1}}{\left(\mu_{ic}\right)^{\sigma_i}}.$$

From (A.50), revenue can be written as a function of l_{icn}:

$$r_{icn} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right) \mu_{ic} l_{icn}.$$

Since $r_{ic} = \sum_n r_{icn}$, we can use (A.59) to write aggregate profits as a function of the mass of workers enmployed in cell ic:

$$M_{ic}\Pi_{ic}(0, \varphi_{ic}) = \frac{(1-\delta_c)(1-\beta_i)}{(1+\rho)(\rho+\delta_c)(\sigma_i - 1)} \mu_{ic} L_{ic}.$$

Substituting this equation into aggregate income (A.58) yields

$$W_c = \sum_{i=1}^{I} \left[L_{ic} w_{ic} + \frac{(1-\delta_c)(1-\beta_i)}{(1+\rho)(\rho+\delta_c)(\sigma_i - 1)} \mu_{ic} L_{ic} \right].$$

As before, we assume that the cost of posting vacancies in city c, k_{ic} is small in all industries. This implies that $\mu_{ic} \approx w_c \; \forall i$. In addition, we now assume small cross-industry differences in $\beta_i \approx \beta$ and $\sigma_i \approx \sigma \; \forall i$.[5] Under these assumptions,

$$W_c \approx \left[1 + \frac{(1-\delta_c)(1-\beta)}{(1+\rho)(\rho+\delta_c)(\sigma - 1)}\right] w_c \sum_{i=1}^{I} L_{ic}. \tag{A.60}$$

Using (A.60), we have:

$$\dot{V}_c = \frac{\dot{W}_c}{\prod_{i=1}^{I} \left(\dot{P}_{ic}\right)^{\alpha_i}} \approx \prod_{i=1}^{I} \left(\frac{\dot{w}_c}{\dot{P}_{ic}}\right)^{\alpha_i} \dot{e}_c. \tag{A.61}$$

From (A.54) and (A.55), the price index can be expressed as:

$$\left(P_{ic}\right)^{1-\sigma_i} = \left(\lambda_{icc}\right)^{-1} M_{ic} \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\frac{\varphi_{ic}}{\mu_{ic}}\right)^{\sigma_i - 1}.$$

Therefore,

$$\left(\frac{\dot{\mu}_{ic}}{\dot{P}_{ic}}\right) \approx \left(\frac{\dot{w}_c}{\dot{P}_{ic}}\right) = \left(\dot{\lambda}_{icc}\right)^{\frac{1}{1-\sigma_i}}. \tag{A.62}$$

Combining (A.62) and (A.61) yields

$$\dot{V}_c \approx \dot{e}_c \prod_{i=1}^{I} \left(\dot{\lambda}_{icc}\right)^{-\frac{\alpha_i}{\varepsilon_i}},$$

where, as we show next, $\varepsilon_i \equiv \sigma_i - 1$ is the trade elasticity in this model.

[5] These assumptions ensure that the share of aggregate profits in aggregate labor income is (approximately) constant across sectors and thus play the same role as macro-level restriction R2(MS) in Arkolakis et al. (2012).

A.10.1 Gravity with Restricted Entry

As in the previous section,

$$\frac{X_{icn}}{R_{ic}} = \frac{A_{in}\left(\tau_{icn}\right)^{1-\sigma_i}}{\sum_{n'} A_{in'}\left(\tau_{icn'}\right)^{1-\sigma_i}}.$$

From (A.59),

$$\left[\sum_{n'} A_{in}\left(\tau_{icn}\right)^{1-\sigma_i}\right]^{-1} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)\frac{M_{ic}}{L_{ic}}\left(\varphi_{ic}\right)^{\sigma_i - 1}\left(\mu_{ic}\right)^{-\sigma_i}.$$

Hence we obtain:

$$\frac{X_{icn}}{R_{ic}} = A_{in}\left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)\frac{M_{ic}}{L_{ic}}\left(\varphi_{ic}\right)^{\sigma_i - 1}\left(\tau_{icn}\right)^{1-\sigma_i}\left(\mu_{ic}\right)^{-\sigma_i}.$$

A.11 Perfect Competition (PC)

A.11.1 An Armington Model with Multiple Sectors and Search Frictions

In this section, we assume that goods are differentiated by country of origin. In particular, each country produces a unique variety of every good i under constant returns to labor. Therefore, consumer preferences are still given by (A.1), although here we interpret the integral sign as a Lebesgue integral to allow for a finite set of varieties in each industry.

A.11.2 Labor Market

The structure of the labor market is unchanged. Under constant returns to labor, however, the value functions of vacancies and workers in any given industry-city cell are independent of the number of workers employed. Without loss of generality, we can therefore assume that all firms are single-worker firms (or jobs).

In a stationary equilibrium, the value of employment in industry i of city c satisfies

$$E_{ic} - U_c = \frac{w_{ic} - \rho U_c}{(\rho + \delta_c)}.$$

The value of unemployment, U_c, can be expressed as:

$$\rho U_c = \theta_c m_c\left(\theta_c\right)\sum_i \eta_{ic}\left(\frac{w_{ic} - \rho U_c}{\rho + \delta_c}\right). \tag{A.63}$$

In turn, the discounted values of filled and vacant position can be written, respectively, as

$$\Pi^f_{ic} = \frac{1}{1+\rho}\left[p_{ic}\varphi_{ic} - w_{ic} + \left(1 - \delta_c\right)\Pi^f_{ic}\right], \tag{A.64}$$

$$\Pi^v_{ic} = \frac{1}{1+\rho}\left[0 + \left(1 - \delta_c\right)m_c(\theta_c)\Pi^f_{ic}\right], \tag{A.65}$$

where p_{ic} and φ_{ic} denote the factory gate price and labor productivity, respectively. Equation (A.65) assumes that a vacancy is open for one period (zero maintenance cost). If the vacancy is filled, workers enter production in the following period. Otherwise, the vacancy is destroyed.

