

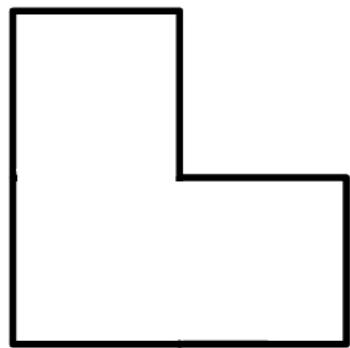
Rep-Tiles



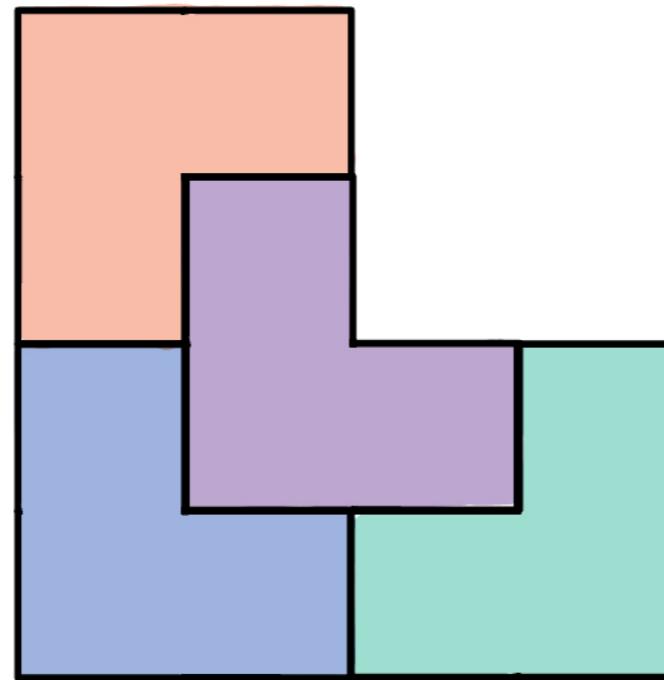
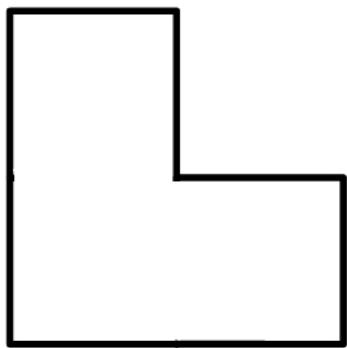
Bena Tshishiku

What is a rep-tile?

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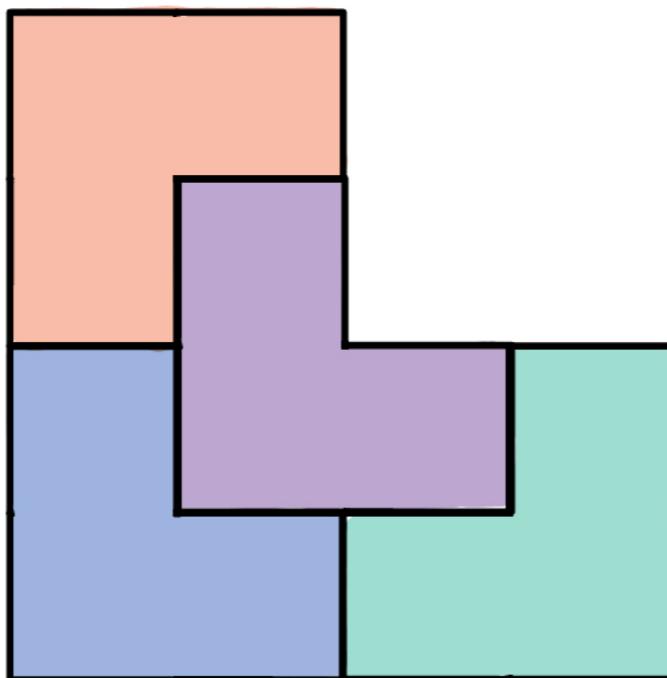
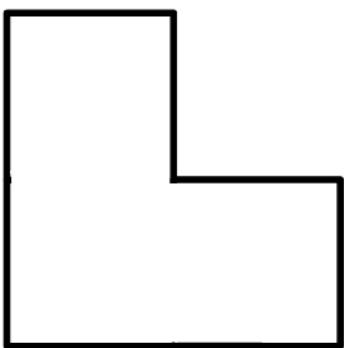


What is a rep-tile?



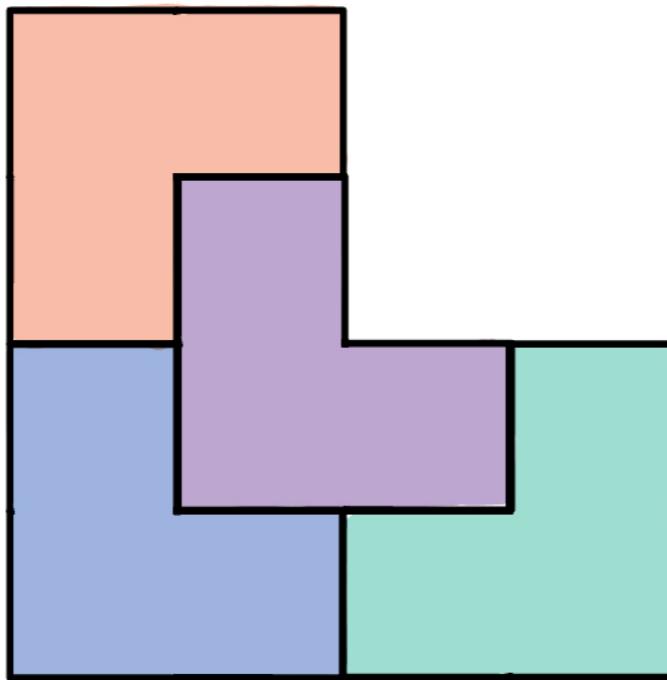
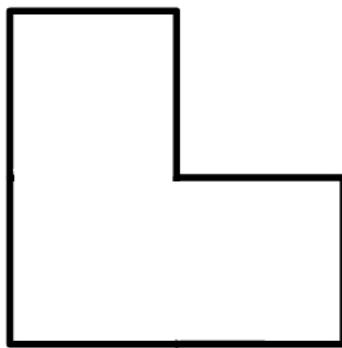
What is a rep-tile?

A replicating tile (rep-tile) is a shape for which multiple copies can form a larger copy of the shape.



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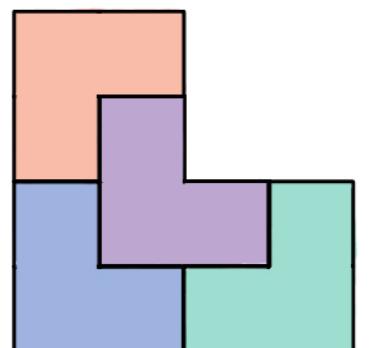
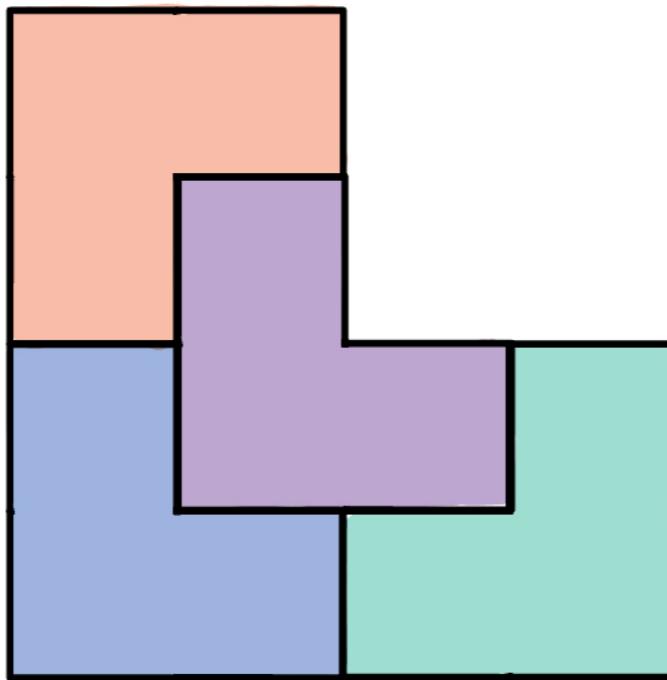
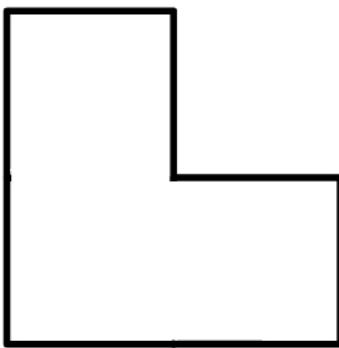
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Rep-tiles in your bathroom

