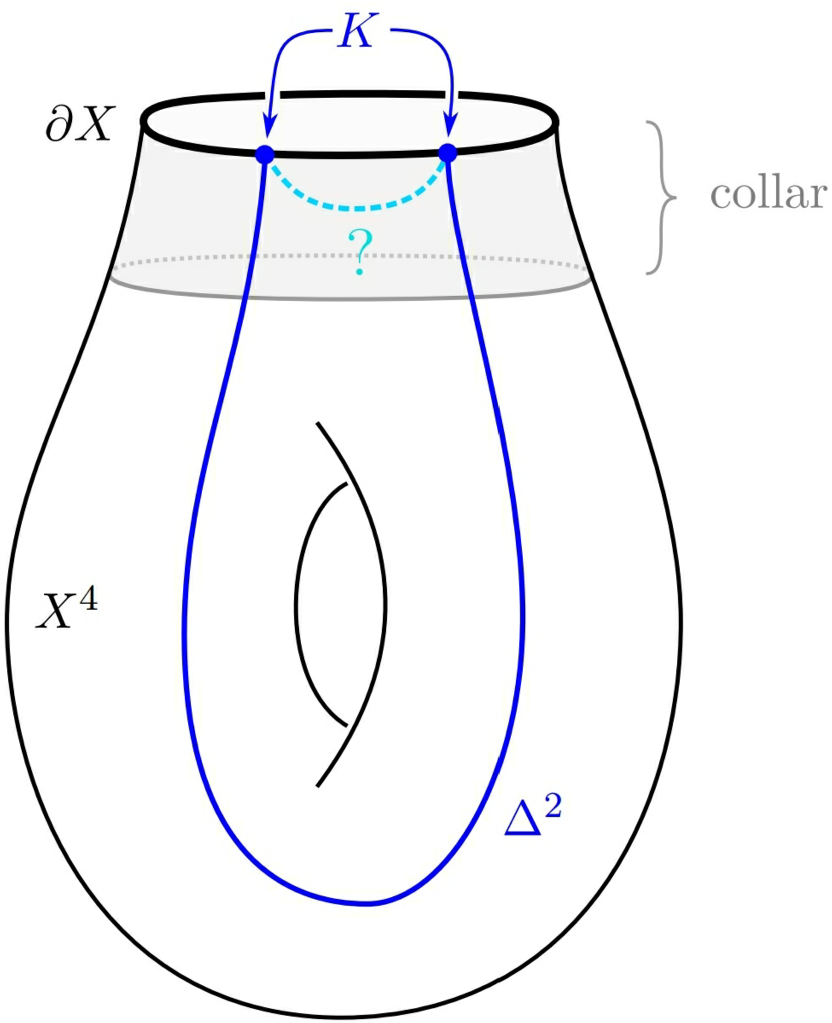
Deep and shallow slice knots in 4-manifolds



Joint work with Michael Klug

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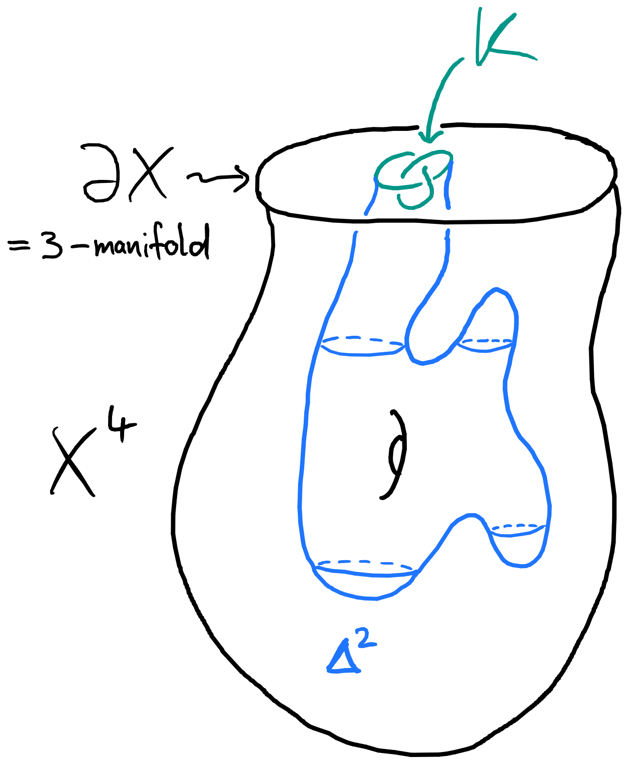