Under free entry, we must have

$$\Pi^v_{ic} = \frac{k_{ic}}{1+\rho}.$$

Together with (A.64) and (A.65), free entry implies

$$\left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{k_{ic}}{m_c(\theta_c)} = p_{ic}\varphi_{ic} - w_{ic}. \tag{A.66}$$

Equation (A.66) has two properties that we will use below. First, $p_{ic}\varphi_{ic} = \mu_{ic}$, where the cost of labor is still defined by (A.32). Second, $w_{ic} \approx p_{ic}\varphi_{ic}$ for small k_{ic}.

A.11.3 Wage Bargaining

The surplus splitting rule that solves the bargaining game can be written as:

$$(1-\beta_i)\left[E_{ic} - U_c\right] = \beta_i \Pi^f_{ic}.$$

$$(1-\beta_i)\left[w_{ic} - \theta_c m_c(\theta_c)\sum_i \eta_{ic}\left(\frac{w_{ic}-\rho U_c}{\rho+\delta_c}\right)\right] = \left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{k_{ic}}{m_c(\theta_c)}.$$

Note that, when k_{ic} is small in all industries of city c, the right-hand side of the previous equation does not vary across industries, and hence $w_{ic} \approx w_c$ for all i.

A.11.4 Gravity

Trade is subject to iceberg variable trade costs only. Due to the structure of preferences, this implies that each location exports to all destinations in any given industry. The value of sales of goods from city c to location n in industry i is $X_{icn} = p_{ic}\tau_{icn}q_{icn}$. Using the CES demand and $p_{ic}\varphi_{ic} = \mu_{ic}$ yields

$$X_{icn} = \tau_{icn}^{1-\sigma_i}\mu_{ic}^{1-\sigma_i}\varphi_{ic}^{\sigma_i-1}X_{in}P_{in}^{\sigma_i-1},$$

where X_{in} denotes location n's expenditure on industry i. Therefore, the trade and cost-of-labor elasticities are both equal to $\sigma_i - 1$. Note, however, that the export share is independent of the cost of labor μ_{ic} conditional on the demand shifter in the destination.

A.11.5 Price Index

Due to the Armington assumption, the consumer price index can be written as

$$P_{in} = \left[\sum_v \left(p_{iv}\tau_{ivn}\right)^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}, \tag{A.67}$$

where v sums across all $C+1$ locations; i.e. $v = 1, ..., C, F$. Taking logs and computing the total differential yields

$$d\,ln\,P_{in} = \frac{1}{1-\sigma_i}\frac{1}{\sum_v \left(p_{iv}\tau_{icn}\right)^{1-\sigma_i}}\left\{\sum_v (1-\sigma_i)\left(p_{iv}\tau_{ivn}\right)^{-\sigma_i}\left(p_{iv}\tau_{ivn}\right)\left[d\,ln\,p_{iv} + d\,ln\,\tau_{ivn}\right]\right\}.$$

Together with (A.67), we obtain

$$d\,ln\,P_{in} = \sum_v \left(\frac{p_{iv}\tau_{ivn}}{P_{in}}\right)^{1-\sigma_i}\left[d\,ln\,p_{iv} + d\,ln\,\tau_{ivn}\right]. \tag{A.68}$$

A.11.6 Trade Share

Trade share can be written as:

$$\lambda_{icn} \equiv \frac{X_{icn}}{X_{in}} = \left(\frac{p_{ic}\tau_{icn}}{P_{in}}\right)^{1-\sigma_i}. \tag{A.69}$$

Therefore,

$$\frac{\lambda_{icn}}{\lambda_{inn}} = \left(\frac{\tau_{icn}p_{ic}}{\tau_{inn}p_{in}}\right)^{1-\sigma_i}.$$

Taking logs, computing the total differential using $w_{ic} \approx p_{ic}\varphi_{ic}$ for small k_{ic} yields

$$d\,ln\left(\frac{\lambda_{icn}}{\lambda_{inn}}\right) \approx (1-\sigma_i)\left[d\,ln\,w_{ic} + d\,ln\,\tau_{icn} - d\,ln\,w_{in}\right]. \tag{A.70}$$

A.11.7 Welfare

From (A.68) and (A.69),

$$d\,ln\,P_{in} = \sum_v \lambda_{ivn}\left[d\,ln\,p_{iv} + d\,ln\,\tau_{ivn}\right].$$

Using $d\,ln\,w_{in} \approx\ d\,ln\,p_{in}$ when k_{in} is small yields

$$d\,ln\,P_{in} \approx \sum_v \lambda_{ivn}\left[d\,ln\,w_{iv} + d\,ln\,\tau_{ivn}\right].$$

Note that $\sum_v \lambda_{ivn} = 1$. Substituting (A.70) into this equation yields

$$d\,ln\,P_{in} \approx -\frac{d\,ln\,\lambda_{inn}}{1-\sigma_i} + d\,ln\,w_{in}. \tag{A.71}$$