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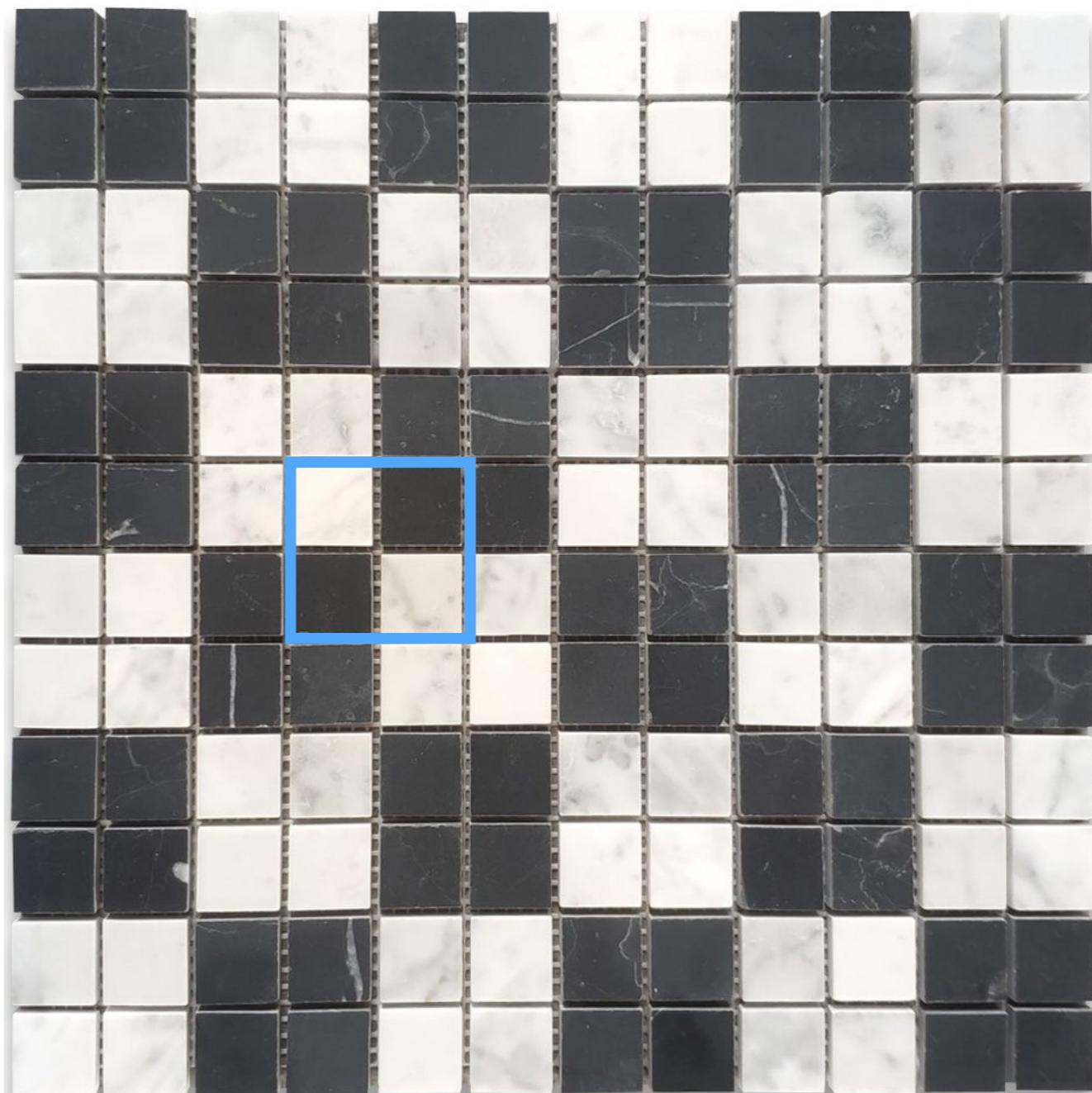
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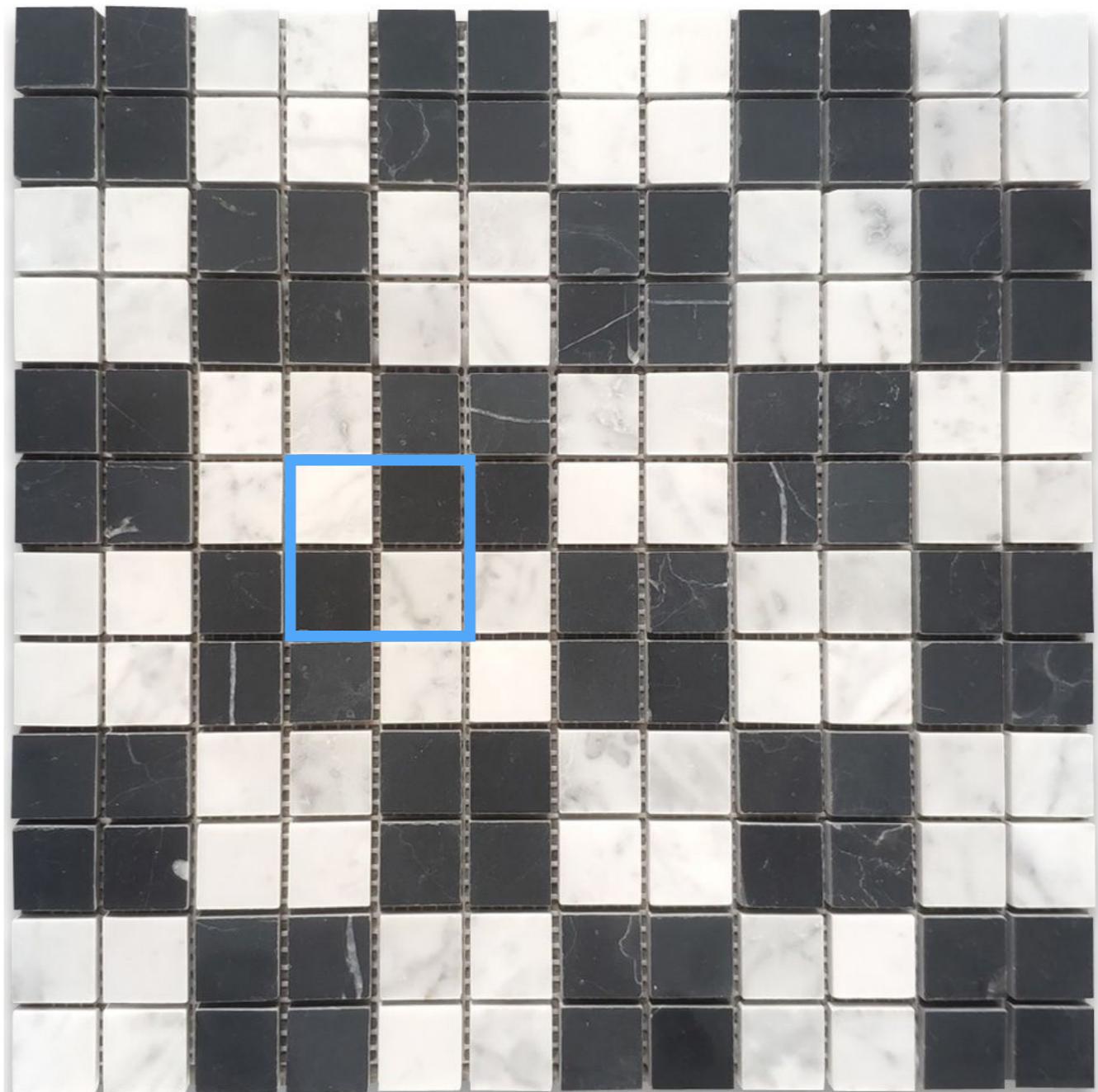
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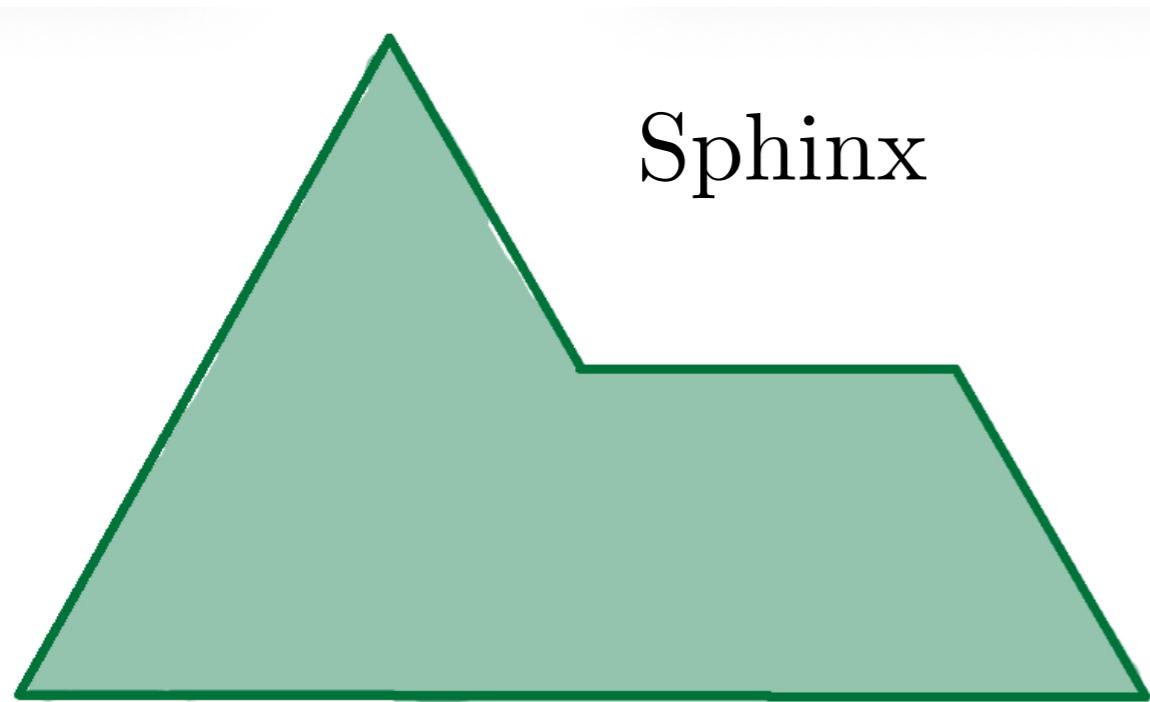
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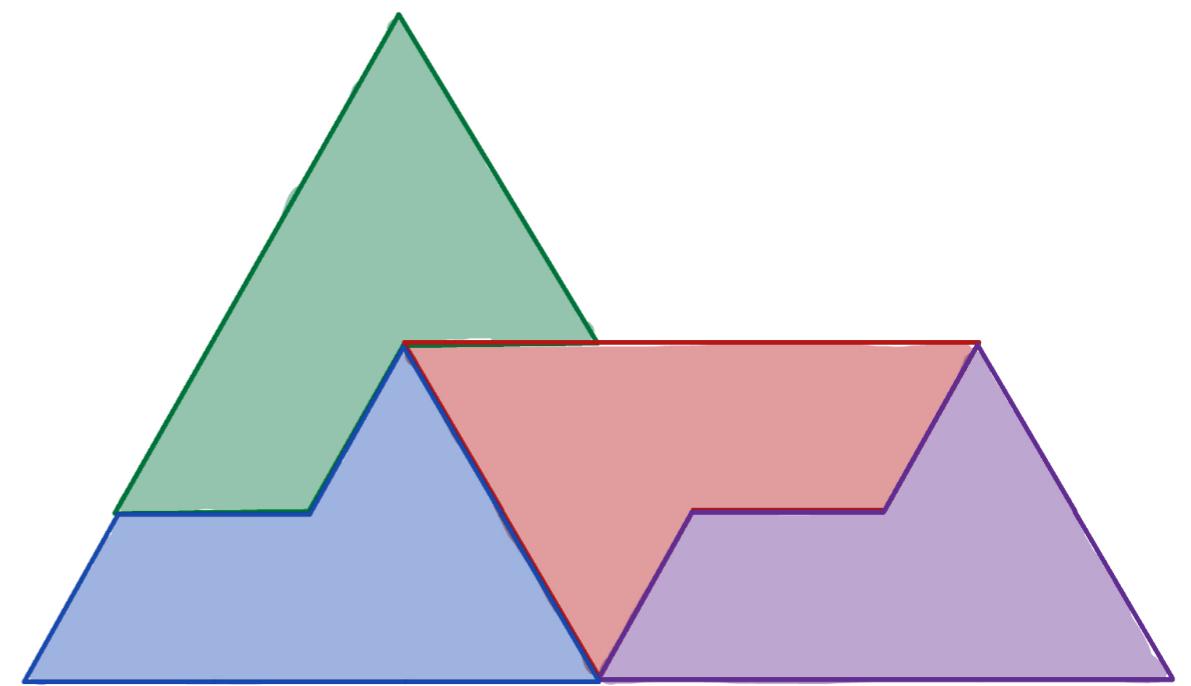
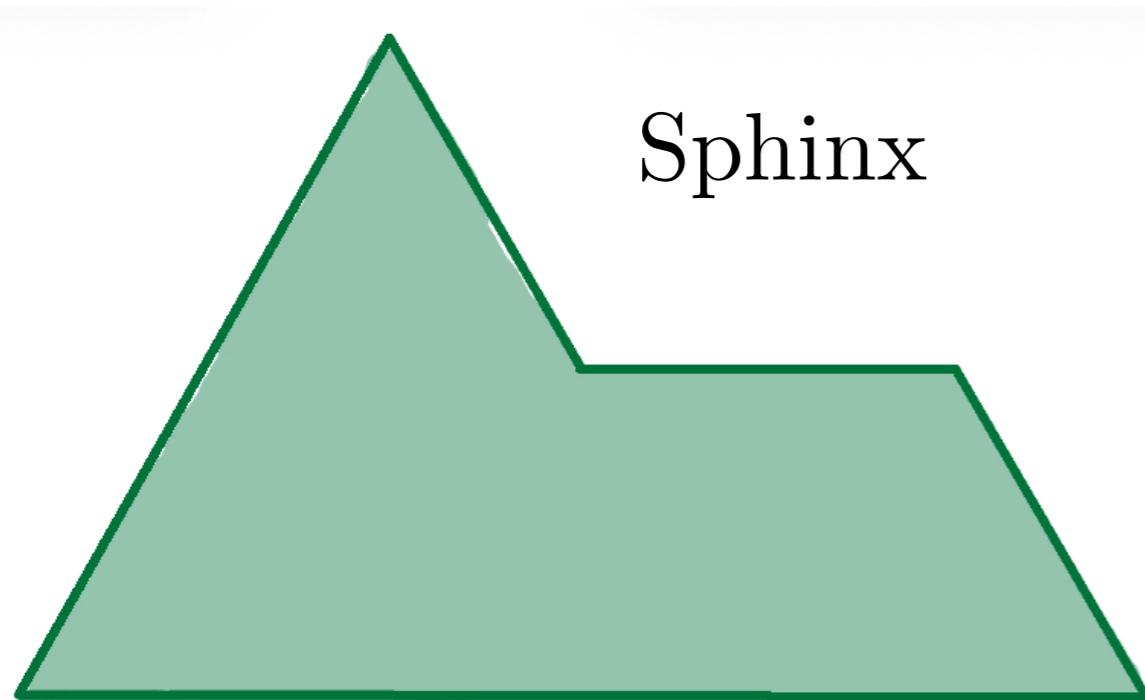
Any reptile gives a way to tile the plane.

