Cardano and the Cubic Equation

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Special Thanks

Thanks to Dr. Markus Hunziker for the book recommendation, and to William Dunham, whose chapter *Cardano and the Solution of the Cubic* in the book *Journey through Genius* provided the primary source for this talk.

Overview

Solving for x

Linear Equation

$$mx + b = 0 \qquad (1)$$

i.e., solve equations that look like this (easy):

$$3x + 2 = 0, \qquad x - 5 = 0, \qquad 42x - 123456789 = 0$$

Quadratic Equation

$$ax^2 + bx + c = 0 \qquad (2)$$

i.e., solve equations that look like this (only a little harder):

$$2x^2 + 3x + 2 = 0, \qquad 4x^2 + x - 5 = 0, \qquad -123x^2 + 42x - 123456789 = 0$$

Solving for x

Linear Equation

$$mx + b = 0 \tag{1}$$

i.e., solve equations that look like this (easy):

$$3x + 2 = 0, \qquad x - 5 = 0, \qquad 42x - 123456789 = 0$$

Quadratic Equation

$$ax^2 + bx + c = 0 \tag{2}$$

i.e., solve equations that look like this (only a little harder):

$$2x^2 + 3x + 2 = 0, \qquad 4x^2 + x - 5 = 0, \qquad -123x^2 + 42x - 123456789 = 0$$

Solving for x

What's the natural next equation to solve? *The cubic:*

Cubic Equation

$$ax^3 + bx^2 + cx + d = 0 \tag{3}$$

i.e., solve equations that look like this (hard):

$$4x^3 - 2x^3 + 3x + 2 = 0,$$
$$100x^3 + x - 5 = 0,$$
$$42x^3 - 21x^2 + 42x - 123456789 = 0$$

Summa de Arithmetica

Luca Pacioli (1494)

In a contemporary mathematics textbook, Pacioli assesses solving the cubic equation as impossible as "squaring the circle", a problem posed by the ancient Greeks (and proven impossible in the 1800s).

Source: from *Wikipedia*, painting often attributed to Jacopo de'Barbari, 1495

- This all changed, when Scipione del Ferro solves the "repressed cubic:"

$$x^3 + mx = n$$

- But, he kept the solution secret.
- On his deathbed in 1526, del Ferro tells his student, Antonio Fior
- A hothead, Fior challenges a noted scholar–Niccolo Fontana

Tartaglia

Tartaglia (1499-1557)

- Niccolo Fontana, better known as "Tartaglia" (meaning "the Stammerer", due to a war wound).
- Fontana had boasted he could solve cubics of another form.

NICOLAVS TARTAGLIA,
BRIXIANVS.

Diuitias patriæ cumulat Tartaglia linguæ,
Euclidem Etrusco dum docet ore loqui.

Tartaglia

The Challenge

- Fior gives Tartaglia 30 "repressed cubics", Tartaglia gave Fior 30 problems of various topics.
- Working furiously, Tartaglia discovers the solution the night before the problems are due, winning 30 to 0.
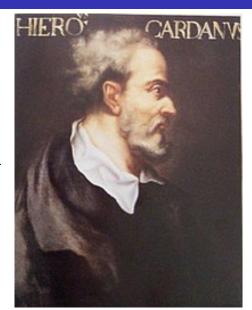- Fior fades from history.

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Cardano

Now enters the strangest character of the story–Gerolamo Cardano (1501-1576). His story is a half-hour talk, so we proceed with a negligent survey.
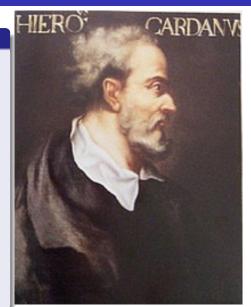

Source: from *Wikipedia*

Cardano

Cardano–Ups and Downs

- A failed and successful doctor with severe psychological issues (he loved the feeling of pain *ending*)

- An avid gambler and one of the first statisticians

- Father of a murderer

- Stellar mathematician, ardent astrologer

- Author of a (dubious) autobiography

- Arrested for heresy, held a pension from the Pope

Source: from *Wikipedia*

Ferrari

- Enter the final player in our story–Ludovico Ferrari.

- *Ars Magna* ("Great Art")

- To include the solution of the cubic, begged Tartaglia.

Ferrari (1522-1565)

- Asked for work from Cardano–on the very day Cardano had received a good omen (the incessant squawking of a magpie)

- Quickly turned to his colleague, by the age of 20 Ferrari was a near equal

Tartaglia's Conditions

- After much begging, Tartaglia gives Cardano the solution–in the form of a poem
  - Ferrari uses the ideas to solve the "next" equation–the quartic!

- The equation must be kept encoded

- This won't do for Cardano–he wants to put it in a book!

- Suddenly, Cardano and Ferrari have an idea.

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Sneaking Around

- Remember del Ferro?

    - Cardano and Ferrari do

    - Obtaining the solution (identical to Tartaglia's) from del Ferro's notes, Cardano and Ferrari publish.

    - del Ferro is credited.

Backlash from Tartaglia

- Accusations and insults fly back and forth between Tartaglia and Ferrari (rather a hot-head)

- A contest ensues on Ferrari's home turf
  - Cardano does not even show

- Tartaglia loses–blaming the "rowdiness and partisanship of the crowd"
  - Given Ferrari's reputation, some historians believe he was lucky to escape with his life.

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The End