Lab report 3: Optical diffraction

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JS Physics

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Abstract

This lab focused on the use of optical diffraction techniques in order to produce various intensity distributions of light on a camera.

Experiment 1 investigated Frauenhofer diffraction(far-field) by a single slit and a pinhole. As expected for the slit, the angular distribution is within quite a narrow range and a large central peak with various subsidiary maxima is apparent. This yielded an average angular deviation from the desired and expected results of $\pm 0.067°$.

The circular aperture has the effect of narrowing this distribution to a central Airy disk which can be observed though with some loss of optical clarity in the data. Indeed there was a difference of $(90 \pm 4.5 \times 10^{-4})$ um between the obtained and theoretical value(200um) of the aperture radius.This quite staggering difference is due to the change in angular distribution of the first airy disk caused by poor filtering of data most likely(i.e external light sources are possibly interfering).

In experiment 3 Fresnel diffraction by a pinhole did qualitatively yield the expected 2 frensel zones as required but for some inaccuracy in the data. The third experiment that dealt with multiple slit diffraction, while qualitatively correct, yielding 3 prominent peaks with the central peak being roughly twice the height of the shoulders had some distortion and a broadening effect in the x direction.

Finally, the technique of spatial filtering was properly implemented by use of a diffraction grating with an aperture a certain distance in the focal length of a 20cm lens to return the desired combinations of m orders(zeroth order only, zero and first order only, first order only). The distance at which these orders were achieved was 20cm, 17cm and 14cm respectively.

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note: references to the bibliography are denoted by |**x**| where x corresponds to the index number of the bibliography entry.

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I. Theory

I shall try to derive the equations used in this experiment and explain the underlying phenomena here.

I. The single slit

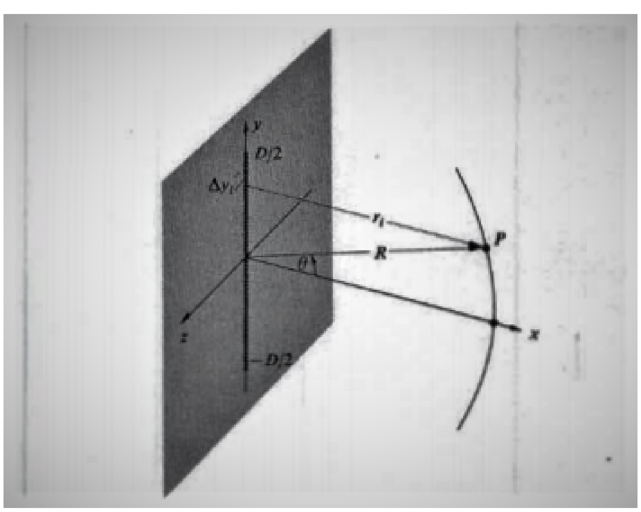

Figure 1: A coherent line source|**2**|

The electric field from a continuous line source such as above is given by the following equation|**2**|

$$E = \varepsilon_L \int_{-D/2}^{+D/2} \frac{Sin(\omega t - kr)}{r}\, dy$$

where ε_L is the source strength per unit length. Therefore, the field at P due to the differential segment of the source dy is

$$dE = \frac{\varepsilon_L}{R} Sin(\omega t - kr)$$

The value r can also be expanded in the form of a maclaurin series like such

$$r = R - ysin(\theta) + (y^2/2R)cos^2(\theta) + ...$$

so...

$$E = \frac{\varepsilon_L}{R} \int_{-D/2}^{+D/2} Sin(\omega t - k(R - ysin(\theta)))\, dy$$

, returning

$$E = \frac{\varepsilon_L D}{R} \frac{sin[(kD/2)sin(\theta)]}{(kD/2)sin(\theta)} sin(\omega t - kR)$$

To simplify the equation and to return intensity instead of electric field it is necessary to let $\beta = (kD/2)sin(\theta)$

and to square the equation. Note that the value of 1/2 is introduced by the $sin^2(\omega t - kR)$ term.

$$I(\theta) = \frac{1}{2}(\frac{\varepsilon_L D}{R})^2(\frac{sin(\beta)}{\beta})^2$$

Then taking the first part of this equation to be the initial intensity we are returned

$$I(\theta) = I_0(\frac{sin(\beta)}{\beta})^2$$

as required. This intensity will have minima equal to zero when $sin(\beta)$ is equal to 0, i.e if $\beta = \pm\pi, \pm 2\pi, \pm 3\pi, ..$ etc.

II. The circular aperture

The equation for the intensity distribution of diffraction through a circular aperture is of a quite similar form to that of the single slit but for the inclusion of a bessel function of the first kind J_1 to account for the circularity introduced.|**3**|

$$I(\theta) = I_0[\frac{2J_1(kasin(\theta))}{kasin(\theta)}]^2$$

where a=hole radius and $\theta = q/R$ with q being the radius of the maximum produced and R being the distance to the screen.

