PHL356H1 - Philosophy of Physics

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Jan 1, 2012

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Chapter 1

Passage of Time

The issue we try to tackle is: Is there a "present" or "now" that moves through our time series? Our intuition, would say 'yes'.

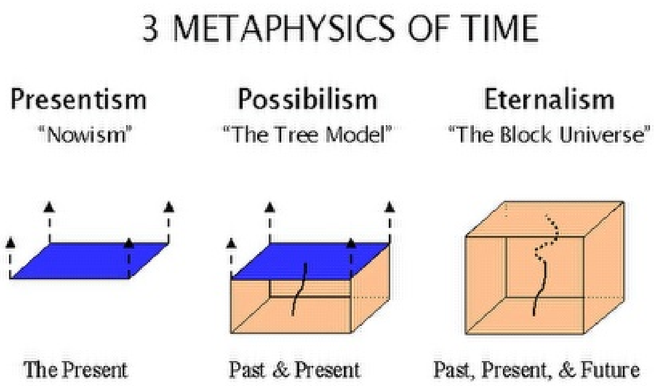

Figure 1.1: Three Possible Models of Time.

Presentism: Only the present events are real, but past & future events are no longer real. The passage of time (the moving of the present) makes events real. These are indicated by the directional arrows on the plane of the present.

Posibilism: Past & present events are real, but future events are now possibilities. Branching could be from free will, quantum indeterminism

Eternalism: All points in time are equally real. Universe is a four-dimensional block, and there is no objective flow of time

1.1 McTaggart's Argument

McTaggart argued that there is in fact no such thing as time, and the temporal order of the world is a mere appearance. He distinguishes two ways in which positions in time can be ordered. First, he says, positions in time can be ordered according to their possession of properties like *being two days future, being one day future, being present, being one day past.* This is known as the **A-series**. Positions can also be ordered in by two-place relations like *two days earlier than, one day earlier than, simultaneous with*, etc. These are known as the **B-series**. The A-series essentially says time consists of non-relative facts about time, and B-series consists of relative facts. McTaggart's argument is roughly as follows:

1. If time exits, events are located in A-series, B-series or both.
2. Events in B-series are eternal and never changing.
3. Change is essential to time. Without change, time would not exist.
4. Events in A-series are always changing relative to the moving present.
5. If time exists, events in B-series must also exist in A-series
6. Events must then be past, present and future at the same time, which is contradictory.
7. Appealing that they are past, future relative to events is unsatisfactory, as it in effect goes back to B-series, which doesn't have change.
8. $\therefore$ Time does not exist

There are several problems with McTagart's argument. He asserts that change is necessary for time to exist, though this may not be true. More problematic, is his notion of change. In reality, we only observe objects that change, rather than the events themselves changing. McTaggart's argument is not sound.

1.2 B-series View

Those who take the B-series view claim that time does not pass. Past, present and future are subjective, and relative to events. Terms such as 'now' or 'the present' aren't actual entities, but are **indexical terms**, which pick up meaning with a specific context, and will change based on the context. All change is in the form of: Entity a has property F at time t and a has property F at time t'. Objects change, rather than events.

1.2.1 Determinism

One common argument against B-series is that it seems to be committed to strict determinism. How does it then account for free will, or quantum indeterminism?

Deterministic: A world W is deterministic iff every world with same laws of nature and initial conditions has the same history as W

Non-Deterministic: A world W is deterministic iff there exits a world W' with same laws of nature and initial conditions has a different history as W

B-series is compatible with both freedom and determinism. A world W' *can exist* with a different history. Finding evidence that the choice is determined however, is a completely different matter. B-series is completely independent of determinism.

1.2.2 Regress Argument

1. If there is a present moving through a time series, then it is moving at some rate.
2. The rate of movement can only be described in terms of some external time: **supertime**
3. If A-series is correct for normal time, it must be correct for supertime.
4. We can recursively ask the same question, about the rate of the present in supertime
5. This regress is infinite, thus the argument should be rejected

1.2.3 Perception Problem

If time does not pass, then why do we perceive that it passes? One speculation is that passage is a **secondary property**, which are properties in the 'mind'. **Primary properties** are of the objective world, such as mass, charge, color, taste, etc ... A rose emits photons which interact with us in such a way as to give rise to the sensation of red. The rose itself, however, has no colour. The red we perceive is a secondary property.

Paul Horwich suggests that this can arise from our growing collection of memories, and anticipations of later events, coupled with features of our language. This somehow gives us the perception that time moves forward. This is of course still highly speculative.

1.2.4 Putnam & Special Relativity

The common man's view of time is that *"All (and only)things that exist now are real"* (presentism). To understand things completely, we have to make a couple assumptions:

1. Instantenous me is real
2. At least one other observer must be real, and he/she can be in motion relative to me
3. **Principle of No Privileged Observers:** If it is the case that all and only the things that stand in a certain relation R to me-now are real, and you-now are also real, then it is also the case that all and only the things that stand in the relation R to you-now are real.

Relation R must be restricted to physical relations independent of the coordinate system, and tense-less. If we take classical physics, and assume relation R to be the relation of simultaneity, we see that R is transitive. Principle 3 is

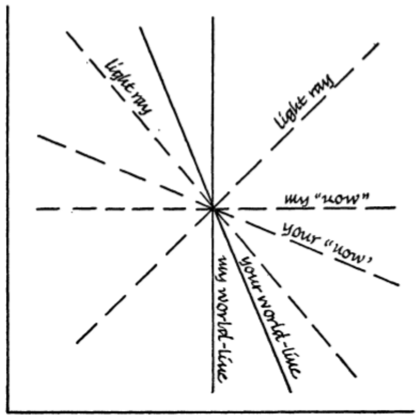

Figure 1.2: Worldlines & Light Cone.

satisfied, so only simultaneous events are real.

Assuming special relativity with the relation of simultaneity, let us have a scenario shown in Figure 1.1. Due to relative time dilation, its possible for an event in my-now's future to be another observer's present time. The problem now seems that R now has a dependency on one's coordinate system; a simultaneous event for me, might not be for you. Now if we assume the laws of nature are invariant, we can claim that relation R will hold for any one "simultaneity line". Now with that my-now's future event, relation R will hold in your-now. And since your-now bears relation R to my-now, then by transitivity, this future event is also real!

Putnam asserts that the deterministic nature of time is solved, and we do reside in a four-dimensional continuum with a particular metric which allows distance $(y, x) = 0$, even when $x \neq y$.

Chapter 2

Space

In *Timaeus* is Plato's threefold distinction of:

1. *that which comes to be*

2. *that in which it comes to be*

3. *that after which the thing coming to be is modeled, and which is its source*

Plato's philosophy is that our the world we live in (1), is a copy of the ideal world of "forms" (3). The dichotomy between the world of physics and the world of forms is the third entity: space (2) which "provides a location for all things that come into being". Space is the collection of all the "somewheres"; "container" of all physical things. Plato believes we must rely on pure reason to learn about the ideal world of forms. We are prisoners in the physical world, and we must attempt to discover the forms of which physical objects are only shadows.

The first question we face is:

> *What kind of thing is space if it is both* ***physical*** *and* ***immaterial****?*

Plato explains that space is the matter of which material objects - copies of forms - are composed. Space is like a piece of clay on which shapes can be stamped, but which also always remains smooth. It can be thought of "as an invisible and characterless sort of thing, one that receives all things ..."

Other questions now pose themselves:

> *If we develop a theory of space, how can we decide whether it is true?*

Questions like these concerning the justification of beliefs are part of **epistemology**: the philosophy of knowledge & belief. How is knowledge possible? Is our knowledge grounded on reason or by experience? Plato seems to be stuck between the two camps. Space is not part of the world of forms to be understood by pure reason; but nut neither is space a physical object.

The third question we deal with is physical:

What role does space play in science, and in particular what interaction (if any) is there between space and material bodies?

According to Plato, as space has the elements impressed upon it, it is disturbed an in turn agitates the elements. Matter acts on space and space reacts on matter.

To summarize, Plato has raised three problems:

1. ***Metaphysical:*** *What kind of thing is space?*
2. ***Epistemological:*** *How can we come to know anything about space?*
3. ***Physical:*** *In what way do space and matter interact?*

2.1 Euclid's Elements

Euclid's *Elements* demonstrates that all geometric knowledge is a deductive consequence of five basic "postulates", then all the rest of geometry is deduced from those truths. All of geometry is "contained" in the *axioms*, waiting to be unpacked as *theorems*.

Euclid's treatment isn't actually complete. David Hilbert showed at the end of the nineteenth century, that Euclidean geometry can be captured in twenty axioms (though Hilbert added a questionable "axiom of completeness"). There are also some points that need to be clarified and updated with some standard modern terminology for the basic elements: *points and lines*.

Euclid defines a point to be "that which has no part", so we take it that points have neither magnitude nor dimensions. Next, he defines lines to be "breadthless length", meaning that lines have only length, and are one-dimensional. Euclid doesn't state that lines are a collection of points, and that on any straight line connecting two points is a third point dividing the line in two: a property known as denseness (from Hilbert's axiomatization). In this translation, "line" refers to any continuous set of points, which we will generally refer to as a "curve". A line is then a straight curve. Intuitevly, "straight" means the shortest path between two points. Also, we will refer to a line between two points as a "segment". Further we can take two segments and construct a tird whose length is equal to the sum of the lengths of segments.

2.1.1 The Theory of Space

Now that we understand the significance of Euclid's work, we can see that it fits the picture of a scientific theory. Then we must ask:

1. *Is it Euclidean geometry logically consistent?*
2. *Does it accurately describe the physical world?*

We can make the **Euclidean hypothesis**: *Euclidean geometry correctly describes physical space.* It is important to note that we are treating space as an entity, distinct from matter, but comprised of all possible locations of material objects. Ultimately, the possible locations of which space is comprised are points. Some collection of these points form lines, and those lines can be the boundaries of certain figures; so objects mentioned are parts of space.

