Domains of simple-stable representations

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April 26, 2025

- Γ hyperbolic group

  $\Gamma = F_n$ free group

  $\Gamma = \pi_1(S)$ closed surface group.

- G semisimple Lie group of non-compact type

  $G = PSL(2,\mathbb{R})$, $PSL(2,\mathbb{C})$,

  $SO^+(n,1) \simeq \mathrm{Isom}^+(\mathbb{H}^n)$

  $G = SL(d,\mathbb{C})$

- $\mathcal{X}(\Gamma, G) = \mathrm{Hom}(\Gamma, G) /\!/ G$ the character variety

- $\mathrm{Out}(\Gamma) = \mathrm{Aut}(\Gamma)/\mathrm{Inn}(\Gamma) \circlearrowright \mathcal{X}(\Gamma, G)$ by pre-composition

S_g closed oriented surface of genus $g \geq 2$

Theorem (Goldman 80)

- $\mathcal{X}(\pi_1(S_g), PSL(2,\mathbb{R}))$ has $4g-3$ connected components, characterized by the Euler number $e \in [2-2g, 2g-2] \cap \mathbb{Z}$

- $e = 2-2g, 2g-2$ corresponding to two copies of $\mathrm{Teich}(S_g)$ with $\mathrm{Out}(\pi_1(S_g)) = MCG(S_g)$-action properly discontinuous.

Goldman's Conjecture

- The $MCG(S_g)$-action on the other components is ergodic.

<u>Theorem</u> (Souto–Storm 06, Lee 15)

- The domain of quasi-Fuchsian (convex-cocompact) representations is a maximal connected domain of discontinuous in $\mathcal{X}(\pi_1(S_g), PSL(2,\mathbb{C}))$.

~~Convex cocompactness~~ Simple - stability

$\rho : \pi_1(S_g) \to \mathrm{Isom}(\mathbb{H}^n) \rightsquigarrow \tau_{\rho,o} : \pi_1(S_g) \to \mathbb{H}^n$

$\gamma \mapsto \rho(\gamma)\cdot o$

orbit map

ρ is ~~convex cocompact~~ simple - stable if

$\tau_{\rho,o}$ is a quasi-isometric embedding along simple closed geodesics of S_g uniformly.

<u>Theorem</u> (Minsky 13, Lee 15, Tholozan-W. 23)

- The set of simple-stable representations is a domain of discontinuous in $\mathcal{X}(\pi_1(S_g), \mathrm{Isom}^+(\mathbb{H}^n))$.

$$\mathcal{EE} \subseteq \mathcal{SS}$$

$SO^+(2,1) \simeq PSL(2,\mathbb{R})$ Goldman's conjecture $\Rightarrow$ "="

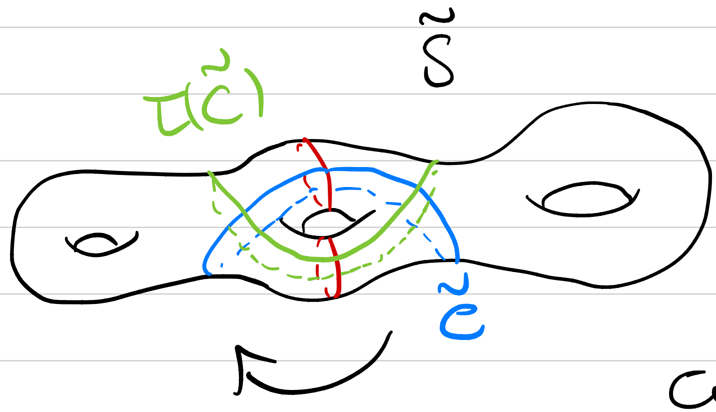
$SO^+(3,1) \simeq PSL(2,\mathbb{C})$ Connected component "=" holds

$SO^+(n,1)$ $n \geq 4$ $\subsetneq$

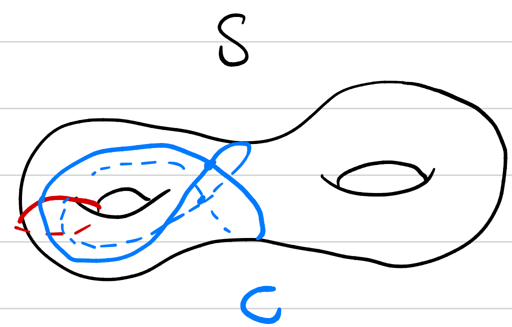
Theorem (Minsky-W. 25)

- There exists a simple-stable representation $\rho : \pi_1(S_g) \to SO^+(4,1)$ on the boundary of domain of convex-cocompact representations.

Construction



$\longrightarrow$



covering of degree 2

$\tau \in \mathrm{Gal}(\tilde{S}/S)$

Find $\tilde{\rho} : \pi_1(\tilde{S}) \to PSL(2,\mathbb{C})$ geometrically finite with parabolicity only at $\tilde{c}$ and $\tau(\tilde{c})$ that can extend to a representation of $\pi_1(S)$.

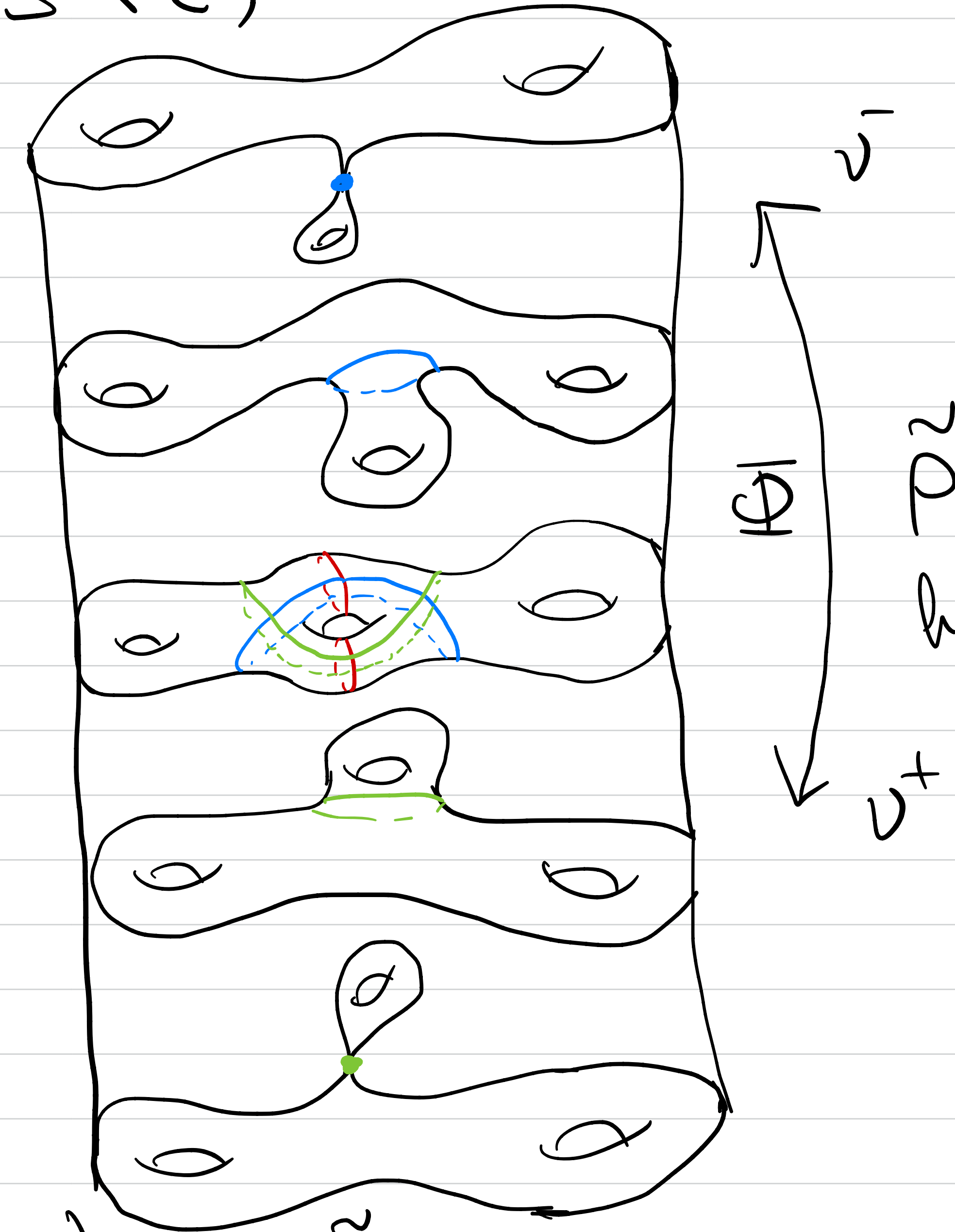
Generalized Simultaneous Uniformization Theorem