Due to the axial symmetry, the central maximum is a circular spot known as an Airy disk. The equation for such an Airy disk is returned by

$$q_1 = 1.22\frac{R\lambda}{2a}$$

,|**3**| putting the equation in terms of θ the angular distribution and solving for a we find

$$a = \frac{1.22\lambda}{2\theta}$$

This equation is quite important and is used to determine the veracity of the results in experiment 1 part 3.

III. Fresnel diffraction by a pinhole

The focusing of light on a screen by a pinhole is given by the equation|**6**|

$$R_m^2 = (r_0 + m\lambda/2)^2 - r_0^2$$

therefore,

$$R_m^2 = mr_0\lambda + m^2\lambda^2/4$$

This second term is negligible while m is not extremely large, returning

$$R_m^2 = mr_0\lambda$$

where R_m is the radius of the intensity distribution produced on the screen and r_0 is the distance between the camera and the pinhole.

IV. Multiple slit diffraction

Once again the intensity distribution equation for the many slits setup bears similarities to the single slit equation, however another term is introduced to account for the division of the light source.

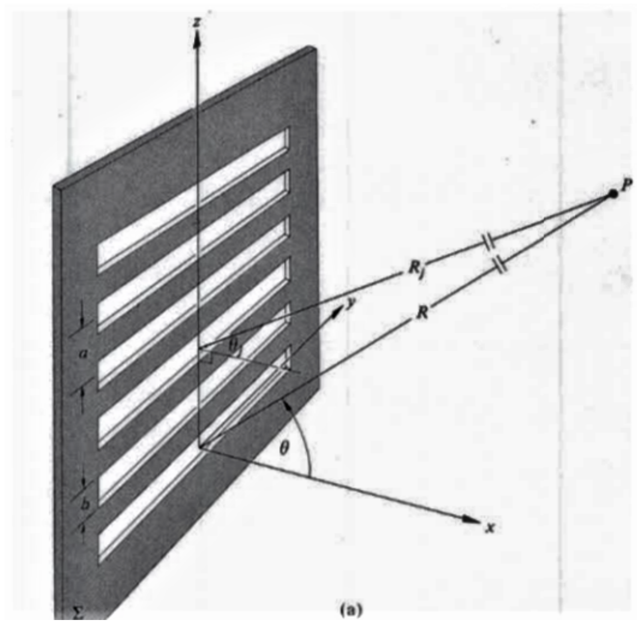

Figure 2: Multiple slit geometry|**4**|

$$I(\theta) = I_0(\frac{sin(\beta)}{\beta})^2(\frac{sin(N\alpha)}{sin(\alpha)})^2$$

where $\beta = kb/2sin(\theta)$ and $\alpha = ka/2sin(\theta)$ with b being the slit width and a being the distance apart as represented in the above figure.

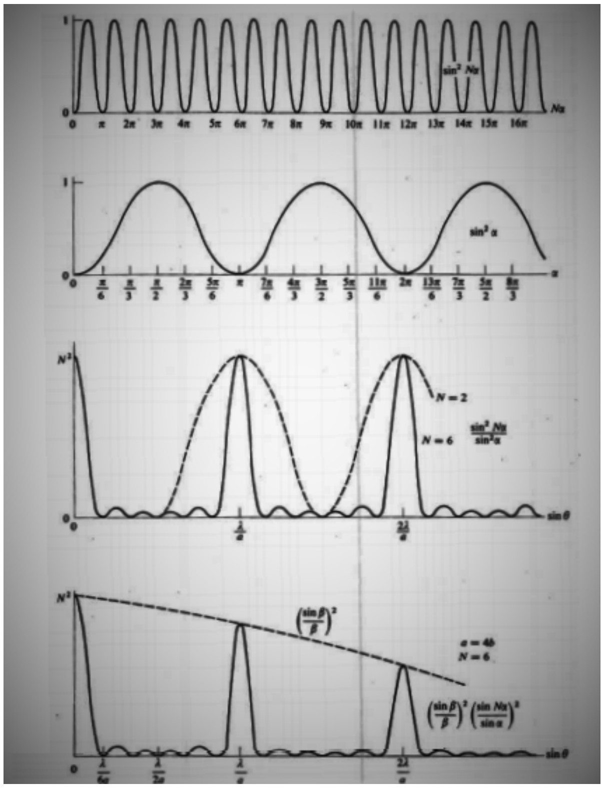

Figure 3: Intensity distributions for various values of N[**4**]

The above image shows the kind of intensity distributions that are expected as one increases N, the number of slits(increasing as one goes down the picture). As N increases, light becomes concentrated in narrower needle-like points with subsidiary peaks becoming less visible. Once can imagine how narrow and tall these peaks would be with N=25 slits as used in the experiment. The ratio between a and b also influences the height difference between the tall peaks, i.e the tall peaks to the right and left of the central peak will be smaller than the central one by some factor. In this case the peaks on the shoulders be half the size of the central peak(b=40um, a=80um, therefore a ratio of 1:2).

