An introduction to probability

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Who am I?

- Statistician working mainly in epidemiology.
- Claim to probability fame: born in the same town where Thomas Bayes lived (Tunbridge Wells, UK).

Course outline

- 9am-10.30am: follow along lecture and problems
- 10.30am-10.45am: refreshments break
- 10.45am-midday: follow along lecture and problems

Resources

- Introduction to probability, Blitzstein and Hwang. Open source book available here:
  `https://projects.iq.harvard.edu/stat110/home`
- Seeing theory, Kunin et al. A beautiful online resource that has lots of creative ways to think about probability.
  `https://seeing-theory.brown.edu/`

Outline

What is probability?

Mathematics is the logic of certainty; probability is the logic of uncertainty

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Bitzstein and Hwang, 2019

Why study probability?

It is used in:

- Statistics: probability is the foundational language of it
- Biology: e.g. inheritance of genes
- Meteorology: e.g. weather forecasts are generated using probabilities
- Epidemiology: e.g. analysing randomised clinical trials and fitting models to epidemiological data
- Physics: our current best explanation of the universe at small scales (quantum theory) is based on probability

The difficulties of probability

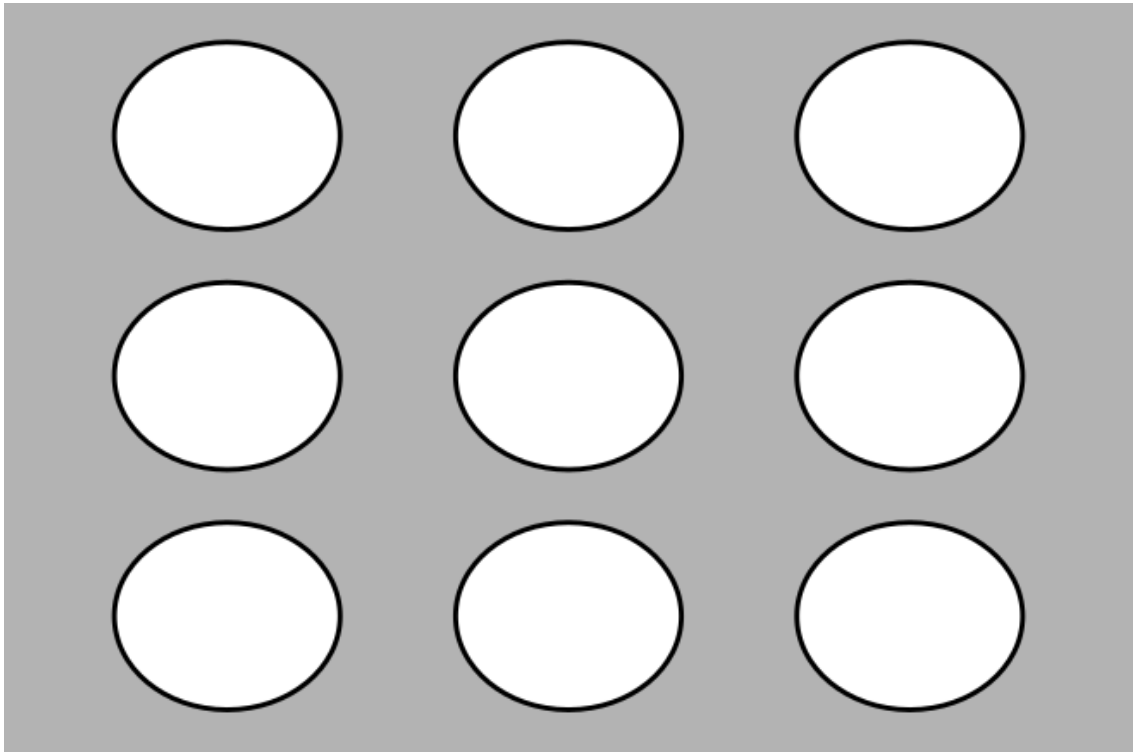
If we rely on our intuitions, it is easy to go awry with probability.

$\Longrightarrow$ need careful mathematical analysis.

Fortunately, *simulation* using R can really help to understand.