We still haven't answered how we can tell whether Euclid correctly describes the properties of space. We need to construct experiments to test the predictions of the theory. There is a problem, with space being immaterial. As we can't observe space, we need to construct experiments that can be observed, and infer what is happening. In the early nineteenth century, the mathematician Carl Friedrich Gauss carried out such an experiment. He assumed that light rays travel along straight lines in space, and measured the internal angles of a spatial traingle whose sides were light rays. Gauss found that the internal angles of his triangle summed to two rights within experimental error.

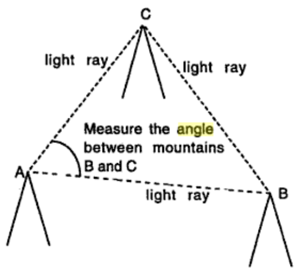

Figure 2.1: Gauss' experiment.

2.2 Zeno's Pardoxes

We will discuss Zeno's work as a challenge to the Euclidean hypothesis. It is inconsistent to both accept the premises of a valid argument and deny its conclusions, so we have to take Zeno's paradoxes seriously, and figure out whether they reveal a contradiction in our beliefs.

2.2.1 The Dichotomy

Zeno's first argument claims that motion is impossible because "that which is in locomotion must first arrive at the halfway stage before it arrives at its goal".

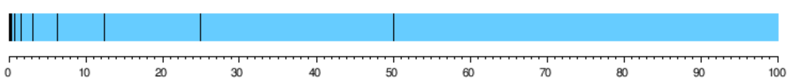

Figure 2.2: The Dichotomy Paradox: the remaining track and the time are ever divided into halves.

Specifically, the root of the paradox is a challenge to the supposition of Euclidean geometry that the finite line sement is a consistent notion.

1. All segments can be divided into two segments

$\therefore$ C1. All segments can be divided into segments without limit.
$\therefore$ *C1'. All segments are composed of an infinity of segments.*

C1'. All segments are composed of an infinity of segments.
2. All segments have finite length.
3. The length of any segment = sum of lengths of segments

$\therefore$ *C2. The length of any segment = an infinite sum of finite lengths.*

C2. The length of any segment = an infinite sum of finite lengths.
4. All infinite sums of finite quantities are infinite.

$\therefore$ *C3. All segments are infintely long.*

All though the argument seems intuitively true, we shall see that it is in fact flawed. The problem lies in premise (4).

Definition: An infinite sequence of increasing positive numbers $\{n_1,n_2,\ ...\}$ converges to a finite limit L if and only if, for all numbers $\epsilon > 0$, there is some number d such that for all integers $c \geq d, |L - n_c| < \epsilon$

Definition: The sum of an infinite series $s_1 + s_2 + ...$ is the limit of the sequence of sums $\{s_1, s_1 + s_2,\ s_1 + s_2 + s_3, ...\}$, if a limit exists

From Cauchy, we see have a precise consistent definition of infinite sums, and showed that infinite sums can consistently be ascribed to finite totals.

2.2.2 The Paradox of Plurality

The next argument is an attempted refutation that a line is composed of points. Part of the problem is to understand how anything with no magnitude can exist.

1. All finite segments are composed of an infinity of identical points
2. The points have either zero length of finite length

$\therefore$ *C1. The total length of the segment is zero.*
OR
$\therefore$ *C2. The total length of the segment is infinite.*

The problem is that we're applying Cauchy's infinite summation to uncountably infinite elements. Any collection with the same number of elements as the sequence of natural numbers is *denumerably infinite* or *countably infinite*. The number of points in a line is *uncountably infinite*.

Each line segment is perpendicular to the segment AB, thus ensuring not only that each segment will pass through the segment AB itself, but that it will

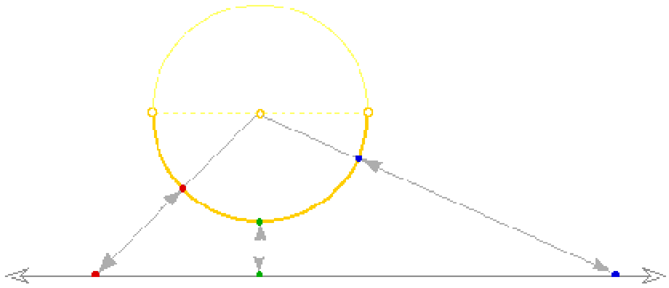

Figure 2.3: Cantor's Proof.

only pass through one point on the semicircle. As a result, the points of the segment AB are matched one-to-one with the points of the semicircle, thus proving that the segment and the semicircle have the same number of points. Now each line segment is drawn from the center of the same semicircle to the line, again ensuring that each segment passes through only one point on the circle and only one point on the line. Thus, the points of the semicircle are now matched one-to-one with the points of the entire line. Therefore, we can conclude that a finite line segment and an infinite line have exactly the same number of points!

We have now showed that the length is not intrinsic to a collection of points, but is logically independent, and then "pasted on" later. Any properties having to do with the distances between points are known as *metrical*.

2.2.3 The Arrow

The final paradox calls into question our geometric understanding of motion. The problem arises because we view both space and time as composed of indivisible points, and because motion is the change of position over time. Zeno thus asks us to consider an arrow supposedly in flight, and its motion at any smallest instant of time.

1. Instants have no parts
2. If the arrow moves during any instant then it has earlier and later parts

---

$\therefore$ *C1. The arrow doesn't move during any instant.*

C1. The arrow doesn't move during any instant.
3. If the arrow doesn't move during any instant, it doesn't move at all

---

$\therefore$ *C2. The arrow doesn't move at all.*

To avoid the paradox, we must reject premise (3). We can do this with the help of Cauchy and Karl Weierstrass. In line with (C1), motion between points p and q occures when if at every instant during the journey the arrow is at the appropriate place along the trajectory. This "at-at theory of motion" requires modern mathematics for it relies on the notion of a *function*. At every

t, the arrow has some position x given as $x(t)$.

Differential calculus exepmlifies the at-at theory. An arrow is moving if its instantenous velocity is non- zero, given by:

$$dx/dt \equiv \frac{x(t+\Delta t)-x(t)}{\Delta t}$$

Thus the arrow is moving at time t as long as it is at an appropriate series of points at the series of subsequent time $t + \Delta t$

Motion is not something that happens during an instant, but rather just a matter of being at the right sequence of places at the right sequence of times.

2.3 Aristotle

We will read Aristotle for the several important questions he poses about space. It is important to note that he also argued that scientific explantion should take the form of logical deductions from first principles. However, Aristotle then states that comphrehension of the basic principles behind our experience was sufficient justification alone without the inclusion of experimentation.

In Aristotle's cosmology, the universe is a finite sphere. At rest at the center is the planet Earth. In an ideal state earth would be a perfect sphere at rest at the center, surrounded by static shells of water, air and fire about which shells of ether would rotate. Any object not in the appropriate place will, as a principle of nature move back. Thus, water poured from a bucket falls, and air under water rises.

Two things are crucial to this account: first the elements move to their appropriate places, which requires a scientific notion of place. Second, objects move in order to reach the places - to achieve goals. Aristotle's theory is *teleological*: it assumes that natural phenomena occur for some purpose.

2.3.1 Place

Aristotle enumerates common beliefs about place, propounds puzzles surrounding it, and tests theories against them: he applies the method of dialectic.

First there are common beliefs:

1. *A place can be seperated from the objects it contains.*
2. *Places don't have places.*
3. *The difference between up & down is absolute.*

Second, there are the puzzles.

- *What kind of thing is space?*

- *How does it interact with matter?*

Aristotle then considers four theories against the common notions and puzzles. He describes place as either:

a *the shape*

b *the matter*

c *an extention between the extremeties*

d *or the extremeties*

[**a**] By "shape", Aristotle means the outer surface of an object. Place cannot be shape though, because the surface of an object cannot be seperated from it, so this theory is incompatible.

[**b**] The matter of an object cannot be seperated from it so it conflicts with (1).

[**c**] By "extremity", Aristotle means the inner surface of whatever contains the object. The "extension" is the volume enclosed. Aristotle views this as an incompatible with the common belief (2). Any finite region of space is contained by other finite region, therefore every place has a place, which begins and infinite regress that (2) avoids. In modern mathematics we are comfortable with this, but Aristotle does not accept this.

[**d**] Refuting all other theories, Aristotle accepts the fourth view. He takes the place of an object to be the boundary of the containing body at which it is in contact. He essentially establishes that place is defined by observable material objects.

If place is a containing surface, then an object is now separable from its place (1). Places as inner surfaces don't themselves have place satisfying (2). This also solves the first puzzles because place itself - a 2-D surface - is only 3-D in the sense of bounding a volume. To show compatibility with (3), we need to consider his theory of motion

2.3.2 Motion

Aristotle asserts that everything has a certain natural "form", that in the ideal course of events it would manifest. The appropriate form is internal, so it does not depend on the external environment. Part of the ideal form is to occupy its natural place. Thus space plays a physical role for Aristotle in that the forms of the elements naturally make the move into their appropriate places. This resolves the causal role of space, and makes it compatible with (3), in that up and down are now absolute. Since earth sits at the center of the universe, and the spherical shell of stars just rotates, two such motionless places exits: the inner boundary of the air and water surrounding earth and the inner boundary of heaven.

Here, Aristotle makes a clear distinction between *kinematics* and *dynamics*: the former refers to study of natural motions (absence of force), and the second to the study of constrained motion (when forces act). This is a basic idea that underlies modern mechanics.

