**ARIMA method**

ARIMA stands for Auto-Regressive Integrated Moving Average while ARMA is integrated with differencing. A nonseasonal ARIMA model is classified as ARIMA(p,d,q) where, p is the order of AR (Auto-Regressive) terms, d is the number of nonseasonal differences needed for stationarity (a series without trend and constant variability i.e., constant mean and variance), and q is the order of MA terms.

In an AR model of a time series the value of X at time t is a linear function of the values of X at time $t-1, t-2 ... t-p$ and is written as:

$$AR(p) : X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} +, ..., +\phi_p X_{t-p} + \epsilon_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \epsilon_t$$

where, $\epsilon_t$ is assumed to be iid (identical and independent of $X_t$ distributed) and is normally distributed i.e., $\epsilon_t$ $N(0, \sigma^2)$.

The model is also written using the backshift operator B ($B^k X_t = X_{t-k}$) as follows:

$$AR(p) : X_t = \sum_{i=1}^{p} \phi_i B^i X_t$$

An alternative notation using $\phi(B)$ as a polynomial of B is:

$$\phi(B) X_t = \epsilon_t$$

where, $\phi(B) = 1 - \phi_1 B - \phi_2 B^2, ..., -\phi_p B^p = 1 - \sum_{i=1}^{p} \phi_i B^i$.

By a MA(q) model of a time series the value of X at time t depends linearly on its own current and previous stochastic terms as follows:

$$MA(q) : X_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} +, ..., +\theta_q \epsilon_{t-q} = \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

Using the backshift operator B, the MA(q) model is simplified as:

$$MA(q) : X_t = (1 + \theta_1 B + \theta_2 B^2 +, ..., +\theta_q B^q) \epsilon_t = (1 + \sum_{i=1}^{q} \theta_i B^i) \epsilon_t = \theta(B) \epsilon_t$$

Thus, an ARMA(p,q) model consists of both autoregressive (AR) part and moving average (MA) part:

$$ARMA(p, q) : X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

or using the backshift operator B, we obtain the equation:

$$ARMA(p, q) : \phi(B) X_t = \theta(B) \epsilon_t$$

In ARIMA models the original series is differenced to get a stationary series i.e., $\nabla X_t = X_t - X_{t-1} = (1 - B) X_t$ where B is the backshift operator. Stationary series requires that all roots of $\phi(z) = 0$ are outside the unit circle thus, the final stationary model of ARIMA(p,d,q) is:

$$ARIMA(p,d,q) : \phi(B)(1-B)^d X_t = \theta(B)\epsilon_t$$

Finally, the ARIMA model can be generalized as follow:

$$ARIMA(p,d,q) : \phi(B)\alpha(B)X_t = \theta(B)\epsilon_t$$

where, $\phi(B)$ is an autoregressive polynomial with all roots are outside unit circle, $\alpha(B)$ is the differencing filter renders the data stationary in which all roots are on the unit circle, and $\theta(B)$ is a moving average polynomial with all roots are outside unit circle (to assure that $\theta(B)$ is invertible). Alternatively, $X_t$ can be expressed as:

$$X_t = \frac{\theta(B)}{\phi(B)\alpha(B)}\epsilon_t$$

Let us define an ARIMA model for the series $X_t^*$ subject to $m$ outliers defined as $L_j(B)$ with weights $w$:

$$X_t^* = \sum_{j=1}^{m} w_j L_j(B) I_t(t_j) + \frac{\theta(B)}{\phi(B)\alpha(B)}\epsilon_t$$

where $I_t(t_j)$ is an indicator variable containing the value 1 at observation $t_j$ where the j-th outlier arises and 0 otherwise, $w_j$ denotes the magnitude of the $j^{th}$ outlier effect, and $L_j(B)$ depends on the type of the $j^{th}$ outlier. Three types of outliers are considered in MODS:
1. Additive Outlier (AO): $L(B) = 1$. This outlier is a one time jump that gets back to the original values after one period.
2. Level Shift (LS): $L(B) = \frac{1}{1-B}$. This outlier is a jump to new levels.
3. Temporary Changes (TC): $L(B) = \frac{1}{1-\delta B}$. This outlier decays back to the original values after the jump in a pace of $\delta$.

Comments:
1. If level shifts are found at consecutive time points, only then point with higher t-statistic in absolute value is kept.
2. If more than one type of outlier exceed a predetermined threshold 'cval' at a given time point, the type of outlier with higher t-statistic in absolute value is kept and the others are removed. Screen example is depicted below:

**References**

Chen, C., and Liu, L. M. (1993). Joint estimation of model parameters and outlier effects in time series. *Journal of the American Statistical Association*, 88(421), 284-297.

# MODS - Multilier Outliers Detecting System

**Choose Market/Series:**

**Foreign Market**
- Representative ER
- High-Low ILS/USD gap
- Nominal effective ER
- Euro USD ER
- ILS Euro ER
- MDC USD ER
- EMR USD ER

**Inflation Expectation**
- 1Y InfExp
- 2Y InfExp
- 10Y InfExp

**Stocks**
- TA-125 stock index
- TA-35 stock index
- TA-VIX
- TA-125 volumes
- Dow stock index
- NASDAQ100 stock index
- SP&500 stock index
- FTSE100 stock index
- DAX stock index
- NIKKEY stock index

**Bond Yields**
- 1Y Makam
- 3M Makam
- 1Y real yield
- 10Y IL GovNom yield

**Frequency of data:**

(•) Daily ( ) Weekly ( ) Monthly

**Range of DAILY observations: 07/05/2017 - 08/07/2018**

1 5,385 — 5,750

**Range of WEEKLY observations:**

1 915 967

**Range of MONTHLY observations:**

1 212 224