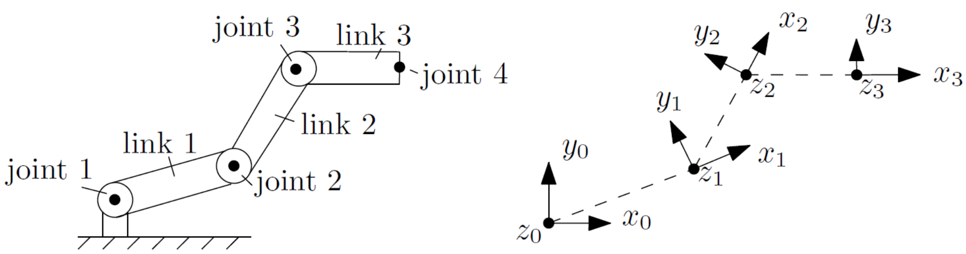
Dynamics and Control of Robotic Systems (4DC00)
Guided self study - Day 3

Answers to Problem 1

Consider the following robot manipulator with three rotary joints.



link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*

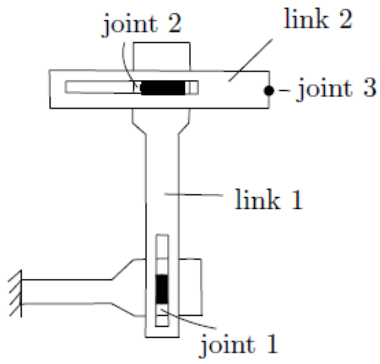
$$\boldsymbol{A}_i^{i-1} = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{for } i \in \{1,2,3\},$$

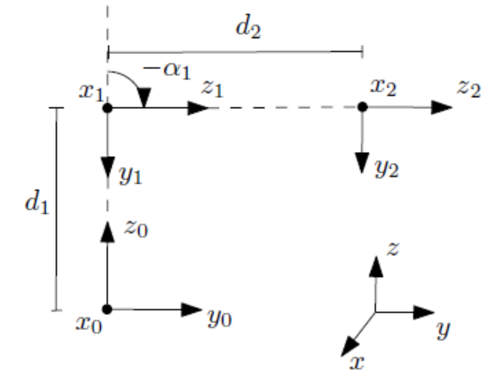
$$\boldsymbol{T}_3^0 = \boldsymbol{A}_1^0 \boldsymbol{A}_2^1 \boldsymbol{A}_3^2 = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ s_{123} & c_{123} & 0 & a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Alternatively, the solution for the forward kinematics can be found by executing the MATLAB script `GSS3_PROBLEM1.m`

Answers to Problem 2

Consider the following robot manipulator with two prismatic joints.





link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	d_1^*	0
2	0	0	d_2^*	0

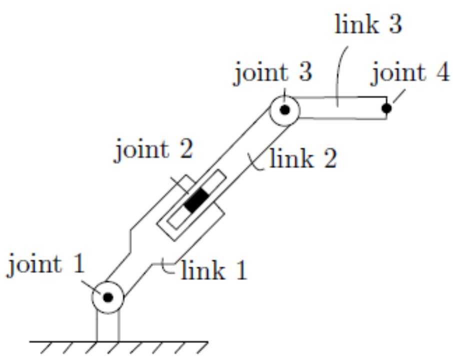
$$\boldsymbol{A}_1^0(d_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \boldsymbol{A}_2^1(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

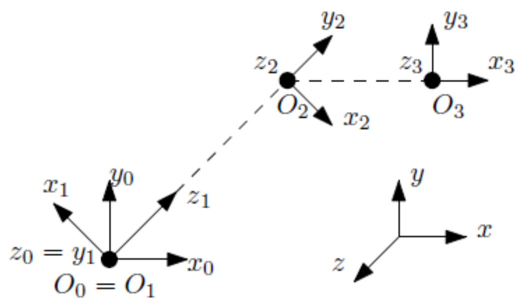
$$\boldsymbol{T}_2^0(d_1, d_2) = \boldsymbol{A}_1^0 \boldsymbol{A}_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Alternatively, the solution for the forward kinematics can be found by executing the MATLAB script `GSS3_PROBLEM2.m`

Answers to Problem 3

Consider the following robot manipulator with one prismatic joints and two rotary joints. Note that the axes $\boldsymbol{z}_0$, $\boldsymbol{y}_1$, $\boldsymbol{z}_2$, and $\boldsymbol{z}_3$ are all normal to the page and they point outwards. From DH convention, we find the following DH table





link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	0	θ_1^*
2	0	$\pi/2$	d_2^*	π
3	a_3	0	0	θ_3^*