2.3.3 Universe

Aristotle argues at length that our world is unique. His argument is that: earth must fall to the center, but if there are two genuine worlds, there are two centers, so earth must be unique. An obvious response is to suggest natural motion is relative. This would then mean that they have distinct proper places, and hence different forms, So by definition it is not truly earth at all, so our world must be unique.

2.4 Descartes

Descartes is best known for his dictum *cogito ergo sum* "I think, therefore I am". To summarize briefly, his philosophical work was a search for secure knowledge. The primary item that Descartes found trustworthy was the *cogito*: the one thing that is impossible to doubt is that you are doubting; but if you are doubting then you are certainly thinking; and if you are thinking then your own existence is certain. From this foothold, he goes about reestablishing an entire system of the world. The central assumption behind this methodology is that knowledge is achieved through an act of mind, not through experience. This is '*rationalism* as opposed to *empiricism* that we found from Aristotle. With regards to space, Descartes touches on two key issues: the nature of space and the role of space in the theory of motion

2.4.1 The Nature of Space

At the base of Descarte's world system is the distinction between mind and matter. Ultimately, everything is made of these two substances. Descartes believes that the defining characteristic of matter is that it is "extended" in three directions. This is important as space too is just pure extension - a volume with no properties. Space and matter in Descarte's view were the same thing.

To address the objection raised by Aristotle, that place and matter must be separable, we now think of space as fixed by its position *relative* to other objects. This is how we usually conceptualize space. Crucial to the generic separable notion of extension is that a place is picked out by its position relative to "bodies we regard as immobile". Places may be made of matter, but are really relative locations; things are not located in a matter-independent space, but in various relations to one another. The question this poses then is are all "frames of reference" truly equal? Behind this claim is the modern picture of an unbounded universe full of objects in permanent motion: a universe with no fixed point. This is known as **relationism**, which commits a complete equality to all reference bodies. Later we will see if this is scientifically true.

2.4.2 The Theory of Motion

The most significant feature of Descartes' Cartesian mechanics is it contains the first modern notion of **inertia**. This is presented in Descartes' first and second laws of motion, and concerns the distinction between *natural* and *constrained* motions.

In Descartes, the notion of proper place is rejected. Instead, it is natural for bodies to keep moving in exactly the way they are, and forces are required to only to change the state of motion. This tendency is their *inertia*. This brings on a conflict with our view of relationism. If an objects relative velocity depends on the reference frame, its velocity is ambiguous. Which velocity is supposed to remain the same?

The problem is that dynamics distinguishes intertial from non-inertial motions. Relationism cannot make this distinction, because what accelerates relative to one object need not accelerate relative to others. This observation is the basis of one of the most influential arguments for matter-independent space. One can define an absolute velocity, and hence make an unequivocal distinction between inertial and non-inertial motion.

2.5 Newton

Newton took the Euclidean hypothesis literally: he accepted that space had a 3-D Euclidean structure, and contrary to the relationist, that it served as a distinct container for material objects.

2.5.1 Absolute Space and True Motion

With Decartes, objects can be at true rest, and yet detectably accelerate, which is contradictory. Also to explain inertial effects, an object's motion must be absolute. Therefore relativism cannot be the whole truth of mechanics. Newton rejects Descartes' true motion as a candidate for absolute motion with mechanics, and subscribes to a "container view", in that space is absolute. he shows that it is in several sense.

Substantivalism: Newton makes clear that he takes space to be a geometric object. It is composed of points, collections of which comprise lines, surfaces and solids, as in geometry.These geometric objects are possible "absolute places" for material objects. The points then exist independently of material objects. Newton's view is that absolute space is "stuff" rather than a "property", but not as substantial as matter. This is known as *substantivalism*: it asserts that absolute space is something as real as matter, and whose existence does not require matter, but which is not the same stuff as matter.

Immutability: Newton believes that space has to be unaffected by matter, or by anything else. Absolute space is an infinite 3-D rigid Euclidean box that exists unchanging thorugh all time.

Absolute Motion: Immutability now allows the specification of one absolute motion, in that we can now define motion relative to unchanging points. Here we should emphasize that absolute motion is motion relative to absolute space, and that its is possible to be a relationist and absolutist in some ways. For example, Leibniz denies that there is substantival matter-independent space, but he did believe that between any two objects have a definite distance independent of measurement.

2.5.2 Relative Spaces

It is important to consider the relational view more carefully. What is relative space? We define a "frame of reference" relative to some object, called the "reference body". Given such a frame, we can introduce relative motion as the change of position relative to the reference body. A body is said to be at rest in a frame if it remains at a fixed distance in a fixed direction. It is in constant relative motion in a frame when it moves in a straight line and covers equal distances in equal times in that frame. Otherwise, if an object changes its speed or travels along a curved directory, it is relatively accelerating.

Four objects at mutual rest - or four distinct points of a single object - can define a relative reference frame. One to define the origin, and the other there for the x,y,z axes. We take the axes to be perpendicular to each other.

Each axis defines a vector of unit length. let unit vector **i** point in the direction of the x-axis, **j** for y-axis, and **k** for z-axis. Between an object at (x,y,z) lies its relative position vector $\mathbf{r} \equiv x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

The relative velocity vector is then $\mathbf{v} \equiv \frac{d\mathbf{r}}{dt} = \frac{dx}{dt} \cdot \mathbf{i} + \frac{dy}{dt} \cdot \mathbf{j} + \frac{dz}{dt} \cdot \mathbf{k}$

The relative acceleration vector is then $\mathbf{a} \equiv \frac{d^2\mathbf{r}}{dt^2} = \frac{d^2x}{dt^2} \cdot \mathbf{i} + \frac{d^2y}{dt^2} \cdot \mathbf{j} + \frac{d^2z}{dt^2} \cdot \mathbf{k}$

If $\mathbf{a} \neq 0$ and $\mathbf{a}$ and $\mathbf{v}$ are parallel, acceleration is linear. If $\mathbf{a}$ and $\mathbf{v}$ are not parallel, trajectory is not straight and the acceleration is non-linear.

If one wants an unambiguous way to describe the location of an object, any relative frame is adequate. However for dynamic relativity, this does not hold generally. This is a direct consequence of the principle of inertia. Inertia cannot hold in every relative frame. The one's that they do hold for are *inertial frames*. Of course, Newton provided an answer to this: absolute space itself is an inertial frame.

One final point: among the problematic features of absolute space is that it cannot be seen, and that its parts are all alike. For this reason, absolutists use relative spaces to describe motions, and postulates an additional absolute space. This grounds the absolute distinction between uniform and accelerated motions.

2.5.3 Inertia

The general point against the relationist is that there seem to be detectable motions that are not comprehensible as relative motions. To follow the logical through: A the heart of Newtonian mechanics is the principle of inertia, according to which accelerations are produces by forces; the effects of forces are generally detectable, so the question of whether an object that accelerates in one relative frame might not accelerate in another, so apparently there is no relational notion of absolute acceleration; thus the principle of inertia does not make sense in relational terms, and objects that are detectably accelerating might not be in relative motion at all. The seems relationists cannot explain inertial effects.

2.6 Leibniz & Clarke

Leibniz claims to derive all knowledge from his *Principle of Sufficient Reason* (PSR): *there ought to be some sufficient reason why things should be so, and not otherwise.*

2.6.1 Leibniz's Relationism

According to Leibniz, relational space is "an order of coexistence. The point is this: in a relative reference frame, one determines the relative locations of all bodies from some reference body. But any object in principle can be a reference body, so collectively one has a collection of objects at various relative distances from one another. *Without matter there would be no "situation of objects" and hence no relational space.*

2.6.2 Leibniz's Arguments

There are two important facts we need to understand Leibniz's arguments. First, every point of absolute space is exactly like the other. Thus there is no way to distinguish experimentally one absolute place from another. Second, there is no way to determine how fast any object is moving in absolute space. You can't distinguish an object from being at rest from an object at constant velocity, since we can't determine what frame is at rest relative to the absolute frame.

The static shift Imagine a second universe like ours except all matter is shifted to another place in absolute space. Since space is Euclidean, their two are exactly alike.

The kinematic shift In a second universe, all matter's absolute velocity is shifted by a constant amount. Since the difference is by a constant, no differences will be seen.

The dynamic shift Imagine a second universe like ours except that the absolute acceleration of every pice of matter differes by a constant amount.

Leibniz's first line of attack is that with a static shift, their is no sufficient reason for the material universe to be located at one absolute palce rather than another. Therefore, if the universe was absolute, something happened without sufficient reason, so from PSR: space is not absolute.

Leibniz's second attack is based on the *Principle of the Identity of Indiscernibles*(PII), which states that "to suppose two things indiscernible, is to suppose the same thing under two names", that is if two things are exactly alike in every way, then they are only one thing. With the kinematic shift, since there is no observable reference body to distinguish between the two, they are then one and the same. By taking on PII, we are reasoning using the philosophy of empiricism, which means at all our knowledge of the world is derived from experience. Any claims that cannot be tested in some way cannot be justified; they are useless.

With dynamic shift however, the relationist's account fails, as they can only explain relative accelerations, so the inertial effects cannot be explained. If the entire universe decelerated, they would have to maintain that "nothing would receive any shock upon the most sudden of stopping of motion".

2.7 Berkeley and Mach

2.7.1 The Principle of Inertia

Mach rejected Newton's absolute space because motion in absolute space is unobservable and hence without scientific significance. The Newtonian might respond, that we do observe acceleration in absolute space with inertial effects. mach rejects this claim, arguing that all that is observed is that inertial effects are produced by the relative rotation with respect to the mass of the earth and other celestial bodies. The only strictly testable statement of the law of inertia is that "objects remain in constant motion relative to the distant stars unless acted upon by a force".