More rep-tiles

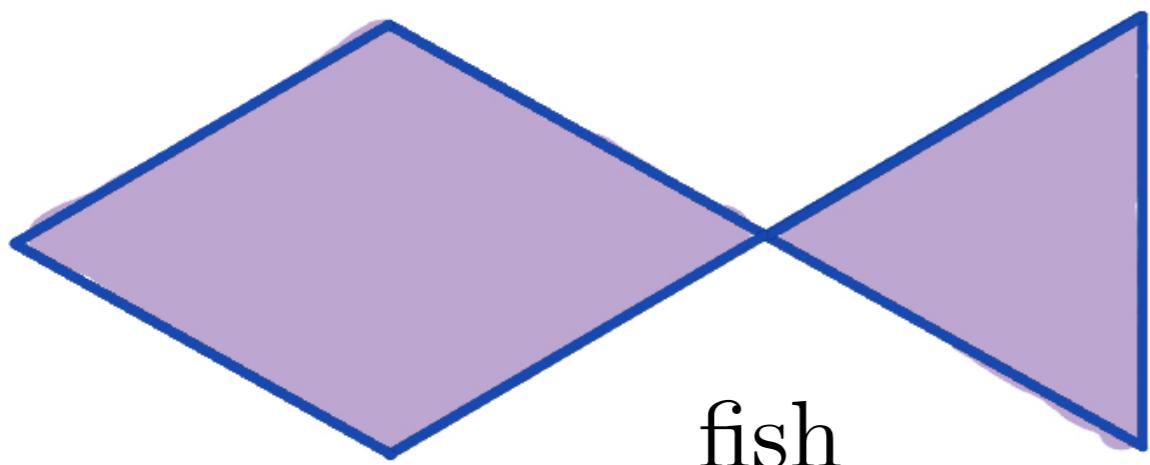
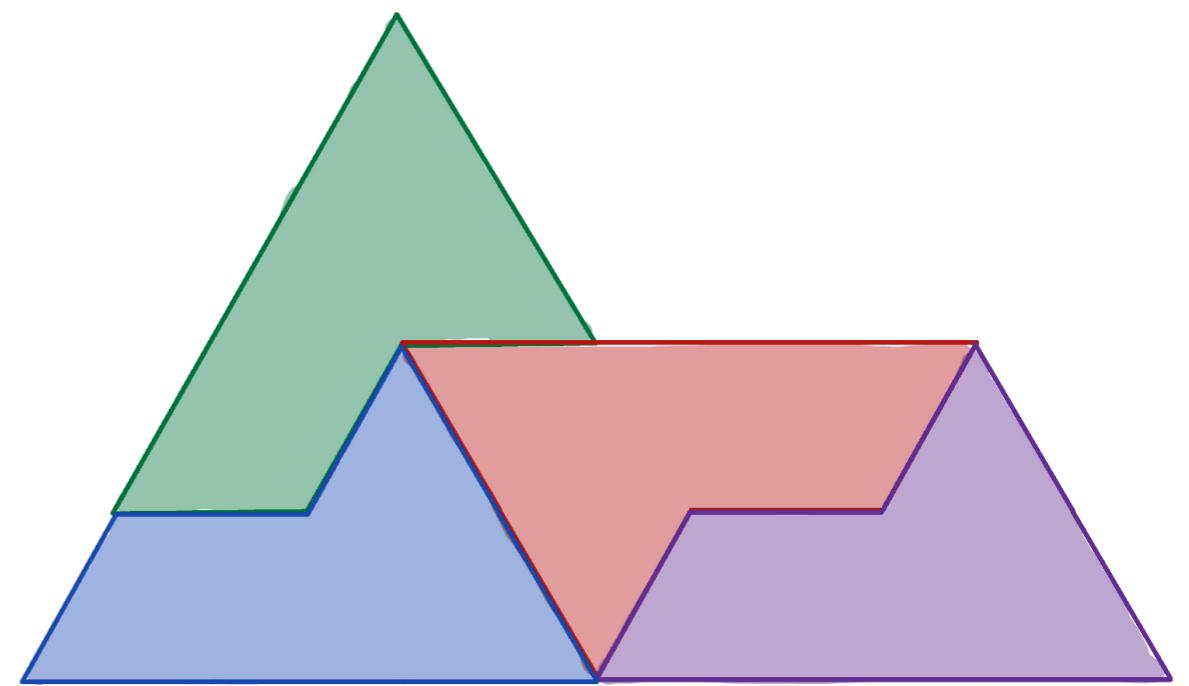
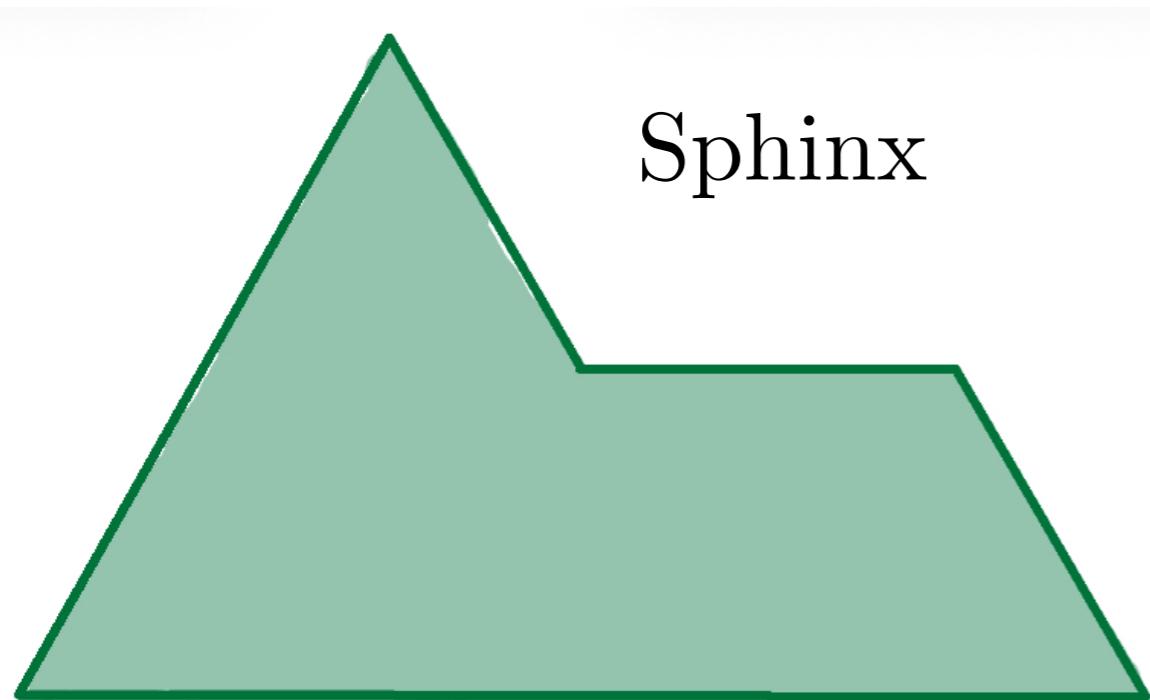
More rep-tiles