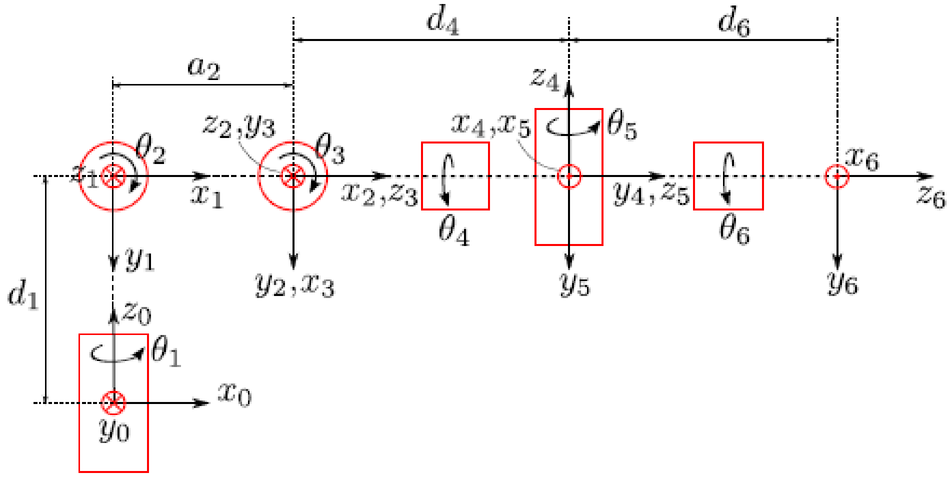
$$\boldsymbol{A}_1^0(\theta_1) = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \boldsymbol{A}_2^1(d_2) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \boldsymbol{A}_3^2(\theta_3) = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\boldsymbol{T}_3^0(\theta_1, d_2, \theta_3) = \boldsymbol{A}_1^0\boldsymbol{A}_2^1\boldsymbol{A}_3^2 = \begin{bmatrix} -c_{13} & s_{13} & 0 & d_2s_1 - a_3c_{13} \\ -s_{13} & -c_{13} & 0 & -d_2c_1 - a_3s_{13} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Alternatively, the solution for the forward kinematics can be found by executing the MATLAB script `GSS3_PROBLEM3.m`

Answers to Problem 4

Consider the following robot manipulator with six rotary joints.

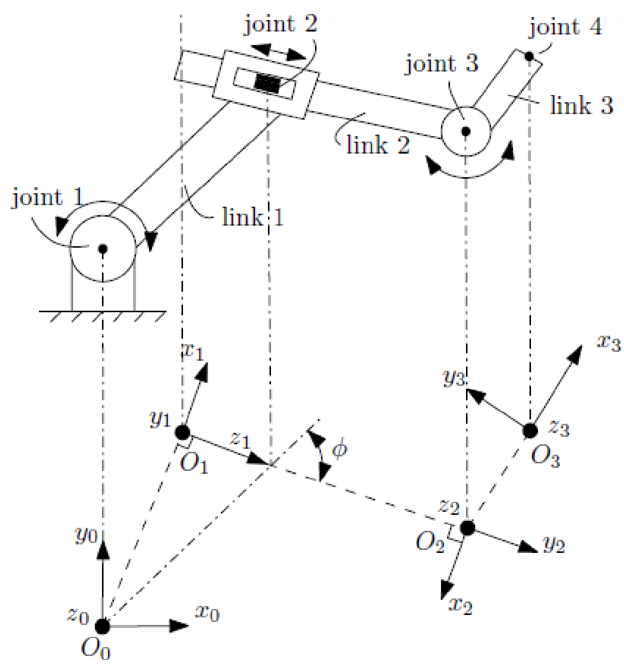


link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	0	$\pi/2$	0	θ_3^*
4	0	$\pi/2$	d_4	θ_4^*
5	0	$-\pi/2$	0	θ_5^*
6	0	0	d_6	θ_6^*

Although the solutions can be derived by hand, due to the complexity of the manipulator, it is recommend to use the Symbolic Toolbox in `Matlab`. The solution for the forward kinematics can be found by executing the `Matlab` script `GSS3_PROBLEM4.m`

Answers to Problem 5

Consider the following robot manipulator with two prismatic joints and one prismatic joint.



link	a_i	α_i	d_i	θ_i
1	$l_1 \sin(\phi)$	$\pi/2$	0	θ_1^*
2	0	$\pi/2$	d_2^*	π
3	l_3	0	0	θ_3^*

For the parameters $\phi = \pi/6$ rad, $l_1 = 0.3$ m, $l_3 = 0.2$ m, we find the following transformation matrix

$$\boldsymbol{T}_3^0(\theta_1, d_2, \theta_3) = \boldsymbol{A}_1^0 \boldsymbol{A}_2^1 \boldsymbol{A}_3^2 = \begin{bmatrix} -c_{13} & s_{13} & 0 & -\frac{1}{5}c_{13} + d_2 s_1 + \frac{3}{20}c_1 \\ -s_{13} & -c_{13} & 0 & -\frac{1}{5}s_{13} - d_2 c_1 + \frac{3}{20}s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$