Using $w_{in} \approx w_n$ for all i and choosing the numeraire $w_n = 1$, we obtain:

$$d\,ln\,P_{in} \approx -\frac{d\,ln\,\lambda_{inn}}{1-\sigma_i}.$$

Welfare in n can be expressed as:

$$V_n = \frac{\sum_i \eta_{in} w_{in}(1-u_n)L_n}{\prod_{i=1}(P_{in})^{\alpha_i}}.$$

Now, consider welfare changes between two equilibriua arising from an arbitrary change in the vector of variable trade costs. Then

$$\begin{aligned}
d\,ln\,V_n = &-\sum_i \alpha_i d\,ln\,P_{in} \\
&+ \frac{1}{Y_n^w}\left[\sum_i e^{ln\eta_{in}+lnw_{in}+ln(1-u_n)} L_n d\,ln\,\eta_{in}\right] \\
&+ \frac{1}{Y_n^w}\left[\sum_i e^{ln\eta_{in}+lnw_{in}+ln(1-u_n)} L_n d\,ln\,w_{in}\right] \\
&- \frac{1}{Y_n^w}\left[\sum_i e^{ln\eta_{in}+lnw_{in}} e^{ln(1-e^{lnu_n})}\left(\frac{e^{lnu_n}}{1-e^{lnu_n}}\right) L_n\right] d\,ln\,u_n,
\end{aligned}$$

where Y_n^w is aggregate labor income. Define the share of labor income in sector i as

$$h_{in} \equiv \frac{\eta_{in} w_{in}(1-u_n)L_n}{\sum_{i'} \eta_{i'n} w_{i'n}(1-u_n)L_n} = \frac{\eta_{in} w_{in}}{\sum_{i'} \eta_{i'n} w_{i'n}}.$$

Now, we can write $d(lnV_n)$ as a function of h_{in} as follows:

$$\begin{aligned} d\,ln\,V_n =& \sum_i \left[-\alpha_i d\,ln\,P_{in} + h_{in} d\,ln\,\eta_{in} + h_{in} d\,ln\,w_{in}\right] \\ & - \frac{1}{Y_n^w}\left[\frac{e^{lnu_n}}{1-e^{lnu_n}} \sum_i \eta_{in} w_{in}(1-u_n)L_n\right] d\,ln\,u_n. \end{aligned}$$

Rearranging yields

$$d\,ln\,V_n = \sum_i \left[-\alpha_i\, d\,ln\,P_{in} + h_{in} d\,ln\,\eta_{in} + h_{in} d\,ln\,w_{in}\right] - \frac{u_n}{1-u_n} d\,ln\,u_n. \tag{A.72}$$

Note that if $w_{in} \approx w_n$ for all i, then $h_{in} = \eta_{in}$. Thus, we obtain

$$\sum_i h_{in} d\,ln\,\eta_{in} + h_{in} d\,ln\,w_{in} = \sum_i d\,\eta_{in} + d\,lnw_n = 0. \tag{A.73}$$

Now, combining (A.73) and (A.72),

$$d\,ln\,V_n \approx \sum_i -\alpha_i d\,ln\,P_{in} - \frac{u_n}{1-u_n} d\,ln\,u_n.$$

Together with (A.71) and (A.73),

$$d\,ln\,V_n \approx \sum_i \frac{\alpha_i}{1-\sigma_i} d\,ln\,\lambda_{inn} - \frac{u_n}{1-u_n} d\,ln\,u_n.$$

Integrating between the inital and final equilibria yields

$$ln\,\dot{V}_n \approx \sum_i \frac{\alpha_i}{1-\sigma_i} ln\,\dot{\lambda}_{inn} + ln\,\dot{e_n}.$$

Exponentiating on both sides,

$$\dot{V}_n = \dot{e}_n \prod_i \left(\dot{\lambda}_{inn}\right)^{\frac{\alpha_i}{1-\sigma_i}}.$$

B Worker Mobility

In the model presented so far, we have assumed that workers are not mobile across cities. This assumption, however, is not essential for the result that local industrial composition matters for wages – which is the basis for our identification strategy – and allows us to greatly simplify the exposition of the main elements of our framework. However, since worker mobility across local labor markets seems like a natural margin of adjustment to local shocks, we briefly discuss how this margin could be incorporated into our model. First, suppose that unemployed workers were occasionally offered the option to move to a different local labor market and, once given this option, could choose the local labor market c' that maximized their indirect utility. This extension would add an additional term to the equation for the value of unemployed search, U_c, capturing the value of this mobility option: $m \cdot \max\{(U_{c'} - \theta^m_{cc'} - U_c), 0\}$, where m is the probability that a worker is offered the mobility option and $\theta^m_{cc'}$ is the location-specific cost of moving to c'.

If the option to move occurs frequently enough (m is sufficiently high), a steady state spatial equilibrium will imply that the value of moving is driven to zero. When incorporating mobility, as in standard spatial equilibrium models, it is appropriate to include housing (or land) costs as an equilibrating mechanism. These costs do not influence wage bargaining, because they have to be incurred whether or not a worker is employed (and wages depend on the difference $[E_{ic}(l;\varphi) - U_c]$). However, housing prices will have to adjust to equate expected utility across locations. Assuming that housing is not perfectly elastically supplied, an improvement of industrial composition in c will make c more attractive and there will be an inflow of workers from other locations. This will simultaneously increase the cost of housing in c and this process will continue until $m \cdot \max\{(U_{c'} - \theta_c^m - U_c), 0\}$ is driven to zero but before wages are equalized. Thus, as long as the mobility friction is small enough, the option to move is *directed*: unemployed workers decide to search locally or move to c' and search.[6] A spatial equilibrium implies that workers must be indifferent between these options. However, if mobility frictions are large, it will prevent the spatial indifference. In this scenario, mobility frictions, importantly, will still imply that local industrial composition is a determinant of local wages, but wages will also be influenced by an additional outside option to move.[7]

In order to simplify the exposition of our model, we assume that mobility frictions are sufficiently low in order for a spatial equilibrium to hold, and thus we ignore the mobility option. We will, however, examine the relevance of mobility as an adjustment mechanism in section 6 of our paper where we examine the impact of trade shocks on local labor market outcomes. To preview those results, we find no evidence that the observed trade shocks over the period that we study had any effect on population sizes of local labor markets. This result is consistent with Autor et al. (2013) for the U.S. and Dauth et al. (2014) for Germany, who also find minimal population adjustments to trade shocks.

