Calculus Guided Inquiry

Relationship Between a Function, First Derivative and Second Derivative

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Objective

1. Experimentally determine an empirical relationship for acceleration, velocity and position as a function of the incline angle for a ball rolling down a ramp.

Materials

1. Ball
2. Ramp
3. Meter stick
4. Protractor
5. Camera

Procedure

1. Place the ramp at an angle of 10° relative to horizontal (using a protractor).
2. Place the meter stick on top of and parallel to the ramp.
3. Set up the camera to face the ramp, make sure the entire meter stick can be captured.
4. Place the ball in the slot on the ramp, in alignment with the top end of the meter stick.
5. Release the ball, capture the entire footage.
6. Repeat steps 1-5, increase the angle by 10° in every capture. Stop after the angle reaches 90°.
7. From the recordings, extract the start time, the time when the ball reaches the end of the meter stick, and calculate the difference. Record the data with reference to the angle in a table.

Experimental Data

θ (°)	10	20	30	40	50	60	70	80	90
Time (s)	1.33	0.90	0.72	0.63	0.50	0.43	0.41	0.40	0.38

Table 1: Experimental data

Analysis of Data

Theoretical

To explain the behavior of the ball's acceleration relative to θ, the theoretical data should first be considered. Since gravity, g, is approximately 9.8 m/s^2 on the surface of the earth, the composite acceleration at any angle relative to horizontal can be found using the following equation:

$$a = 9.8 \sin\theta \tag{1}$$

θ (°)	10	20	30	40	50	60	70	80	90
Acceleration (m/s^2)	1.70	3.35	4.90	6.30	7.51	8.49	9.21	9.65	9.80

Table 2: Theoretical Data

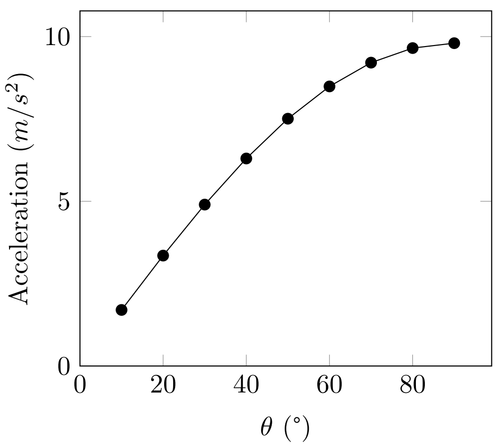

Figure 1: Acceleration vs θ

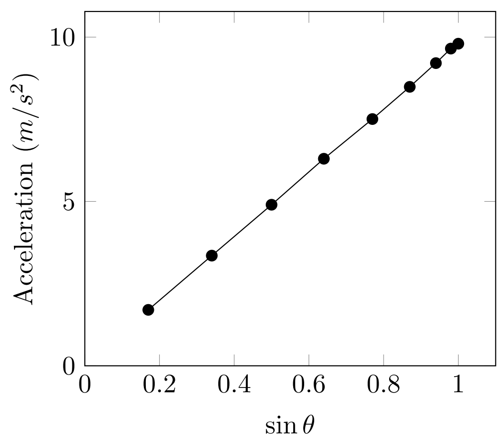

Figure 2: Acceleration vs $\sin\theta$

When the acceleration vs θ graph is plotted (Figure 1), the resulting curve looks like a portion of a sinusoidal function. A relationship between acceleration and $\sin\theta$ can then be established (Figure 2):

$$a \propto \sin\theta \tag{2}$$

Experimental

With the above relationship, a discussion about the experimental data can now take place. Given Equation:

$$d = v_0 t + \frac{1}{2}at^2 \tag{3}$$

Above equation can be re-ordered to find the composite acceleration as a function of time ($v_0 t$ is 0 because the object starts at rest):

$$a(t) = \frac{2d}{t^2} \tag{4}$$

θ (°)	10	20	30	40	50	60	70	80	90
Time (s)	1.33	0.90	0.72	0.63	0.50	0.43	0.41	0.40	0.38
Acceleration (m/s^2)	1.13	2.47	3.86	5.04	8.00	10.82	11.90	12.50	13.85

Table 3: Experimental data with calculated acceleration

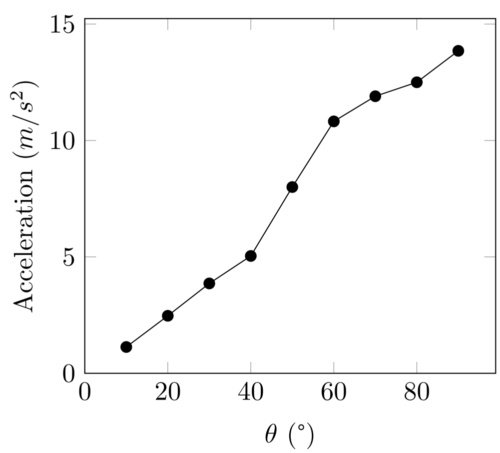

Figure 3: Acceleration vs θ

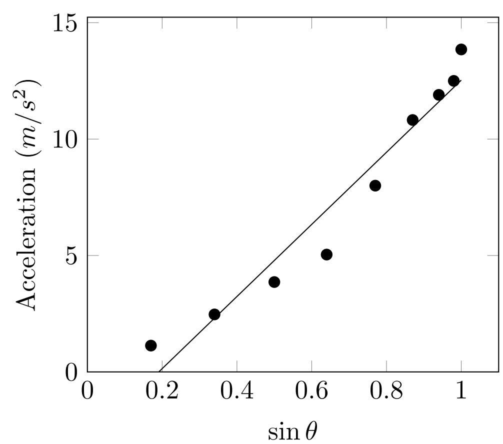

Figure 4: Acceleration vs $\sin\theta$

This relationship can still faintly be noticed:

$$a \propto \sin\theta \tag{5}$$

For a complete equation, a constant k is needed:

$$a = k\sin\theta \tag{6}$$

$$k = \frac{a}{\sin\theta} \tag{7}$$

A table of values of k:

θ (°)	10	20	30	40	50	60	70	80	90
Acceleration (m/s^2)	1.13	2.47	3.86	5.04	8.00	10.82	11.90	12.50	13.85
k	6.51	7.22	7.72	7.84	10.44	12.49	12.66	12.69	13.85

Table 4: Experimental Constant

Friction prevents the number to stay consistent. An average of these constants will be taken to represent the trend.

$$\frac{6.51 + 7.22 + 7.72 + 7.84 + 10.44 + 12.49 + 12.66 + 12.69 + 13.85}{9} = 10.16 \tag{8}$$

Therefore, the experimental value of k is 10.16:

$$a = 10.16 \sin\theta \tag{9}$$

Instantaneous Velocity and Position At any angle, $a = 10.16 \sin\theta$ will be a constant. This allows the addition of a new variable—time, t.

Velocity is the anti-derivative of acceleration, therefore:

$$v = 10.16 \sin\theta t \tag{10}$$

Position is the anti-derivative of velocity:

$$s = 5.08 \sin\theta t^2 \tag{11}$$

Conclusion

The experimentally determined empirical relationships are as follows:

- Acceleration as a function of the incline angle:

$$a = 10.16 \sin\theta \tag{12}$$

- Instantaneous Velocity as a function of the incline angle and time:

$$v = 10.16 \sin\theta t \tag{13}$$

- Position as a function of the incline angle and time:

$$s = 5.08 \sin\theta t^2 \tag{14}$$