V. Spatial filtering

Spatial filtering is a method by which one can alter the diffraction of light to finetune the intensity distribution to a desired value. I.e if one wants the zero order to vanish then the imaging aperture in front of the camera can be moved such that destructive interference between beams occurs at the first mmaximum. Every beam has a definitive phase relationship that is taken into account by the shape of the grating. The light travels in such a manner that it cancels out within certain locations of the converging lens focal length. Consider the following image

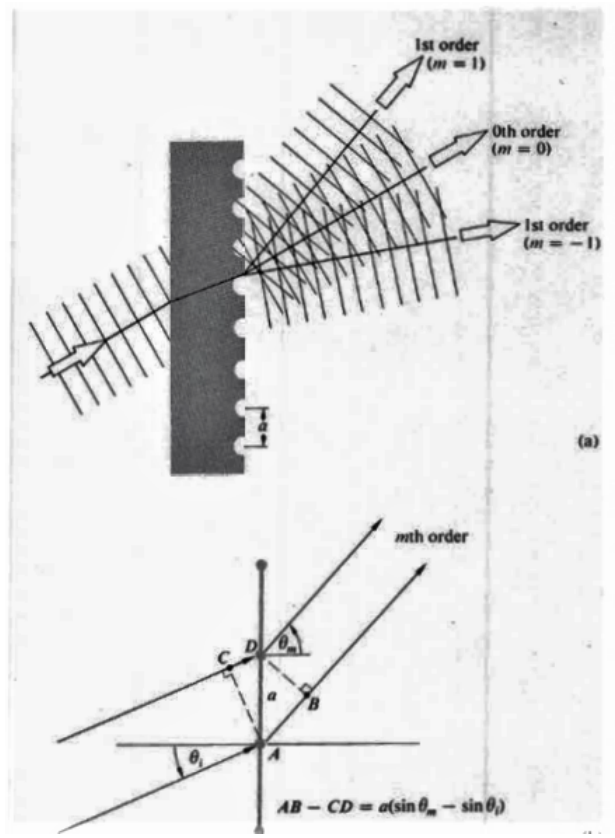

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Figure 4: Light traveling through a diffraction grating geometry[**5**]

The grating equation is then

$$asin(\theta)_m = m\lambda$$

II. Experimental procedure

It should be noted that the apparatus was placed on two conjoined meter sticks, therefore 2 metres of length. Taking the position of the laser then to be 0cm and upwards from that verging to 200cm at the end. Some measurements will also be given in terms of distance to the camera lens(which is on the second metre length from the laser) but these instances will be made clear.

.1 Part 1 - Cleanup setup

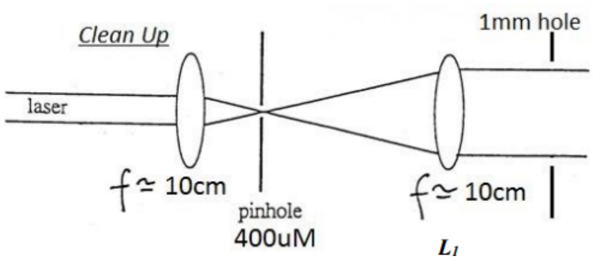

Figure 5: Preliminary cleanup apparatus setup|**1**|

Apparatus component	position(from laser)(cm)	error(cm)
10cm lens	30	$\pm 0.1cm$
400um pinhole	40	$\pm 0.1cm$
10cm lens	50	$\pm 0.1cm$
1mm hole	75	$\pm 0.1cm$

.2 Part 2 - Fraunhofer diffraction by a single slit and a pinhole

The cleanup apparatus is setup as specified previously. It is the position of lens, slit and circular aperture that is relevant here.

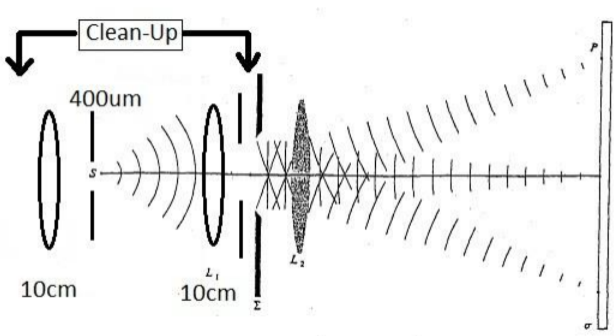

Figure 6: Experiment 1 general setup|**1**|

The first task was to record the intensity distribution produced by a single slit of width 100um without the use of a lens. The single slit was placed at a position of 85cm(from the laser) and the resulting intensity distribution was recorded. This result was then to be compared to a theoretical result produced by the equation

$$I(\theta) = I_0(Sin\beta/\beta)^2$$

where $\beta = (kb/2)sin(\theta)$, k=$2\pi/\lambda$ and b=slit width.