(Ahlfors 60, Bers 06, Kra 72, Marden 74, Maskit 70, Mostow 68, Prasad 73, Thurston 82)

$$\left\{ \begin{array}{c} \text{geometrically finite} \\ \rho : \pi_1(S) \to PSL(2,\mathbb{C}) \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \overbrace{P^+, P^-}^{\text{multicurves}}, \\ \nu^+ \in \mathrm{Teich}(S \setminus P^+) \\ \nu^- \in \mathrm{Teich}(S \setminus P^-) \end{array} \right\}$$

$\nu^+ \in \mathrm{Teich}(\tilde{S} \setminus \tilde{c})$

$\tilde{\rho}$



$\tilde{\rho} \rightsquigarrow \rho$ by Φ

$\nu^- = \tau(\nu^+) \in \mathrm{Teich}(\tilde{S} \setminus \tau(\tilde{c}))$

$$
\begin{tikzcd}
\tilde{\rho} : \pi_1(\tilde{S}) \arrow[r] & PSL(2,\mathbb{C}) \simeq SO^+(3,1) \arrow[d, squiggly] & + \quad \Phi \\
\rho : \pi_1(S) \arrow[r] & O^+(3,1) \subset SO^+(4,1) &
\end{tikzcd}
$$

Graph Products of Morse Local-to-Global Groups

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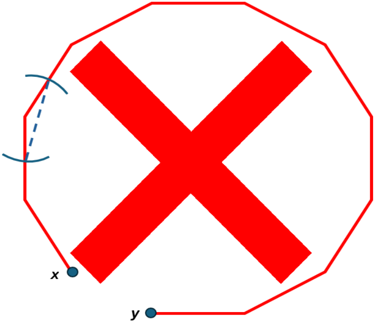
April 26, 2025

Motivation: Generalizing Hyperbolic Groups

In a hyperbolic space, all quasi-geodesics are

Morse



"local-to-global"

Morse Local-to-Global Groups and Spaces

Defined by Russell, Spriano, Tran (2022)

- Non-hyperbolic spaces can have Morse quasi-geodeiscs
- If all the Morse quasi-geodesics have the "local-to-global" property, the space is Morse local-to-global

$$\begin{array}{c}\text{locally Morse}\\+\\\text{locally quasi-geodesic}\end{array}\quad\Longrightarrow\quad\begin{array}{c}\text{globally Morse}\\+\\\text{globally quasi-geodesic}\end{array}$$

Examples of Morse Local-to-Global Groups

From Russell, Spriano, Tran (2022):

- Direct Products of Infinite Groups
- Hierarchically Hyperbolic Groups
- CAT(0) Groups
- Hyperbolic Groups Relative to MLTG Groups

Non-Examples?

Theorem (Russell, Spriano, Tran 2022)

There exists groups which are not Morse local-to-global.

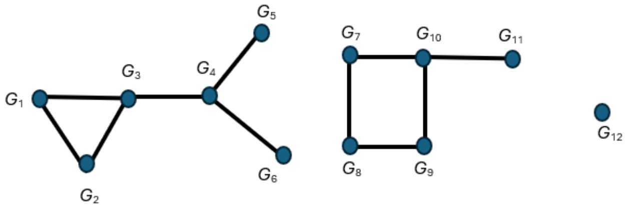
All known examples are infinitely presented!

Graph Products

Definition (Graph Products)

Let Γ be a finite simplicial graph with vertex set $V(\Gamma)$ and edge set $E(\Gamma)$. Assign a group G_v to each $v \in V(\Gamma)$. The <u>graph product G_Γ</u> is defined to be

$$G_\Gamma = \left(\mathop{\ast}_{v \in V(\Gamma)} G_v \right) \Big/ \langle\langle [g_v, g_w] \mid g_v \in G_v,\ g_w \in G_w,\ \{v, w\} \in E(\Gamma) \rangle\rangle .$$

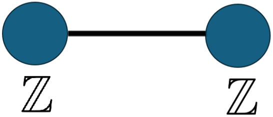


Examples of Graph Products

Right-angled Artin groups are graph products (all vertex groups being $\mathbb{Z}$)

Examples:

$$\Gamma = F_2 = \langle a, b \rangle$$



$$\Gamma = \mathbb{Z}^2 = \langle a, b \mid ab = ba \rangle$$

Technique

Tools from hierarchically hyperbolic groups

$+$

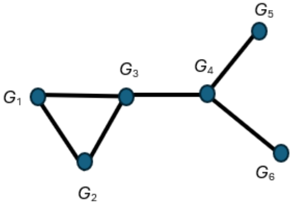
Theorem (Berlyne, Russell 2022)

Graph products are relatively hierarchically hyperbolic groups.

Main Result

Theorem (P, in progress)

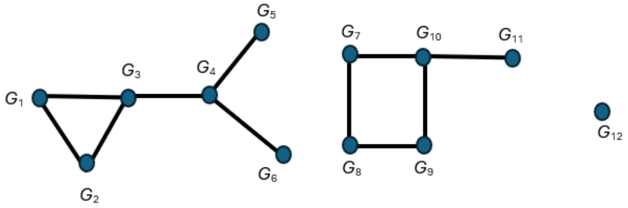
Multi-vertex connected graph products of infinite groups are Morse local-to-global.



Main Result (Continued)

Corollary (P, in progress)

Graph Products of infinite Morse local-to-global groups are Morse local-to-global.



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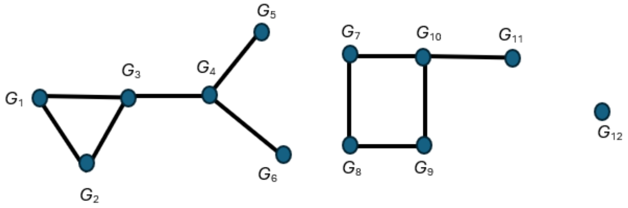

Thank You

Singularity of Stationary measures and Patterson–Sullivan measures for Mapping Class Groups

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GATSBY 2025 Spring

Let

S : closed conn. ori. surface of genus ≥ 2

$\mathrm{Mod}(S)$: Mapping class group of S $(= \mathrm{Homeo}^+(S)/\sim)$

Let

$$\mu : \text{ probability on } \mathrm{Mod}(S) \text{ with finite support}$$

For each $n \in \mathbb{N}$,

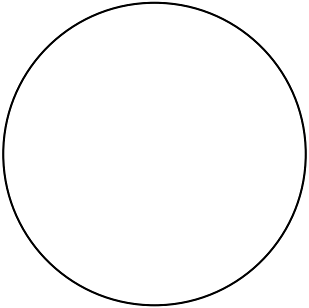
$$w_n := g_1 \cdots g_n, \qquad g_i \sim \mu \quad \text{i.i.d.}$$

$$\rightsquigarrow (w_n)_{n \in \mathbb{N}} : \text{ "Random walk on } \mathrm{Mod}(S)\text{"}$$

$\mathcal{T}(S)$: Teichmüller space of S (equipped with the Teichmüller distance)

For a fixed $o \in \mathcal{T}(S)$,

$$\{w_n o \in \mathcal{T}(S) : n \in \mathbb{N}\} \quad \rightsquigarrow \quad \text{random walk in } \mathcal{T}(S)$$



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Theorem (Kaimanovich–Masur, 1996)

Almost every $(w_n o)_{n \in \mathbb{N}}$ converges to $\mathcal{PMF}$ (= Thurston's boundary of $\mathcal{T}(S)$).