C Derivation of the Estimating Equations

C.1 Linear Approximation of the Unobservable Cost of Labor

In this section we derive the linear approximation to the unobservable cost of labor. Ultimately, the objective of the approximation is to express the gravity and revenue equations as a linear function of industry-city log wages. We take the linear approximation around the point where the cost of posting vacancies is constant across cities. Let $x_0 = (w_{ic} = w_0, k_{ic} = k_{i0}, e_c = e_0)$ denote that point. The approximation is given by:

$$\begin{aligned}
\mu_{ic} &\approx (w_0 + \psi k_{i0}) + (w_{ic} - w_0) \frac{\partial \mu_{ic}}{\partial w_{ic}}\Big|_{x_0} + (k_{ic} - k_{i0}) \frac{\partial \mu_{ic}}{\partial k_{ic}}\Big|_{x_0} + (e_c - e_0) \frac{\partial \mu_{ic}}{\partial e_c}\Big|_{x_0} \\
&\approx (w_0 + \psi k_{i0}) + (w_{ic} - w_0) + (k_{ic} - k_{i0}) \left(k_{i0} \cdot \frac{\partial \psi_c}{\partial k_{ic}}\Big|_{x_0} + \psi \right) + (e_c - e_0) \cdot k_{i0} \cdot \frac{\partial \psi_c}{\partial e_c}\Big|_{x_0} \\
&\approx \tilde{\psi}_{i0} + w_{ic} + \tilde{\psi}_{i1} k_{ic} + \tilde{\psi}_{i2} e_c, \qquad \text{(C.1)}
\end{aligned}$$

where ψ is evaluated around x_0, the point where k_{ic} is constant; i.e. $\psi = \left(\frac{\rho+\delta}{1-\delta}\right) \frac{1}{m(\theta)}$, where the vacancy-filling rate $m(\theta)$ is constant around x_0. In addition, $\tilde{\psi}_{i0} = -k_{i0} \left(k_{i0} \cdot \frac{\partial \psi_c}{\partial k_{ic}}\Big|_{x_0} + e_0 \cdot \frac{\partial \psi_c}{\partial e_c}\Big|_{x_0} \right)$,

[6] Random search across cities has no impact on our framework because the outside option in random search doesn't depend on c, and thus can be captured by an intercept in an empirical specification.

[7] Moreover, the forces we emphasize in our model also have implications for worker mobility and housing costs. See for example, Beaudry et al. (2012, 2014); Green et al. (2017) who examine worker mobility in a setting of frictional local labor markets.

$\tilde{\psi}_{i1} = \left(k_{i0} \cdot \frac{\partial \psi_c}{\partial k_{ic}}\Big|_{x_0} + \psi \right)$ and $\tilde{\psi}_{i2} = k_{i0} \cdot \frac{\partial \psi_c}{\partial e_c}\Big|_{x_0}$.

Subtracting an arbitrary constant $\overline{w}$ (e.g. the average wage in an arbitrary industry), dividing by $\overline{w}$ and adding the log of that constant, equation (C.1) can be rewritten as follows:

$$\ln \overline{w} + \frac{(\mu_{ic} - \overline{w})}{\overline{w}} \quad \approx \quad \frac{\tilde{\psi}_{i0}}{\overline{w}} + \ln \overline{w} + \frac{(w_{ic} - \overline{w})}{\overline{w}} + \frac{\tilde{\psi}_{i1} k_{ic}}{\overline{w}} + \frac{\tilde{\psi}_{i2} e_c}{\overline{w}} \tag{C.2}$$

Finally, noting that $\ln x \approx \ln z_0 + \frac{x - z_0}{z_0}$, where z_0 is an arbitrary constant around which the approximation is taken, we can relate the log unit cost to log industry-city wages:

$$\ln \mu_{ic} \quad \approx \quad \psi_{i0} + \ln w_{ic} + \psi_{i1} k_{ic} + \psi_{i2} e_c, \tag{C.3}$$

where $\psi_{i0} = \frac{\tilde{\psi}_{i0}}{\overline{w}}$, $\psi_{i1} = \frac{\tilde{\psi}_{i1}}{\overline{w}}$ and $\psi_{i2} = \frac{\tilde{\psi}_{i2}}{\overline{w}}$.

C.2 Linear Approximation of the Wage Equation

The goal of the linear approximation is to link the industry-city wage to the industrial composition of the local labor market. Substituting equation (A.63) in equation (7) yields:

$$w_{ic} = \tilde{\gamma}_{1c} \bar{w}_c + \gamma_{2c} k_{ic}, \tag{C.4}$$

where $\bar{w}_c = \sum_i \eta_{ic} w_{ic}$, $\tilde{\gamma}_{1c} = \frac{\theta_c m_c(\theta_c)}{\rho + s + \theta_c m_c(\theta_c)}$, $\gamma_{2c} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{\rho+s}{1-\delta}\right)\frac{1}{m_c(\theta_c)}$, and both parameters are dependent on the tightness of the labor market.

