Contents

Introduction

These notes are a collection of various things that I wanted to write down for myself to use as a reference.

Homotopy theory

CW-complexes

Definition 1. A map $f\colon X \to Y$ is called a *weak homotopy equivalence* if it induces isomorphisms

$$\pi_n(X, x_0) \to \pi_n(Y, f(x_0))$$

for all $n \geq 0$ and all choices of basepoints x_0 in X.

Theorem 1 (Whitehead's Theorem). *A weak homotopy equivalence between CW-complexes is a homotopy equivalence.*

Proposition 1 (Geometric interpretation of n-connectedness). *If (X, A) is an n-connected CW-pair, then there exists a CW-pair $(Z, A) \sim_{\mathrm{rel}\, A} (X, A)$ such that all cells of $Z \setminus A$ have dimension greater than n.*

Homology

Definition 2 (Acyclic). A space X is called *acyclic* if $\widetilde{H}_i(X) = 0$ for all i, i.e. if its reduced homology vanishes.

Example 1. Removing a point from a homology sphere yields an acyclic space. If the dimension was at least 3 this does not change the fundamental group, so if we started with a nontrivial homology sphere (i.e. $\pi_1 \neq 1$) this will give an example of an acyclic, but non-contractible space.

This example for the Poincaré homology sphere is described in (Hatcher, 2002, Example 2.38). TODO Insert proof.

Knot Theory

Constructions & Definitions

Definition 3. If K is an oriented knot, then

- the *reverse* $\overline{K}$ is K with the opposite orientation
- the *obverse* rK is the reflection of K in a plane
- the *inverse* $r\overline{K}$ is the concordance inverse of K.

Proposition 2. *For $K \subset \mathbb{S}^3$ we have that $K \# r\overline{K}$ is slice, even ribbon.*

Definition 4 (Homotopically unlinked, (Rolfsen, 2003, 3.F.9.))**.** If $L = L_1 \cup \ldots \cup L_n$ is a link with n components, we say that L_i is *homotopically unlinked* from the remaining components if there is a homotopy h_t from the embedding of L_i to the constant map such that the images of h_t and L_j are disjoint at all times $t \in \mathbb{I}$ and for all other components $j \neq i$.

Example 2. In the Whitehead link both components are homotopically unlinked from each other.

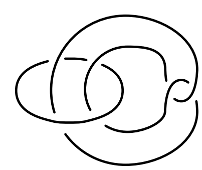

Figure 1: Positive Whitehead link, picture from (Meier, 2015).

Remark 1. Homotopic linking (for two component links) is **not** a symmetric relation.

Definition 5. A link $L = L_1 \cup L_2$ of two components in $\mathbb{R}^n$ is *splittable* if there are disjoint, topological n-balls $\mathbb{D}^n_1, \mathbb{D}^n_2 \subset \mathbb{R}^n$ such that L_i lies in the interior of $\mathbb{D}^n_i$.

Proposition 3. *If a link is splittable, then each component is homotopically unlinked from the other.*

The converse of 3 is not true, an example of van Kampen and Zeeman is discussed in (Rolfsen, 2003, 3.K.5.).

Definition 6. The lower central series of a group G is defined inductively by

Observe that $[G, G_{i-1}] = [G_{i-1}, G]$.

$$G_0 := G$$
$$G_i := [G, G_{i-1}] = \langle [g, h] \mid g \in G, h \in G_{i-1} \rangle$$

Proposition 4. *This satisfies:*

- $G_0 \supseteq G_1 \supseteq G_2 \supseteq \dots$
- *each* G_i *is normal in* G
- *the quotient* G_i/G_{i+1} *is in the center of* G/G_{i+1}.

Lemma 1. *If* F *is a free group, then* $\bigcap_{i=1}^{\infty} F_i$ *is the trivial group.*

Heuristic idea: The length of nonempty words in G_i increases with i, and so only the identity (which is the empty word in a free group) survives in all steps.