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Theorem (Kaimanovich–Masur, 1996)

Almost every $(w_n o)_{n \in \mathbb{N}}$ converges to $\mathcal{PMF}$ (= Thurston's boundary of $\mathcal{T}(S)$).

Moreover, the hitting measure

$$\nu_{Hit}(E) = \mathrm{Prob}\left((w_n)_{n \in \mathbb{N}} : \lim_{n \to \infty} w_n o \in E\right) \quad \textit{for} \quad E \subset \mathcal{PMF}$$

is the unique μ-stationary measure on $\mathcal{PMF}$.

Another natural measure on $\mathcal{PMF}$:

Patterson–Sullivan (or, conformal) measure of $\langle \operatorname{supp} \mu \rangle < \mathrm{Mod}(S)$

(e.g. Athreya–Bufetov–Eskin–Mirzakhani (2012): $\exists$ PS-meas. in Leb. class)

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Theorem (Gadre, 2014)

The μ-stationary meas. ν_{Hit} is singular to Leb. on $\mathcal{PMF}$.

Proof: construction of a singular subset in $\mathcal{PMF}$

Moment condition was relaxed by Gadre–Maher–Tiozzo (2017)

For general $\langle \operatorname{supp} \mu \rangle < \operatorname{Mod}(S)$,

$$\exists \text{ PS-measure } \notin \text{ Leb. class.}$$

(e.g. Masur (1986), Kerckhoff (1990): Leb(limit set of Handlebody group) = 0)

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Theorem (K.–Zimmer, 2025)

If $\langle \operatorname{supp} \mu \rangle$ is relatively hyperbolic and contains a multitwist, then

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Example

1. When $\operatorname{genus}(S) \geq 4$, $\exists$ finite index $\Gamma < \pi_1(\mathbb{S}^3 \setminus \text{figure-8})$ such that

$$\Gamma \hookrightarrow \operatorname{Mod}(S)$$

   with multitwists (Kent–Leininger, 2024)

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2. Combination theorem for Veech subgroups (Leininger–Reid, 2006)
3. More generally, “parabolically geometrically finite” subgroups (Dowdall–Durham–Leininger–Sisto, 2024) + (Udall, 2025)

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Proof Idea.

Establish

Tukia's generalization of Mostow's rigidity

for a wide class of group actions.

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for a wide class of group actions.

Crucial facts:

1. Contracting property of the axis of a p-A in $\mathcal{T}(S)$ (Minsky, 1996)
2. Linear growth in $\mathrm{Cayley}(\mathrm{Mod}(S))$ (Farb–Lubotzky–Minsky, 2001)

$\square$

Metric Structures on Groups

Robbie Lyman
Rutgers Univ-Newark

(GATSBY 2025)

We like group actions on metric spaces!

Problem: Want to study "all" $G \curvearrowright (X,d)$ (continuously) by isometries for a fixed G.

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Problem: Want to study "all" $G \curvearrowright (X,d)$ (continuously) by isometries for a fixed G.

$\forall x \in X$, define $d_{X,x} : G \times G \longrightarrow \mathbb{R}$ by

$$d_{X,x}(g,h) = d_X(g \cdot x, h \cdot x)$$

this is a continuous, left-invariant pseudo-metric on G!

Solution: Consider

$$\mathrm{PMet}(G) = \{ d : G \times G \to \mathbb{R} : d \text{ cts, left-inv. pseudometric} \}$$

Observe that $\mathrm{PMet}(G)$ is preordered: say

$$d \gtrsim d' \quad \text{when} \quad \mathrm{id} : (G, d) \longrightarrow (G, d')$$

is coarsely Lipschitz.

NB: $d + d' \in \mathrm{PMet}(G)$, so if $\mathrm{PMet}(G)$ has a maximal (pseudo)-metric, it has a maximum, which is an invariant of G.

Notice that if $A \subset G$ is d-bounded and $d \geq d'$, then A is d'-bounded.

Def (Rosendal) $A \subset G$ is **coarsely bounded** if it is d-bounded $\forall\, d \in PMet(G)$.

Prop (Rosendal, Milnor–Schwarz) Suppose $G \curvearrowright X$ geodesic is cobounded and $\forall\, x \in X$, $\varepsilon > 0$, the set

$$\{ g \in G : g.B_\varepsilon(x) \cap B_\varepsilon(x) \neq \emptyset \}$$

is coarsely bounded in G. Then $d_{X,x}$ is maximal in $PMet(G)$.

Prop (Rosendal, Serre) If $G_0 \lneq G_1 \lneq \cdots$ is a proper ascending chain of open subgroups s.t. $G = \bigcup_{i=0}^{\infty} G_i$.
Then $PMet(G)$ has no maximal metric.

Pf (Sketch) $G \curvearrowright T$ w/ quotient

but each d_{T,v_0}-bounded subset of G lives in some G_n, so no coarsely bdd set generates G!

Finite covers and A-polynomials

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joint work with Tam Cheetham-West

Apr 26th 2025

Profinite Completion

- Let M be a finite-volume hyperbolic 3-manifold.
- Consider the profinite completion

$$\widehat{\pi_1(M)} = \varprojlim \pi_1(M)/N$$

with $[\pi_1(M) : N] < \infty$.

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Question

What properties of M are determined by $\widehat{\pi_1(M)}$?

A-polynomials

- Let M be a compact 3-manifold with a single torus boundary.
- For $\rho \in R(M) = \mathrm{Hom}(\pi_1(M), SL(2,\mathbb{C}))$, it can be conjugated so that

$$\rho(\mathcal{M}) = \begin{pmatrix} m & * \\ 0 & m^{-1} \end{pmatrix}, \ \ \rho(\mathcal{L}) = \begin{pmatrix} l & * \\ 0 & l^{-1} \end{pmatrix}$$

where $\langle \mathcal{M}, \mathcal{L} \rangle$ is a basis of $\pi_1(\partial M)$.

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  where $\langle \mathcal{M}, \mathcal{L} \rangle$ is a basis of $\pi_1(\partial M)$.
- Projection map

$$\begin{aligned} p : R(M) &\to \mathbb{C}^2 \\ \rho &\mapsto (l, m) \end{aligned}$$

- The *A-polynomial* $A^M(l, m)$ is a two-variable polynomial generating the Zariski closure of $\mathrm{im}\, p$ excluding 0-dim components.

Main Result

Theorem (Cheetham-West-Y)

Let M, N be one-cusped finite-volume hyperbolic 3-manifolds. Suppose there is a regular profinite isomorphism

$$\Phi : \widehat{\pi_1(M)} \cong \widehat{\pi_1(N)}$$

Then:

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Then:

- Φ *induces an isomorphism between* $\pi_1(\partial M) \to \pi_1(\partial N)$ *mapping the basis* $\langle L, M \rangle$ *to the basis* $\langle L', M' \rangle$ *such that*

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  under these bases.

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  under these bases.
- *There is a bijection between the strongly detected boundary slopes from the* $SL(2, \mathbb{C})$*-character varieties.*

Main Result

In the multi-cusp case, there is a similar result.

Theorem (Cheetham-West-Y)

Let M, N be multi-cusped finite-volume hyperbolic 3-manifolds. Suppose there is a regular profinite isomorphism

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there is a bijection between the strongly detected boundary curves from the $SL(2, \mathbb{C})$-character varieties.

Main Result

Thank you!

On Matroids and the 1- System of Curves on Surfaces.