Solving for $\bar{w}_c$, we obtain:

$$\bar{w}_c = \frac{\gamma_{2c}}{1 - \tilde{\gamma}_{1c}} \sum_i \eta_{ic} k_{ic},$$

and substituting back into equation (C.4), the reduced-form wage equation can be written as:

$$\begin{aligned} w_{ic} &= \frac{\tilde{\gamma}_{1c} \cdot \gamma_{2c}}{1 - \tilde{\gamma}_{1c}} \sum_i \eta_{ic} k_{ic} + \gamma_{2c} k_{ic}, \\ &= \gamma_{1c} \sum_i \eta_{ic} k_{ic} + \gamma_{2c} k_{ic}, \end{aligned}$$

where $\gamma_{1c} = \frac{\tilde{\gamma}_{1c} \cdot \gamma_{2c}}{1 - \tilde{\gamma}_{1c}}$.

In order to take the linear approximation of w_{ic}, it is useful to decompose the vacancy posting cost k_{ic}, without loss of generality, as follows:

$$k_{ic} = \tilde{k}_i + \tilde{k}_c + \tilde{\xi}_{ic},$$

where $\tilde{k}_i$ represents a common (across cities) industry component, $\tilde{k}_c$ represents city-specific component, and ξ_{ic} is an idiosyncratic component that sums to zero across industries, within cities (i.e. $\sum_i \xi_{ic} = 0$).

Using this decomposition of the vacancy posting cost, one can rewrite the wage equation as:

$$w_{ic} = \gamma_{1c} \tilde{K}_c + (\gamma_{1c} + \gamma_{2c}) \tilde{k}_c + \gamma_{1c} \sum_i \eta_{ic} \tilde{\xi}_{ic} + \gamma_{2c} \tilde{k}_i + \gamma_{2c} \tilde{\xi}_{ic},$$

where $\tilde{K}_c = \sum_i \eta_{ic} \tilde{k}_i$ captures the weighted city-average of national-level recruiting costs.

Let $w_{ic} = w_{ic}(\tilde{K}_c, \tilde{k}_i, \tilde{k}_c, \{\xi_{ic}\}_i, e_c)$ describe the reduced-form equation. We take a linear approximation of the wage equation around the point where recruitement costs are constant and the employment rate is the same across cities, i.e. around $x_0 = (\tilde{K}_c = \tilde{k}^i, \tilde{k}_i = \tilde{k}^i, \tilde{k}_c = \tilde{k}^c, \{\tilde{\xi}_{ic}\}_i = 0, e_c = e_0)$. Around that point, cities have an identical industrial composition (i.e. $\eta_{ic} = 1/I$). The linear approximation is given by:

$$\begin{aligned} w_{ic} &\approx (\gamma_1+\gamma_2)\left(\tilde{k}^i+\tilde{k}^c\right) + \left(\tilde{K}_c - \tilde{k}^i\right)\frac{\partial w_{ic}}{\partial \tilde{K}_c}\Big|_{x_0} \\ &+ \left(\tilde{k}_c - \tilde{k}^c\right)\frac{\partial w_{ic}}{\partial \tilde{k}_c}\Big|_{x_0} + \left(\tilde{k}_i - \tilde{k}^i\right)\frac{\partial w_{ic}}{\partial \tilde{k}_i}\Big|_{x_0} + \sum_i \left(\tilde{\xi}_{ic} - 0\right)\frac{\partial w_{ic}}{\partial \tilde{\xi}_{ic}}\Big|_{x_0} + (e_c - e_0)\frac{\partial w_{ic}}{\partial e_c}\Big|_{x_0} \\ &\approx \gamma_1 \tilde{K}_c + (\gamma_1+\gamma_2)\tilde{k}_c + \gamma_2 \tilde{k}_i + \gamma_1 \frac{1}{I}\underbrace{\sum_i \tilde{\xi}_{ic}}_{=0} + \gamma_2 \tilde{\xi}_{ic} + (e_c - e_0)\left(\frac{\partial \gamma_{1c}}{\partial e_c}\Big|_{x_0} + \frac{\partial \gamma_{2c}}{\partial e_c}\Big|_{x_0}\right)\left(\tilde{k}^i + \tilde{k}^c\right), \end{aligned} \tag{C.5}$$

where we have used the property that $\sum_i \tilde{\xi}_{ic} = 0$, and where γ_1 and γ_2 are constant parameters corresponding to γ_{1c} and γ_{2c}, evaluated at x_0, respectively.

Collecting terms, equation (C.5) can be rewritten as:

$$w_{ic} \approx \tilde{\gamma}_0 + \gamma_1 \tilde{K}_c + \gamma_2 \tilde{k}_i + \gamma_3 \tilde{k}_c + \tilde{\gamma}_4 e_c + \gamma_2 \tilde{\xi}_{ic}, \tag{C.6}$$

where each of the $\tilde{\gamma}$s are constant parameters obtained from the linear approximation. Specifically, $\tilde{\gamma}_0 = -e_0\left(\frac{\partial \gamma_{1c}}{\partial e_c}\Big|_{x_0} + \frac{\partial \gamma_{2c}}{\partial e_c}\Big|_{x_0}\right)\left(\tilde{k}^i + \tilde{k}^c\right)$, $\gamma_3 = (\gamma_1+\gamma_2)$ and $\tilde{\gamma}_4 = \left(\frac{\partial \gamma_{1c}}{\partial e_c}\Big|_{x_0} + \frac{\partial \gamma_{2c}}{\partial e_c}\Big|_{x_0}\right)\left(\tilde{k}^i + \tilde{k}^c\right)$.