Characterizing the unknot

Proposition 5. *A knot is trivial if and only if its longitude represents the trivial element of the knot group.*

Proof. This follows from Dehn's Lemma and the loop theorem. □

Lemma 2 (Dehn's Lemma (Rolfsen, 2003, 4.A.1))**.** *Suppose* M^3 *is a* 3*-manifold and* $f\colon \mathbb{D}^2 \to M^3$ *is a piecewise-linear map of a disk with no singularities on the boundary, i.e.*

$$x \in \partial\mathbb{D}^2, x \neq y \in \mathbb{D}^2 \Rightarrow f(x) \neq f(y).$$

Then there exists an embedding $g\colon \mathbb{D}^2 \to M^3$ *with* $g(\partial\mathbb{D}^2) = f(\partial\mathbb{D}^2)$.

Invariants

Fundamental group of knot and link complements

Proposition 6. *Knot complements* $\mathbb{S}^3 \setminus K$ *are aspherical.*

An space X is called *aspherical* if all its higher homotopy groups vanish, i.e. $\pi_n(X) = 0$ for $n \geq 2$.

For a CW-complex X this is equivalent to the universal covering $\widetilde{X}$ being contractible.

By definition, an aspherical space in an Eilenberg-MacLane space of type $K(\pi_1(X), 1)$.

Proof. Uses the Sphere theorem to show that π_2 is trivial. H_3 of the universal cover vanishes because it is non-compact. Since the universal cover is a 3-dimensional manifold we conclude that all its homotopy groups are trivial, so it is contractible. TODO More details □

Corollary 1. *Fundamental groups of knot complements are torsion-free.*

Proof. The classifying space $K(\pi_1(\mathbb{S}^3 \setminus K), 1)$ has a finite dimensional model. Now a standard argument using that the group homology of cyclic groups is nontrivial in infinitely many degrees. TODO □

Proposition 7. *If* L *is a non-splittable link,* $\mathbb{S}^3 \setminus L$ *is aspherical.*

Corollary 2. *The fundamental group of a link complement is torsion free.*

Proof. The link group is a free product of the groups of the non-splittable parts of L. Now use that the free product of torsion free groups is torsion free. TODO □

In general, all torsion in a free product is conjugated to torsion in one of the summands of the free product.

Alexander polynomial

Definition 7. L oriented link with Seifert matrix A, then the first homology of the infinite cyclic covering of the link complement, $H_1(X_\infty;\mathbb{Z})$, has square presentation matrix $tA - A^T$.

The *Alexander polynomial* of L is given by

$$\Delta_L(t) \doteq \det(tA - A^T)$$

where $\doteq$ means "up to a multiplication with a unit $\{\pm t^{\pm n}\}$ of the Laurent ring $\mathbb{Z}[t, t^{-1}]$".

Remark 2. $\mathbb{Z}[t^{\pm 1}]$ is **not** a PID.

Definition 8. The *tunnel number* $t(K)$ of a knot $K \subset \mathbb{S}^3$ is the minimal number of arcs that must be added to the knot (forming a graph with three edges at a vertex) so that its complement in $\mathbb{S}^3$ is a handlebody. The same definition is valid for links.

The boundary will be a minimal Heegaard splitting of the knot complement (The knot complement is a manifold with boundary, so what is the definition of a Heegaard splitting in that case?).

Remark 3. Every link has a tunnel number, this can be seen by adding a "vertical" tunnel at every crossing in a link diagram. This shows that the tunnel number of a knot is always less than or equal to the crossing number, $t(K) \leq c(K)$.

Example 3.
- The unknot is the only knot with tunnel number 0. (Why?)
- The trefoil knot has tunnel number 1.
- The figure eight knot has tunnel number 1.

Arf invariant

Theorem 2. *The Arf invariant of a knot K is related to the Alexander polynomial by*

$$\mathrm{Arf}(K) = \begin{cases} 0 & \textit{if } \Delta_K(-1) \equiv \pm 1 \textit{ modulo } 8 \\ 1 & \textit{if } \Delta_K(-1) \equiv \pm 3 \textit{ modulo } 8. \end{cases}$$

Remark 4. If K is a slice knot, we know that its determinant $|\Delta_K(-1)|$ is an odd square integer. Thus we have $\Delta_K(-1) \equiv \pm 1$ modulo 8 and as such $\mathrm{Arf}(K) = 0$; Arf is a well defined concordance invariant.

$(2k+1)^2 = 4k^2 + 4k + 1 = 4\underbrace{k(k+1)}_{\text{even}} + 1 \equiv 1$ modulo 8

Tristram-Levine ω-signatures

Definition 9 ((Lickorish, 2012, Definition 8.8), (Kauffman, 1987, Definition 12.5))**.** Let $L \subset \mathbb{S}^3$ be an oriented link and $\omega \in \mathbb{S}^1 \subset \mathbb{C}$ a unit complex number with $\omega \neq 1$.

