**Dynamics and Control of Robotic Systems (4DC00)**

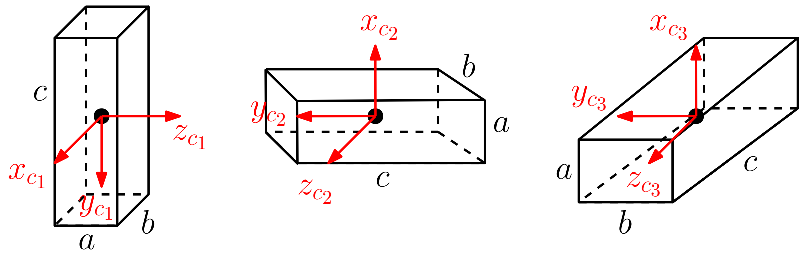
Guided self study - Day 12

# Answers to Problem 1

Take the transfer function $F(s) = 2s^2 + s$ and $H(S) = K_p + K_d s$ with the control gains $K_p = 2\omega^2$ and $K_d = 4\zeta\omega - 1$.

# Answers to Problem 2

Consider a robot manipulator with three prismatic joints. Let the generalized coordinate vector of the robot be $q(t) = [d_1,\, d_2,\, d_3]^\top = [q_1,\, q_2,\, q_3]^\top$. We denote the centers of mass of each joints $c_1$, $c_2$, and $c_3$ which are schematically represented in the figure below. The reference frames for the links can be obtained by deriving the forward kinematics for the robot manipulator. The dimensions of the links are $a = b = 1/4$ and $c = 1$ (m); and masses $m_1 = m_2 = m_3 = 1$ (kg).




This leads to the following moment of inertia matrices for links, respectively

$$J_1 = \frac{m}{12}\begin{bmatrix} a^2+c^2 & 0 & 0 \\ 0 & a^2+b^2 & 0 \\ 0 & 0 & c^2+b^2 \end{bmatrix}; \quad J_2 = \frac{m}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+b^2 & 0 \\ 0 & 0 & c^2+a^2 \end{bmatrix};$$

$$J_3 = \frac{m}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}. \tag{1}$$

From the forward kinematics, we can analogously derive the (linear) velocities components of each link. That is,

$$v_{c_1} = J_{v_{c_1}} \dot{q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{q}; \qquad v_{c_2} = J_{v_{c_2}} \dot{q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{q};$$

$$v_{c_3} = J_{v_{c_3}} \dot{q} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{q}. \tag{2}$$

It shall be clear that we only have linear velocities and no angular velocities. This arises from the fact that the robot only has prismatic joints; and therefore it holds that $J_{\omega_{c_1}} = J_{\omega_{c_2}} = J_{\omega_{c_3}} = 0_{3\times 3}$. Given the linear velocities of the center of masses, we can compute the total kinetic energy of the system as

$$K = \frac{1}{2} \left[ m_1 {v_{c_1}}^\top v_{c_1} + m_2 {v_{c_2}}^\top v_{c_2} + m_3 {v_{c_3}}^\top v_{c_3} \right] \tag{3}$$

$$= \frac{1}{2} \dot{q}^\top \left( \sum_{i=1}^{3} m_i J_{v_{c_1}}^\top J_{v_{c_1}} \right) \dot{q} = \dot{q}^\top \underbrace{\begin{bmatrix} m_1 + m_2 + m_3 & 0 & 0 \\ 0 & m_2 + m_3 & 0 \\ 0 & 0 & m_3 \end{bmatrix}}_{D(q)} \dot{q} \tag{4}$$

where $D(q) = D$ is the inertia matrix. Since the inertia matrix is a constant matrix (meaning change in $q$ do not affect $D$), it follows that the Coriolis matrix must be zero matrix $C(q, \dot{q}) = 0_{3\times 3}$. This also simply follows from the Christoffel symbols

$$c_{ijk} = \left( \frac{\partial\, d_{kj}}{\partial\, q_i} + \frac{\partial\, d_{ki}}{\partial\, q_j} - \frac{\partial\, d_{ij}}{\partial\, q_k} \right), \tag{5}$$

where it follows that $c_{ijk} = 0$ for all $i, j, k \in \{1, 2, 3\}$ since $\partial\, d / \partial\, q_i = 0$. Lastly, the potential energy is given by $P(q) = P_1(q) + P_2(q) + P_3(q) = (m_1 + m_2 + m_3) g q_1$. Hence, the force vector related to the potential energy is described by

$$G(q) = \frac{\partial\, P}{\partial\, q} = \begin{bmatrix} (m_1 + m_2 + m_3) g \\ 0 \\ 0 \end{bmatrix}. \tag{6}$$

Therefore, the equation of motion for the robot manipulator can be compactly written as $D(q)\ddot{q} + G(q) = u(t)$ with the actuator forces $u$ in Newton.

# Answers to Problem 3

Execute the Matlab script `GSS12_EXERCISE3.m`

# Answers to Problem 4

Execute the Matlab script `GSS12_EXERCISE4.m`