To compare the results it was necessary to convert the obtained data along the x-axis into angular distribution. Since each pixel represented a distance of 14um, the distance of orders was given by (center - pixel number)×14um. The angle created is therefore|1|

$$\theta = Tan^{-1}(\frac{x}{r})$$

where r is the distance from the camera to the slit. These distributions could then be mapped on top of each other and compared.

For the second part of this experiment, a similar process was underwent but with a lens of focal length 40cm added to the apparatus. The only difference here analytically is that in determining the angular distribution, the focal length of lens 2 was used instead of r i.e $\theta = Tan^{-1}(x/f)$.

For the final part of this experiment, Frauenhofer diffraction by a pinhole of diameter 400um was observed without the use of a lens. The pinhole was placed at a position of 80cm(from the laser, therefore 1.2m from the camera). Since a lens was not used, the value r(the distance from the camera to the hole) was used to determine the angular distribution. Since circular apertures behave differently to slits, a different equation was utilised to map the theoretical results. $I(\theta) = I_0[2J_1(kaSin(\theta))/(kaSin(\theta))]^2$ where a= the hole radius(200um) and J_1= the first order Bessel function. The following equation was then used to extract meaning from the results, the derivation of which is discussed in the theory section

$$a = \frac{1.22\lambda}{2\theta}$$

.3 Part 3 - Fresnel diffraction by a pinhole

Once again, the cleanup setup from previous experiments was utilised with naught but the relevant pinhole position being relevant here. Three pinholes of diameter 1.2mm,1.4mm and 1.6mm were utilised.

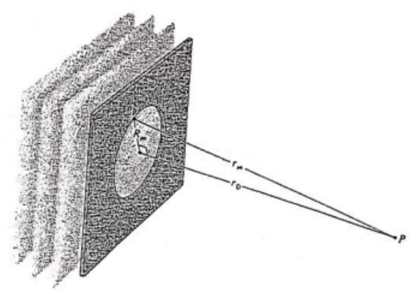

Figure 7: Experiment 2 pinhole and relation between distance measurements|**1**||**6**|

The goal of this experiment was to verify the position at which 2 Fresnel zones would occur, this position r_0 being the distance between the camera and the pinhole. This varies for each pinhole diameter and the way to determine the veracity of these results is to use the equation

$$R_m^2 = m\lambda_0 r_0$$

where R_m is the radius of the intensity distribution in question. These were placed at position 60cm from the camera.

.4 Part 4 - Multiple slit diffraction

As previously, the cleanup setup was maintained and the setup takes much the same form as figure 5, however, in place of a single slit was the grating (in copper foil) provided N = 25, b = 40 m and a = 80 m and using a lens of focal length 20cm this time. The goal was to observe the pattern produced and compare it to the theoretical value described by the equation|1|

$$I(\theta) = I_0(Sin\beta/\beta)^2(SinN\alpha/Sin\alpha)^2$$

.5 Part 5 - Spatial filtering

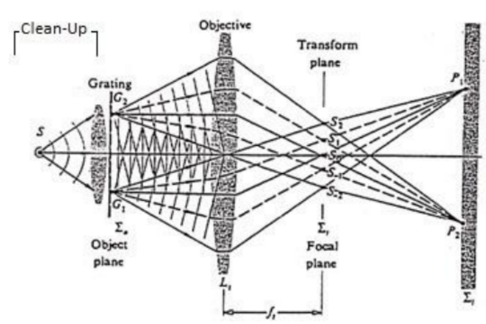

Figure 8: Experiment 4 general setup|**1**|

The aim of this experiment was to understand the principle by which spatial filters function. Spatial filtering is the process of fine tuning the intensity distribution caused by diffraction in order to zone in on certain specific areas. The experiment was set up as above with an image grating and lens with a 20cm focal length. The aperture provided was placed in the focal length of the lens at various distances from said lens in order to filter out various combinations of intensity distribution orders. The three desired combinations of orders were:

(a) Zeroth order diffraction only(aperture distance=20cm)
(b) Zero and first order diffraction only(aperture distance=17cm)
(c) First order diffraction only(aperture distance=14cm)

III. Results+Discussion

I. Part 1 - Fraunhofer diffraction by a single slit and a pinhole

I.1 Frauenhofer diffraction with a single slit (no lens)