The *ω-signature* is sometimes called the *equivariant signature* or *Tristram-Levine-signature*.

The ω-signature $\sigma_\omega(L)$ of L is defined to be the signature of the Hermitian matrix[1]

$$(1-\omega)A + (1-\overline{\omega})A^T$$

where A is any Seifert matrix for L.

[1] Any Hermitian matrix is diagonalizable with real eigenvalues, and the signature is defined as the number of positive minus the number of negative eigenvalues.
Sylvester's Law of Inertia states that the signature of a Hermitian matrix B is not changed by congruence $C \cdot B \cdot C^T$.

Definition 10. $\sigma_{-1}(L) = \sigma(A + A^T)$ is known as **the** signature of L or the *Murasugi signature* .

Theorem 3. *The ω-signature $\sigma_\omega(L)$ is a well defined link invariant, i.e. it does not depend on the choice of Seifert surface.*

Proof. Directly check that the signature does not change under S-equivalence. TODO Define S-equivalence □

TODO The ω-signature can jump at the zeros of the Alexander polynomial because at those an eigenvalue can cross zero (changes sign) and is constant in between.

See this answer on MathOverflow: `https://mathoverflow.net/questions/85976/why-tristram-levine-signature-jumps-at-the-zeros-o`

There are some notes on the ω-signatures (also relating them to the 0-surgery $\mathbb{S}^3_0(K)$ on the knot K) at (Conway, 2018).

Concordance

Slice knots

Definition 11 (Surgery on a knot in $\mathbb{S}^3$)**.** The notation $\mathbb{S}^3_0(K)$ denotes the 0-*surgery on a knot* $K \subset \mathbb{S}^3$, i.e. removing a tubular neighborhood $\mathbb{S}^1 \times \mathbb{D}^2$ of K and gluing in $\mathbb{D}^2 \times \mathbb{S}^1$ via a homeomorphism of the boundaries, which are both $\mathbb{S}^1 \times \mathbb{S}^1$. TODO

Definition 12 (Trace of a knot)**.** For $n \in \mathbb{Z}$ the *n-trace of a knot* $K \subset \mathbb{S}^3$ is the 4-manifold $X_n(K)$ obtained by attaching an n-framed 2-handle to the 4-ball along K, i.e.

$$X_n(K) = \mathbb{D}^4 \cup_{K \times \text{framing}\colon \mathbb{S}^1 \times \mathbb{D}^2 \hookrightarrow \mathbb{S}^3} (\mathbb{D}^2 \times \mathbb{D}^2).$$

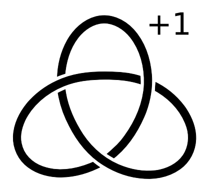

Figure 2: A Kirby diagram for $X_n(K)$ is given just by the knot K with the framing n written next to it. For example, here is a Kirby diagram representing the 1-trace X_1(right handed trefoil). The boundary of this 4-manifold is the $+1$-surgery $\mathbb{S}^3_{+1}$(right handed trefoil), a possible description of the Poincaré homology sphere.

Theorem 4 ((Miller and Piccirillo, 2018, Thm. 1.8))**.**
- *K is smoothly slice if and only if $X_0(K)$ smoothly embeds in $\mathbb{S}^4$.*
- *Similarly, K is topologically slice if and only if $X_0(K)$ topologically embeds in $\mathbb{S}^4$.*

Remark 5 (Exotic $\mathbb{R}^4$ from a topologically, but not smoothly slice knot). References: (Davis, 2011) TODO

Concordance of Links

Definition 13. A *smooth link cobordism* between the links $L_0, L_1 \subset \mathbb{S}^3$ is a smooth, compact, oriented surface Σ generically embedded in $\mathbb{S}^3 \times \mathbb{I}$ such that $\partial\Sigma = \overline{L_0} \coprod L_1$, where $\partial\Sigma \subset \mathbb{S}^3 \times \{0, 1\}$.