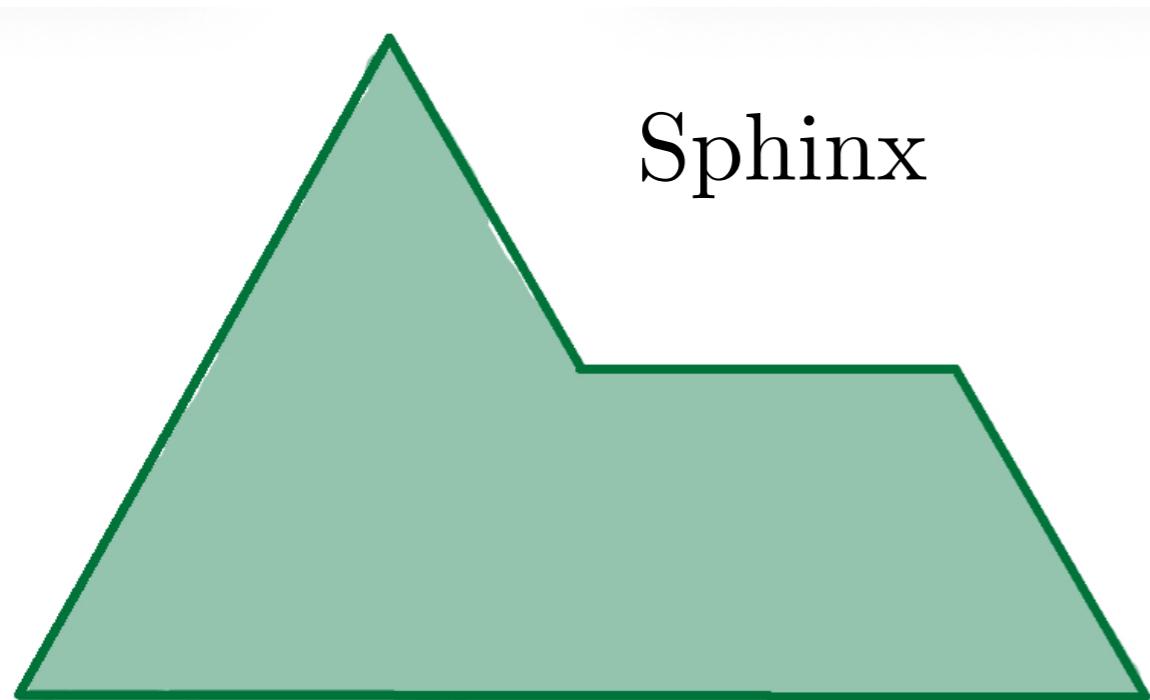
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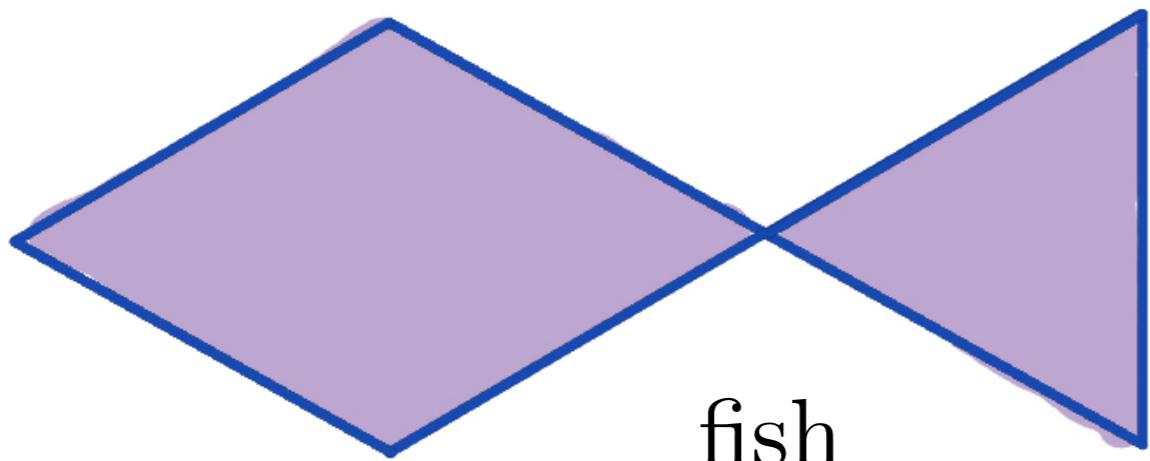
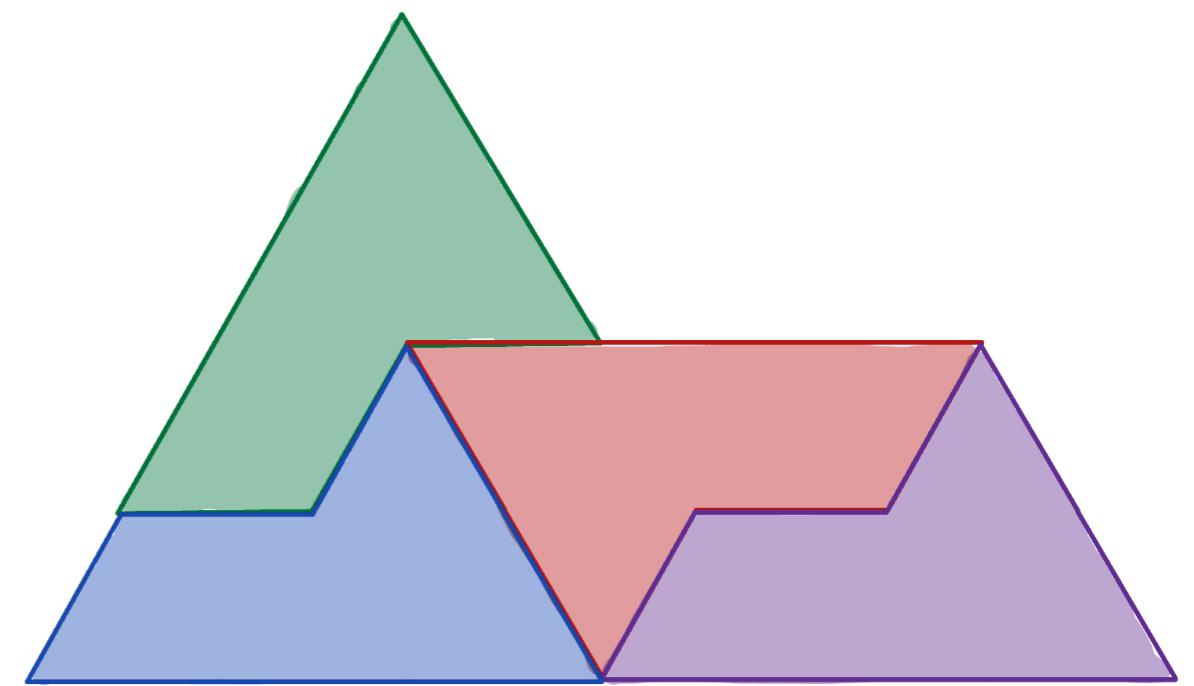
More rep-tiles



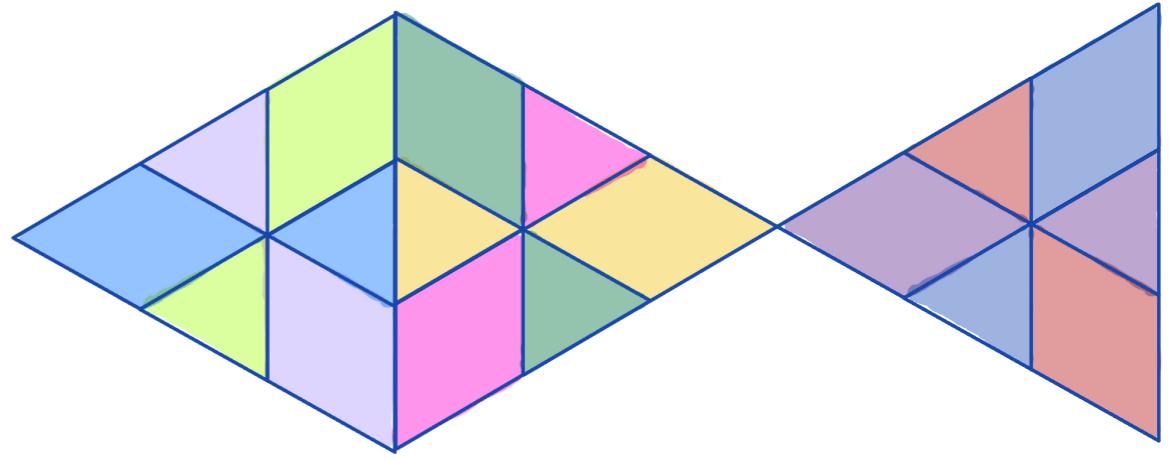
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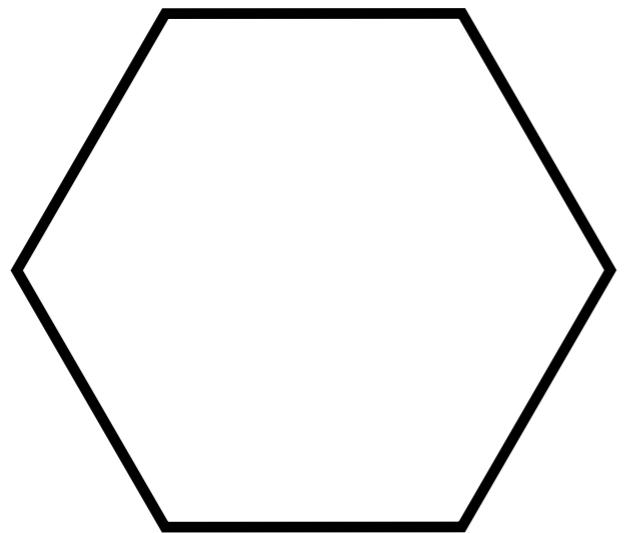
Sphinx