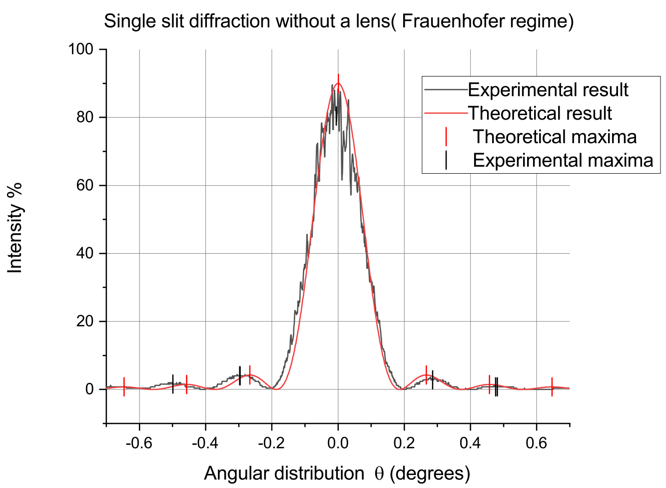

Figure 9: Exp1 part 1: Diffraction by 100um slit(no lens)

Theoretical peak centers(°)	Experimental peak centers(°)	error(°)
-0.46	-0.499	$\pm 112 \times 10^{-7}$
-0.266	-0.2957	$\pm 112 \times 10^{-7}$
0	0.004	$\pm 112 \times 10^{-7}$
0.266	0.2825	$\pm 112 \times 10^{-7}$
0.45	0.4757	$\pm 112 \times 10^{-7}$

Standard error between Experimental and Theoretical angles(°)
0.02344(°)

Theoretical peak intensity(%)	Experimental peak intensity(%)	error(%)
1.48	1.6	±0.0488
4.247	4	±0.0488
89	79.6	±0.0488
4.247	2.8	±0.0488
1.48	0.8	±0.0488

Standard error between Experimental and Theoretical intensity%
4.266%

As can be observed above the theoretical results are largely in agreement with the experimental results but for some fuzziness of the experimental data obtained. The experimental data is also slightly slanted to the left. This may account for the slight differences in angular distribution. If the light was entering the camera at a slight angle, some light would be lost, this on the right side of the above figure. This would cause maxima to slightly shift where the light is absent.

I.2 Frauenhofer diffraction by single slit with lens

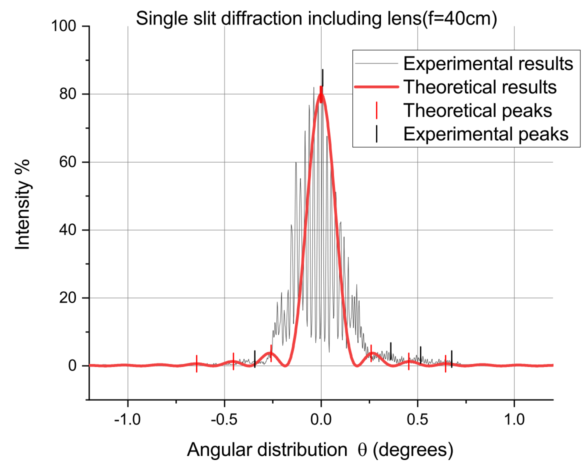

Figure 10: Exp1 part 2: Diffraction by 100um slit(with lens)

Theoretical peak centers(°)	Experimental peak centers(°)	error(°)
-0.25	-0.34	$\pm 350 \times 10^{-7}$
0	0	$\pm 350 \times 10^{-7}$
0.2586	0.36	$\pm 350 \times 10^{-7}$
0.4532	0.51	$\pm 350 \times 10^{-7}$
0.6437	0.67	$\pm 350 \times 10^{-7}$

Standard error between Experimental and Theoretical angles(°)
0.067(°)

Theoretical peak intensity(%)	Experimental peak intensity(%)	error(%)
3.7	2	±0.0488
80	84.8	±0.0488
3.701	4.4	±0.0488
1.3	2.3.2	±0.0488
0.655	2	±0.0488

Standard error between Experimental and Theoretical intensity%
2.52%

This experiment deviates more from the theoretical values than the previous one though the peaks are still recognisably close. There is some distortion around the shoulders of the zeroth order maximum. This may be accounted for by an off angle in some component of the optical setup or perhaps the filter used was dirty(i.e the increments in the zeroth order peak where data suddenly goes to zero in close needle like points are dust particles blocking the light).