DEFINITION: Let masses $m_1, m_2, ...$ be located at position vectors r_1, r_2,... respectively from the origin of some reference frame: the distance from the origin to the center of mass, the *center of mass displacement*, $R = \left|\frac{\sum_{i=1}^n m_i r_i}{\sum_{i=1}^n m_i}\right|$.

The displacement speed is $\frac{dR}{dt}$ and the displacement acceleration is $\frac{d^2R}{dt^2}$. Thus $\left|\frac{d^2(\left|\frac{\sum_{i=1}^n m_i r_i}{\sum_{i=1}^n m_i}\right|)}{dt^2}\right| = 0$ implies that the displacement varies constantly in the given frame.

MACH PROPOSES: Any object defines a reference frame and hence a center of mass displacement for the entire universe, R_u. If $\frac{d^2 R_U}{dt^2} = 0$ holds, then the center of mass displacement changes constantly; equivalently, the displacement of the object from the center of mass of the universe changes constantly. The principle of inertia states that $\frac{d^2 R_U}{dt^2} = 0$ for all objects unless acted upon by some force. $\frac{d^2 R_U}{dt^2} = 0$ determines an inertial frame.

In a Machian theory, then, the deep distinction between forced and unforced, natural and unnatural motions is erased. Material objects simply interact in various ways, sometime to produce constant inertial motion, and sometime to produce absolute acceleration. However we still require a theory that will specify the intertial motion of each object at a time. Furthermore it will have to address the matter of the universe will act on any object not moving constantly to produce inertial effects in it. Mach does not give us such a theory.

2.8 Conventionality of Geometry

Kant proposed many of the most profound and influential ideas in Western philosophy since Aristotle. So far we have seen some ways in which both empiricist and rationalist philosophy contributed to the development of the philosophy of space. Kant saw the strengths and weaknesses of both programs, and he attempted to formulate a third alternative.

Kant explains that every experience of the physical world can be split into two components: sensations of things, and the framework in which these sensations are organized. We find that sensations are always organized spatially. Space is the framework within which all our experiences of the physical world occur. From this Kant draws two conclusions: First, the framework is required for us to have experiences, and so it cannot come from experience but is provided, in advance, by us. Second, since the frame work is provided prior to our experiences, our experience will always be spatial and we can be certain in our knowledge of space.

To clarify his philosophy of space, Kant emphasized two distinctions. First he distinguishes two kinds of knowledge: the *a posteriori* and the *a priori*. The first kind of knowledge is empirical, and hence obtained from experience. All other knowledge is *a priori*, and does not rely on experience at all for justification. Kant will argue that our knowledge of space is *a priori*. Later we will see that this view is untenable.

The second distinction is between *analytic* and *synthetic* statements. Analytic statements are those that are true in virtue of definition alone. For example, “newspapers contain news”. An analytic statement does not telly you anything new or informative: it only makes explicit a part of the definition. Analyticity also explains why “newspapers contain news” is *a priori*. Experience cannot be the justification of analytic belief, because no experience can undermine such a belief. All other statements are synthetic, and involve ascribing some new non-definitional property to their subject. Since all analytic statements are a priori, it follows that all a posteriori statements are synthetic. This scheme contains one further possibility: a priori synthetic knowledge. Such knowledge would be both necessary and universal and genuinely new.

2.8.1 Handedness

Kant offers another argument in favor of absolute space based on the familiar phenomena of reflection. He argues that left and right handed mirror images are different, for they are non-congruent, but they do not differ in any intrinsic spatial relations. There is no way to distinguish them by internal distance relations. Kant's challenge to the relationist is to account somehow for handidness even though the distinction is a spatial distinction.

The relationist can answer this through the notion of fitting. For example, the difference between left and right hands rests on such fasts as that left hands fit correctly into left-handed gloves. This is definetly a relational distinction. But does this just postpone the problem? What actually makes a glove left-handed? A right handed glove could be left handed, if you simultaneously reflected everything else in the universe. But that means the right and left handed gloves have the same handedness. However, why should the relationist accept that everything has a reversed handedness in a mirror image? *Within a world*, the handedness of an object is the opposite of the mirror image.

2.8.2 Space

Kant claims that Euclidean geometry is a system of synthetic a priori truths. When we have a visual experience of the world, Kant analyzes such an apperance into two parts. First, it contains a host of sensations caused by the objects and parts of objects we're looking at. Second, there is the matter in which this multitude of appearance is organized into a single, coherent visual field. Kant calls this - whaterver it is that order the many components of an experience into a whole - the form of the experience. Kant believes that the form itself is not something that we learn from experience, but something that is required in advance in order to have any experience at all. Then when objects act on us to produce sensations, the mind is activated to arrange them within the form of experience. Space is known a priori. Kant believes that since all experience is Euclidean, any conceptualization of the world is also in Euclidean space. An immediate consequence of this view that experience involves an essential subjective component is that we can never take an entirely objective view of the world. Every experience involes the form of apperance, as well as the sensations produced by objects, and so we never have direct access to the things themselves. The price of certainty of geometry is that it is only applicable to experience, not to the mind-independent world. An important aspect of this point is that space, in a sense, is not real at all. It is soemthing added into experience by the mind, not something that exists in itself.

2.8.3 Non-Euclidean Geometry

We will consider two non-Euclidean axiomatic geometries: *elliptical* and *Bolyai-Lobachevskian.* The axioms of elliptic geometry are:

1. Any two points lie on a line
2. Any segment can be continued indefinitely

3. Any two points (the center and point on the circumference) define a circle
4. All right angles are equal
5. Whatever interior angles are made by a line falling on two lines, the two lines will intersect if continued indefinitely

To demonstrate that this system is consistent we will show that it correctly describes geometry on a sphere.

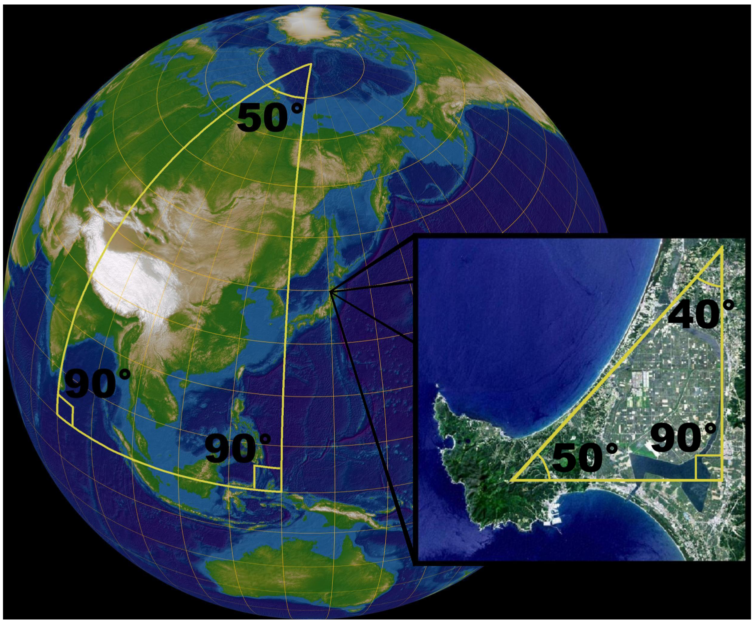

Figure 2.4: Internal angles of a triangle on a sphere don't add up to 180

1. On a sphere, the shortest curve between two points are segments of "great circles". On any sphere of radius R and center C, any circle of radius R and center C is a great circle.
2. To extend a line segment in spherical geometry, you continue it further along the great circle. If continued long enough, it will close back on itself, but this just means that the begining of the line has been hit, so you can continue the line.
3. One point is the center, and one point is the circumference, determining the radius
4. By inspection we can see that all right angles are congruent.
5. Given any two great circles G and H, we pick any point P on H. P' the antipodal of P will be on the other hemisphere defined by G. The great circle of PP' must cross G

In Lobachevski geometry, we have the following axioms:

1. Any two points lie on a unique line
2. Any segment can be continued indefinitely
3. Any two points define a circle
4. All right angles are equal
5. For any line and any point, there are infinitely many lines through the point parallel to the line.

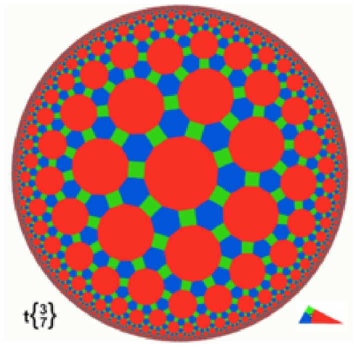

Figure 2.5: Poincare's disk

2.8.4 Conventionalism

Inhabitants of a world with BL geometry or spherical geometry would realize that they are living in a world with such a geometry. It is also entirely possible that their world could exist within a Euclidean world, as we have constructed above, but there is no way of them realizing this. Hence, their geometry is a conventional choice.

This is a problem of **underdetermination**. For any body of data (O_1, O_2, O_3, ...) there are indefinitely many other theories that can account for it. The realist will argue that one of there theories is actually correct. The anti-realist would say that since we can't tell in principle which theory is true, then the concept of truth does not apply, and none of them are actually true.

Chapter 3

Special Relativity

3.1 Space-Time

In modern physics, space and time are not considered distinct, but as forming a single entity "space-time". On the space-time conception important new facts are revealed about the nature of intertia and the absolute/relative debate.