Subtracting an arbitrary constant $\overline{w}$ (e.g. the average wage in an arbitrary industry), dividing by $\overline{w}$ and adding the log of that constant, we obtain the following approximation for log industry-city wages:

$$\ln w_{ic} \approx \gamma_0 + \gamma_1 \sum_i \eta_{ic} k_i + \gamma_2 k_i + \gamma_3 k_c + \gamma_4 e_c + \gamma_2 \xi_{ic}, \tag{C.7}$$

where $\ln w_{ic} \approx \ln \overline{w} + \frac{(w_{ic} - \overline{w})}{\overline{w}}$, $\gamma_0 = \ln \overline{w} + \frac{(\tilde{\gamma}_0 - \overline{w})}{\overline{w}}$, $\gamma_4 = \frac{\tilde{\gamma}_4}{\overline{w}}$, $k_i = \frac{\tilde{k}_i}{\overline{w}}$, $k_c = \frac{\tilde{k}_c}{\overline{w}}$ and $\xi_{ic} = \frac{\tilde{\xi}_{ic}}{\overline{w}}$.

Importantly, equation (C.7) shows that, at the national level, inter-industry wage differentials are given by $\gamma_2 k_i$, which expresses the average wage in industry i relative to an omitted group. Letting $\nu_i = \gamma_2 k_i$ denote the national industry wage premium, we finally express log industry-city wages as a function of industrial composition:

$$\ln w_{ic} \approx \gamma_0 + \frac{\gamma_1}{\gamma_2} K_c + \gamma_2 k_i + \gamma_3 k_c + \gamma_4 e_c + \gamma_2 \xi_{ic}, \tag{C.8}$$

where $K_c = \sum_i \eta_{ic} \nu_i$ is an index that captures industrial composition at the city level.

D Additional Results

D.1 Instruments Relevance

To check for the relevance of our instruments, we also perform an analysis of variance (ANOVA) of the regressions used to construct industrial national-wage premia and employment growth. The rationale for this test is that our instruments are functions of base-period local employment shares, national sector growth rates, and national industrial premia. Therefore, variation in our instruments stems from cross-sector, over-time variation in these national-level objects.

For our instruments to be relevant, we require the national-component ν_{it} in (24) to be a significant determinant of wages. Similarly, we require the national-growth rate component g_{it} in (26) to be a strong predictor of local industry growth. Results from the ANOVA test are shown in Table D.1. The first and second columns regress $\Delta \ln L_{ict}$ and $\ln w_{ict}$ on an entire set of city-year and industry-year fixed effects, respectively. The results show that national-level components explain a considerable part of the variation in industry-city adjusted wages and employment growth.

Table D.1: ANOVA of industry-city growth rates and adjusted wages

Dependent variable	Δ ln L_{ict}	ln w_{ict}
Model SS	94.21	804.76
Industry-year FE SS	49.01	509.30
City-year FE SS	12.74	268.19
Residual	115.26	79.22
Observations	16884	17822
R-squared	0.450	0.91

D.2 Additional Specification Checks

In this section, we investigate additional robustness exercises to probe the validity of our Bartik-type instruments by performing several specification checks suggested by Goldsmith-Pinkham et al. (2017). First, we assess the correlation between our instruments and characteristics of cities in the base year. In particular, we compute, by city, several variables aimed at capturing labour market conditions and city-skill in the base year. We then investigate the relationship between these variables and the base-period industrial structure. The idea is that if the instruments (through initial industry shares) are correlated with city characteristics in the base year, then any trend or shock that is correlated to those city characteristics could also be correlated with the instruments, therefore potentially violating the exclusion restriction we require for our instruments to be valid.

Table D.2 contains the results of this exercise. In the first two columns, we regress the value of our within- and between-instruments in 1996 (the first year we can calculate the instruments) on shares of college-educated, female, and German workers and the log employment rate and size of the workforce, average over the period 1992-1993. In these two columns, only one coefficient is statistically significant but the variables are jointly significant.

Since our identification strategy rests on the assumption that the initial industrial structure is not correlated with the residual in our second-stage regressions, we investigate the relationship between the initial industrial structure and city characteristics. In column 3, we compute the first principle component of our 58 industrial categories (i.e. the component that explains most of the variance in industry shares) in the base year. The idea is simply to reduce the dimension of our industrial categories into a single dimension that we can regress on our vector of city characteristics. Finally, in columns 4 and 5 we repeat these exercises by simply splitting industries into durables and non-durables. While the base-year characteristics are rarely individually significant, we cannot reject that they are jointly correlated with initial industrial structure.