I.3 Fraunhofer diffraction pattern produced by a 400 m pinhole (without lens L2)

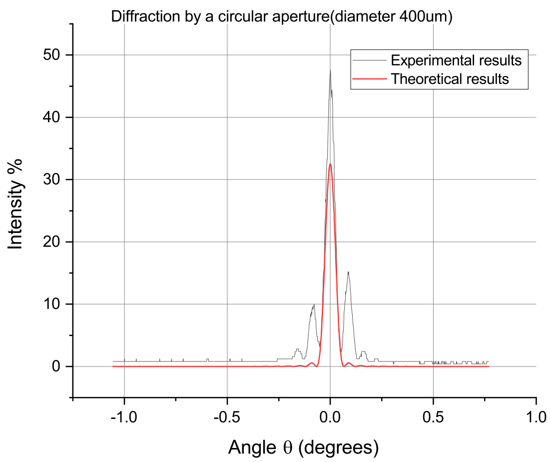

Figure 11: Exp1 part 3: Diffraction by 400um pinhole(no lens)

left shoulder minimum (°)	Right shoulder minimum (°)	error(°)
0.022	0.025	$\pm 1.6 \times 10^{-5}$
Average angle=	$(0.024 \pm 1.6 \times 10^{-5})°$	

Theoretical angle(°)	$\lambda(nm)$	Obtained a(um)	Theoretical a(um)
0.035	650	$290 \pm (4.5 \times 10^{-10})$	200

There is a striking difference between the expected value of a and the value obtained here. It is a difference of nearly half the hole radius. There is also a difference in how the graphs turn out. The experimental data exhibits peaks at roughly the same place as the theoretical, however, those are taller and generally not in line with what is expected, hence the central maximum is raised and the angle decreased returning a different answer. This could be due to a component of the apparatus being at an angle when it should have been straight or it could be that not enough filters to dampen the light going through and hence we have exaggerated peaks.

II. Part 2 - Fresnel diffraction by a pinhole

II.1 Fresnel diffraction by 1.2mm pinhole

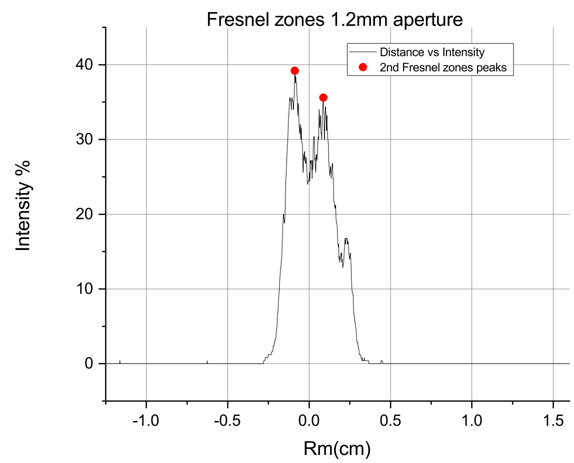

Figure 12: Exp2 part 1: Fresnel zones 1.2mm aperture

left peak(cm)	right peak(cm)	error(cm)
(0.0882)	0.0868	(14×10^{-4})
$Average R_m =$	$0.088 \pm 14 \times 10^{-4}$	

m(order)	$\lambda(nm)$	$r_o(cm)$	Rm(cm)
2	650	60 ± 0.1	$0.088 \pm 14 \times 10^{-4}$

$R_m^2(m^2)$	$mr_o\lambda(m^2)$	difference(m^2)
$(7.7 \times 10^{-7}) \pm (19.6 \times 10^{-11})$	$7.8 \times 10^{-7} \pm (1.3 \times 10^{-9})$	(0.1×10^{-7})

There is a slight difference between the expected and obtained values and one finds that the value of r_o that perfectly matches the equation is 59.23cm or rather 0.77cm different from the measured value. Since these values are reasonably close but still not within experimental error range, this was probably just an incorrect distance reading or perhaps the camera was at a slightly different position than thought.

II.2 Fresnel diffraction by 1.4mm pinhole

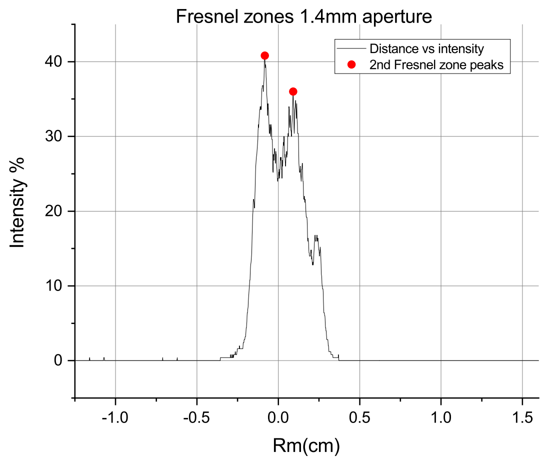

Figure 13: Exp2 part 2: Fresnel zones 1.4mm aperture

left peak(cm)	right peak(cm)	error(cm)
(0.091)	0.0826	(14×10^{-4})
$Average R_m =$	$0.0868 \pm 14 \times 10^{-4}$	

m(order)	$\lambda(nm)$	$r_o(cm)$	Rm(mm)
2	650	60 ± 0.1	$0.0868 \pm 14 \times 10^{-4}$

$R_m^2(m^2)$	$mr_o\lambda(m^2)$	$difference(m^2)$
$(7.5 \times 10^{-7}) \pm (19.6 \times 10^{-11})$	$7.8 \times 10^{-7} \pm (1.3 \times 10^{-9})$	(0.3×10^{-7})

These values obtained are very nearly in line with what is expected for but for a slight difference. If one reverse engineer the equation $R_m^2 = mr_o\lambda$ to find r_o the perfect fit value is 57.69cm, in other words a 2.31cm difference between the measurement value. This difference is for the same reason as for the 1.2mm hole. While the data is qualitatively returning what is desired, a level of accuracy was lost.