fish

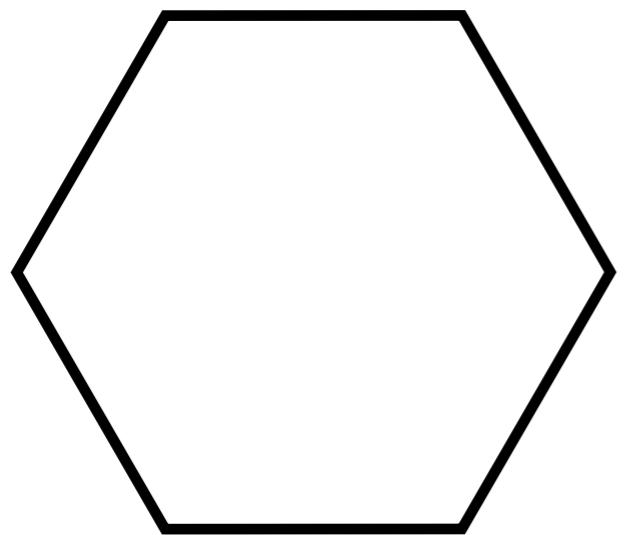


Non-examples

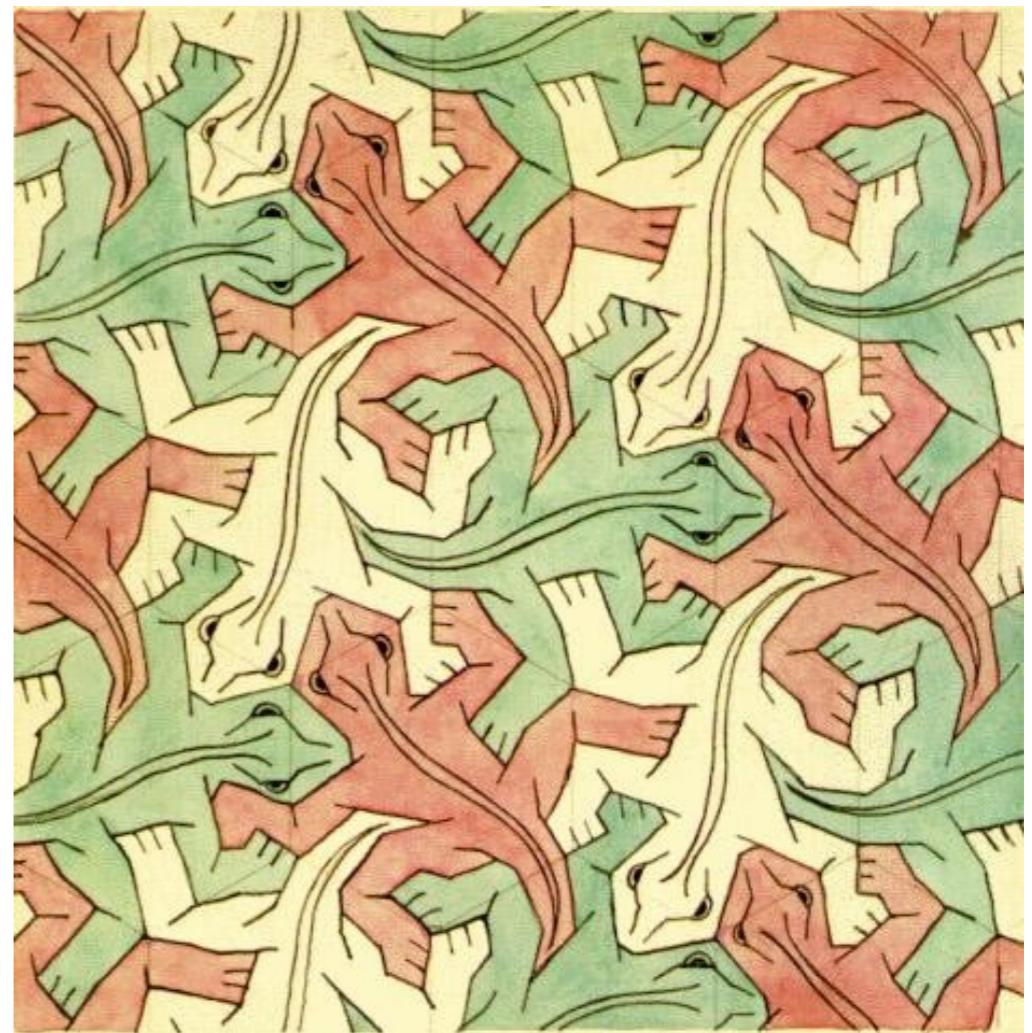


regular hexagon

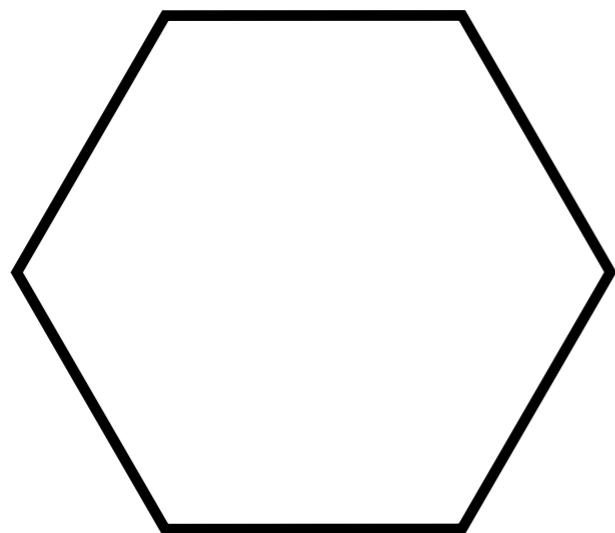
Non-examples



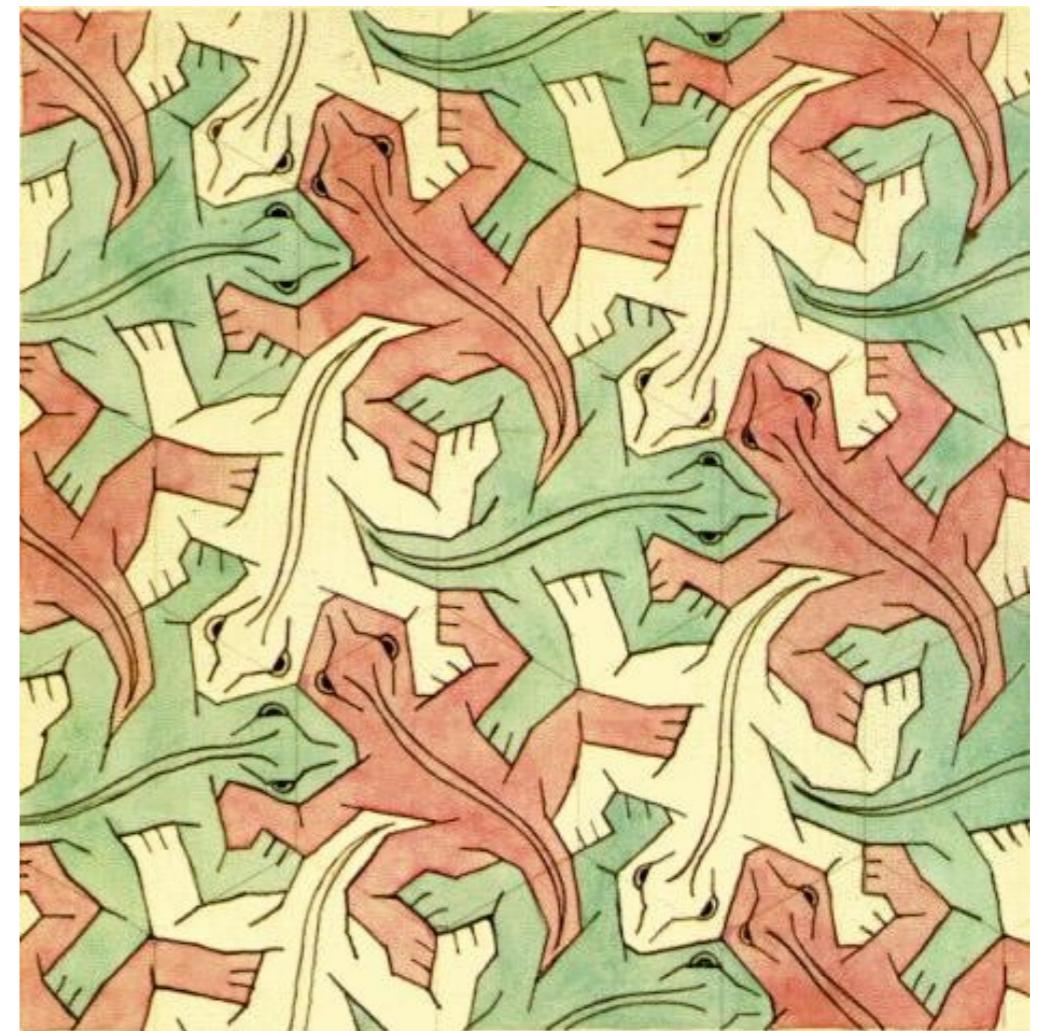
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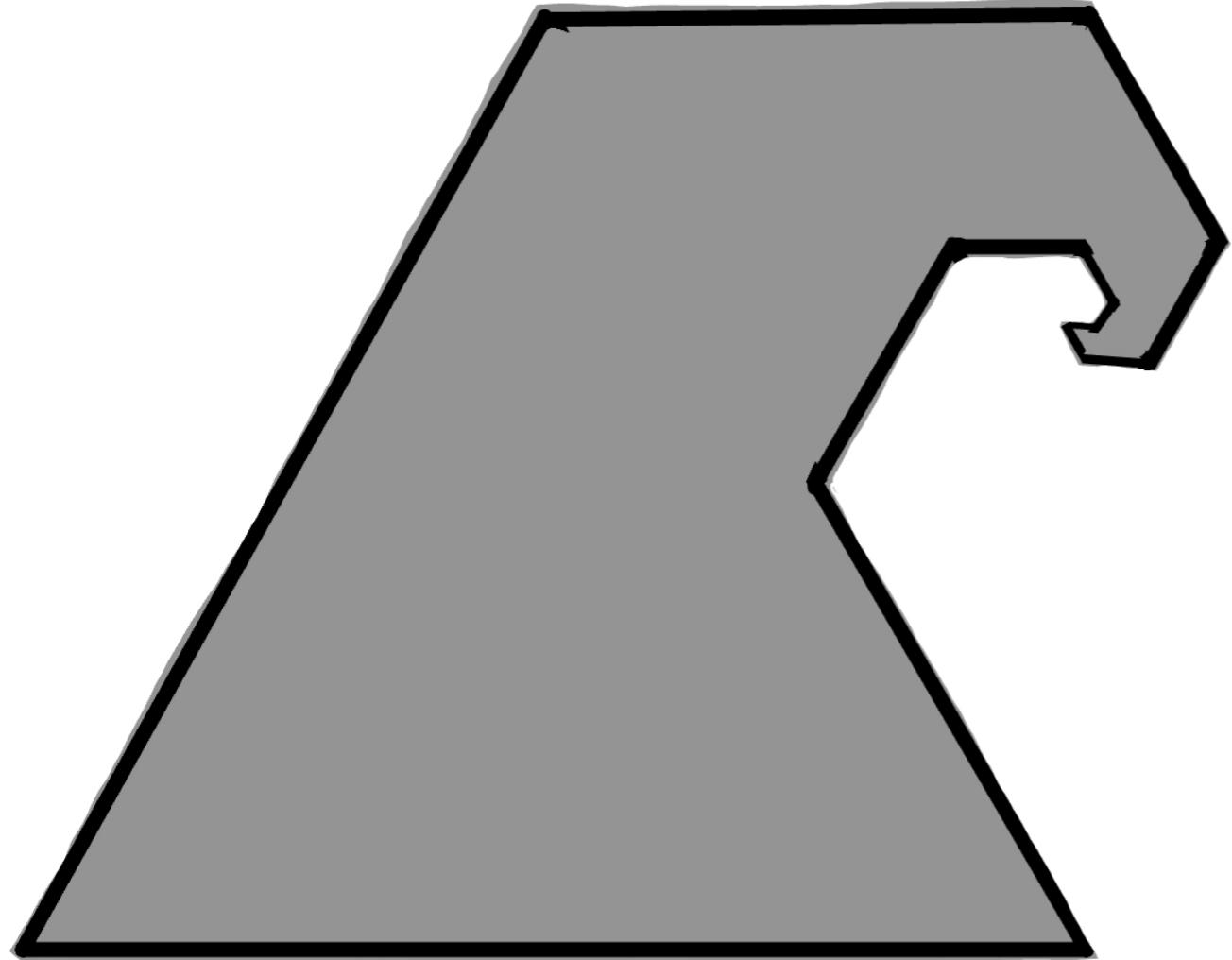