Table D.2: Relationship between industry shares and city-specific characteristics

	IVB$_{ct}$ **1996**	**IVW$_{ct}$** **1996**	**Component 1** **1992-1993**	**Non-durables** **1992-1993**	**Durables** **1992-1993**
City-specific characteristics in 1992-1993:					
Share of college graduates	0.005	0.008	-0.194	0.771	-0.771
	(0.003)	(0.008)	(0.134)	(1.034)	(1.034)
Share of females	-0.004	-0.017**	-0.044	-0.347	0.347
	(0.003)	(0.006)	(0.144)	(0.784)	(0.784)
Share of Germans	0.001	-0.005	-0.056	0.639	-0.639
	(0.002)	(0.005)	(0.146)	(0.643)	(0.643)
Log employment rate	0.005	-0.009	0.748***	-0.571	0.571
	(0.003)	(0.007)	(0.181)	(0.889)	(0.889)
Log workforce	-0.0001	0.0002	-0.081	0.027	-0.027
	(0.0002)	(0.0002)	(0.105)	(0.028)	(0.028)
Observations	24	24	24	24	24
R-squared	0.618	0.519	0.909	0.450	0.450
F-stat	5.82	3.88	35.94	2.94	2.94
p-val > F-stat	0.00	0.01	0.00	0.04	0.04
Proportion			0.27		

Notes: IVW$_{ct}$ corresponds to $\sum_i \hat{\eta}_{ict-1}\Delta\hat{\nu}_{it}$ and IVB$_{ct}$ to $\sum_i \hat{\nu}_{it}\Delta\hat{\eta}_{ict}$. The term 'Component 1' refers to the first principal component of industry shares in 1992-1993. In column 3, the first principal component and the city-specific characteristics are standardized to have unit standard deviation. The term 'Proportion' refers to the proportion of the variance of industry shares explained by the first principal component. The term 'Durables' ('Non-durables') refers to the share of employment in industries that produce durable (non-durable) goods in 1992-1993. Standard errors in parentheses.

D.3 Additional Tables

Table D.3: Trade Exposure and Other Outcomes

	Population			Wages		
	(1) EE	(2) China	(3) Both	(4) EE	(5) China	(6) Both
Δ Import Exposure	-0.017**	-0.014**	-0.0028	-0.00091	-0.0024	-0.000060
	(0.0052)	(0.0068)	(0.0021)	(0.0011)	(0.0025)	(0.00026)
Δ Export Exposure	0.0065**	0.0069	-0.00078	0.0012	-0.024**	0.00050
	(0.0032)	(0.0082)	(0.0012)	(0.00086)	(0.0071)	(0.00033)
Constant	0.14**	0.21**	0.18**	0.025**	0.20**	0.028**
	(0.046)	(0.049)	(0.047)	(0.0060)	(0.044)	(0.0049)
Observations	652	652	652	652	652	652
R^2	0.413	0.390	0.446	0.951	0.442	0.952
Predicted Impact:						
Mean	0.97	0.98	0.98	1.00	0.96	1.00
Med	0.98	0.98	0.98	1.00	0.97	1.00
10th pct.	0.95	0.95	0.96	1.00	0.93	1.00
90th pct.	0.99	0.99	0.99	1.01	0.98	1.01

Notes: Standard errors, in parentheses, are clustered at the level of 50 larger labor markets areas. (***), (**), and (*) denote significance at the 1%, 5% and 10% level, respectively. The Table presents regression results of (28), estimated on different city outcome variables over 326 cities of West Germany. The dependent variables are population growth (columns 1-3) and the growth of wages (column 4-6) at the city level. Δ Import Exposure (IPW_{ct}) and Δ Export Exposure (EPW_{ct}) are observed decadal changes (1988-1998 and 1998-2008) in import and export exposure, respectively.Specifically, $IPW_{ct} = \sum_i \frac{E_{ict}}{E_{it}} \frac{\Delta M^{G \leftarrow East}_{i(t+10)}}{E_{ct}}$, where Δ denotes a decadal time difference, $\frac{E_{ict}}{E_{it}}$ is city c's share of industrial employment and E_{ct} is city c manufacturing employment. $\Delta M^{G \leftarrow East}_{i(t+10)}$ denotes the change in imports from the East between t and $t+10$. Δ Export Exposure (EPW_{ct}) is computed similarly using exports. Each specification includes a set of region-time fixed effects and city-specific controls (the share of employment in tradable goods industries, the share of high-skilled, foreign and female workers, as well as the percentage of routine/intensive occupations). In each column, we instrument import exposure using $IVIPW_{ct} = \sum_i \frac{E_{ic(t-10)}}{E_{(it-10)}} \frac{\Delta M^{Others \leftarrow East}_{i(t+10)}}{E_{c(t-10)}}$, where $\Delta M^{Others \leftarrow East}_{i(t+10)}$ denotes changes in imports from the East to other high income countries. We instrument export exposure in a similar way using exports. We weigh our regressions by the share of the population in year 1978. In columns 1 and 4 (2 and 5), IPW_{ct} and EPW_{ct} are computed using imports from and exports to Easter Europe (China) only. In columns 3 and 6, IPW_{ct} and EPW_{ct} reflect trade exposure with both Eastern Europe and China.

E Consistency, Heterogeneity and the Over-identification Test

Our instruments can be written as:

$$
\begin{aligned}
IV_{ct}^{W} &= \sum_{j\in S} \hat{\eta}_{jct-1}\Delta\hat{\nu}_{jt}, \\
IV_{ct}^{B} &= \sum_{j\in S} \hat{\nu}_{jt}\Delta\hat{\eta}_{jct} = \sum_{j\in S} \hat{\eta}_{jct-1}\left[\frac{1+g_{jt}}{\sum_{j\in S}\hat{\eta}_{jct-1}(1+g_{jt})} - \frac{1}{\sum_{j\in S}\hat{\eta}_{jct-1}}\right]\nu_{jt} \\
&= \sum_{j\in S} \hat{\eta}_{jct-1} g_{jc}^{*}\nu_{jt}
\end{aligned}
$$