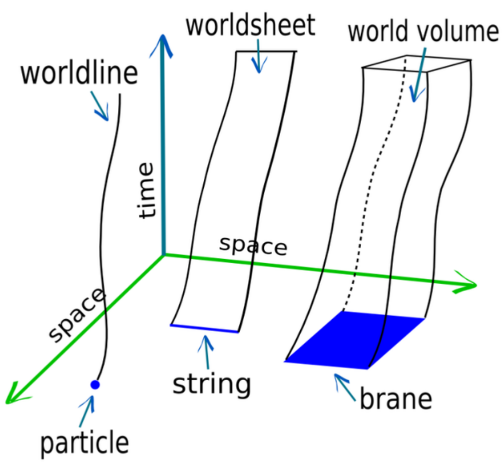

Figure 3.1: Space-time diagrams

Since all occurences take place somewhere at some time, points of space-time are known as *events*. Now since objects exist over a period of time, they sweep out various curves.

3.1.1 Newtonian Space-Time

As we shall see, spacetiem can take many forms. The first that we will consider is the form that Newton might have took. First of all, we expect that, like absolute space, it would be a substance-like object independent of matter. Any static shift in space-time would be located idfferently in a substantial space-time. Newton would also take that it would allow for definition of absolute motion. More concretely:

Newtonian space-time is defined by the following properties: between any two events p and q, let there be a definite, Euclidean spatial distance, R(p,q) and temporal interval T(p,q)

We still inherit the problems concerning PII that Leibniz raised earlier. We do however allow for absolute acceleration, so we can formulate a principle of inertia like before. This view is konwn as *substantival Newtonian space-time.*

3.1.2 Galilean Space-Time

An alternative formulation is Galilean space-time, or neo-Newtonian space-time, which is defined as: the is a definite tiem interval between any two points of space-time, but only if two points are simultaneous is there any spatial distance between them. Further, for any segment of a space-time curve it is a definite fact whether it is staight or not. Concretely:

Galilean space-time is defined by the following properties: between any two events p and q, (i) let there be a definite temporal interval T(p,q);(ii) if they are simultaneous($T(p,q) = 0$), let there be a definite spatial distance $D(p,q)$; (iii) given any curve c through p, let c have a definite curvature at p, $S(c,p)$

Let us take a substantival view of Galilean space-time, postulating that space-time exists independently of matter, so that static shifts lead to distinct but undetectable states of affairs. However a kinematic shift in Galilean space-time does not change anything. There is no definite distance between points at different times on a world line, and hence have no definite speed. It is thus impossible for two Galilean universes to differ just beacuse their material contents have motions that differ by a fixed amount. Also, our space-time diagrams misrepresent Galilean space-time. They show world lines to have definite slopes, when they do not. A trajectory going up is the same as a world line that is sloped. A kinematic shift has no real effect, as Leibniz claimed. However we can differentiate curved world lines from straight line: Absolute acceleration is acceleration relative to any straight trajectory. Therefore one now avoids Leivniz's kinematic shift problem and can explain inertial effects. Every inertial frame is now equivalent, and we have removed Newton's absolute frame. The principle that all inertial frames are equivalent is known as the **principle of relativity**.

3.2 Lorentz Transformation

There are two postulates in special relativity:

Principle of Relativity The laws of nature have the same form in all inertial frames of reference

Invariant Speed The speed of light is the same in every frame.

3.2.1 Relative Character of Simultaneity

In classical physics, we have the assumption that *"The events A and B occured simultaneously"* can be given meaning independent of a reference frame. In order for this to have any significance, we must be able to devise an experiment to test its validity. We setup an apparatus at the midpoint of two events A and B. Each event emits a light signal, and we will call them simultaneous if they arrive at the midpoint at the same time. We assume that simultaneity as defined by this experiment is transitive.

Is our definition invariant for all different frames of reference? Consider two frames of reference, one connected to the Earth (S), and another with a long train (S*) moving along a straight track at a constant speed. We shall have two observers, one stationed on the ground by the track and one on the train. Let us assume now that two thunderbolts strike, each hitting the train as well as the ground. The recording apparatus was stationed in between the two strikes. At the instant that the thunderbolt strikes at A and A*,these two points coincide. The same is true for B and B*. Because of the finite time needed by the light signals to reach C and C*, the signal from A, A* reaches C* first, then C. The signal from B, B* reaches C first, then C*. As a result, the train observer observes that the light from A, A* reaches his apparatus before the B, B* signal. We can conclude that two events that are simaltaneous in one frame of reference may not be in another.

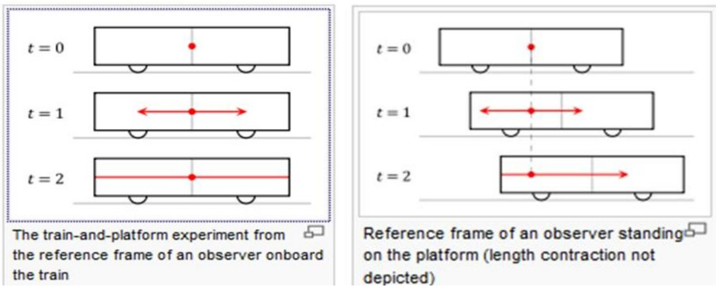

Figure 3.2: Train vs Platform Frame of Reference

This conclusion will also effect our notion of length measurment. If two distances are parallel to the direction of relative motion, and if they are travelling along the same straight line, the two distances are equal if the start and end coincidences occur *simultaneously*. In the case of the thunderbolts, the two distances AB and A*B* appear equal to the ground observer. The train observer on the other hand finds A coincides with A* before B coincides B*, and concludes that A*B* is longer than AB.

We also cannot tell whether two clocks at a considerable distance agree with each other. Considering the same scenario with clocks D and D*, let us assume they agree at the moment when D* passes D. After a while D* and D will be a considerable distance apart. Since an observer from the ground and train will have a different view of simultaneity, we find time is also relative to frames of reference.

3.2.2 Simple Derivation

Consider two inertial frame S and S*. S* moves reltively to S at the constant rate v along the x-axis; at the S-time $t = 0$, the points of origin coincide.

$$x^* = \alpha(x - vt)$$
$$y^* = y$$
$$z^* = z$$

We can compare the lengths of rods in differents states of motion in an invariant manner if they are paralllel to each other and orthogonal to the direction of motion. To complete the transformation, we have to formulate an equation to connect t^*. t^* must depend on t,x,y, and z linearly.

$$t^* = \beta t + \gamma x$$

Now assume that at time $t = 0$, an electromagnetic spherical wave leaves the point of origin of S and S^*. Since the speed of light is constant:

$$x^2 + y^2 + z^2 = c^2 t^2$$
$$x^{*2} + y^{*2} + z^{*2} = c^2 t^{*2}$$

Substituting to replace the starred quantities we get:

$$c^2(\beta t + \gamma x)^2 = \alpha^2(x - vt)^2 + y^2 + z^2$$

Rearranging the terms:

$$(c^2\beta^2 - v^2\alpha^2)t^2 = (\alpha^2 - c^2\gamma^2)x^2 + y^2 + z^2 - 2(v\alpha^2 + c^2\beta\gamma)xt$$

To be equivalent to S frames equation, the coefficients of t^2 and x^2 must be the same, and the coefficient of xt must vanish so:

$$c^2\beta^2 - v^2\alpha^2 = c^2$$
$$\alpha^2 - c^2\gamma^2 = 1$$
$$v^2\alpha^2 + c^2\beta\gamma = 0$$

Eliminating α^2 with (1) (3) and (2) (3) we get:

$$\beta(\beta + v\gamma) = 1$$
$$c^2\gamma(\beta + v\gamma) = -v$$

Eliminating γ by substitutions we get:

$$\beta^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$
$$\gamma = \frac{1-\beta^2}{\beta v} = -\frac{\beta v}{c^2}$$
$$\alpha^2 = -\frac{c^2\beta\gamma}{v} = \beta^2$$

Substituting all these values we get the new transformation equations:

$$x^* = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$y^* = y$$
$$z^* = z$$
$$t^* = \frac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

3.2.3 Kinematic Effects

If we now compare S-time interval $(t_2 - t_1)$ is related to t_2^*, t_1^* by:

$$t_2 - t_1 = (t_2^* - t_1^*)/\sqrt{1 - v^2/c^2}$$

We see that the rate of the clock slows down from S by a factor $\sqrt{1 - v^2/c^2}$. This is known as *time dilation*. We see that it also has an effect on length in each frame of reference:

$$x_2 - x_1 = (x_2^* - x_1^*)\sqrt{1 - v^2/c^2}$$

This effect is called *Lorentz contraction*. To summarize, every clock appearts to go at its fastest rate when it is at rest relative to the observer. If it moves relative to the observer, its rate is slowed down by $\sqrt{1 - v^2/c^2}$. Every rigit body appearts to be longest when at rest relative to the observer. When it is not it appears contracted by a factor of $\sqrt{1 - v^2/c^2}$, while dimensions perpendicular to the direction of motion are unaffected.

3.2.4 Proper Time Interval

Consider two events at the space and time coordinates (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2). The difference between the squared time interval and the squared distance

3.3 Conventionality in Special Relativity

Conventionalism: Theories do not correspond to any fact of the matter. They are conventionally accepted, for practicality or because we lack a choice.

Verificationism is the doctrine that a proposition has meaning if and only iff there is an empirical method of testing it.

Our definition of simultaneity depends on the constant speed of light. How do we know this to be true?

3.3.1 The Light Principle

One-way Principle The speed of light is c in every direction

Two-way Principle On a round trip, the average speed of light is c

The two-way principle has been empirically verified, but the one-way speed cannot. If we accept the one-way speed, it will be a convention as it is not an established fact. We find that we can't measure the one-way speed of light independent of any clock synchronization scheme.

3.3.2 Reichenbach

The two-way speed of light is empirical fact, but the one-way speed is convention. *"To determine the simultaneity of distant events we need to know a velocity, and to measure a velocity we require knowledge of the simultaneity of distant events. The occurrence of this [...] logical circle shows that a knowledge of simultaneity is impossible in principle."*(Philosophy of Space and Time, 126f)

1. A statement has truth value if and only if it is empirically testable.
2. It is impossible to test the one-way speed of light.
3. $\therefore$ There is no fact of the matter about the one-way speed of light.

