Dynamics and Control of Robotic Systems (4DC00)
Guided self study - Day 2

Answers to Problem 1

$$\boldsymbol{H}_1^0 = \begin{bmatrix} \boldsymbol{R}_1^0 & \boldsymbol{d}_1^0 \\ \boldsymbol{0}_3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_2^0 = \begin{bmatrix} \boldsymbol{R}_2^0 & \boldsymbol{d}_2^0 \\ \boldsymbol{0}_3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_2^1 = \begin{bmatrix} \boldsymbol{R}_2^1 & \boldsymbol{d}_2^1 \\ \boldsymbol{0}_3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_1^0\boldsymbol{H}_2^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boldsymbol{H}_2^0.$$

Answers to Problem 2

$$\boldsymbol{H}_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \boldsymbol{H}_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \boldsymbol{H}_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Answers to Problem 3

After the rotation of the coordinate frame $\boldsymbol{o}_3\boldsymbol{x}_3\boldsymbol{y}_3\boldsymbol{z}_3$, we obtain the frame $\boldsymbol{o}_4\boldsymbol{x}_4\boldsymbol{y}_4\boldsymbol{z}_4$

$$\boldsymbol{H}_4^3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_4^0 = \boldsymbol{H}_3^0 \boldsymbol{H}_4^3 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{H}_2^4 = \boldsymbol{H}_3^4 \boldsymbol{H}_2^3 = \mathbf{Rot}_{z,-\frac{\pi}{2}} \boldsymbol{H}_2^3$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Answers to Problem 4

After the rotation and translation of the coordinate frame $\boldsymbol{o}_2\boldsymbol{x}_2\boldsymbol{y}_2\boldsymbol{z}_2$, we can obtain the frame $\boldsymbol{o}_5\boldsymbol{x}_5\boldsymbol{y}_5\boldsymbol{z}_5$, that is,

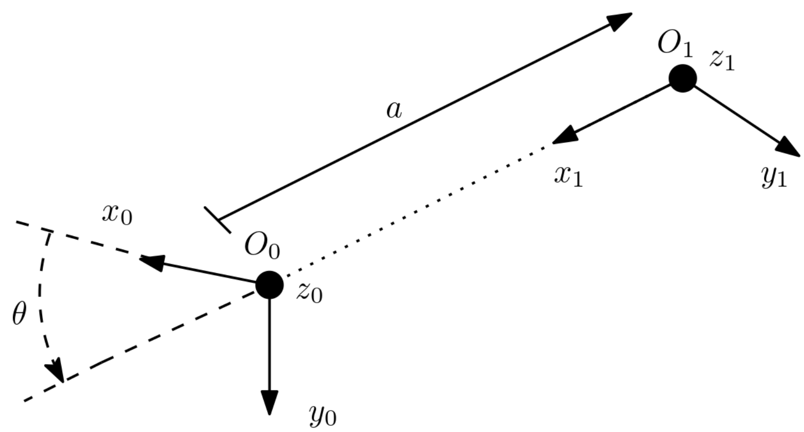
$$\boldsymbol{H}_5^2 = \mathbf{Trans}_{x,0.5}\mathbf{Trans}_{y,0.3}\mathbf{Rot}_{z,\frac{\pi}{2}} = \begin{bmatrix} 0 & -1 & 0 & 0.5 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\begin{aligned} \boldsymbol{H}_5^3 = \boldsymbol{H}_2^3\boldsymbol{H}_5^2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0.5 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & -1 & 0 & 0.5 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} \boldsymbol{H}_5^4 = \boldsymbol{H}_2^4\boldsymbol{H}_5^2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0.5 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 & 0.5 \\ -1 & 0 & 0 & -0.3 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} \boldsymbol{H}_5^0 = \boldsymbol{H}_2^0\boldsymbol{H}_5^2 &= \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0.5 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1.8 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

Answers to Problem 5



Pay attention to the **sign conventions** for the angles and displacements!

$$\boldsymbol{H}_1^0 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & -5/200 \\ \sqrt{3}/2 & 1/2 & 0 & -5\sqrt{3}/200 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Answers to Matlab problem

Answers to the Matlab problem can be found in the file GSS2_MATLAB_PROBLEM1.m