# MBRA 2.3.0 Tool Window Calculations Tab

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The MBRA 2.3.0 Tool Window Calculations tab performs an advanced set of equations on the user-provided data in order to determine the most useful set of allocations for Prevention and Response Budgets, and the most likely allocation for an attacker's budget. As the equations can be difficult to understand from the implementation itself, they are explained below. To save typing, the $n$ nodes and $m$ links are referred to together as nodes, and where $i = [1, n + m]$.

Each set of equations is based on values determined by the other formula. That is, the value of each variable can only be determined by finding the values of the other two variables first. Because of this, the calculations are performed for a specified number of iterations, with each iteration calculating new vulnerability, consequence, and threat values for each node based upon the values calculated in the previous iteration. The results come closer to their "true" value with each iteration, and most eventually converge on that "true" value. Some few data sets can cause results to diverge, however, and fail to find the true values. The algorithm currently assumes that if a value has not converged after the designated number of iterations, it will not ever converge. This pattern of iterating the values of the variables in order to get closer to the true answers is referred to in the original algorithm description as a "Stackleberg game".

## General Variables

Let:

| | |
|---|---|
| $n_i$ | represent some node $i$ |
| $c_\infty$ | represent the minimum consequence for any node as defined by the user in the MBRA general preferences window |
| $v_\infty$ | represent the minimum vulnerability for any node as defined by the user in the MBRA general prefences window |
| $d_i$ | represent the calculated prevention allocation for $n_i$ |
| $r_i$ | represent the calculated response allocation for $n_i$ |
| $a_i$ | represent the calculated attack allocation for $n_i$ |
| $v_{i_0}$ | represent the user-input vulnerability for $n_i$ |

$c_{i_0}$ represent the user-input consequence for $n_i$

$t_{i_0}$ represent the user-input threat for $n_i$

$v_i$ represent the calculated vulnerability for $n_i$

$c_i$ represent the calculated vulnerability for $n_i$

$t_i$ represent the calculated threat value for $n_i$

$w_i$ represent the calculated network weight for $n_i$

$\beta_i = -ln(c_\infty)$ for all nodes

$\gamma_i = -ln(\frac{v_\infty}{v_i})$ for each node

## Calculating Prevention Allocation and Vulnerability

Let:

$e_i$ be the user-input prevention cost for $n_i$

$d_i$ be the calculated prevention allocation for $n_i$

$\lambda_v$ be the correction factor for vulnerability calculations

$P$ be the total prevention budget

Then:

$$ln(\lambda_v) = \frac{-\sum_i [\frac{e_i}{\gamma_i} ln(\frac{e_i}{w_i t_i v_{i_0} c_i \gamma_i})] - P}{\sum_i \frac{e_i}{\gamma_i}}$$

$$d_i = -\frac{e_i}{\gamma_i}[ln(\lambda_v) + ln(\frac{e_i}{w_i t_i v_{i_0} c_i \gamma_i})]$$

Having determined the prevention allocation, vulnerability for $n_i$ can be calculated:

$$v_i = v_{i_0} e^{-\gamma_i (\frac{r_i}{e_i})}$$

## Calculating Response Allocation and Consequence

Let:

$h_i$ be the user-input response cost for $n_i$

$r_i$ be the calculated prevention allocation for $n_i$

$\lambda_c$ be the correction factor for consequence calculations

$R$ be the total prevention budget

Then:

$$ln(\lambda_c) = \frac{-\sum_i [\frac{h_i}{\beta_i} ln(\frac{h_i}{w_i t_i v_{i_0} c_i \beta_i})] - R}{\sum_i \frac{h_i}{\beta_i}}$$

$$r_i = -\frac{h_i}{\beta_i}[ln(\lambda_c) + ln(\frac{hi}{w_i t_i c_{i_0} v_i \beta_i})]$$

Having determined the response allocation, consequence for $n_i$ can be calculated:

$$c_i = c_{i_0} e^{-\beta_i(\frac{r_i}{h_i})}$$

## Calculating Attack Allocation and Threat

Let:

| | |
|---|---|
| $e_i$ | be the user-input prevention cost for $n_i$ |
| $a_i$ | be the calculated prevention allocation for $n_i$ |
| $\lambda_t$ | be the correction factor for threat calculations |
| $A$ | be the total attack budget. This is currently set to be the same amount as the prevention budget. |

Then:

$$ln(\lambda_t) = \frac{-\sum_i [\frac{e_i}{\gamma_i} ln(\frac{e_i}{w_i v_i c_i \gamma_i})] - P}{\sum_i \frac{e_i}{\gamma_i}}$$

$$a_i = -\frac{e_i}{\gamma_i}[ln(\lambda_t) + ln(\frac{e_i}{w_i v_i c_i \gamma_i})]$$

Having determined the attack allocation, threat for $n_i$ can be calculated:

$$t_i = 1 - e^{-\gamma_i(\frac{a_i}{e_i})}$$