Proposition 8. *Linking numbers are concordance invariants.*

Remark 6 (The Hopf link is "the most non-slice link", (Krushkal, 2015)). Any link in $\mathbb{S}^3$ bounds immersed smooth disks $\coprod^n \mathbb{D}^2 \looparrowright \mathbb{D}^4$. TODO

Definition 14 (Boundary link)**.** A link $L^n \subset \mathbb{S}^{n+2}$ whose components bound disjoint Seifert surfaces is called a *boundary link* .

It is possible that each component of a link bounds a Seifert surface missing the other components, but still they do not bound disjoint surfaces.

Example 4. The untwisted Bing double of any knot is a boundary link. TODO Insert definition of Bing double TODO Insert picture TODO Proof that Bing doubles are boundary links (draw the taco shells, need 4 copies of the Seifert surface of the knot)

Proposition 9 ((Rolfsen, 2003, 5.E.1))**.** *If any two components of $L^1 \subset \mathbb{S}^3$ have nonzero linking number, then L is **not** a boundary link.*

Proof. Use the definition 15 of linking number where you count the intersection points of one component with a Seifert surface for the other. □

Definition 15 (Linking number via intersections with a Seifert surface, (Rolfsen, 2003, 5.D.(2)))**.** Let J and K be two disjoint oriented knots (e.g. link components). Pick a PL Seifert surface M^2 for K, with a bicollar (N, N^+, N^-) of the interior $\mathring{M}$. Make J transverse to M, i.e. assume after a small homotopy of J in $\mathbb{S}^3 \setminus K$ that J meets M in a finite number of points and that at each point J passes locally from N^+ to N^- or from N^- to N^+. Corresponding to this direction, weight the intersection types with $+1$ or -1. The signed sum of these intersection points is the linking number $\operatorname{lk}(J, K) \in \mathbb{Z}$.

On first sight this seems to depend on the choice of Seifert surface M.

Proposition 10 ((Rolfsen, 2003, 5.E.8))**.** *If a link L is a boundary link, then each component represents an element in the second commutator subgroup*[2] *of the fundamental group of the complement of the remaining component(s).*

[2] Also called *second derived subgroup* or $G^{(2)}$, it is generated by elements of the form $[[x, y], [z, w]]$.

The Cappell-Shaneson way to slice a knot

(Teichner et al., 2010, 4.1)

Definition 16. A knot $K\colon \mathbb{S}^1 \subset \mathbb{S}^3$ which is slice in a homology 4-ball is called *homologically slice*.

Open question ((Still open?)). Find an obstruction which is able to tell the difference between homologically slice and actually slice.

Heegaard Floer homology

The concordance invariant $\tau(K) \in \mathbb{Z}$ for a knot $K \subset \mathbb{S}^3$ is defined in (Ozsváth and Szabó, 2003), this yields a homomorphism $\mathcal{C} \to \mathbb{Z}$.

TODO Explain this definition

Definition 17 (Ozsváth and Szabó's τ-invariant)**.**

$$\tau(K) = \min\{i \in \mathbb{Z} \mid H_*(\mathcal{F}(K,i)) \to \widehat{HF}(\mathbb{S}^3) \text{ is nontrivial}\}$$

Properties of τ:

- It provides a lower bound for the four-ball genus,

$$|\tau(K)| \leq g_4(K)$$

- TODO

Definition 18 ((Hom, 2011))**.** An *L-space* is a rational homology 3-sphere[3] with the simplest possible Heegaard-Floer homology,

$$\dim \widehat{HF}(Y) = |H_1(Y;\mathbb{Z})|.$$

[3] Y^3 is a rational homology sphere iff $H_1(Y;\mathbb{Q}) = 0$, which means that the first homology with integer coefficients is a finite group.
For any rational homology 3-sphere $\operatorname{rk} \widehat{HF}(Y) \geq |H_1(Y;\mathbb{Z})|$, and L-spaces are those for which this bound is sharp.

A knot admitting a non-trivial L-space surgery is called an *L-space knot* .

Example 5.
- Lens spaces are L-spaces (and this is also where the name comes from).
- Torus knots are L-space knots, as $pq \pm 1$ surgery on the (p,q)-torus knot yields a lens space.

TODO Why?

Satellite operators

(Cochran et al., 2014)

Let P be an oriented knot in the solid torus $\mathbb{S}^1 \times \mathbb{D}^2$, this is called a *pattern knot* . For K an oriented knot in $\mathbb{S}^3$ we denote by $P(K)$ the (untwisted) *satellite* of K with pattern P. The knot K is sometimes called a companion[4] of $P(K)$ (Lickorish, 2012, p. 10).

