ELE8088: Control & Estimation Theory **QUB, 2023/24**

Altitude Hold of Quadcopter

1 Problem statement

In this lab we will design an estimation and control system for *bzzz* — the quadcopter shown in Figure 1. The quadcopter is equipped with a number of sensors. In particular, there is a downward-facing time-of-flight (ToF) sensor, which is an infrared distance sensor that measures the distance from the ground with a standard error of $\pm 6\,\mathrm{cm}$. The quadcopter carries a barometer which by measuring the barometric pressure can give an estimate of the altitude (where the altitude is zero at sea level); the standard error of this sensor is $\pm 25\,\mathrm{cm}$. The quadcopter is also equipped with a GNSS (global navigation satellite system) sensor, which uses the ZED-FP9 module. The GNSS module is couples with a base GNSS station, which allows it to produce accurate position estimates. The GNSS can estimate the altitude of the vehicle (with respect to the sea level) with a standard error of $\pm 5\,\mathrm{cm}$.

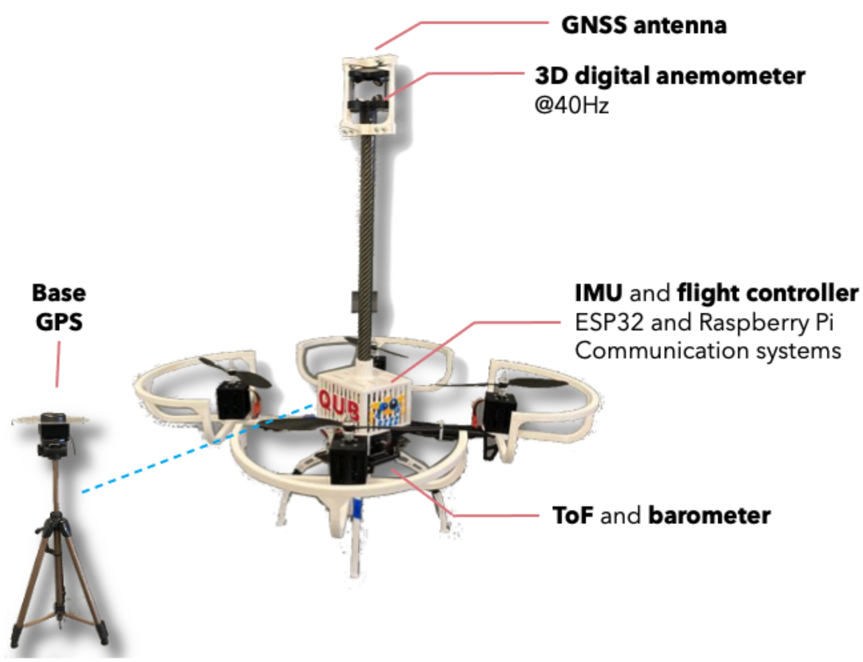

Figure 1: Bzzz quadcopter

It should be noted that the ToF sensor measures the distance from the ground, while the GNSS and the barometer measures the altitude. In principle the ground is not perfectly flat, and the barometric pressure at a given altitude can (very slowly) change. In case the quadcopter doesn't have a clear view of the sky, it is possible that the GNSS connection gets interrupted.

2 Modelling

The two main forces acting on the quadcopter are the weight, mg, and the force from the propellers, $F_{\rm prop}$, which has been found to depend linearly on the throttle reference signal $\tau \in [0, 1]$. The throttle reference signal is a signal that is sent to the electronic speed controllers (ESCs) of the four motors; at $\tau = 0$ the motors do not spin, whereas $\tau = 1$ corresponds to the maximum rotation speed.

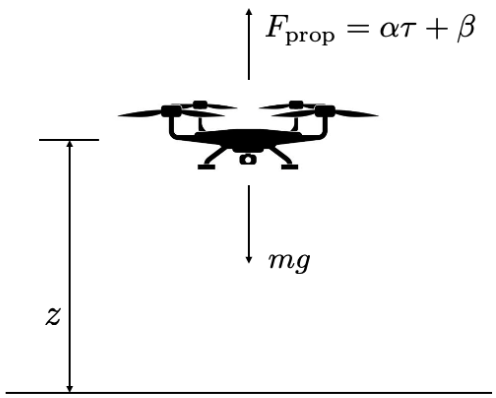

Figure 2: Forces acting on *bzzz*: the weight, mg, and the total force from the propellers, $F_{\rm prop} = \alpha\tau+\beta$, where α and β are (constant) coefficients and τ is a throttle signal. Here z denotes the *altitude* of the quadcopter.

In an experiment, the quadcopter was placed on digital scales and the lift (in g) was measured for different values of τ. The experimental results are shown in Figure 3, from which it seems that a reasonable model for the lifting force is

$$F_{\rm prop} = \alpha\tau + \beta, \tag{1}$$

where $\alpha > 0$ and $\beta < 0$ are constants, which depend on the level of charge of the battery. Although the values of α and β can be estimated from the data shown in Figure 3, their exact value is unknown while flying.

From the model of Equation (1), the total acceleration is

$$a = \frac{F_{\rm prop} - mg}{m} = \frac{\alpha\tau + \beta - mg}{m} = \frac{\alpha}{m} + \frac{\beta - mg}{m} = \alpha^1\tau + \alpha^0, \tag{2}$$

where $\alpha^1 = \alpha/m$ and $\alpha^0 = (\beta - mg)/m$.