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April 26, 2025

Definition & Problem Setup

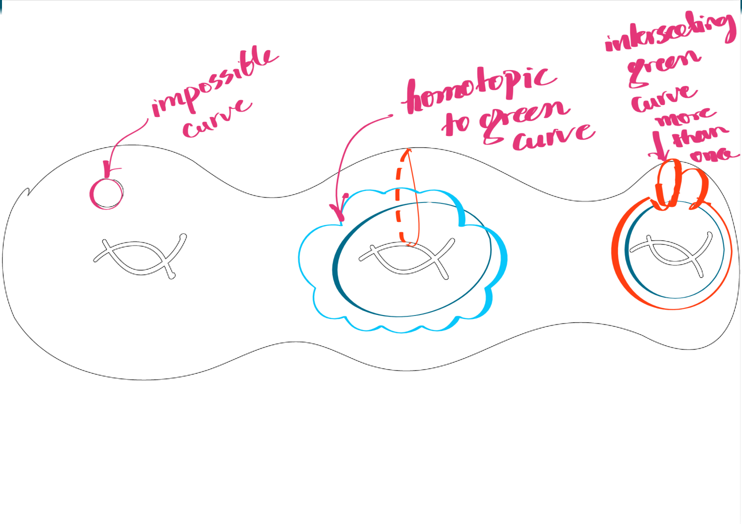
- Let S_g be a connected, closed, oriented surface of genus g. Let Γ be a set of essential, simple closed curves on the surface S_g, such that:

  - No two curves are homotopic (or isotopic we are not strict about this yet).

  - Each two curves intersect at at most one point.

  Call such a set of curves a 1-System of curves on S_g.

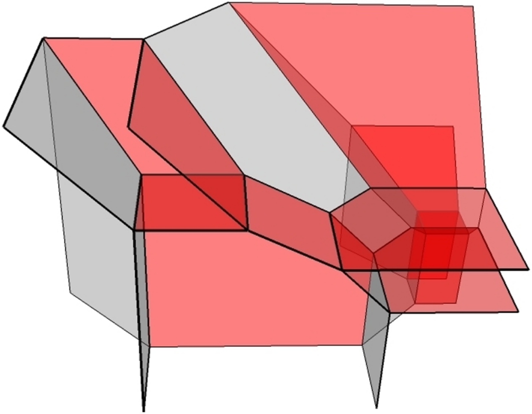
Inessential Curves & curves intersecting more than once are not allowed



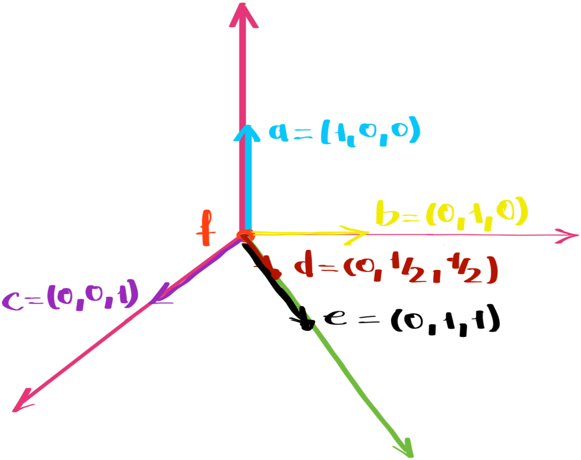
Open Problem

Determine or Estimate how large a 1-System of curves on a surface can be?

Tropical Manifolds & Bergman Fan of a matroid



Some Motivating Examples for Matroids



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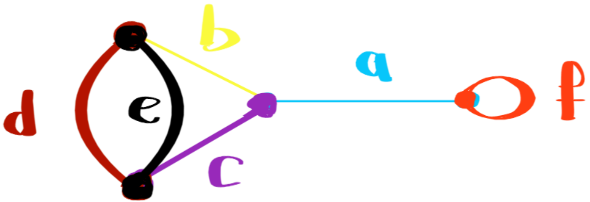


Independent Sets in case of the V.C. & the Graph

Matroid theory will give us a background to attack both problems at the same time as one matroid. The list of independent sets in case of V.C. is:

- $\emptyset a, b, c, d, e$
- ab, ac, ad, ae, bc, bd, be, ad, ce
- abc, abd, abe, acd, ace

Notice that this independent list is the same for the previous V.C. and Graph.

Definition

A matroid M is an ordered pair $(E, \mathcal{I})$ consisting of a finite (not necessarily but for the sake of our purposes it is finite) set E and a collection $\mathcal{I}$ of subsets of E (its members are the independent sets of M) having the following three properties:

- (I1) $\emptyset \in \mathcal{I}$
- (I2) If $I \subset J$ and $J \in \mathcal{I}$, then $I \in \mathcal{I}$.
- (I3) If $I, J \in \mathcal{I}$, and $|I| < |J|$, then there is $j \in J - I$ such that $I \cup \{j\}\mathcal{I}$."Exchange Axiom"

Edmonds' Matroid Intersection Theorem

- If M_1 and M_2 are two matroids on the same ground set E. We attempt to define a new matroid $M_1 \cap M_2$ as follows: $I \subseteq E$ is independent in $M_1 \cap M_2$ if $I = I_1 \cap I_2$, where I_1 is independent in M_1 and I_2 is independent in M_2. Note, in general, the intersection of two matroids is not necessarily a matroid but still if

$$\mathcal{I}_1 \cap \mathcal{I}_2 := \{I_1 \cap I_2 |\, I_1 \in \mathcal{I}_1 and\ I_2 \in \mathcal{I}_2\},$$

it is an important problem to find the largest independent set in $\mathcal{I}_1 \cap \mathcal{I}_2$ (In our problem this refers to the total number of colors).

- Let $I \in \mathcal{I}_1 \cap \mathcal{I}_2$ and let $A \subseteq E$, then we can show that

$$|I| \leq r_1(A) + r_2(E \setminus A)$$

where r_1 is the rank function (the size of the largest independent subset) for M_1 and r_2 is the rank function for M_2.

Edmonds' Matroid Intersection Theorem

- And from the previous bullet we can conclude that

$$\max_{I \in \mathcal{I}_1 \cap \mathcal{I}_2} |I| \leq \min_{A \subseteq E} (r_1(A) + r_2(E \setminus A))$$

  The equality above is Edmonds' Matroid Intersection Theorem.

- we are actually using the following version in our research, for every $A \subseteq E$

$$\max_{I \in \mathcal{I}_1 \cap \mathcal{I}_2} |I| \leq (r_1(A) + r_2(E \setminus A)) \qquad (*)$$

  where total number of colors t $= \max_{I \in \mathcal{I}_1 \cap \mathcal{I}_2} |I|$.

T. Aougab, T. Huynh & The Matroid Intersection Theorem

- T. Aougab *Existence of a connected total rainbow subgraph*.
- T. Huynh *Matroid Intersection Theorem*
  by defining a set to be independent in the first matroid M_1 iff it contains at most one edge of each color and a set to be independent in the second matroid M_2 iff it does not contain a cycle.

Our Main Question

Given t *and* K_n *for some* n, ,
when do edge colored complete graphs satisfy relation $(*)$ *above?*

Thanks For Your Attention!

Questions?

Skinning Maps of Geometric Limits

Sungjin Park (Yale)

April 2025

Brief Deformation Theory of Kleinian Groups

Let M be a compact oriented hyperbolizable 3-manifold with incompressible boundary components $S_1, \cdots, S_k$ with $\chi(S_i) < 0$ for all i.

Theorem (Ahlfors, Bers, Marden, Sullivan, $\cdots$)

The set of convex cocompact hyperbolic structures on the interior of M is parametrized by $\prod_{i=1}^{k} \mathcal{T}(S_i)$.

Example (quasi-Fuchsian group, Bers parametrization)

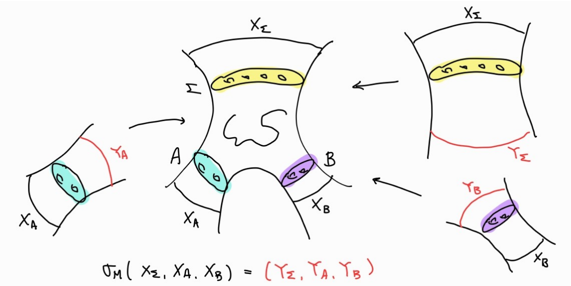
If $M = S \times [0,1]$, $\mathrm{QF}(S) \cong \mathcal{T}(S) \times \mathcal{T}(\overline{S})$.

