Numerical methods assignment 4 - Random numbers

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1 Theory:

The purpose of this experiment was to analyse the dart problem using various probabilistic methods. The poisson distribution allows one to calculate the number of darts that hit certain areas of the board given some pre-established mean value, in our case being($<n>$=1,5 and 10). The poisson equation calculates how many times we hit a certain area of the board over N trials(N=50 for us). It is normally given in the form:

$$P(N) = \frac{<n> e^{-<n>}}{N!}$$

It is possible to use this equation for values of N from 0 to 50 and to then map a distribution of this.
$\sum_{n=0}^{N} P(n)$ allows us to determine if the distribution is normalised by seeing that their sum adds up to 1.
$\sum_{n=0}^{N} nP(n)$ allows us to determine the mean of the distribution which should match the input value $<n>$ if it is actually correct.
$\sum_{n=0}^{N} n^2 P(n)$, the second moment, represents the variance of the curves obtained. A large value of variance means a broad curve with a large spread between values, a smaller value means a narrow curve with small spread between values.
The standard deviation of a poisson distribution is simply measured by taking the square root of the mean. This is a measurement of the spread between points similar to variance. A large standard deviation means points are on average further from each other and vice a versa.

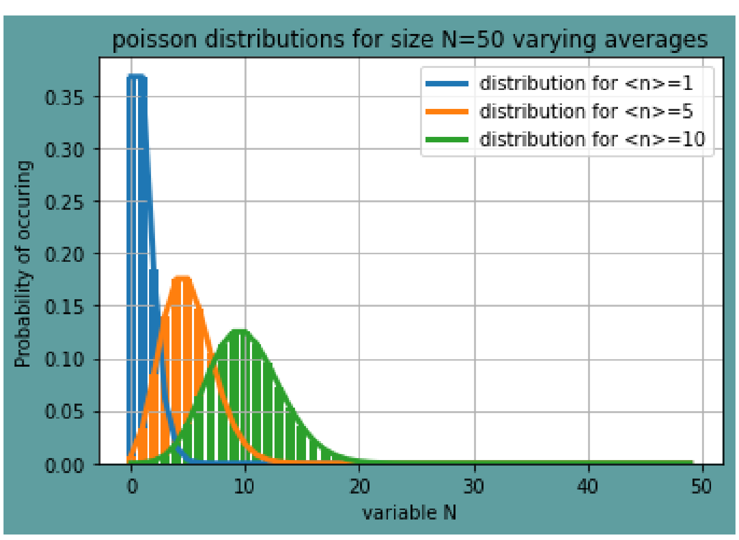
The second part of this experiment was to simulate this dart problem and see if we could return something following the poisson solution obtained. This was done by labellng the regions of the dart board from L=0 to L=100 and seeing how many darts hit certain areas over a certain number of throws and trials. The number of darts hitting a certain over for each trial was given by H(n). A normalised distribution of this could then be obtained using the equation :

$$P_{sim}(n) = \frac{H(n)}{\sum_n H(n)}$$

This could then be compared to the poisson equation in order to see for what values our simulation will probe the poisson equation.

2 Results + Discussion:

Part 1:



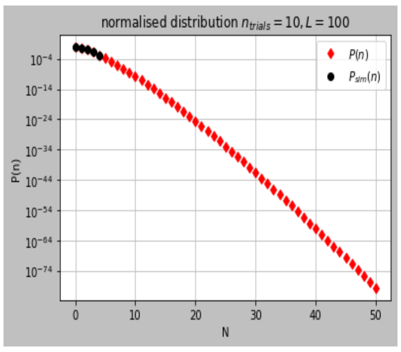
for $<n>=2$: $\sum_{n=0}^{N} P(n) = 1$,——$\sum_{n=0}^{N} nP(n) = 1$,—— $\sum_{n=0}^{N} n^2P(n) = 2$ —,standard deviation=1

for $<n>=5$: $\sum_{n=0}^{N} P(n) = 1$, ——— $\sum_{n=0}^{N} nP(n) = 5$ ——— $\sum_{n=0}^{N} n^2P(n) = 30$ —, standard deviation=2.23

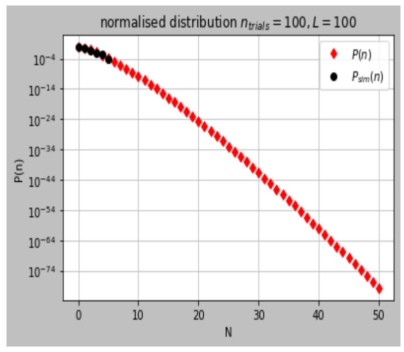
for $<n>=10$: $\sum_{n=0}^{N} P(n) = 1$,——$\sum_{n=0}^{N} nP(n) = 10$,—— $\sum_{n=0}^{N} n^2P(n) = 110$ —,standard deviation=3.16

These values make sense. The sum of all the probabilities returns 1, showing the distributions are normalised. The value of the first moment(the mean) is exactly the same as the input value, and the value of the second moment(the variance) increases as the value of the average increases which makes sense given the plot above. As the poisson distribution moves further out, the curve becomes broader as described here. The standard deviation demonstrates a similar relationship which agrees with the graph(individual points are further from each other than in the other graphs).

