E parts: $k=0.5$, $P_0=0.6$, $P_1=0.1$

let x be the random variable of the outcomes

z be the random variable for the coin be chosen

calculate the responsibility w

$$P(z=C_0, x=HHH \mid P_0) = 0.5 \cdot (0.6)^3 \cdot (0.4)^0 = 0.108$$

$$P(z=C_1, x=HHH \mid P_1) = 0.5 \cdot (0.1)^3 \cdot (0.9)^0 = 0.0005$$

$$P(z=C_0, x=HHT \mid P_0) = 0.5 \cdot (0.6)^2 \cdot 0.4 = 0.072$$

$$P(z=C_1, x=HHT \mid P_1) = 0.5 \cdot (0.1)^3 \cdot 0.9 = 0.0045$$

$$P(z=C_0, x=TTT \mid P_0) = 0.5 \cdot (0.6)^0 \cdot (0.4)^3 = 0.032$$

$$P(z=C_1, x=TTT \mid P_1) = 0.5 \cdot (0.1)^0 (0.9)^3 = 0.3645$$

$$w_0 = \frac{0.108}{0.108+0.0005} \doteq 0.9954 \qquad w_2 = \frac{0.032}{0.032+0.3645} = 0.0807$$

$$w_1 = \frac{0.072}{0.072+0.0045} \doteq 0.9412$$

M step:

the weighted log likelihood:

$$J = \sum_{i=1}^{n} w_i \ln\left(k \cdot P_0^{m_i} \cdot (1-P_0)^{n-m_i}\right) +$$

$$\sum_{i=1}^{n} (1-w_i) \ln\left((1-k) P_1^{m_i} \cdot (1-P_1)^{n-m_i}\right)$$

where m is the times showing H in one trial

$$\frac{\partial J}{\partial k} = \sum w_i \cdot \frac{1}{k} + \sum (1-w_i) \cdot \frac{-1}{1-k} = 0$$

$$(1-k)\sum w_i = k \sum (1-w_i)$$

$$\sum w_i - k\sum w_i = k \cdot n - k \sum w_i$$

$$k = \frac{\sum w_i}{n}$$

$$\frac{\partial J}{\partial P_0} = \sum w_i \cdot \frac{k \cdot m_i \cdot P_0^{m_i-1}(1-P_0)^{n-m_i} + k \cdot P_0^{m_i}(n-m_i)(1-P_0)^{n-m_i-1} \cdot (-1)}{k \cdot P_0^{m_i} \cdot (1-P_0)^{n-m_i}}$$

$$= \sum w_i \frac{m_i(1-P_0) - (n-m_i)P_0}{P_0(1-P_0)} = 0$$

$$\sum m_i w_i - P_0 \sum m_i w_i = nP_0 \sum w_i - P_0 \sum m_i w_i$$

$$P_0 = \frac{\sum m_i w_i}{n \sum w_i}$$

$$\frac{\partial J}{\partial P_1} = \sum (1-w_i) \frac{m_i(1-P_1) - (n-m_i)P_1}{P_1(1-P_1)} = 0$$

$$\sum \left(m_i(1-P_1) - (n-m_i)P_1\right) = \sum w_i \cdot \left(m_i(1-P_1) - (n-m_i)P_1\right)$$

$$\sum m_i - n^2 P_1 = \sum m_i w_i - nP_1 \sum w_i$$

$$\sum m_i - \sum m_i w_i = n(n - \sum w_i)P_1$$

$$P_1 = \frac{\sum m_i - \sum m_i w_i}{n(n - \sum w_i)}$$

$$\therefore\ k = \frac{2.0173}{3} \doteq 0.6724$$

$$P_0 = \frac{4.8686}{3 \cdot 2.0173} \doteqdot 0.8045$$

$$P_1 = \frac{5 - 4.8686}{3 \cdot (3 - 2.0173)} \doteqdot 0.0446$$

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