Skinning Map

Definition (Skinning map)

The **skinning map** $\sigma_M : \prod_{i=1}^{k} \mathcal{T}(S_i) \to \prod_{i=1}^{k} \mathcal{T}(\overline{S_i})$ *is defined as follows: Let* $M(X)$ *be the hyperbolic structure of* M *determined by* $X \in \prod_{i=1}^{k} \mathcal{T}(S_i)$. *Then,* $\sigma_M(X)$ *is the tuple of conformal structures of 'inward-pointing' end of covers of* $M(X)$ *associated to each* $\pi_1(S_i)$.

The cover associated to $\pi_1(S_i)$ is quasi-Fuchsian by Thurston's covering theorem.



Skinning Map

Theorem (Bounded Image Theorem, Thurston, Kent)

If M is acylindrical, the image of σ_M has compact closure.

Conjecture (Minsky)

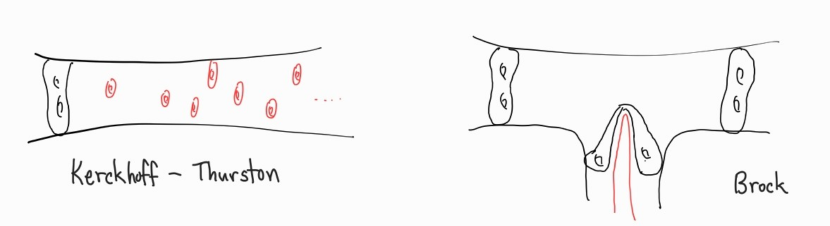
The Teichmüller diameter of the image of σ_M is bounded by the constant only depends on the topology of ∂M.

Geometric Limit

Definition

A sequence (M_n, p_n) of manifolds with base points converges geometrically to (M, p) if there exist $K_n \to 1$, $r_n \to \infty$ such that there exist smooth maps $\psi_n : (B_M(p, r_n), p) \to (M_n, p_n)$ which is K_n-bilipschitz to the image.

Examples (geometric limits of Kleinian surface groups)



Setting for the theorem

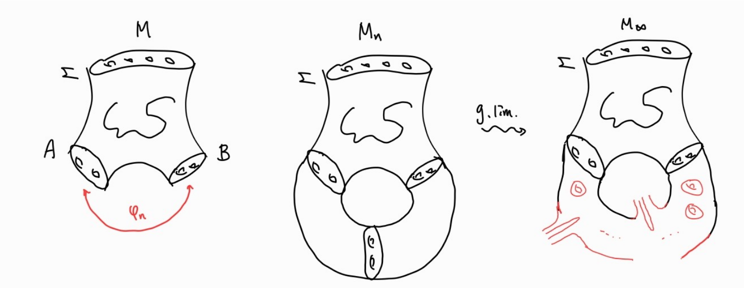
Setting

M : compact oriented hyperbolizable 3-manifold with three incompressible boundary components Σ, A and B, where A and B have same genus.

φ_n : gluing maps between A and B.

M_n : the manifold constructed from M by gluing A and B by φ_n.

M_∞ : a geometric limit of $M_n(X)$ with a fixed conformal structure X on Σ.



Main Result (In Progress)

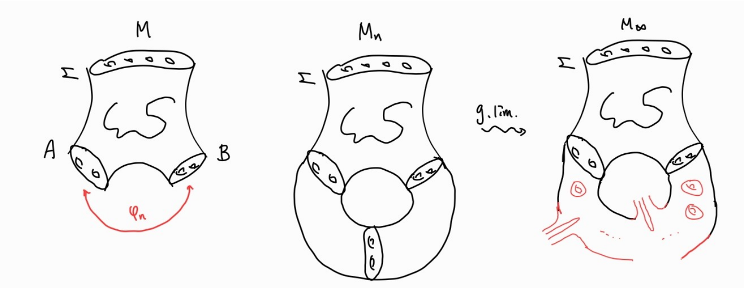
Theorem

Assume that $\sigma_{M_n}, \sigma_{M_\infty} : \mathcal{T}(\Sigma) \to \mathcal{T}(\overline{\Sigma})$ are 'well-defined'. Then,

(a) *σ_{M_n} converges pointwise to σ_{M_∞}.*

(b) *If M is acylindrical, $\overline{\mathrm{image}(\sigma_{M_n})} \to \overline{\mathrm{image}(\sigma_{M_\infty})}$.*

(c) *If M is a book of I-bundle and each M_n is acylindrical, then $\overline{\mathrm{image}(\sigma_{M_n})} \to \overline{\mathrm{image}(\sigma_{M_\infty})}$.*



The Mapping Class Group of Graphs

From Tree Branches to Spiderwebs

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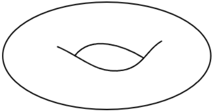
April 26, 2025

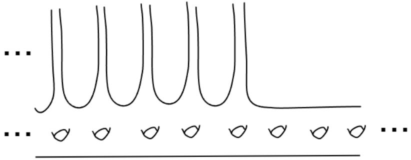
Definition (Mapping class group for surfaces)

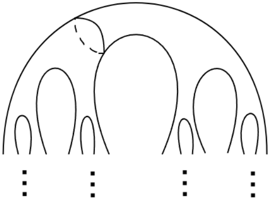
The *mapping class group* of a closed orientable surface S is

$$\mathrm{Mod}(S) = \mathrm{Homeo}^{+}(S)/\mathrm{Homeo}_0(S)$$

Orientation-preserving homeomorphisms modulo isotopy.





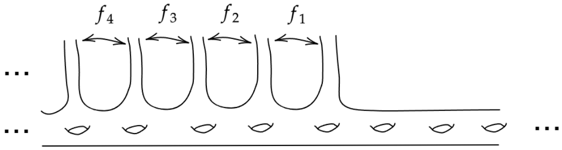


Topology

Definition

We endow $\mathrm{Homeo}^+(S)$ with the compact-open topology, and equip $\mathrm{Mod}(S)$ with the quotient topology.

Elements of $\mathrm{Maps}(S)$ are closer together if they agree on larger compact sets.



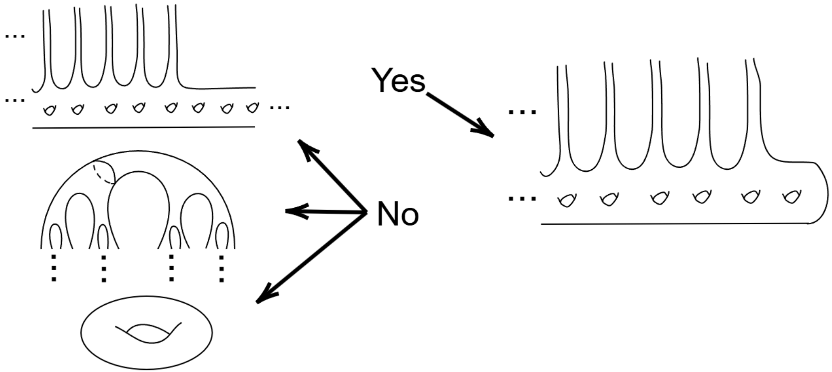
$f_i \to id$ since the ends swapped get further away.

Dense Conjugacy Classes of Surfaces

Question

Given S, does $Mod(S)$ have a dense conjugacy class?

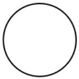
A complete classification was obtained by Lanier and Vlamis, and also by Hernández Hernández, Hrušák, Morales, Randecker, Sedano, and Valdez.

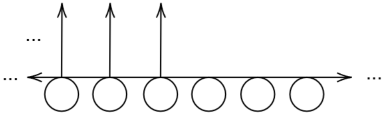


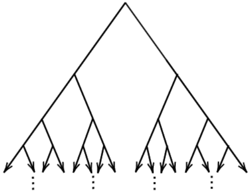
Mapping class group for **graphs**

Definition (Mapping class group for graphs)

Let X be a locally finite graph. The *mapping class group of X*, denoted $\mathrm{Maps}(X)$, is the group of proper homotopy equivalences of X, modulo proper homotopy.