II.3 Fresnel diffraction by 1.6mm pinhole

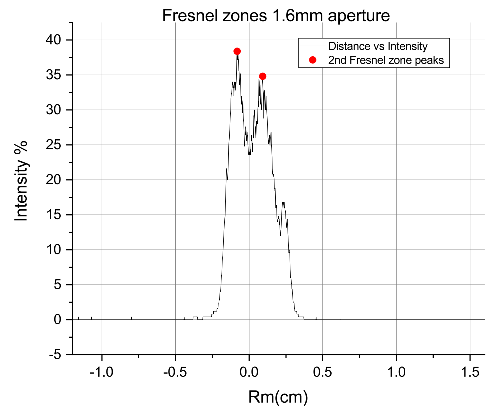

Figure 14: Exp2 part 3: Fresnel zones 1.6mm aperture

left peak(cm)	right peak(cm)	error(cm)
(0.0924)	0.0812	(14×10^{-4})
$Average R_m =$	$0.087 \pm 14 \times 10^{-4}$	

m(order)	$\lambda(nm)$	$r_o(cm)$	Rm(mm)
2	650	60 ± 0.1	$0.087 \pm 14 \times 10^{-4}$

$R_m^2(m^2)$	$mr_o\lambda(m^2)$	$difference(m^2)$
$(7.6 \times 10^{-7}) \pm (19.6 \times 10^{-11})$	$7.8 \times 10^{-7} \pm (1.3 \times 10^{-9})$	(0.2×10^{-7})

These results are reasonably close and demonstrate qualitatively the two fresnel zones though there is some loss of accuracy. This could have been avoided by applying more rigor and doing proper research on what results were to be expected before the lab took place.

III. Part 3 - Multiple slit diffraction

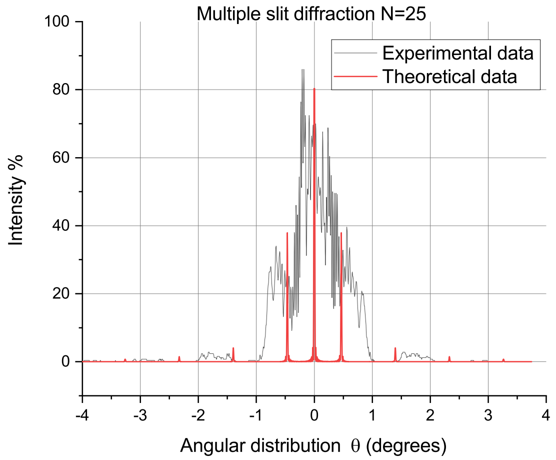

Figure 15: Exp3: Multiple slit diffraction N=25, b=40um ,a=80um,,f=20cm

While this figure has peaks in roughly the same areas as the theoretical value there is some stretching or distortion of the image. This could be due to dust on the filter or possibly due to not using a filter of a high enough threshold value. That is to say, light from the surrounding laboratory could have entered the receiver and caused this broadening. A ratio of roughly 1:2 can also be observed between the left and right peaks in relation to the height of the central peak. as required.

IV. Part 4 - Spatial filtering

IV.1 zero order diffraction only

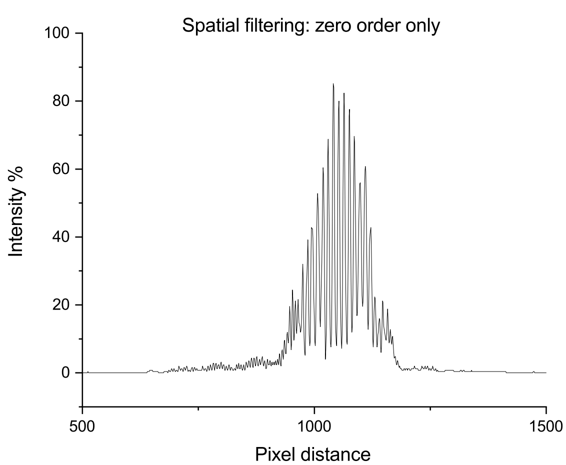

Figure 16: Exp4: spatial filtering zeroth order only

This image was taken by placing the aperture just on the edge of the focal length of the 20cm lens. There is a very clear broad peak in the middle of the distribution corresponding to the zeroth order.