where $g_{jc}^{*} = \left[\frac{1+g_{jt}}{\sum_{j\in S}\hat{\eta}_{jct-1}(1+g_{jt})} - \frac{1}{\sum_{j\in S}\hat{\eta}_{jct-1}}\right]$. As stated in the text, we are interested in the cross-city covariance between our instruments and the error term:

$$
\frac{1}{I}\frac{1}{C}\sum_{c}\sum_{i\notin S}\left(\sum_{j\in S}\hat{\eta}_{jct-1}\Delta\hat{\nu}_{jt}\right)\Delta u_{ict}^{G} = \frac{1}{I}\frac{1}{C}\sum_{i\notin S}\sum_{j\in S}\Delta\hat{\nu}_{jt}\sum_{c}\hat{\eta}_{jct-1}\Delta u_{ict}^{G}
$$

$$
\frac{1}{I}\frac{1}{C}\sum_{c}\sum_{i\notin S}\left(\sum_{j\in S}\hat{\eta}_{jct-1}g_{jc}^{*}\hat{\nu}_{jt}\right)\Delta u_{ict}^{G} = \frac{1}{I}\frac{1}{C}\sum_{i\notin S}\sum_{j\in S}\hat{\nu}_{jt}\sum_{c}\hat{\eta}_{jct-1}g_{jc}^{*}\Delta u_{ict}^{G}
$$

Thus, a sufficient condition for our instruments to be valid is that $\frac{1}{C}\sum_{c}\hat{\eta}_{jct-1}\Delta u_{ict}^{G} \to^{p} 0$ as $C\to\infty$ as stated in the text. Note that each instrument weights this condition differently; either $\Delta\hat{\nu}_{jc}$ or $\nu_{jc}\cdot g_{jc}^{*}$.

Let Z_c represent an instrument, and let $\tilde{Z}_c$ represent the residual from regressing Z_c on industry dummies and city-employment rate (as in our main specification). Our 2SLS estimate of the wage coeffiicent in the gravity equation is given by (ignoring the t subscript for simplicity):

$$
\begin{aligned}
\hat{\phi}_{1}^{Z_c} &= \frac{\widehat{\mathrm{Cov}}\left(\tilde{Z}_c, \Delta\ln X_{icF}/R_{ic}\right)}{\widehat{\mathrm{Cov}}\left(\tilde{Z}_c, \Delta\ln w_{ic}\right)} \\
&= \frac{\sum_c\sum_i \tilde{Z}_c\Delta\ln X_{icF}/R_{ic}}{\sum_c\sum_i \tilde{Z}_c\Delta\ln w_{ic}} \\
&= \frac{\sum_c\sum_i \tilde{Z}_c\phi_{1i}\Delta\ln w_{ic}}{\sum_c\sum_i \tilde{Z}_c\Delta\ln w_{ic}} + \frac{\sum_c\sum_i \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_c\sum_i \tilde{Z}_c\Delta\ln w_{ict}} \\
&= \sum_i \phi_{i1}\cdot\frac{\sum_c \tilde{Z}_c\Delta\ln w_{ic}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ic}} + \frac{\sum_i\sum_c \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ict}} \\
&= \sum_i \phi_{i1}\cdot\frac{\sum_c \tilde{Z}_c\frac{\hat{\gamma}_1}{\hat{\gamma}_2}Z_c}{\sum_i\sum_c \tilde{Z}_c\frac{\hat{\gamma}_1}{\hat{\gamma}_2}Z_c} + \frac{\sum_i\sum_c \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ict}} \\
&= \sum_i \phi_{i1}\cdot\frac{\frac{\hat{\gamma}_1}{\hat{\gamma}_2}\sum_c \tilde{Z}_c Z_c}{\frac{\hat{\gamma}_1}{\hat{\gamma}_2}\sum_i\sum_c \tilde{Z}_c Z_c} + \frac{\sum_i\sum_c \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ict}} \\
&= \sum_i \phi_{i1}\cdot\frac{1}{I} + \frac{\sum_i\sum_c \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ict}} \\
&= \phi_1 + \frac{\sum_i\sum_c \tilde{Z}_c\Delta u_{ic}^{G}}{\sum_i\sum_c \tilde{Z}_c\Delta\ln w_{ict}},
\end{aligned}
$$

where $\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G \to^p 0$ as $C \to \infty$ under the assumption that $\frac{1}{C}\sum_c \hat{\eta}_{jct-1}\Delta u_{ict}^G \to^p 0$ as $C \to \infty$. Note that $\phi_1 = \frac{1}{I}\sum_i \phi_{1i}$ is an average of an industry specific effect.

The over-identification test is:

$$\hat{\phi}_1^{IV^W} - \hat{\phi}_1^{IV^B} = \frac{\sum_i \sum_c \tilde{IV}_c^W \Delta u_{ic}^G}{\sum_i \sum_c \tilde{IV}_c^W \Delta \ln w_{ict}} - \frac{\sum_i \sum_c \tilde{IV}_c^B \Delta u_{ic}^G}{\sum_i \sum_c \tilde{IV}_c^B \Delta \ln w_{ict}} = 0$$

This condition will hold under any of the three conditions:

1. The exogeneity conditions for the instruments hold as stated in the text,
2. $\tilde{IV}_c^W = \tilde{IV}_c^B$ or are proportional. Since both instruments are based on the base-period industrial structure, this would occur if the national-level shocks used in each instrument were the same. This is easily rejected in our data: the correlation between our instruments is 0.32 after taking out year-effects, as we do in all of our estimations.
3. (1) and (2) don't hold, but the two terms just happen to balance. We view this as an unlikely 'knifes-edge' scenario.

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