SECTION A

Choose the *best* answer for each question, and record it on the supplied test answer sheet. Each question is worth one mark. Answer all thirty (30) questions.

QUESTION 1

Consider the following formula:

$$\log_a \left(\frac{a^4 b^x}{(a^2)^3 b^y} \right)$$

Which of the following is equal to the formula above?

(a) $\frac{x-y+\log_b a^{-2}}{\log_b a}$

(b) $\frac{(x-y)(\log_b a^{-2})}{\log_a b}$

(c) $\frac{xy+\log_b a^{-3}}{\log_b a}$

(d) $\frac{2-\log_a b^{x-y}}{\log_b a}$

QUESTION 2

Consider the following made-up definition:

A *thingamabob* is an integer whose binary representation (with no leading zeros) is the NOT of itself when reversed. For example, 12 is a thingamabob since its binary representation is 1100 and when reversed this becomes 0011, the NOT of 1100.

Which of the following is a thingamabob?

(a) 48

(b) 56

(c) 32

(d) None of the above

QUESTION 3

Recall that a divisor of z is an integer x such that $x|z$, and a prime number x is a number whose only positive divisors are itself and 1.

Which of the following sets is equal to the set of all positive prime divisors of 100?

(a) $\{x \in \mathbb{Z} : x > 1 \wedge x \,|\, 100 \wedge \forall y \in \mathbb{Z}^+ y \,|\, x \wedge y > 1 \wedge y \neq x)\}$

(b) $\{x \in \mathbb{Z} : x > 1 \vee x \,|\, 100 \vee \neg(\exists y \in \mathbb{Z}^+ y \,|\, x \vee y > 1 \vee y \neq x)\}$

(c) $\{x \in \mathbb{Z} : x > 1 \wedge x \,|\, 100 \wedge \neg(\exists y \in \mathbb{Z}^+ y \,|\, x \wedge y > 1 \wedge y \neq x)\}$

(d) $\{x \in \mathbb{Z} : x > 1 \wedge x \,|\, 100 \wedge \{y \in \mathbb{Z}^+ : y \,|\, x\} = \emptyset\}$

QUESTION 4

Suppose that a Boolean formula A is a tautology. Then:

(a) Any other formula B logically implies A.

(b) A logically implies any other satisfiable formula B.

(c) A is logically equivalent to any formula B that is not a contradiction.

(d) A is also contingent.

QUESTION 5

Consider the following proposition:

$$p = \exists e \in S\ \forall x \in S\ ex = x$$

Which of the following is logically equivalent to p above?

(a) $e = 1$

(b) $\neg(\forall e \in S\ \neg(\exists x \in S\ ex \neq x))$

(c) $\neg(\exists e \in S\ \neg(\forall x \in S\ ex = x))$

(d) $\neg(\forall e \in S\ \exists x \in S\ ex \neq x)$

QUESTION 6

Consider the following proof:

Proof. Suppose that there are a finite number (say n) of primes, $p_1, p_2, \ldots p_n$. Let $q = p_1 p_2 \ldots p_n + 1$. Now for any $j \in \{1 \ldots n\}$ we have

$$q - 1 = (p_1 \ldots p_{j-1} p_{j+1} \ldots p_n) p_j$$

so $q \equiv 1 \pmod{p_j}$. So q has no prime divisors and hence is prime. But clearly $q \geq p_j$ for all j, so q is a new prime that is not in our list of all primes.
Hence there is an infinite number of primes. □

This proof is *best* described as:

(a) A proof by contradiction

(b) A proof by contrapositive

(c) A proof by counterexample

(d) A proof by construction

QUESTION 7

Suppose that we have the following block of code:

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x = x + 1
y = x * x
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Supposing that the block has a pre-condition $\{x \geq 10\}$, which of the following is *not* a possible post-condition of the block?

(a) $\{y \geq 121\}$

(b) $\{y = (x+1)^2\}$

(c) $\{x \geq 10 \wedge y \geq 10\}$

(d) $\{y \geq x\}$

QUESTION 8

The *inverse* of a relation $R \subseteq A \times A$ is given by :

$$R^{-1} = \{(s, t) : (t, s) \in R\}$$

To distinguish between this definition of inverse and that usually applied to functions, we will call this the *relational inverse*, and call the inverse of a function the *functional inverse*.

Which of the following *not* true about the relationship between relational inverses and functional inverses?

(a) Since a function is a relation, it has a relational inverse. Thus every function has a functional inverse.

(b) The functional inverse of a function f, when viewed as a relation, is also the relational inverse of f viewed as a relation.

(c) Since a function is a relation, we can always find the relational inverse for any function, but it is not necessarily the functional inverse.

(d) Given a function f, if f's relational inverse is also a function, then it is the functional inverse of f.

QUESTION 9

A *clique* in a graph $G = (V, E)$ is a set of vertices $C \subseteq V$ such that every vertex in C is adjacent to every other vertex in C. Which of the following is true about cliques?

(a) In every graph G, every clique in G is a connected component

(b) Every subset of a clique is also a clique

(c) If the number of vertices in C is even then the induced subgraph on C is bipartite

(d) Cliques cannot have a cycle of odd length

QUESTION 10

Suppose that we have a finite state automaton given as a state change diagram. Recall that a state change diagram is like a directed graph, but it has labels on the edges and loops (edges from a vertex to itself) are allowed.

Which of the following is *not* true?

(a) If there is a sequence of inputs that will take the FSA from state s to state t then there is a directed path in the state change diagram from s to t.

(b) If the FSA is in some state s and receives an input, the resulting state will be adjacent to s.

(c) If there exists an input that causes the FSA to accept, then the state change diagram is acyclic.

(d) If the FSA is in some state s and after receiving some inputs, it is again in state s then the state change diagram contains a directed cycle or loop.

END OF PAPER

SECTION Appendix

Answers: a, b, c, a, d, a, b, a, b, c

Base-10	Hexadecimal	4-bit binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

θ (in degrees)	$\sin\theta$	$\cos\theta$
0	0	1
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
90	1	0