A REVIEW OF QUANTUM MACHINE LEARNING

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1. Introduction

2. Speedup Techniques

2.1. **HHL Algorithm.** One such application of Phase Estimation is with respect to solving linear systems of equations. This is the so-called HHL algorithm [1].

The general problem statement of a linear system is if we are given matrix A and unit vector $\vec{b}$, then find $\vec{x}$ satisfying, $A\vec{x} = \vec{b}$.

However, assume that instead of solving for x itself, we instead solve for an expectation value $x^T M x$ for some linear operator M. Hence, one can show that our algorithm has a runtime bound of $O(\log(N)\kappa^2)$, if we can further assume that the linear system is sparse and has a low condition number κ.

So, assume that A in our linear system is an $N \times N$ Hermitian matrix. Notice that this is an "unrestrictive" constraint on A because we can always take non-Hermitian matrix A' and linear system $A'\vec{x} = \vec{b}$ and instead solve $\begin{bmatrix} 0 & A' \\ A'^{\dagger} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ x \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$. Hence, we we will assume that A is Hermitian from here on.

Recall that because A is hermitian $\Rightarrow$ we can perform quantum phase estimation using e^{-iAt} as the unitary transformation. This can be done efficiently if A is sparse.

So, we first prepare $|b\rangle$ (the representation of $\vec{b}$). We assume that this can be done efficiently or that $|b\rangle$ is supplied as an input.

Denote by $|\psi_j\rangle$ the eigenvectors of A with associated eigenvalues λ_j. Hence, we can express $|b\rangle$ as $|b\rangle = \sum_j \beta_j |\psi_j\rangle$. So, we initialize a first register to state $\sum_j \beta_j |\psi_j\rangle$ and second register to state $|0\rangle$. After applying phase estimation, we then have the joint state $\sum_j \beta_j |\psi_j\rangle \left|\widetilde{\lambda}_j\right\rangle$, where $\widetilde{\lambda}_j$ is an approximation of λ_j. We'll assume that this approximation is perfect from here on.

Next we add an ancilla qubit and perform a rotation conditional on the first register while now holds $|\lambda_j\rangle$. The rotation transforms the system to

$$\sum_j \beta_j |\psi_j\rangle |\lambda_j\rangle \left(\sqrt{1 - \frac{C^2}{\lambda_j^2}} |0\rangle + \frac{C}{\lambda_j} |1\rangle \right)$$

for some small constant $C \in \mathbb{R}$ that is $O(1/\kappa)$.

Hence, we can undo phase estimation to restore the second register to $|0\rangle$.

Now, if we measure the ancillary qubit in the computational basis, we'll evidently collapse the state to $|1\rangle$ with some probability.

Finally, we can make a measurement M whose expectation value $\langle x| M |x\rangle$ corresponds to the feature of x we wish to evaluate.

2.2. **Amplitude Amplification.**

References

[1] Seth Lloyd. Quantum algorithm for solving linear systems of equations. In *APS March Meeting Abstracts*, 2010.

[2] Michael A Nielsen and Isaac L Chuang. *Quantum computation and quantum information*. Cambridge university press, 2010.