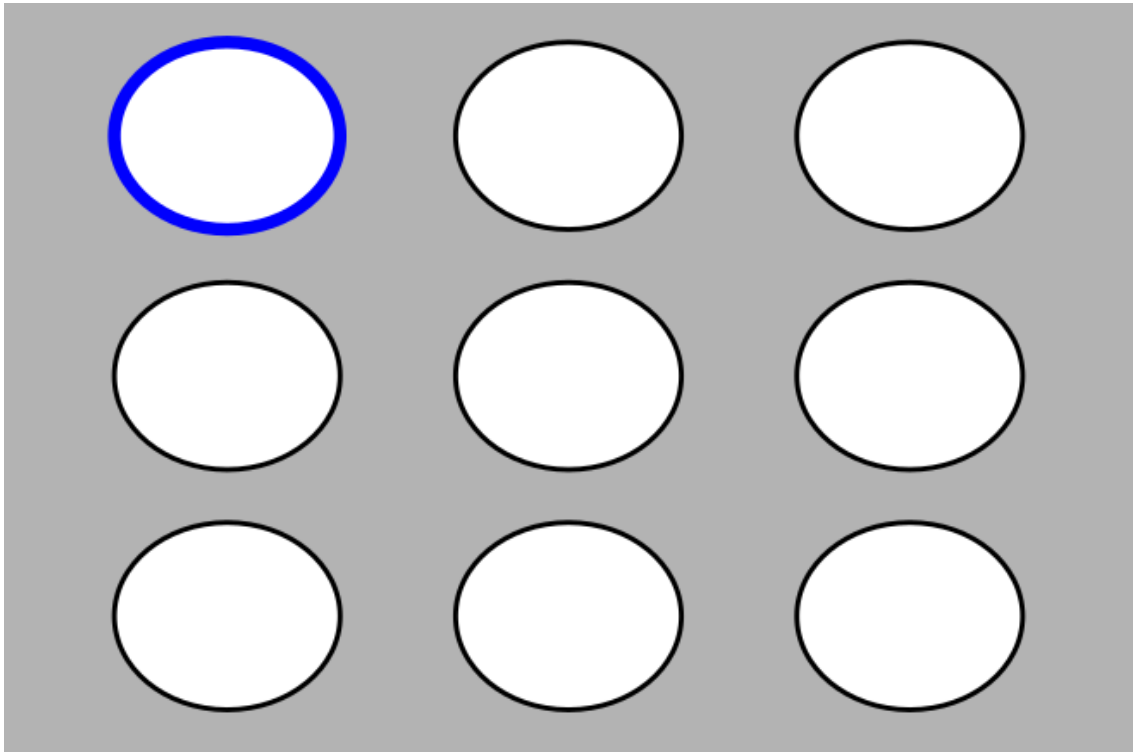
Blitzstein and Hwang's Pebble World

As an example, consider reaching into a bag to pull out one of nine pebbles: we call this *pebble world*.



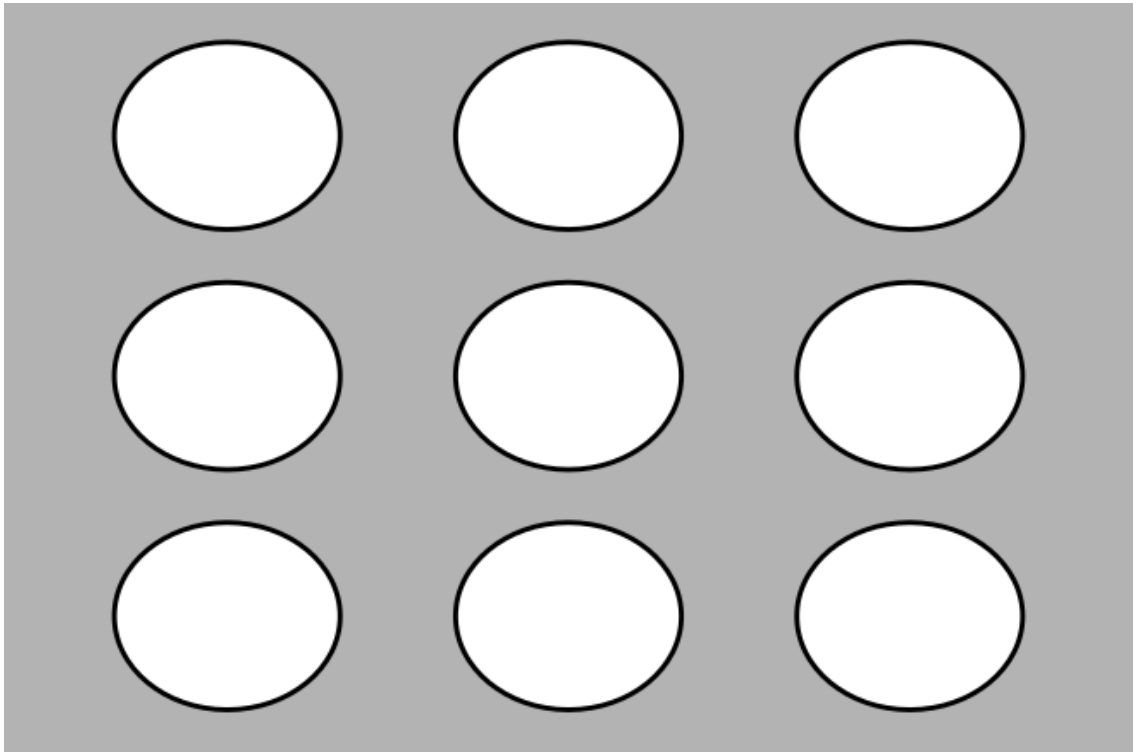
Outcomes

An *outcome* is a possible result of some activity. Here pulling one particular pebble out of the bag would be an outcome.



Sample spaces

The *sample space* is the *set* of all possible outcomes of an experiment. Here, it is the set of all pebbles.



Events

An *event* is a *set* of possible outcomes. For example, below event A corresponds to selecting one of five pebbles; event B to selecting one of two.

As we can see, two or more events can happen at once.