As a result, a dynamical model of the system is

$$\ddot{z} = a \Leftrightarrow \ddot{z} = \alpha^1\tau + \alpha^0, \tag{3}$$

where z denotes the altitude of the quadcopter. Note again, that the exact values of the coefficients α^0 and α^1 are not known while flying (but we will estimate them). We can write this model as

$$\dot{z} = v, \tag{4a}$$
$$\dot{v} = \alpha^1\tau + \alpha^0, \tag{4b}$$

where v is the quadcopter's vertical velocity. By discretising, using Euler's discretisation, with sampling time T_s, we have

$$z_{t+1} = z_t + T_s v_t. \tag{5a}$$
$$v_{t+1} = v_t + T_s(\alpha^1 \tau_t + \alpha^0). \tag{5b}$$

The sampling time is $T_s = 100\,\mathrm{ms}$. Note that this system is at equilibrium whenever $\alpha^1 \tau_t + \alpha^0 = 0$, that is, equivalently, $\tau_t = -\alpha^0/\alpha^1$. This defines the *hovering throttle signal*, $\tau^{\mathrm{eq}} = -\alpha^0/\alpha^1$.

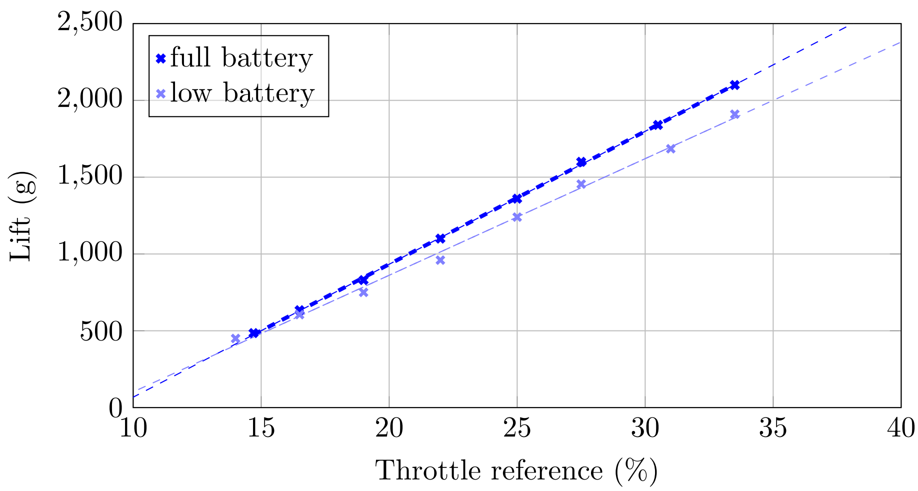

Figure 3: Static lift (g) plotted against throttle reference (%).

3 Estimation

3.1 System dynamics

In order to estimate the parameters α and β we will use the Kalman filter. Firstly, we write the model of Equation (6) as

$$z_{t+1} = z_t + T_s v_t + w_t^v, \tag{6a}$$
$$v_{t+1} = v_t + T_s(\alpha_t^1 \tau_t + \alpha_t^0) + w_t^z, \tag{6b}$$
$$\alpha_{t+1}^0 = \alpha_t^0 + w_t^0, \tag{6c}$$
$$\alpha_{t+1}^1 = \alpha_t^1 + w_t^1. \tag{6d}$$

3.2 Outputs

Here we are considering three outputs. Firstly, the measurement from the GNSS sensor is

$$y_t^{\mathrm{gnss}} = z_t + v_t^{\mathrm{gnss}}, \tag{7}$$

where z_t is the actual altitude of the quadcopter and v_t^{gnss} is a measurement noise term. Secondly, the ToF sensor measures the distance from the ground, which is equal to the altitude plus a bias term plus the measurement noise; overall,

$$y_t^{\mathrm{tof}} = z_t + d_t^{\mathrm{tof}} + v_t^{\mathrm{tof}}, \tag{8}$$

where we assume that d_t^{tof} is described by a simple model of the form

$$d_{t+1}^{\mathrm{tof}} = d_t^{\mathrm{tof}} + w_t^{\mathrm{tof}}. \tag{9}$$

This way, the variable d_t^{tof} becomes a state of the system. Thirdly, the barometer measurements are again subject to a bias and measurement error, that is,

$$y_t^{\mathrm{bar}} = z_t + d_t^{\mathrm{bar}} + v_t^{\mathrm{bar}}, \tag{10}$$

with

$$d_{t+1}^{\mathrm{bar}} = d_t^{\mathrm{bar}} + w_t^{\mathrm{bar}}. \tag{11}$$

3.3 Overall system

The overall system has state $x_t = (z_t, v_t, \alpha_t^0, \alpha_t^1, d_t^{\mathrm{tof}}, d_t^{\mathrm{bar}})$ and output $y_t = (y_t^{\mathrm{gnss}}, y_t^{\mathrm{tof}}, y_t^{\mathrm{bar}})$. It can be written as

$$x_{t+1} = \underbrace{\begin{bmatrix} 1 & T_s & 0 & 0 & 0 & 0 \\ 0 & 1 & T_s & T_s\tau_t & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{A_t} \bar{x}_t + w_t, \tag{12}$$

$$y_t = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} x_t + v_t. \tag{13}$$

Note that A_t depends on the throttle reference signal, τ_t, which is the control variable.

4 Today's lab and your assignment

In today's lab we will design a Kalman filter to estimate the state of the system based on the above formulation.

Following the lab you need to write a lab report where you will be proposing an estimation system. In addition, you should design a control system that will allow the quadcopter to fly at a constant altitude. Your controller should use the altitude estimates from your Kalman filter. In your report you should test your estimator in a number of different realistic scenarios: for example, (i) the quadcopter flies over a step, so the measurement of the ToF sensor increases suddenly, (ii) the GNSS sensor does not give any measurements for a few seconds, (iii) the battery drains gradually, so the value of α^1 decreases slowly, (iv) there is a slow change in the bias of the barometer.

In all cases, you will need to choose certain parameters such as the variance-covariance matrices of the process and sensor noises. Discuss and justify your choices.