[4] You can imagine that the satellite knot is orbiting its companion.

An important special case of satellites are

Definition 19 ((p,q)-cables)**.** The (p,q)*-cable* of a knot K , denoted by $K_{p,q}$ is the satellite knot with pattern the (p,q)-torus knot. p is the number of times $K_{p,q}$ transverses the longitudinal direction of K, and q the meridional number.

We can view a pattern as a function

$$P \colon \mathcal{K} \to \mathcal{K}$$

on the set of isotopy classes of knots. These descent to yield functions (*satellite operators*)

$$\mathcal{K}/\sim \to \mathcal{K}/\sim$$

$\mathcal{K} = \frac{\{\text{knots } \mathbb{S}^1 \hookrightarrow \mathbb{S}^3\}}{\text{isotopy}}$

for various equivalence relations $\sim$ on $\mathcal{K}$, for example concordance.

If K and J are concordant, their satellites will be concordant via a concordance which "follows along" the concordance between K and J.

Braid groups

Exercise 1. Show that there is a presentation for the braid groups with just two generators.

TQFTs - Topological Quantum Field Theories

TODO Write down axioms

A monoidal functor is supposed to preserve the identity objects for the tensor product. Since the empty set is the identity for the tensor product in the bordism category (given by disjoint union of the bordisms), the TQFT should send this to the identity object for $\otimes_R$, which is just the ground ring R.

Open questions

Open question 1. Is the crossing number of a satellite knot bigger than that of its companion?

Open Questions in Knot concordance

A list of knots whose sliceness status is (supposedly) not known:

1. $(2, 1)$-cable on the figure eight
2. Negative Whitehead double of the right handed trefoil
3. Positive Whitehead double of the left handed trefoil
4. Any Whitehead double of the figure eight
5. ...

Current developments/Recently solved

- The Conway knot $11n34$ is not slice, as shown in (Piccirillo, 2018)
- Kronheimer and Mrowka found an error in their paper (Kronheimer and Mrowka, 2013) which (incorrectly?) claimed that Rasmussen's s-invariant could not detect an exotic 4-ball.
  In particular the methods of (Freedman et al., 2009) could possibly lead to an exotic 4-sphere.

4-Manifolds

Bordism

Definition 20. The n-dimensional oriented *cobordism group* over the space X is

$$\Omega_n[X] = \frac{\{f \colon M^n \to X \mid M^n \text{ oriented, closed, } n\text{-dim. manifold}\}}{\text{bordism}}$$

If $\alpha \colon M^n \to X$ represents a bordism class, M^n is allowed to have more than one component.

Proposition 11 ((Kauffman, 1987, 13.15, p. 319)).

- *Pushing forward a fundamental class*

$$\begin{aligned} \Omega_n[X] &\to H_n(X; \mathbb{Z}) \\ [f \colon M^n \to X] &\mapsto f_*([M]) \end{aligned}$$

  is an isomorphism for $n \leq 3$.

- *The sequence*

$$\Omega_4[*] \to \Omega_4[X] \to H_4(X; \mathbb{Z})$$

  is exact.

Mazur manifolds

References:

- Homology spheres: (Saveliev, 2013)

- (Akbulut et al., 1979)

  TODO

Andrews-Curtis Conjecture

(SCP)

Definition 21. A *balanced presentation* is a presentation

$$\langle g_1, \ldots, g_n \mid r_1, \ldots, r_n \rangle$$

with the same number of generators and relations.

The Andrew-Curtis moves on a balanced presentation are

(i) **1-handle slides:** Replace a pair of generators $\{x, y\}$ by $\{x, xy\}$

(ii) **2-handle slides:** Replace a pair of relations $\{r, s\}$ by $\{grg^{-1}s, s\}$, where g is any word in the generators

(iii) **1-2-handle cancellations:** Add a generator together with a new relation killing it

In particular, you are <u>not</u> allowed to make a copy of a relation to keep for later use (which would correspond to **2-3-handle creation/cancellation**).

Conjecture (Andrews-Curtis conjecture (False?)). Any balanced presentation of the trivial group can be transformed by Andrews-Curtis moves to the trivial presentation.