Knot K in the boundary of a 4-mfld. X^4

is <u>slice in X</u> if there exists a

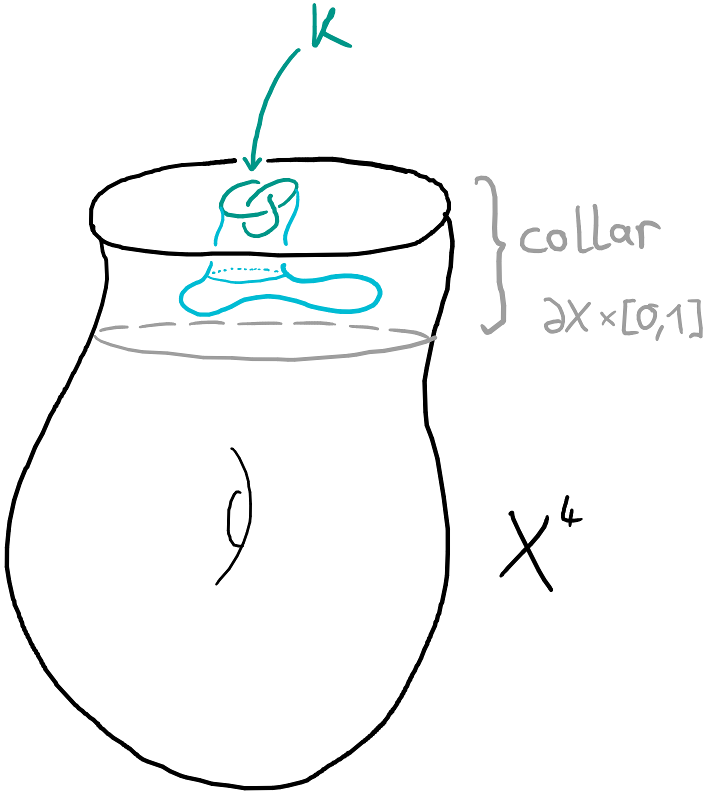
slice disk $\Delta^2 : \mathbb{D}^2 \hookrightarrow X$

with $\partial \Delta^2 = K \subset \partial X$

Some authors require the slice disk to be null-homologous ($\Leftrightarrow [\Delta, \partial\Delta] = 0 \in H_2(X, \partial X)$)

This is called <u>H-slice</u> in [Manolescu, Marengon, Picirillo: Relative genus bounds in indefinite four-manifolds]

We won't put this condition on our slice disks here.