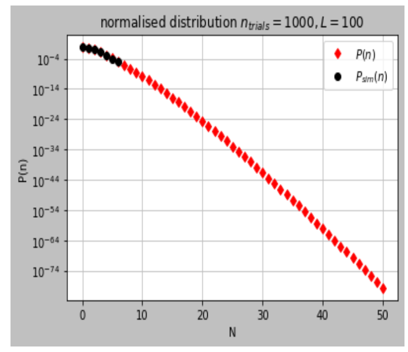
Part 2:



(a) Minimum value is 0.001

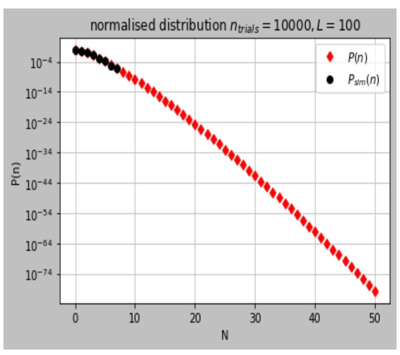


(b) Minimum value is 0.0001

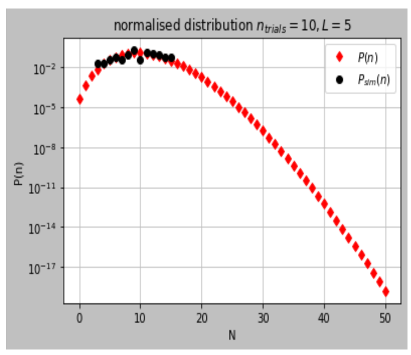


(c) Minimum value is 1e-05

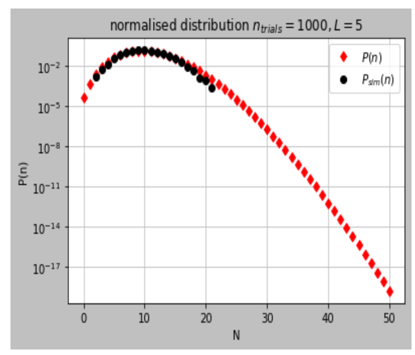
Figure 1: (Trials=10,L=100),(Trials=100,L=100),(Trials=1000,L=100)



(a) Minimum value is 1e-06

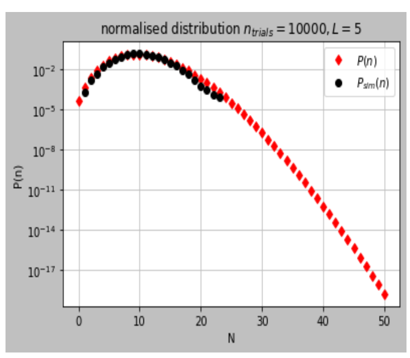


(b) Minimum value is 0.02



(c) Minimum value is 0.0002

Figure 2: (Trials=10000,L=100), (Trials=10,L=5), (Trials=1000,L=5)



(a) Minimum value is 8e-05

Figure 3: Trials =10000, L=5

Darts were thrown at random in the 100 element array comprising regions of the board for varying numbers of trials and regions. The results of all these(in black icons) were plotted against the poisson distribution values that would be obtained).

It was found that for lower numbers of regions such as L=5, a higher curvature was obtained and for higher numbers regions such as L=100, a much less pronounced curve was obtained according to the poisson distribution(red curve).This is because the graphs that map distributions of more regions(L=5) have already gone past the turning point of the curve and are reaching much lower values than in the simulation for L=5.

As the number of trials increases e.g trials =(10, 100,1000,10000), the accuracy of the simulation increased and was able to reach lower and lower values on the poisson equation curve and thereby return greater accuracy. The minimum value that was able to be reached for all these varying number of regions and trials is listed at the bottom of each graph. It was found that for 10000 trials, for L=5 and L=100, the minimum value for the L=100 trial was 80 times smaller than that reached by the L=5 simulation(i.e $(8 \times 10^{-5}/1 \times 10^{-6})$.