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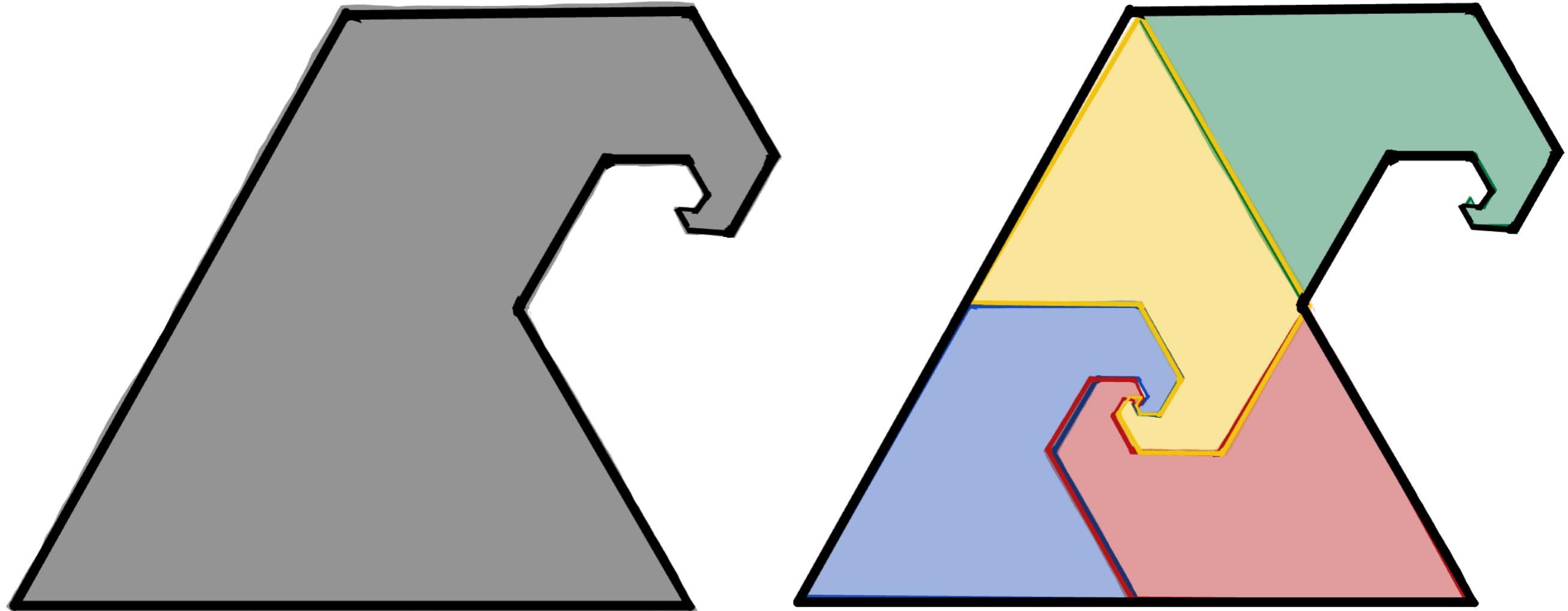
Tiling by reptiles,
but the reptiles
are not rep-tiles.

A fractal rep-tile



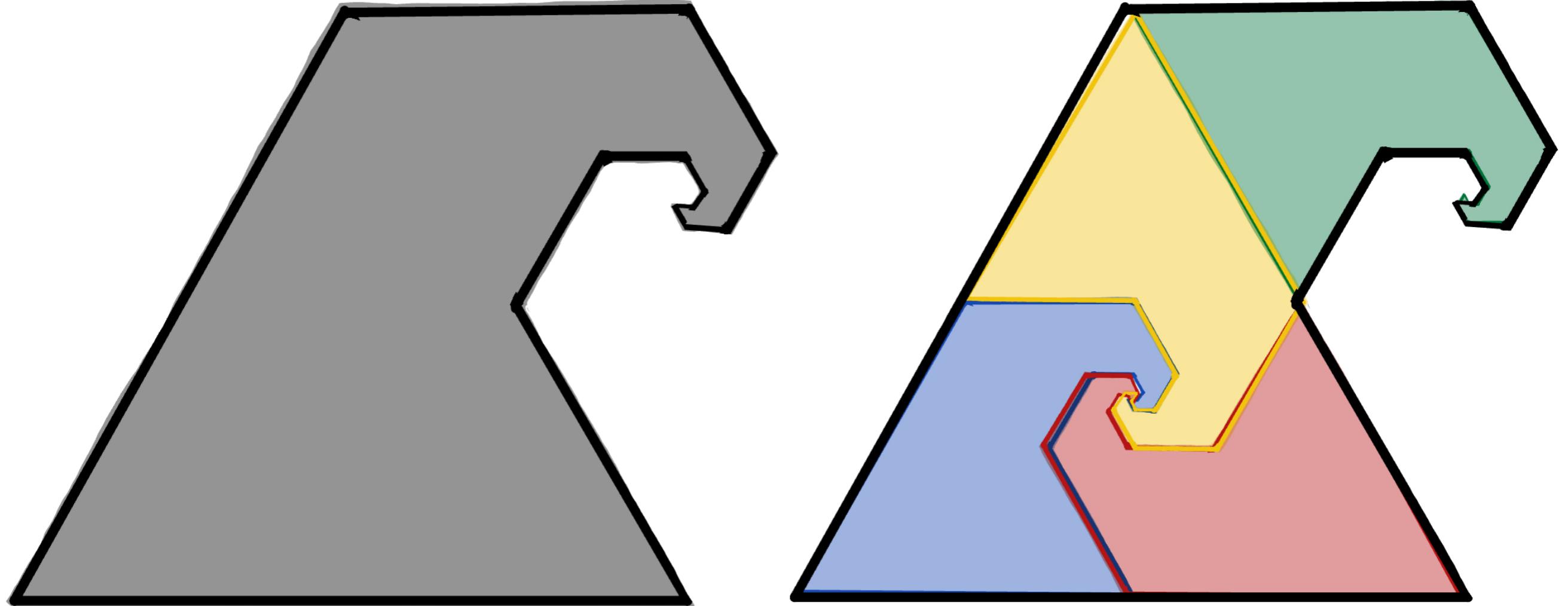
elephant

A fractal rep-tile



elephant

A fractal rep-tile



elephant

?!

*How do we begin to organize and understand
the zoo of rep-tiles?*

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Multiplicity of a rep-tile = # of copies needed to replicate

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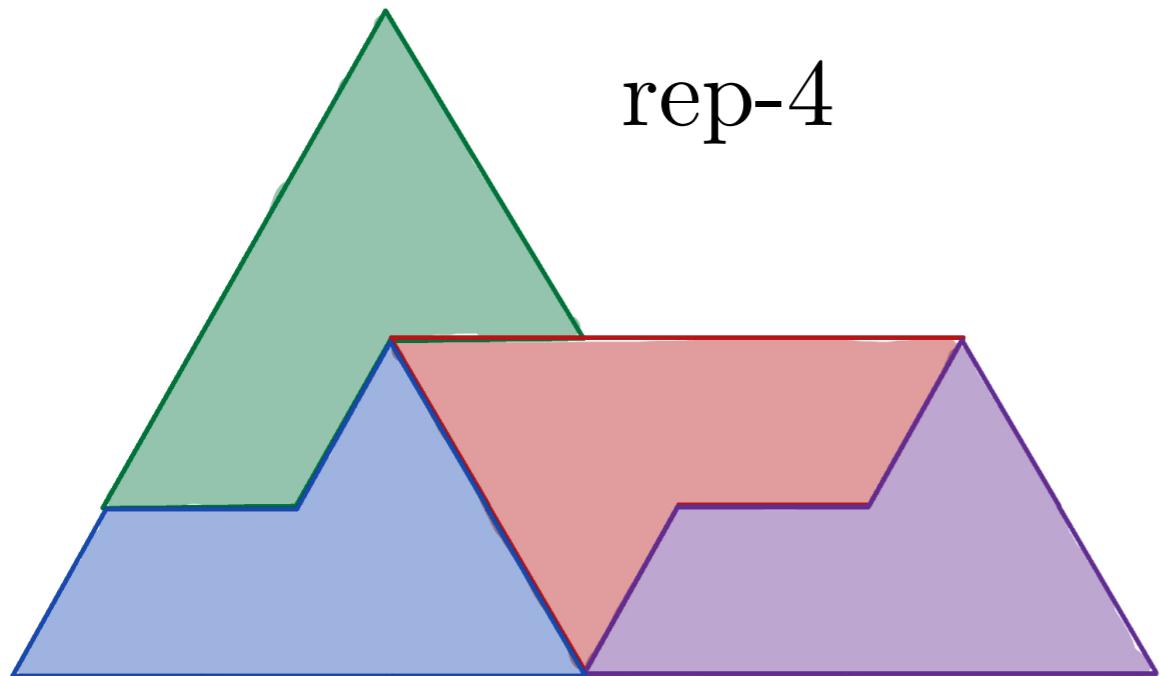
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A shape is rep- k if it is a rep-tile with multiplicity k