There are some typical ways to critisize this argument:

1. Find a method that will measure one-way speed of light.
2. Reject verificationism
3. Appeal to some other principle, e.g.: Simplicity
4. Link to a well established principle in physics that is well-established, e.g.: space is isotropic

3.3.3 Grünbaum

Grünbaum argues for the same conclusion, but based on the causal theory of time order.

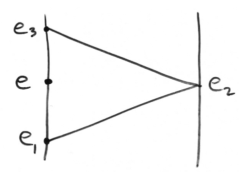

Figure 3.3: Causal ordering of events

1. All temporal relations reduce to physical processes
2. There are causal chains that link events e_1 to e_2 and e_2 to e_3, so the temporal must be $e_1 < e_2 < e_3$
3. There is no link between e and e_2

4. $\therefore$ There is no physically definable ordering of events e and e_2. Any ordering must then be a convention

3.3.4 Malament

Malament makes a counter-argument, claiming that there is a natural definition of simultaneity, where $\epsilon = \frac{1}{2}$

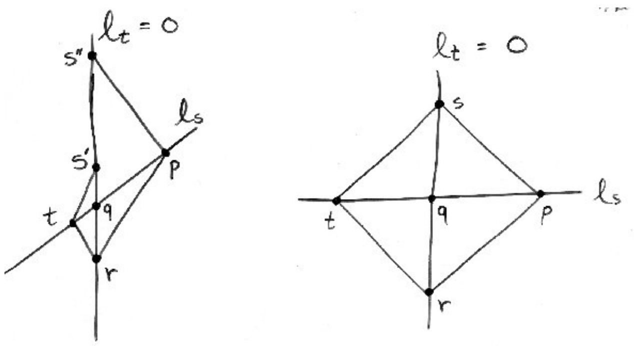

Figure 3.4: Malament's Construction

1. l_t is a time-like line and l_s is a space-like line, intersecting at event q
2. Some event r earlier that q.
3. r is causally connected to t and p, via time relation κ
4. Light cones from t and p will intersect l_t at some point. When they intersect the same point s, then l_t is orthogonal to l_s
5. Simultaneity is just orthogonality in some frame O, and is the only definable simultaneity relation from κ

If Malament is correct, we would have a non-conventional way to define simultaneity within a single frame.

Chapter 4

General Relativity

There are two basic building blocks of modern relativistic cosmology: a manifold of events, and the fields defined on it. We define our world in a four-dimensional manifold of events. Just having the events however doesn't define anything about the events themselves. There is no conception of which events are in the past or future, or any relation between the events for that matter. We need to define a **metric field** to provide such information. The matter of our universe is represented by matter fields.

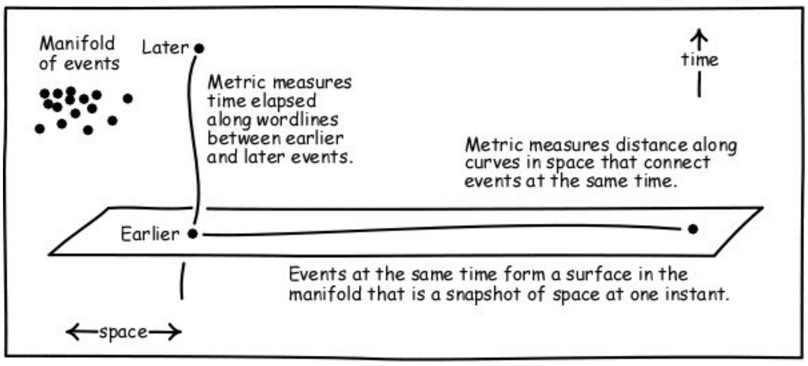

Figure 4.1: Metric Field

Principle of Equivalence Gravitational force and accelerated motion are indistinguishable

Geodesic Principle The natural motion of an object is free fall. If an object is in free fall, its path follows a geodesic

Principle of General Relativity The laws of nature are the same in all frames of reference

4.0.5 Gravity

Gravity is actually caused by the curvature of spacetime. The curvature of spacetime is caused by the the presence of matter-energy. This is described by the **Einstein field equations**:

$$R_{\mu\nu} \; - \; \tfrac{1}{2} R \, g_{\mu\nu} \; = \; \tfrac{8\pi G}{c^4} T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $R = Tr(R\mu\nu)$ is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, and $T_{\mu\nu}$ is the stress-energy tensor. A metric tensor would be solutions for the EFE's, as the Ricci and curvature tensors actually depend on the metric tensor. When fully written out, the EFE are a system of 10 coupled partial differential equations. Einstein would later make a modification with the cosmological constant $\Lambda g_{\mu\nu}$ to scale the metric tensor, to allow for a static universe, which Einstein later calling it the "biggest blunder [he] ever made". We can make a simplification to:

$$G_{\mu\nu} \; = \; 8\pi \, T_{\mu\nu}$$

Crudely speaking, spacetime tells how matter how to move, and matter tells how spacetime how to curve.

4.0.6 Spacetime Substantivalism

Since we first posited the existence of a manifold of events, it seems natural to define this manifold as spacetime. This however, specifically excludes the metric field, which contains information on spatial distances and time. Shouldn't that be included in spacetime? In general relativity, we find that the metric field also includes the gravitational field. The gravitational field itself carries energy and momentum, which is a natural distinguishing characteristic of matter. The metric field appears to be part of the container, and part of the matter contained!

If we were to take the formulation of our universe as described above literally, spacetime is a manifold of events with certain fields defined on the manifold. It seems as though this manifold is an independent structure that bears properties.

4.0.7 The Hole Argument

Essentially, our model of our universe consists of M the manifold, $g_{\mu\nu}$ the metric tensor, and $T_{\mu\nu}$ the stress-energy tensor. We can define a **diffeomorphism**, an invertible function that maps one differentiable manifold onto another, that can map the metric tensor and stress-energy tensors.

Any two definitions of our universe with such a spreading will still agree on all invariant properties. These invariant properties include distance along spatial curves, time along worldlines, rest mass, and number of particles. This is important, as we find that all observables can be reduced into these invariants.Therefore, since the two spreadings or distributions of metric and matter fields of a hole transformation agree on invariants, they also agree on all observables. ***They are observationally indistinguishable.***

We now define a mapping that is identical to the original except for some region, known as 'the hole'. Within the hole, any particle would obtain a different trajectory. Physically however, there would be no discernible difference in velocity or time of the trajectory.

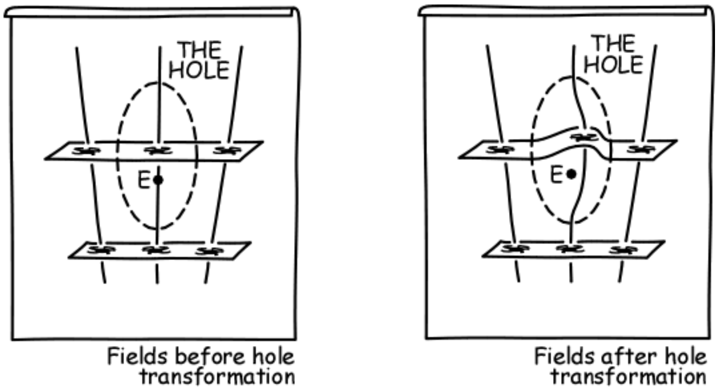

Figure 4.2: The Hole Transformation: Does Event E pass through the worldline?

Chapter 5

Quantum Mechanics

5.1 Formalism

Physical states are represented by **state vectors** $(|A\rangle, |B\rangle, ...)$. Each state must have a length of 1. Every physical system is then associated with some vector space. Measurable properties, known as **observables**, are represented as linear operators on the vector spaces of the physical systems. If some physical state happens to be an eigenvector , with eigenvalue a, of an operator of a certain property, then that state has the value a for that particular measurable property. Sums of vectors are then **superpositions** of physical states.

For example lets take:

$$|hard\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} |soft\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\langle hard|\ soft\rangle = 0$, so the two vectors will constitute a basis of two-dimensional space.

$$\text{Hardness Operator} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

where we stipulate that '+1' is hard and '-1' is soft. We can see that $|hard\rangle$ and $|soft\rangle$ are eigenvectors of the hardness operator. We can represent color in this space too:

$$|black\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} |white\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

$$\text{Color operator} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ '+1' is black and '-1' is white}$$

From this we can see the principles of superposition and incompatibility.

$$|black\rangle = \frac{1}{\sqrt{2}}|hard\rangle + \frac{1}{\sqrt{2}}|soft\rangle$$
$$|white\rangle = \frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

$$|hard\rangle = \frac{1}{\sqrt{2}}|black\rangle + \frac{1}{\sqrt{2}}|white\rangle$$
$$|soft\rangle = \frac{1}{\sqrt{2}}|black\rangle - \frac{1}{\sqrt{2}}|white\rangle$$

States of color are superpositions of hardness, and states of hardness are superpositions of color. Moreover, hardness states aren't eigenvectors of the color operator and vice-versa. Hardness and color operators are **incompatible**. Measurable physical properties are said to be incompatible if the measurment of one will always necessarily disrupt the other.

Given the state of any physical system at any "initial" time with any forces and constaints the system is subject to, the state of the system at a later time can be calculated. For nonrelativistic systems, the dynamics of the system are determined by the **Schrödinger equation**. Quantum-mechanical dynamic systems are **linear**. Given $|A\rangle, |A'\rangle, |B\rangle, |B'\rangle$, then $a|A\rangle + b|B\rangle \rightarrow a|A'\rangle + b|B'\rangle$.