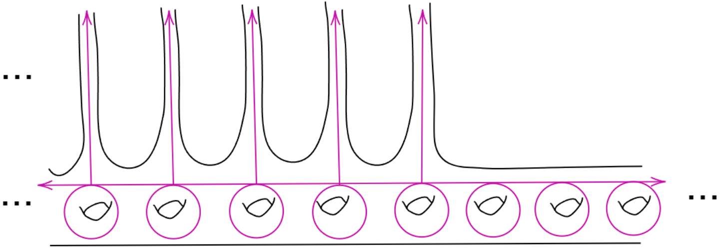




The topology is similar to a quotient of the compact-open topology, but a little more work is required.

Locally Finite Graphs Mirror Surfaces and Out(F_n)

Surfaces: “A 1-D version of $Mod(S)$”



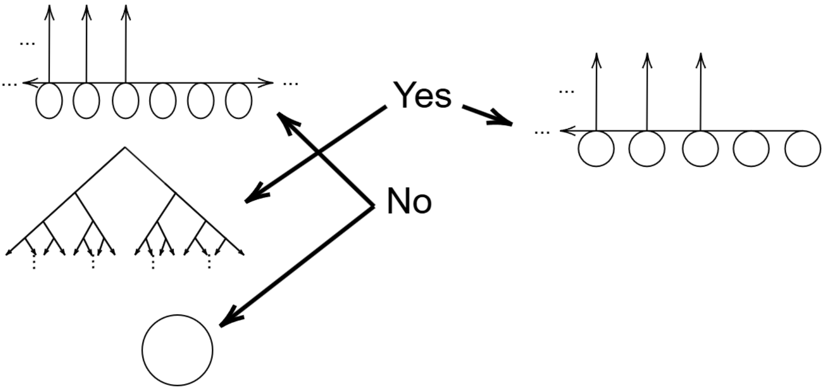
Out(F_n): “A BIG version of Out(F_n)” (Colloquially named Big Out)

If a graph X is finite, then $\mathrm{Maps}(X) \cong \mathrm{Out}(F_{\mathrm{rk}(\pi_1(X))})$

Dense Conjugacy Classes of Graphs

Question

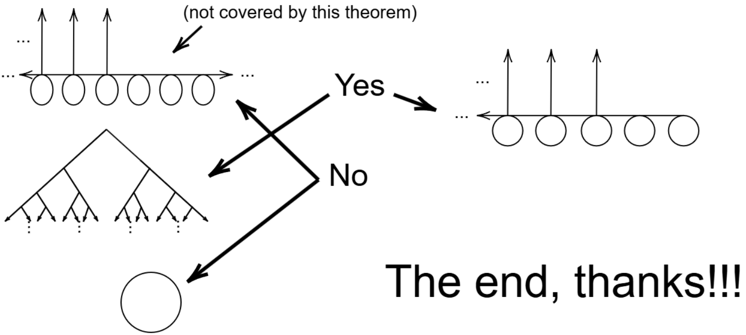
Given X, does $\mathrm{Maps}(X)$ have a dense conjugacy class?



Given X, does Maps(X) have a dense conjugacy class?

Theorem (K, trying to generalize)

Let X be a stable self-similar locally finite graph. Then Maps(X) has a dense conjugacy class if and only if X is proper homotopy equivalent to the Cantor tree, or both the genus of X is zero or infinite, and the graph X has a unique maximal end.



The transitivity of pure Hurwitz classes of dynamical branched covers of the sphere

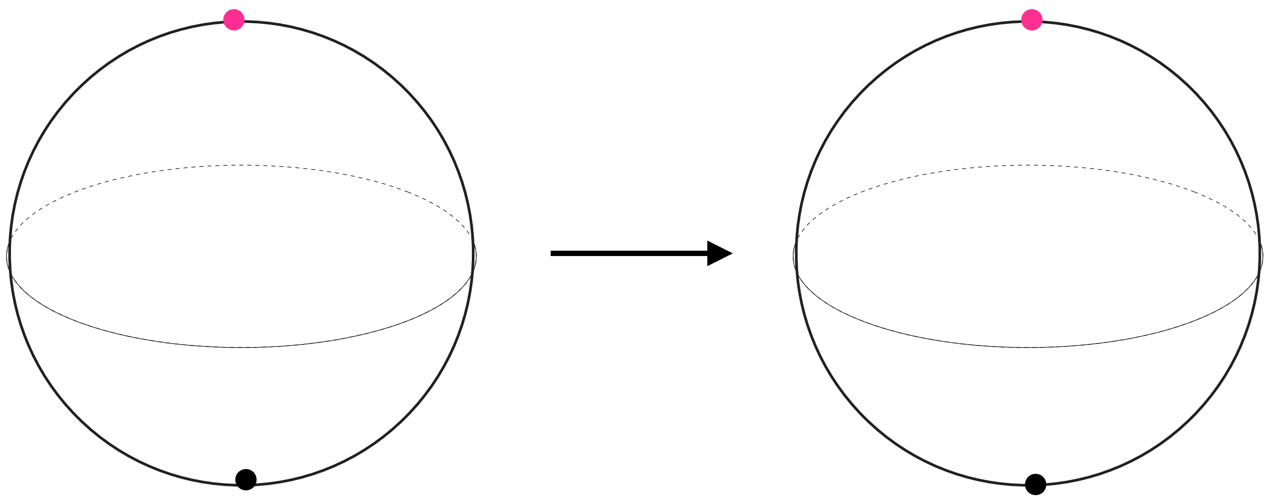
Becca Winarski
College of the Holy Cross

Joint with Yvon Verberne

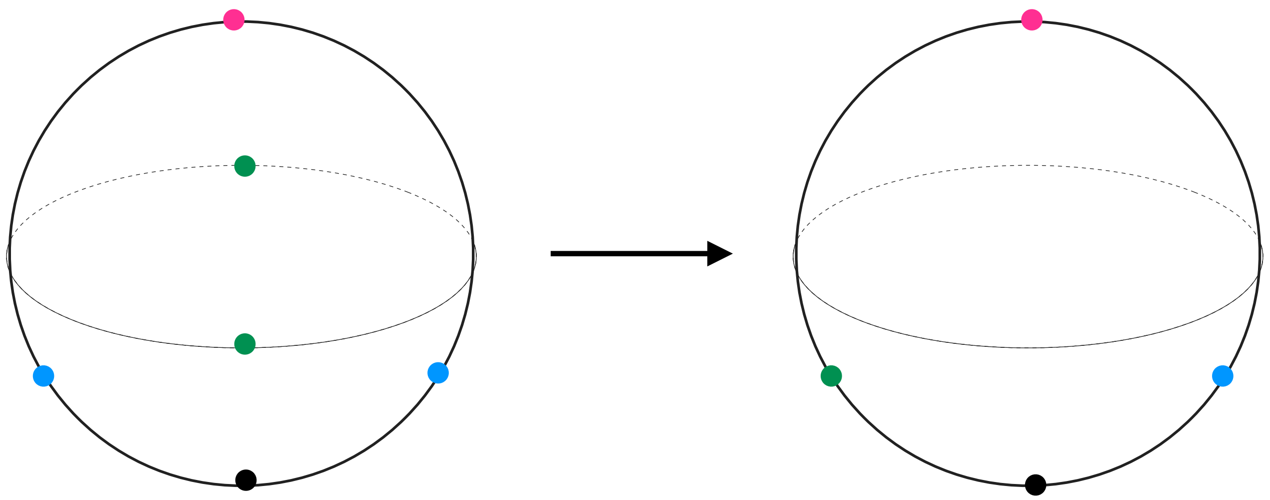
Goal

Invitation: Translate your favorite questions and techniques about mapping classes and branched covers to a dynamical setting