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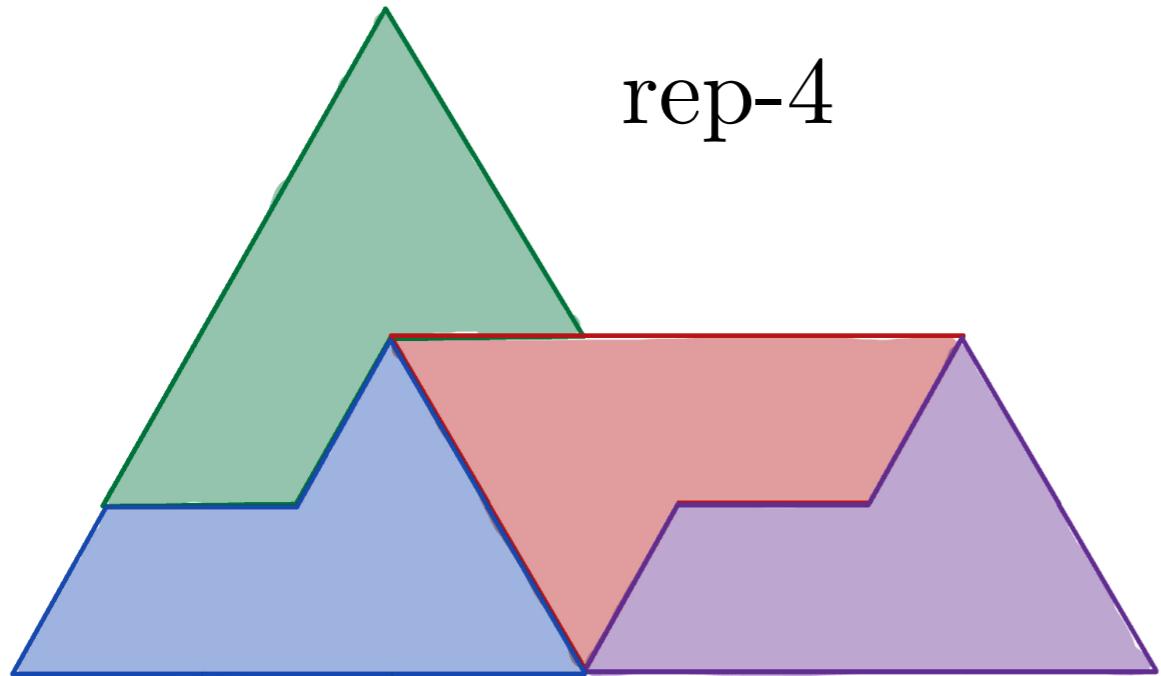
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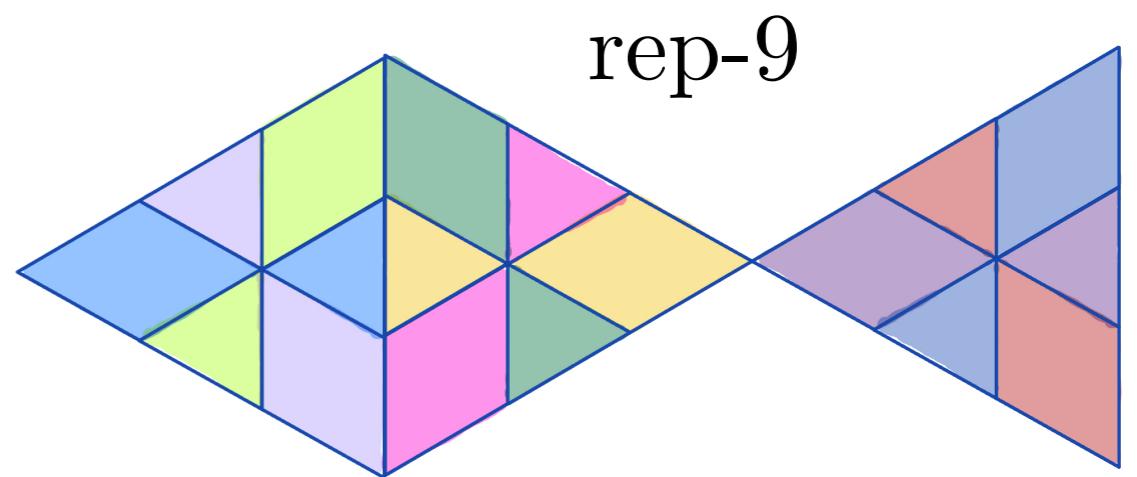
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rep-4

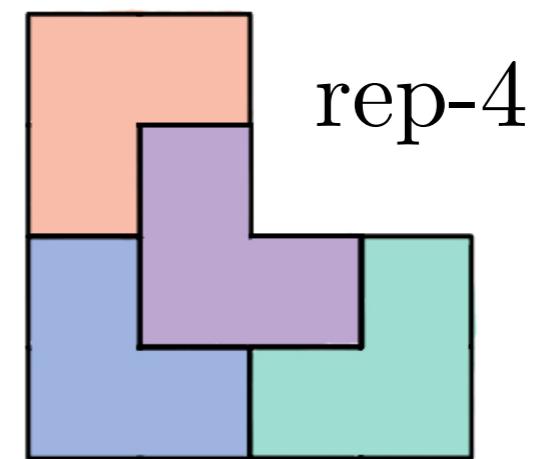
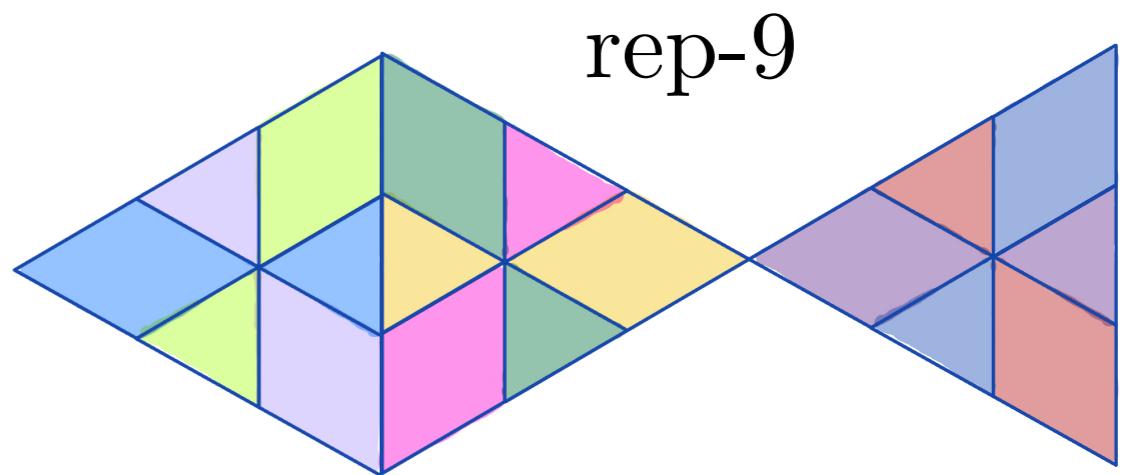
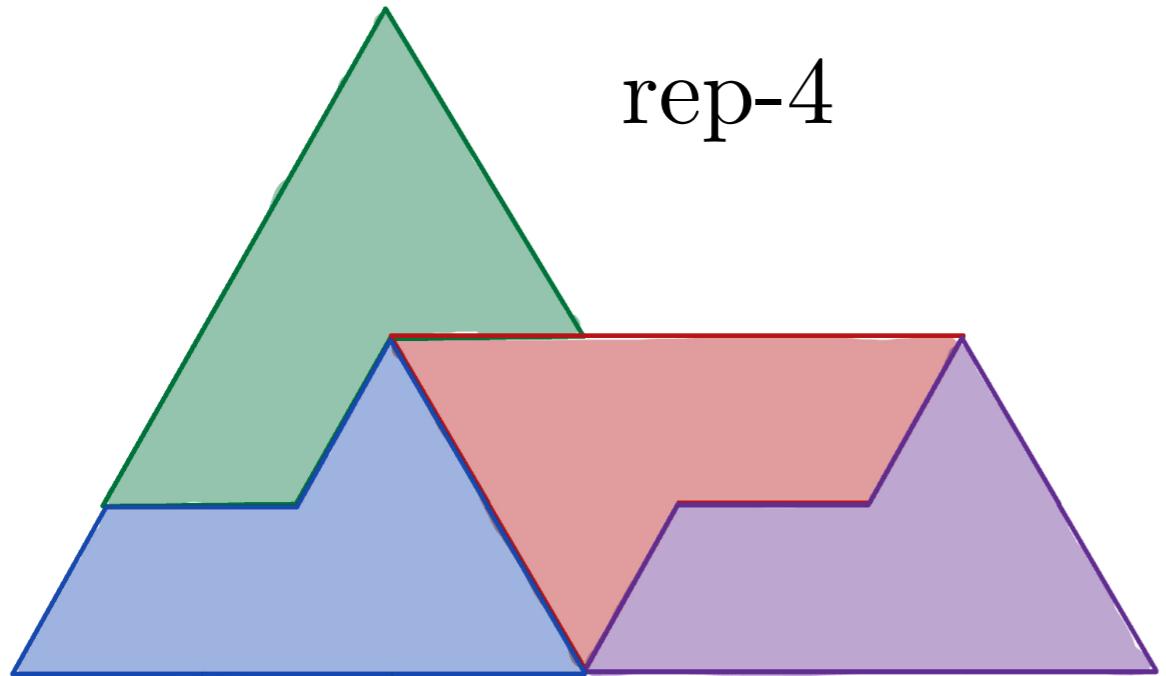


