Konig's Thm proof. $G = (X \sqcup Y, E)$

M max matching

$e = \{x, y\} \in M$ if $\exists$ alt path from

unsat. $u \in X$ containing e

put $y \in Q$. Else $x \in Q$.

<u>Claim</u> Q is vertex cover

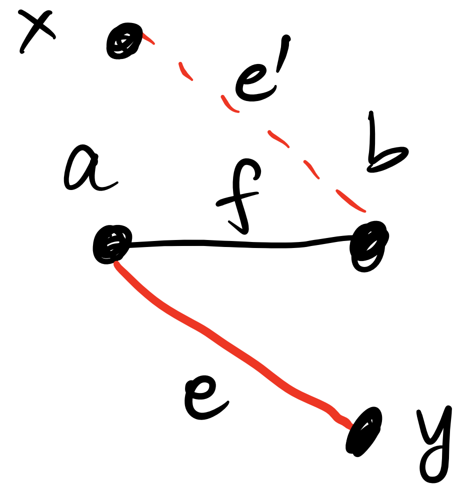
Fix edge $f = \{a, b\}$ $a \in X$ $b \in Y$.

<u>WTS</u> either $a \in Q$ or $b \in Q$.

If $f \in M$, done. Assume $f \notin M$.

Either a or b saturated (M max).

Case 1 a saturated



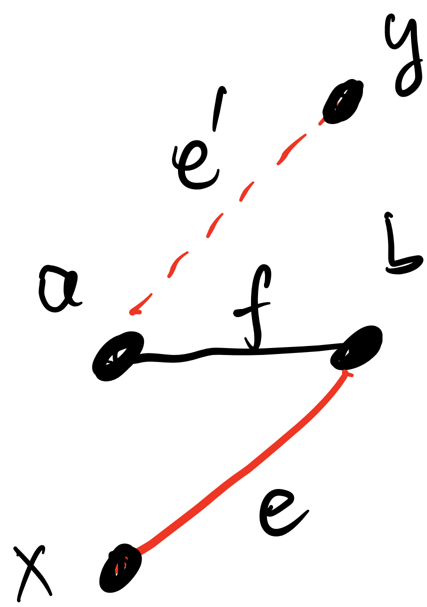
- if $\exists$ alt path P from unsaturated $u \in X$ containing e, then $y \in Q$

  Observe then that b is saturated

  (either P contains f or we can add f to P. M has no augmenting paths)

  Conclude e' part of alt path from unsaturated $\Rightarrow b \in Q$.

- if $\nexists$ path from unsat. u that contains e. then $a \in Q$ by definition of Q.



<u>Case 2</u> b saturated

- if $\exists$ alt path from unsaturated $u \in X$ to e, then $b \in Q$ by definition of Q.
- if $\nexists$ alt path from unsat $u \in X$ to e then a is saturated

(otherwise f & e make alt path...)

Now if $e' = \{a, y\}$ part of alt path... then so is e. contrary to our assumption. Then e' not part of alt path... $\Rightarrow$ $a \in Q$

$\square$