Now suppose we have a system whose state vector is $|a\rangle$ and we measure property B with eigenvectors $|B = b_i\rangle$ and eigenvalues b_i. The **probability** that the outcome of the measurement will be $B = b_i$ is equal to : $(\langle a| B = b_i\rangle)^2$. Now since $(1x)^2 = (-1x)^2$, the probability of $|a\rangle$ and $-|a\rangle$ is the same. As they have the same observable consequence, $|a\rangle$ and $-|a\rangle$ represents the same physical state.

If we make a measurement of an observable O of a system S, and the outcome is $O = @$, then *the state vector of S after that measurement must necessarily be an eigenvector of O with eigenvalue* $@$. The effect of measuring an observable must necessarily be to "collapse" it to some eigenvector of the measured observable. The outcome is determined by the probability, as described above.

So how can we predict the behaviour of some particular physical system? We firt identify the vector space associated with that system, and then the operators associated with various measurable properties of the system. we can then build specific correspondences between individual physical states and individual vectors. The present state vector can then be ascertained by measurements. The state vector of future times can be calculated by the dynamics of the system (Schrödinger equation).

The vectors spaces used in quantum mechanics are actually **complex**. The norm of any vector is instead: $\sqrt{\langle A| A\rangle}$. The probability of an outcome is then $|\langle a| B = b_i\rangle|^2$. This ensures the results are postive real numbers. More generally now $|A\rangle$ and $@|A\rangle$ represent the same state where $|@| = 1$.

Elements of the linear operators can now be complex numbers too. Operators that have all of their eigenvectors associated only with *real* eigenvalues are known as **Hermitian operators**. As measurable quantities are always real numbers, operators associated with measurable properties must necessarily be Hermitian. Hermition operators have some certain properties:

1. If two vectors are both eigenvectors of the same Hermitian operator, and their eigenvalues are different, then the vecters are necessarily orthogonal.

2. Any Hermitian operator on an N-dimensional space will always have at least one set of N mutually orthogonal eigenvectors. It is always possible to form a basis of the space out of the eigenvectors of the Hermitian operator.

3. If a Hermitian operator on an N-dimensional space happens to have N different eigenvalues, then there is a unique vector in the space associated with each of those eigenvalues. Operators like that are called *complete* or *nondegenerate* operators

4. Any Hermitian operator on a given space will be associated with some measurable property

5. Any vector in a given space will be an eigenvector of some complete Hermitian operator on that space. Any quantum state will be associated with some definite value of some measurable property of that system.

From this, it turns out that every quantum-mechanical system necessarily has infinitely many mutually incompatible measurable properties. By (3) and (5), every state in this space is necessarily the only eigenstate associated with a certain particular eigenvalue of a particular complete operator. By fact (1), none of the continuous infinity of states which aren't orthogonal to the state in question can possible be eigenstates of the same complete operator. The complete operators of which those other states are eigenstates clearly can't even be *compatible* with the operator in question. And so there must be necessarily be *a continuous infinity of mutually incompatible complete measurable properties.*

The **commutator** of two matrices A, B, denoted by $[A, B] = AB - BA$. Operator matrices of incompatible observables can't possibly share any complete basis of eigenvectors, since such eigenvectors would correspond to definite value states of both observables at the same time. *The commutators of incompatible observable matrices are non-zero.* It turns out that the commutator can be used to assesed the degree of their compatibility.

5.1.1 Coordinate Space

We expect that from quantum theory, we can reproduce classical physics. This is known as the **principle of correspondence**. To do so, we need to calculate the commutators of quantum observables. It happens that momentum and position of a particle are incompatible observables.

$$[p, x] = i\hbar$$

The x and p operators will have bases that do not share the same eigenvectors. A state of some momentum will be some superposition of different position states, and vice-versa.

Now consider a particle confined to a one-dimensional coordinate space. Let $|X = 1\rangle$ represent the sate in which the particle is located at point 1. So $X\,|X = 5\rangle = 5\,|X = 5\rangle$. The possible eigenvalues of the position operator X will extend from $-\infty$ to $+\infty$. As the different eigenvectors of X must form a basis of the state space of this particle, and since X has an infinity of different eigenvalues, and since the eigenvectors $|X = @\rangle$ must necessarily all be orthogonal. It then follows that *the state space of this particle must be infinite-dimensional*.

Any vector in that infinite-dimensional space can be expanded in terms of X eigenstates, like:

$$|\psi\rangle = a_5\,|X = 5\rangle + a_7\,|X = 7\rangle + a_{72.93}\,|X = 72.93\rangle + ...$$

where $a_x = \langle\psi|\ X = x\rangle = \psi(x)$. This **wave function**, serves to pick the unique vector $|\psi\rangle$ out of the infinite-dimensional space. Location probabilities can be read off from the wave functions. THe probability that the particle is located at $x = x_1$ is $|\psi(x)|^2$. It turns out that any measurable property of particles is representable as an operator on the wave function, rather than on the state vectors.

5.1.2 Systems of More than a Single Particle

Consider a pair of particles, number one in the state $|\psi_a\rangle$ and number two in the state $|\psi_b\rangle$. The quantum state vector of a pair of particles is written as:

$$|\psi_a\rangle_1\,|\psi_b\rangle_2\,or\,\left|\psi_a^1, \psi_b^2\right\rangle$$

Now that we have two particles, we have to deal with joint probabilities. If the particles do not interact, we can use the law of composition of independent probabilities, where we multiply the probabilities together. Multiplying vectors now becomes:

$$\left\langle\psi_c^1, \psi_d^2\right|\,\psi_a^1, \psi_b^2\right\rangle = \left\langle\psi_c^1\right|\,\psi_a^1\right\rangle \times \left\langle\psi_d^2\right|\,\psi_b^2\right\rangle$$

Now suppose the vectors $\left|\psi_1^1\right\rangle ... \left|\psi_n^1\right\rangle$ forms a basis for particle 1, and $\left|\psi_1^2\right\rangle ... \left|\psi_n^2\right\rangle$ forms a basis for particle 2. So:

$$\left\langle\psi_i^1, \psi_j^2\right|\,\psi_k^1, \psi_l^2\right\rangle = 0 \text{ for i=k and j=l}$$

The dimensionality of the two-particle space is N^2. The entire set of vectors will be of the form:

$$\left|\psi_i^1, \psi_j^2\right\rangle \text{ for i,j = 1...N}$$

They will form the basis of the two-particle system, and any linear combination of these basis vectors will be another vector of the space.

Now consider a state:

$$|Q\rangle = \frac{1}{\sqrt{2}}\left|\psi_1^1, \psi_1^2\right\rangle + \frac{1}{\sqrt{2}}\left|\psi_2^1, \psi_2^2\right\rangle$$

We find that it can't be decomposed into a well-defined state of particle 1 and particle 2. States like these are called **nonseperable**. In states like these, no measurable property of particle 1 alone, and particle 2 alone have any definite value.

5.1.3 Field Theory

In relativistic field quantum theories, one imagines that there is an infinitely tiny physical system permanently at every mathematically defined point in space. Each one of these systems is stipulated to be a quantum mechanical system. The complete array of these systems forms the field.

5.1.4 Uncertainty Principle

The **Heisenberg Uncertainty Principle** states that there is a fundamental limit to the accuracy with which certain pairs of physical properties of a particle, such as position and momentum, can be simultaneously known. Operators that are non-commutable, then are mutually complementary. The most common example is position and momentum as described earlier:

$$[x, p] = i\hbar$$

One way to understand the complementarity between position and momentum is by wave-particle duality. If a particle described by a plane wave passes through a narrow slit in a wall like a water-wave passing through a narrow channel, the particle diffracts and its wave comes out in a range of angles. The narrower the slit, the wider the diffracted wave and the greater the uncertainty in momentum afterward.

5.2 EPR Paradox and Bell's Inequality

5.2.1 "God Does not Play Dice"

Suppose we prepare two spin-1/2 particles a and b in the singlet spin state

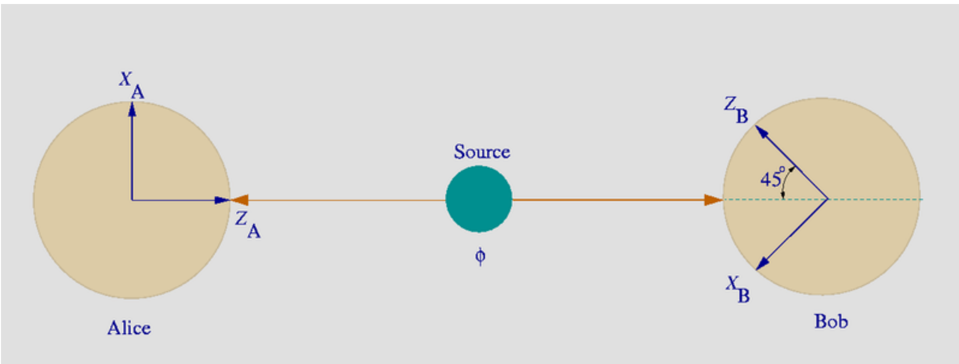

Figure 5.1: EPR experiment

$$|\psi_s\rangle \frac{1}{\sqrt{2}}(|a:+z;b:-z\rangle - |a:-z;b:+z\rangle)$$

Particle a is measured along u_a and particle b is measured along u_b. If they measure along the same axis $u_a = u_b = e_z$, then with with probability of 1/2, Alice will measure particle a as $+\hbar/2$ and Bob will find $-\hbar/2$ or vice-versa. There is perfect anti-correlation between the results.

EPR then assumes the following three principles:

Completeness: Every element in reality has a counterpart in the theory

Criterion of Reality: If without disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity

Locality: A measurment that is space-like seperated from the system, does not disturb the system.