Example 6. TODO

$$\langle x, y \mid xyx = yxy, x^5 = y^4 \rangle$$

is a balanced presentation of the trivial group, but until now nobody was able to find a sequence of Andrews-Curtis moves to transform it into the trivial presentation

$$\langle x, y \mid x, y \rangle$$

TODO Explain how to construct homotopy 4-spheres from balanced presentations of the trivial group (Akbulut and Kirby)

In (Kirby, 2006, I.3) Kirby suspected that the *Dolgachev surface* $E(1)_{2,3}$ (an exotic copy of the rational elliptic surface $\mathbb{CP}^2 \# 9\overline{\mathbb{CP}^2}$) might require 1- and/or 3-handles. But in (Akbulut, 2008) Akbulut found a handlebody presentation without those.

Open question ((Still open?)). Does a simply connected, closed, smooth 4-manifold need 1-handles and/or 3-handles?

Observation 1. If exotic $\mathbb{S}^4$ exist, their handle decomposition must contain 1- or 3-handles.

For this suppose you have a handle decomposition of a manifold with the homology of $\mathbb{S}^4$ and no 1- and 3-handles, then there could not be any 2-handles either because these would give nontrivial second homology.

Definition 22. Use the following notation to denote the abelian group

$$\Gamma_n = \frac{\text{orientation preserving diffeomorphisms of } \mathbb{S}^{n-1}}{\text{those that extend to a diffeomorphism of } \mathbb{D}^n}$$

- $\Gamma_1 = \Gamma_2 = 0$
- Munkres, Smale: $\Gamma_3 = 0$
- Cerf: $\Gamma_4 = 0$
- Kervaire, Milnor: $\Gamma_5 = \Gamma_6 = 0$, $\Gamma_7 = \mathbb{Z}/28$

Proposition 12. *For $n \geq 5$ we can identify Γ_n with the set of oriented smooth structures of the topological n-sphere. I.e. in dimension ≥ 5 all exotic spheres can be obtained by using a diffeomorphism of $\mathbb{S}^{n-1}$ to glue two n-disks along their boundary.*

Theorem 5 (Cerf, (Geiges and Zehmisch, 2010))**.** *Any diffeomorphism of the 3-sphere $\mathbb{S}^3$ extends over the 4-ball $\mathbb{D}^4$, in other words*

$$\Gamma_4 = 0.$$

Observation 2. Cerf's theorem implies that there are no exotic structures on $\mathbb{S}^4$ that can be obtained by gluing two 4-disks along their boundary.

Trisections

"Trisections are to 4-manifolds as Heegaard splittings are to 3-manifolds"

References

- Original paper: (Gay and Kirby, 2016)
- Lecture notes: (Gay, 2019)

Definitions

Definition 23. • The standard genus g surface is

$$\Sigma_g = \#^g(\mathbb{S}^1 \times \mathbb{S}^1)$$

- The standard genus g solid handlebody is

$$H_g = \natural^g(\mathbb{S}^1 \times \mathbb{D}^2)$$

with $\partial H_g = \Sigma_g$

- The standard 4-dimensional 1-handlebody (of "genus k") is

$$Z_k = \natural^k(\mathbb{S}^1 \times \mathbb{D}^3)$$

i.e. a 4-ball to which we attach k-many 4-dimensional 1-handles.

- $\#^n A$ is the connected sum of n copies of A, with $\#^0 A = \mathbb{S}^m$
- TODO

$\partial(A \natural B) = (\partial A)\#(\partial B)$

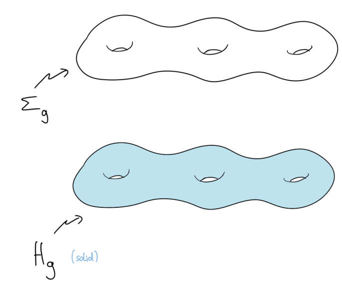

Figure 3: Standard manifolds of dimension 2, 3

Topics to study and Reading List

Questions

- TODO

Reading List

- List of open problems concerning quantum invariants is at (Ohtsuki et al., 2002)
- Vassiliev knot invariants, for this (Bar-Natan, 1995)
- Khovanov homology (Bar-Natan, 2005)
- Kaufman's books, for example (Kauffman, 2001)
- Baez's book on gauge theory (Baez and Muniain, 1994)

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