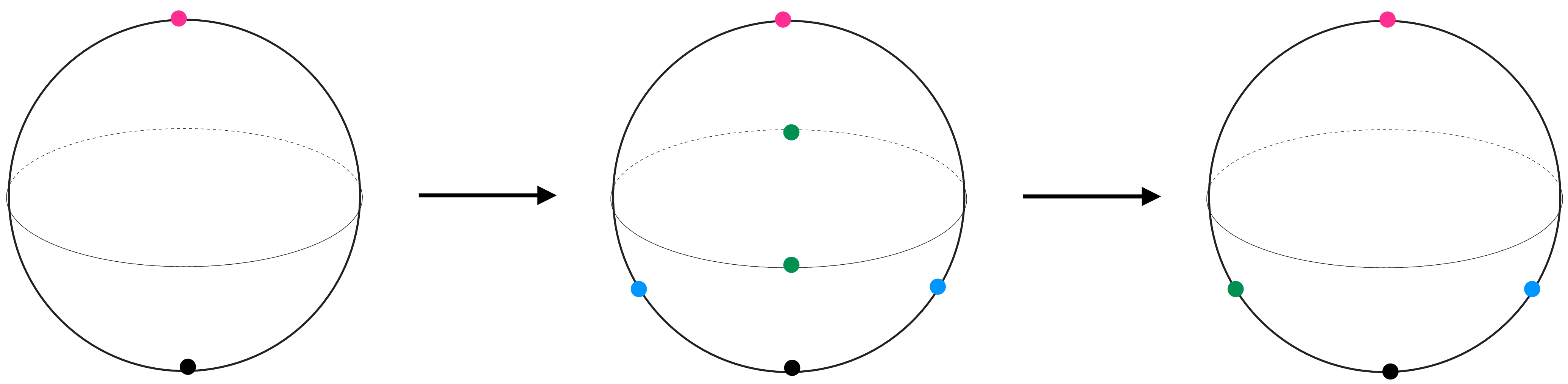
Dynamical Branched Covers



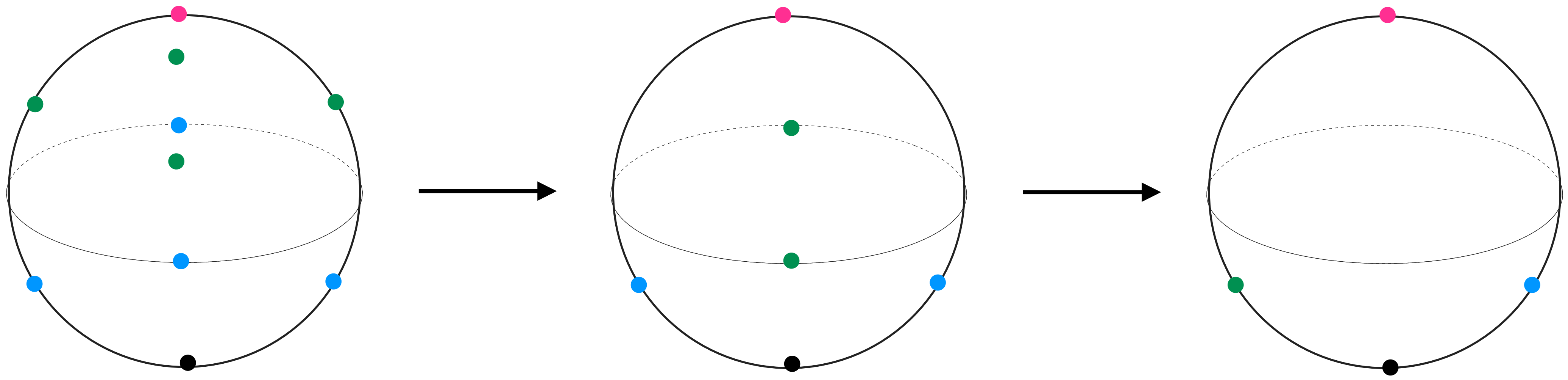
Dynamical Branched Covers



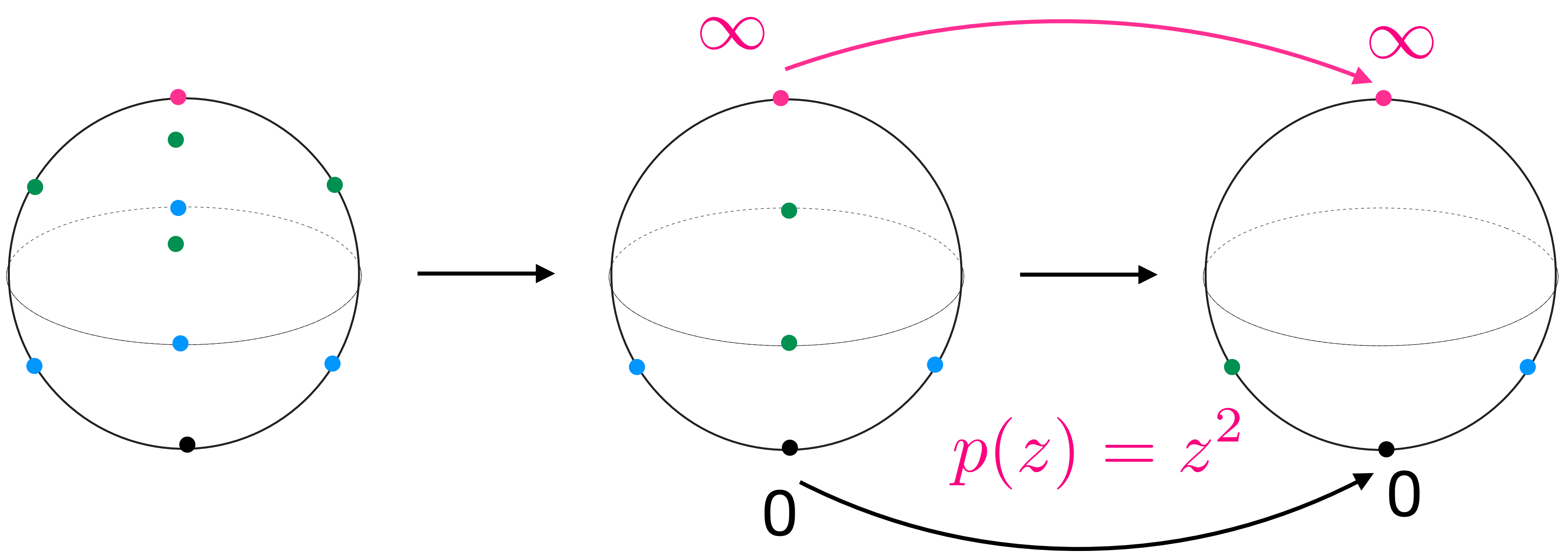
Dynamical Branched Covers



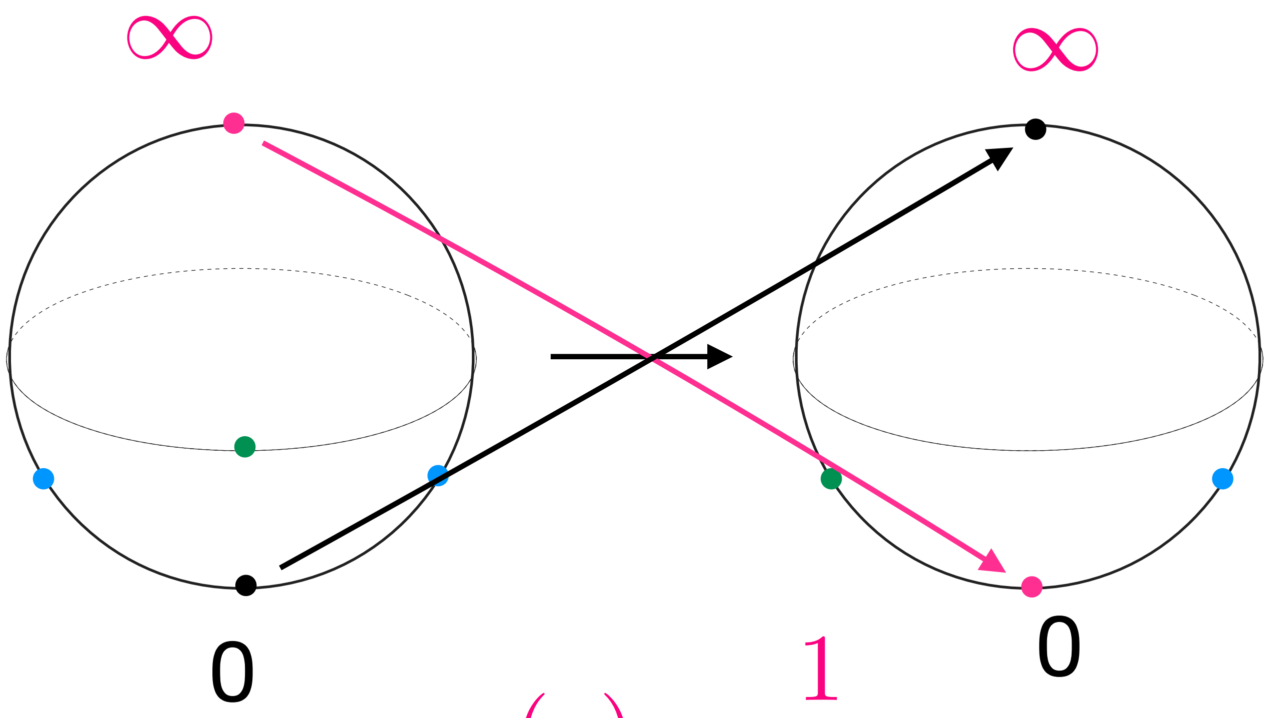
Dynamical Branched Covers



More examples: polynomials!



More examples: rational maps

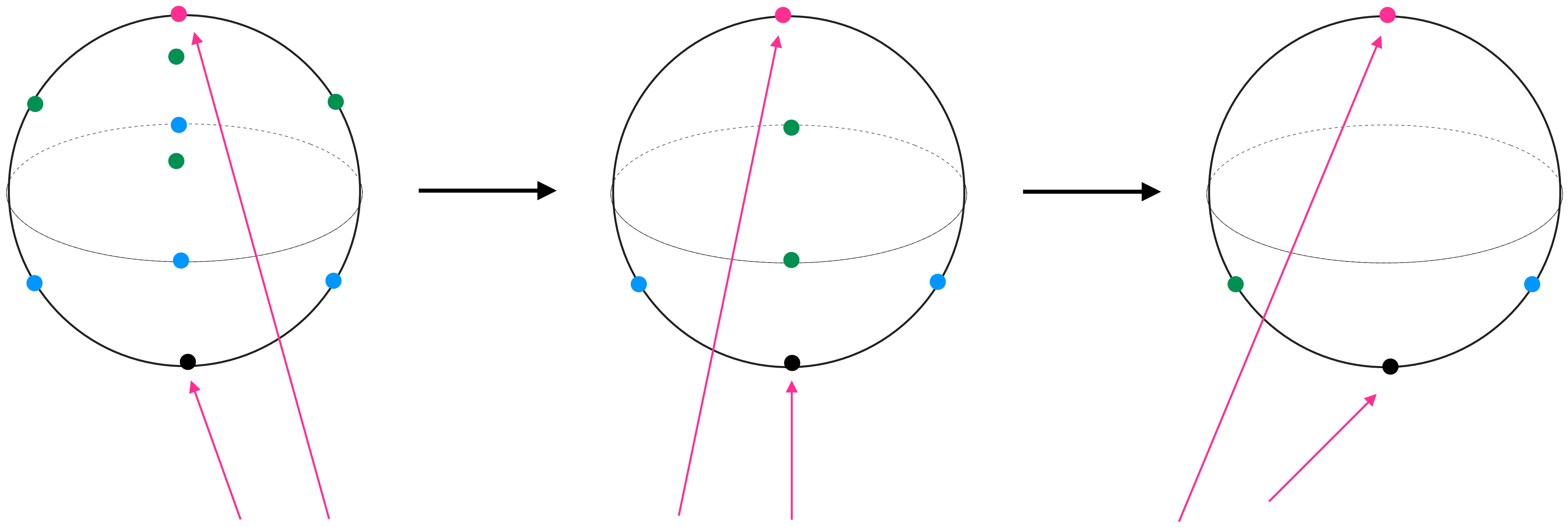


$$p(z) = \frac{1}{z^2}$$

$$\infty \mapsto 0$$

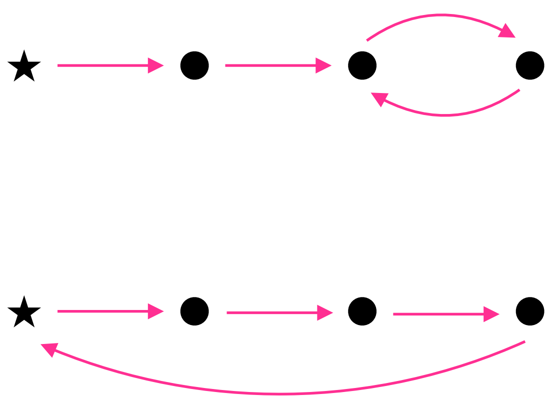
$$0 \mapsto \infty$$

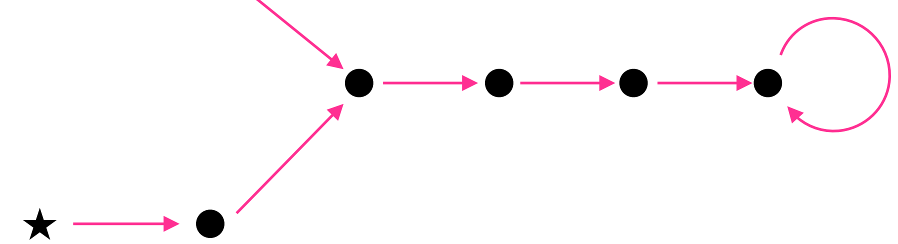
Dynamical Branched Covers

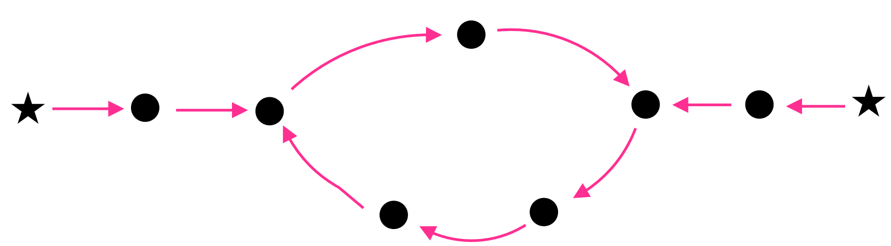


Post-critical set=(entire) forward orbit of critical (ramified) points

Dynamical Portraits







Main Result

Theorem (Verberne–W)

$f, g : S^2 \to S^2$ degree 2 branched covers. Then there exists some finite set of points M and mapping class $\phi \in \mathrm{Mod}(S^2, M)$ so that g and $f\phi$ have the same portrait.

Main Result

Theorem (Verberne–W)

$f, g : S^2 \to S^2$ degree 2 branched covers. Then there exists some finite set of points M and mapping class $\phi \in \mathrm{Mod}(S^2, M)$ so that g and $f\phi$ have the same portrait.

In particular, we develop an algorithm to find M and ϕ

What’s new?

Complex dynamics usually focuses on the action of *pure* mapping classes on post-critically finite branched covers because they don’t change the portrait.

This is a tool to pursue the study of the action of *all* mapping classes on post-critically finite branched covers.

This is the beginning

Our result: Composing by mapping classes allows you to move between (all) portrait classes.

Purely combinatorial (symmetric groups not braid groups)

Questions:

If you track the braid, not just the permutation, can you move between <u>combinatorial</u> classes (a finer relation)?

Which portraits (sometimes/never) have obstructions?

Are there certain twists that lead to obstructions – can we predict them?