The EPR argument is then as follows.

1. By locality the measurment on b was not disturbed by the measurment of a
2. We could predict the outcome of b with certainty so by the criterion of reality, the spin component is an actual element of reality
3. The initial description of $|\psi_s\rangle$ does not account for this reality

We are now left with two options. We can stick with our orignal quantum description, and take it that non-local characteristics are possible. Particles a and b do not have individual realities, and only have a well-defined value after measurement. Alternatively, we can adopt the view of EPR where quantum mechanics is insufficient.

5.2.2 Bell's Inequality

If the super-theory that Einstein was hoping for existed, it will involve for the pair (a, b) some parameter λ which will determine complete the results obtained by Alice and Bob. We denote by Λ the manifold in which the parameter λ evolves. There must exists a function $A(\lambda, u_a) = \pm\hbar/2$ for Alice and $B(\lambda, u_b) = \pm\hbar/2$ for Bob. These results depend on *lambda* where if λ pertains to some subset $\Lambda_+(u_a)$ then $A(\lambda, u_a) = \hbar/2$. If λ corresponds to the complementary subset $\Lambda - \Lambda_+(u_a)$ then $A(\lambda, u_a) = -\hbar/2$ The function only depends on the direction of analysis and the value of λ. The parameter is a *hidden variable*, not accessible to quantum mechanics.

We now introduce the correlation function $E(u_a, u_b)$ which is equal to the expectation value of the product of results of Alice and Bob.

$$E(u_a, u_b) = \tfrac{4}{\hbar^2} \int P(\lambda) A(\lambda, u_a) B(\lambda, u_b) d\lambda \text{ where } |E(u_a, u_b)| \leq 1$$

The function $P(\lambda)$ describes the unknown distribution of λ. We can then express the expectation as:

$$E(u_a, u_b) = \tfrac{4}{\hbar^2} \langle\psi_s| S_a \cdot u_a \otimes S_b \cdot |\psi_s\rangle = -u_a \cdot u_b$$

Bell's theorem then states:

1. For a local hidden-variable theory, the quantity

$$S = E(u_a, u_b) + E(u'_a, u_b) + E(u_a, u'_b) - E(u'_a, u'_b)$$

always satisfies the inequality

$$|S| \leq 2$$

2. This inequality is violated by the predictions of quantum mechanics

First we introduce:

$$S(\lambda) = A(\lambda, u_a)B(\lambda, u_b)+A(\lambda, u'_a)B(\lambda, u_b)+A(\lambda, u_a)B(\lambda, u'_b)-A(\lambda, u'_a)B(\lambda, u'_b)$$

$$S(\lambda) = A(\lambda, u_a)(B(\lambda, u_b) + B(\lambda, u'_b)) + A(\lambda, u'_a(B(\lambda, u'_b) - B(\lambda, u_b))$$

So S becomes:

$$S = \frac{4}{\hbar^2} \int P(\lambda)S(\lambda)d\lambda$$

We see the $S(\lambda)$ is always equal to $\pm\hbar^2/2$. Substituting and integrating, we arrive at the inequality stated in 1: $|S| \leq 2$
However consider the case of:

$$u_b \cdot u_a = u_a \cdot u'_b = u'_b \cdot u'_a = -u_b \cdot u'_a = \frac{1}{\sqrt{2}}$$

From quantum mechanics:

$$E(u_a, u_b) = -u_b \cdot u_a$$

So S evaluates to $2\sqrt{2}$, violating our previous inequality.

This result was later verified experimentally, in particular the Aspect experiment(1982). This means the EPR is false, and nature is actually non-local.

5.3 Delayed Choice Experiment

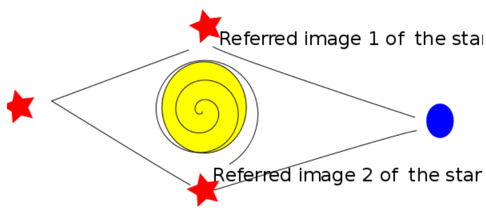

Figure 5.2: Delayed Choice experiment

John Archibald Wheeler proposed a variation of the famous double-slit experiment of quantum physics, one in which the method of detection can be changed after the photon passes the double slit, so as to delay the choice of whether to detect the path of the particle, or detect its interference with itself. Since the measurement itself seems to determine how the particle passes through the double slits, and thus its state as a wave or particle.

An implementation of the experiment in 2007 showed that the act of observation ultimately determines whether the photon will behave as a particle or wave, verifying the unintuitive results of the thought experiment. This is unintuitive as it seems if our present choices create the reality of our past.

5.4 The Measurement Problem

The measurment problem is the conflict between the postulate of collapse and the dynamics of a given system. The dynamics of physical systems in general are fully deterministic. The collapse postulate is not. It is unclear to us how the two can both be consistent.

5.4.1 Schrödinger's cat

Schrödinger's famous thought experiment poses the question, when does a quantum system stop existing as a superposition of states and become one or the other? Can macroscopic systems get into a superposition of states?

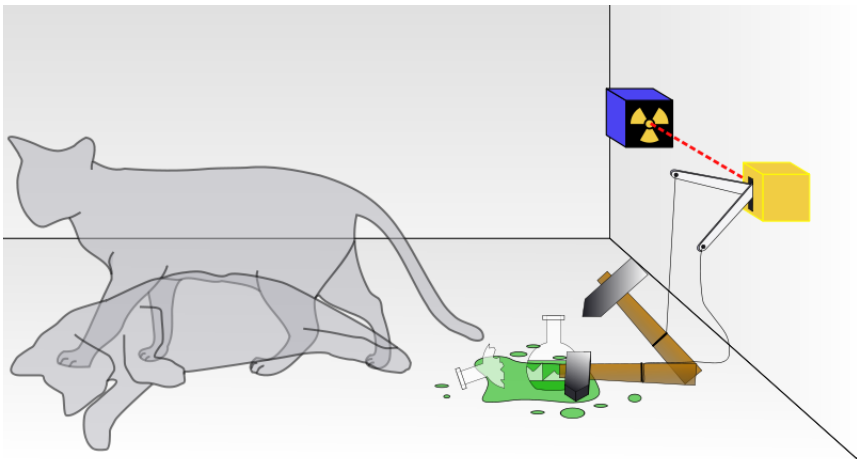

Figure 5.3: Schrödinger's cat

A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small that perhaps in the course of the hour, one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges, and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would *express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.*

Different interpretations of QM have different replies to this problem. This thought experiment is a manifestation of the measurement problem.

5.4.2 Collapse of the Wave Function

So what is exactly required of a wave function collapse?

1. A measurement must always have an outcome.
2. The Born rule should hold from a measurement.
3. It should be consistent with experimental results of the dynamics of physical systems.

The issue is how can we guarantee an outcome from the measurment? One theory of collapse is the **GRW theory**. The theory states that particles can undergo spontaneous collapses. The collapses happen probabilistically and at some given rate. The measurment is no longer a special act that collapses the wavefunction. Spontaneous collapses are said to be extremely rare, such that single-particle experiments are consistent.

An issue with GRW is with the conservation of energy. Normal collapses leave particles in perfect eigenstates, like for position. Due to position and momenta being fourier-transform pairs, the particle's momenta will be completely uncertain. This could leave particles to accquire momenta and energy after a collapse, which is invalid. The resolution is the collapse is a convolution with a bellcurve of an infinitely small variance.

5.4.3 von Neumann Chains

How do we even know that macroscopic objects cannot become entangled in a quantum superposition?

In the Schrödinger's cat thought experiment, macroscopic objects like the cat can become part of the superpostion. Macroscopic measuring devices should also follow the same laws of dynamics, and become similarily entangled. If this is the case, at what point does the wavefunction collapse?

Wigner suggests that *consciousness* makes the measurement and collapses the wavefunction. This would require **mind-body dualism**: that some mental phenomena of our consciousness is non-physical

5.5 Interpretations

5.5.1 Ensemble Interpretation

This is Born's original interpretation. The wave function describes the probability density of an ensemble of particles. Particles actually have a definite position and momenta at all times. The wave function is not physically real. There is no requirement for superposition of states to physically exist.

This interpretation breaks down as motion of single particles are also predicted by it's wavefunction. Rejecting this would mean rejecting a quantum description of individual systems.

5.5.2 Cophenhagen Interpretation

The Copenhagen Interpretation is currently the most commonly accepted interpretation. There is no definite definition of the Copenhagen Interpretation, but it consists of several key principles shown by Bohr:

1. A system is completely described by its wave function.
2. **Born Rule**: Description of nature is probabilistic, with the probability of an event related to the amplitude of its wave function.
3. **Uncertainty Principle**: It is not possible to know the value of all properties of the system at the same time
4. Matter exhibits **wave-particle duality**
5. Measuring devices are classical and only measure classical properties
6. **Correspondence principle**: Quantum mechanical principles of large systems will closely approximate the classical description.

Similarily, we take a subjective view where the wavefunction is just a mathematical tool for calculating probabilities.

Measuring devices conduct a finite quantum of action on the micro-world. This quantum of action links the micro-system and the macro-measuring device into an indivisible and uncontrollable unity. There is no explanation of the mechanism of how this conversion happens, and is just accepted.

With Schrodinger's cat experiment, the wave function reflects our knowledge of the system. The wave function means that, once the cat is observed, there is a 50% chance it will be dead, and 50% chance it will be alive.

5.5.3 Many-Worlds Interpretation

Glossary

Eternalism All points in time are equally real. Universe is a four-dimensional block, and there is no objective flow of time, 1

Possibilism Past & present events are real. Events in the future are possible. This allows for indeterminateness., 1

Presentism Only present events are real. The passage of time makes future events real., 1