IV.2 zero and first order diffraction only

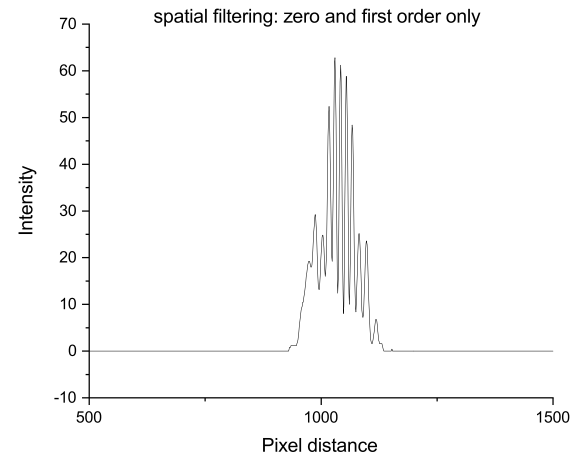

Figure 17: Exp4: spatial filtering zero and first order

The aperture was moved further within the focal length of the 20cm lens to a distance of 17cm to the lens. As can be seen above there is a large peak in the middle with two subsidiary(first order) peaks alongside it. It would seem that moving the aperture within the focal length of the lens within the transform plane returns different combinations of orders.

IV.3 First order diffraction only

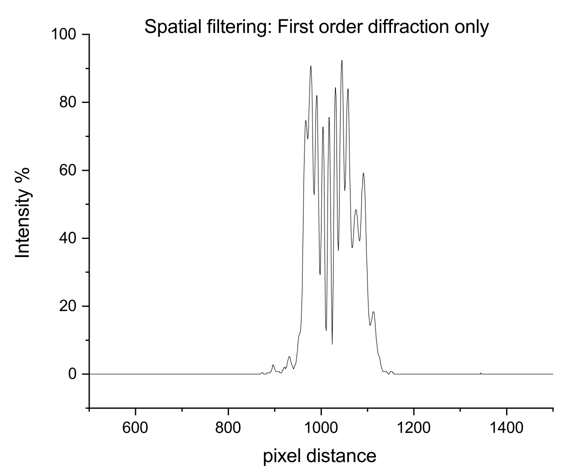

Figure 18: Exp4: First order only

The aperture was then moved to a distance of 14cm from the 20cm focal lens. One observes two distinguishable albeit fuzzy peaks with a recognisable cavity between them. This corresponds to the first order peaks exclusively.

It is clear then that the spatial filters works by moving the aperture within the focal length of the lens used to obtain distinct orders.

IV. Error Analysis

The error analysis for this experiment was quite straightforward with each unit of measurment having an error of ± 14um. This could easily be converted into angular form by taking $Tan^{-1}(x/f)$ for the relevant focal length or distance of each experiment setup.

The error in the measuring stick was $\pm 1mm$ which again was easy to account for. Other sources of error were harder to quantify and could only be observed qualitatively. For example, one can observe that a distribution is skewed somewhat and that therefore, some component of the apparatus is at a slight angle but the exact angle is unknown. One can observe a fuzziness/distortion caused by dust but cannot quantify an absence of data.

V. Conclusion

In conclusion the various types of optical diffraction and the ways in which it could be fine tuned were observed qualitatively but with some loss in the clarity of the data and accuracy of measurement. Experiment 1 was for the most part quite in line with expectations but for uncertainty in the use of the circular aperture. Experiment 2 was a success with minor differences to expectations.Experiment 3 was qualitatively alright but the distortion in the data leads one to question their validity, while experiment 4 was also a success. This lab serves to demonstrate the rigor and precision required in optical experiments.

References

[1] TrinityCollegeDublin/Schoolofphysics/JSlabs[*"Optical diffraction" 2020*]

[2] E Hecht [*Optics 3rd edition "chapter 10 Diffraction seciton 10.2 Frauenhofer diffraction part 10.2.1 The single slit"*] pages 442-443

[3] E Hecht [*Optics 3rd edition "chapter 10 Diffraction seciton 10.2 Frauenhofer diffraction part 10.2.5 The Circular aperture"*] pages 459-461

[4] E Hecht [*Optics 3rd edition "chapter 10 Diffraction seciton 10.2 Frauenhofer diffraction part 10.2.3 Diffraction by Many Slits"*] pages 451-453

[5] E Hecht [*Optics 3rd edition "chapter 10 Diffraction seciton 10.2 Frauenhofer diffraction part 10.2.7 The Diffraction grating"*] pages 451-453

[6] E Hecht [*Optics 3rd edition "chapter 10 Diffraction seciton 10.3 Fresnel diffraction part 10.3.3 Plane waves"*] page 485