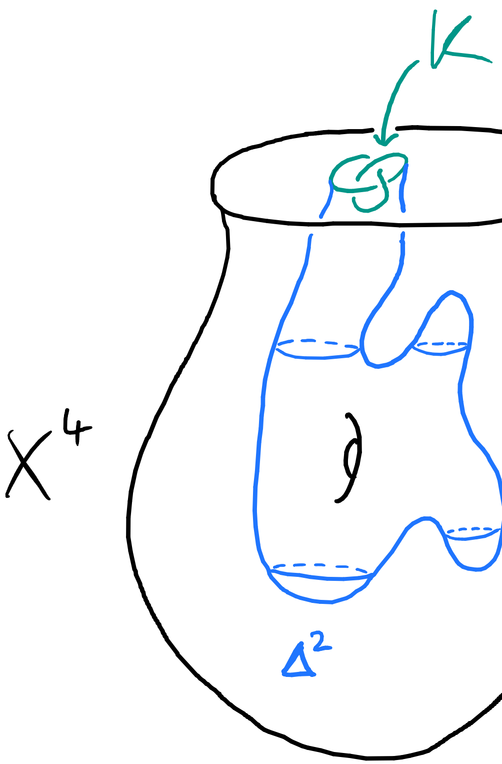


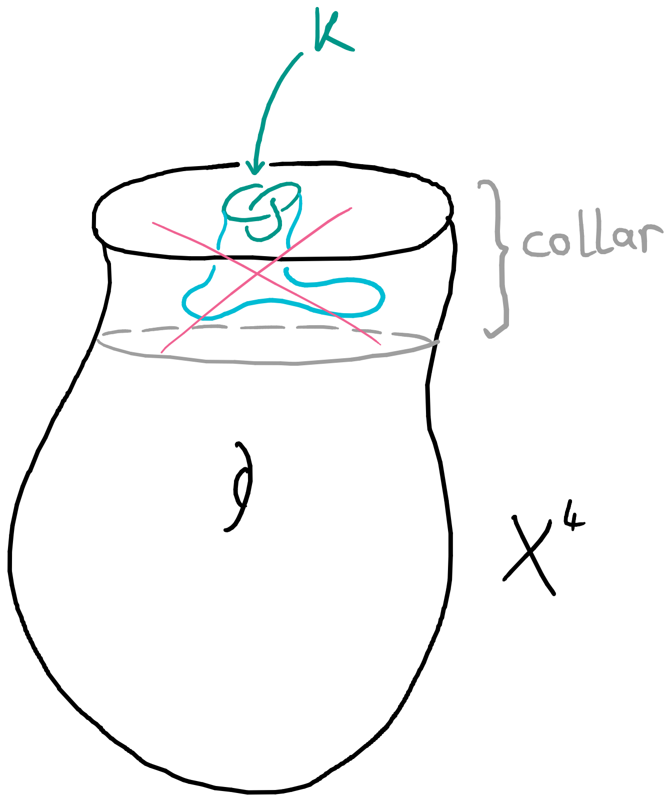
K is <u>shallow slice in X</u> if there is a slice disk in a collar neighborhood $\partial X \times [0,1]$ of the boundary.

K is <u>deep slice in X</u> if every slice disk for it

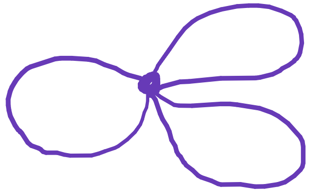
"needs to use the extra topology of X",

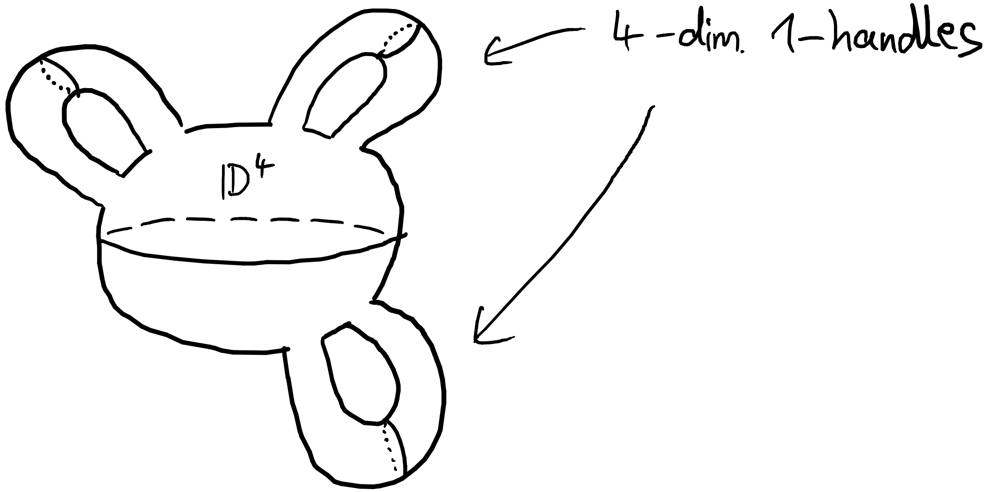
i.e. if K is slice in X, but <u>not</u> shallow slice.

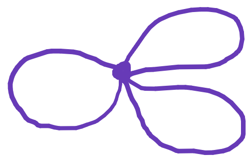




Non-example: There are no deep slice knots in $\natural^k S^1 \times D^3$.

$\natural^k S^1 \times D^3$ = thickening of 

= 

Any slice disk generically avoids the spine 

$\Rightarrow$ disk lives in a collar neighborhood of the boundary

Example: $X^4 = \underset{\text{0-handle}}{\mathbb{D}^4} \cup \underset{\text{at least one 2-h.}}{(2\text{-handles})}$ has deep slice knots in boundary (which are nullhomotopic in ∂X, but not contained in a 3-ball)

Two cases

$\pi_1(\partial X) = \{1\}$ and thus $\partial X \cong S^3$

We use a theorem of Rohlin on the genus of embedded surfaces representing 2-dim. homology classes in $\hat{X} = X \cup (4\text{-handle})$

$\pi_1(\partial X)$ non-trivial

Use Wall's self-intersection number with values in $\dfrac{\mathbb{Z}[\pi_1(\partial X)]}{\langle g = g^{-1}, 1 \rangle}$ of the track of a homotopy in $\partial X \times [0,1]$