1 Knot Theory

2 Definitions

2.1 Alexander polynomial

Definition 2.1. L oriented link with Seifert matrix A, then the first homology of the infinite cyclic covering of the link complement, $H_1(X_\infty;\mathbb{Z})$, has square presentation matrix $tA - A^T$.

The *Alexander polynomial* of L is given by

$$\Delta_L(t) \doteq \det(tA - A^T)$$

where $\doteq$ means "up to a multiplication with a unit $\{\pm t^{\pm n}\}$ of the Laurent ring $\mathbb{Z}[t, t^{-1}]$".

Remark 2.2. $\mathbb{Z}[t^{\pm 1}]$ is **not** a PID.

2.2 Invariants

Definition 2.3. The tunnel number $t(K)$ of a knot $K \subset \mathbb{S}^3$ is the minimal number of arcs that must be added to the knot (forming a graph with three edges at a vertex) so that its complement in $\mathbb{S}^3$ is a handlebody.
The boundary will be a minimal Heegaard splitting of the knot complement (??).

3 Open questions

Open question 1. Is the crossing number of a satellite knot bigger than that of its companion?

4 4-manifolds

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