

## Help on Univariate (cubic) spline method

The cubic spline method utilizes third-order equations between each pair of points that follow two conditions: Condition (1): Go through the non-NA points. Condition (2): Match first, second, and third derivatives at the interior points.

Each equation goes through two consecutive data points so that the  $i^{th}$  spline function ( $x_{i+1} > x > x_{i-1}$ ) can be written as:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

Then, for any interval there are four equations (one for the original points - condition (1) and three for the derivatives - condition (2)). For  $n$  data points  $(t_1, \dots, t_n)$ , there are  $n-1$  intervals and thus  $4(n-1)$  unknowns ( $a_i, b_i, c_i, d_i$ ) to evaluate in order to solve all the spline function coefficients. Additionally, there is no 'one equation' that can represent the whole spline function on the domain.

While data of a particular size presents many options for the order of spline functions, cubic splines are preferred because they provide the simplest representation that exhibits the desired appearance of smoothness.

Screen example is depicted below. In all imputation figures, the non-NA observations are the colored lines while the imputed values ( $Y_{it}^*$ ) are the colored dots.