rep-9

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Multiplicity

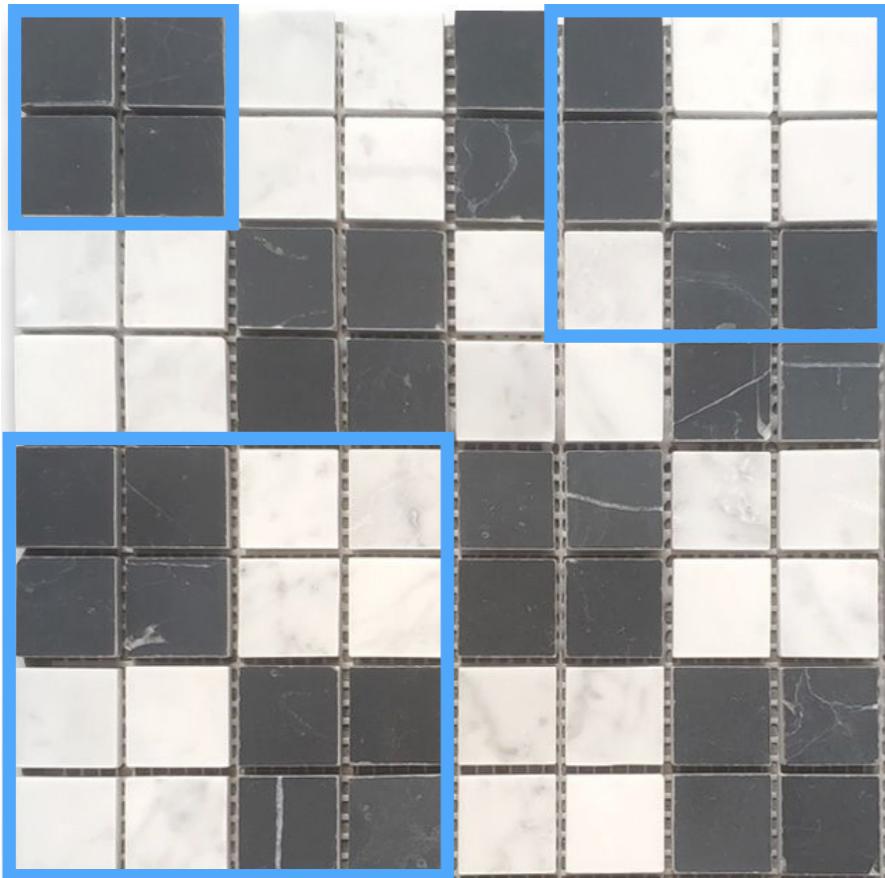
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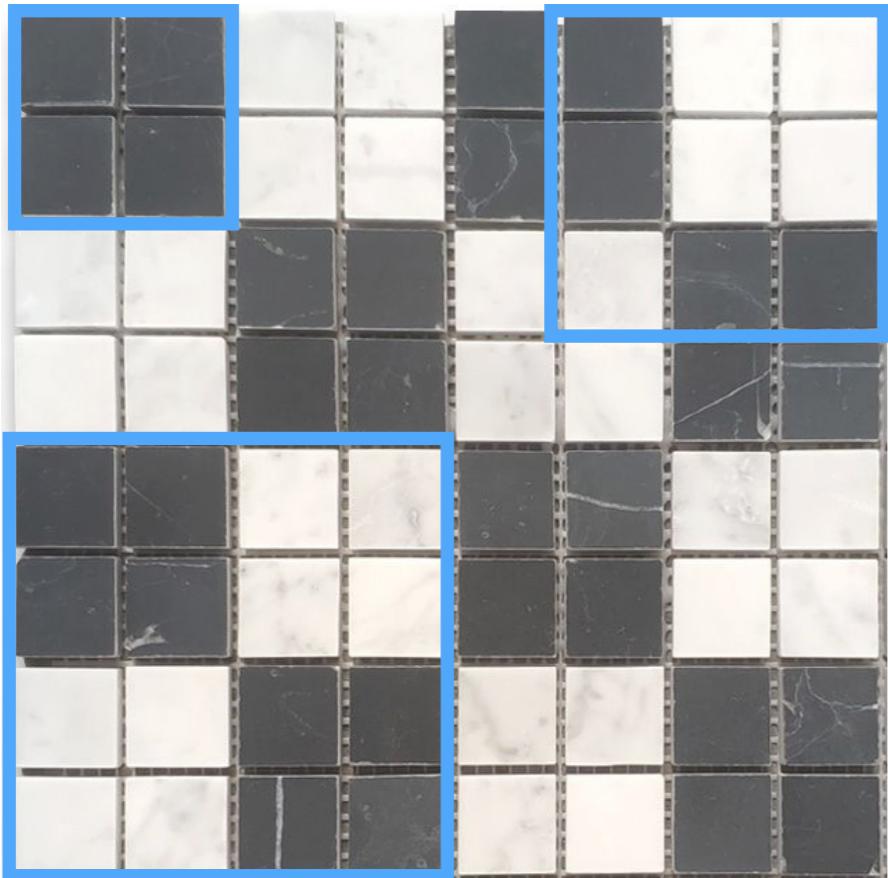


A square is rep-4, rep-9, rep-16,
... and rep- n^2 for every n .

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Question. Given k , is there a rep-tile with multiplicity k ?

Multiplicity

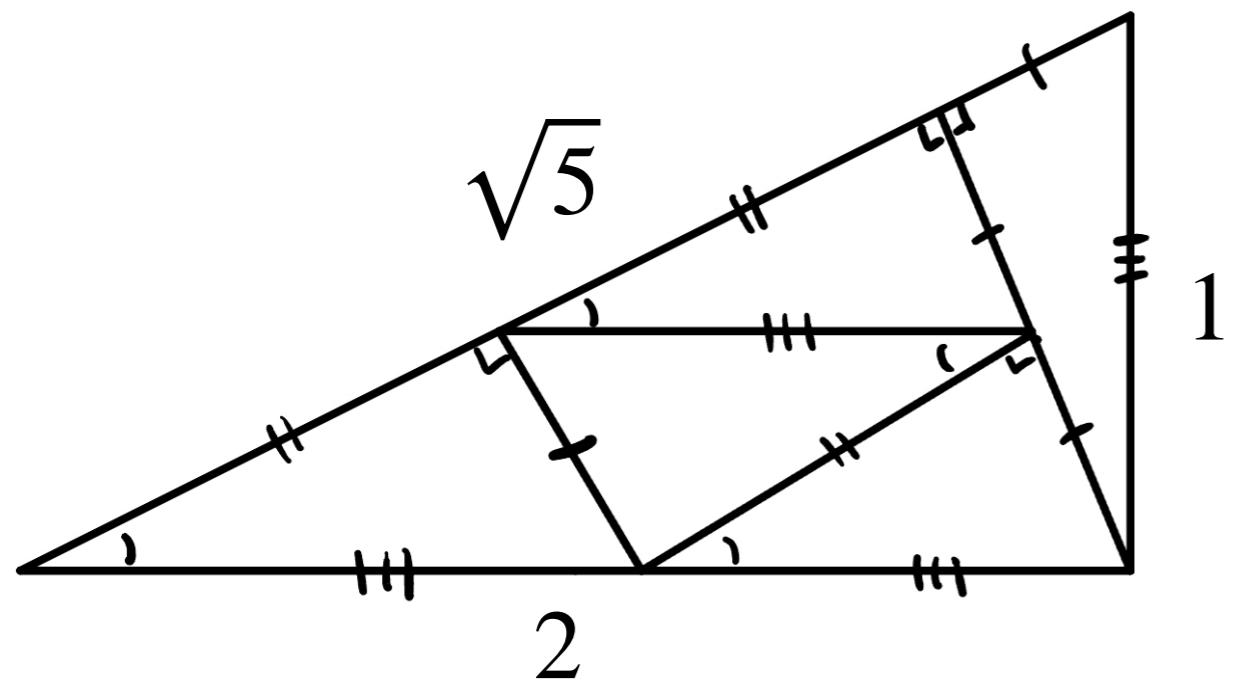
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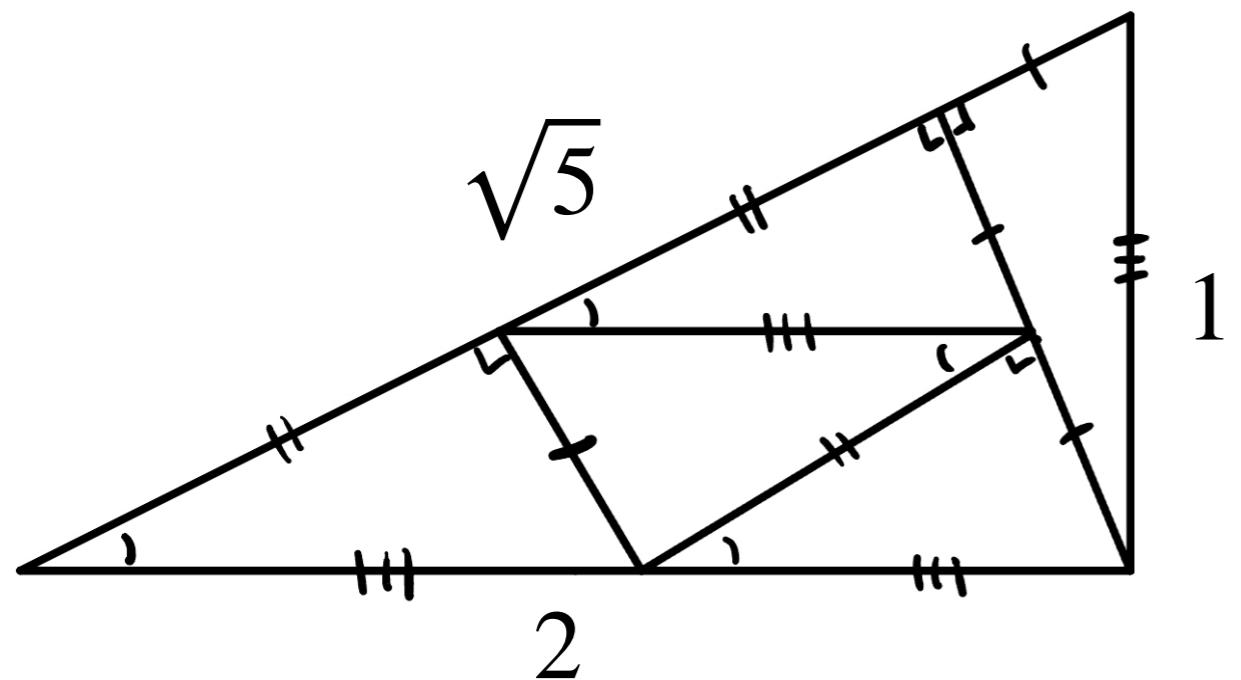
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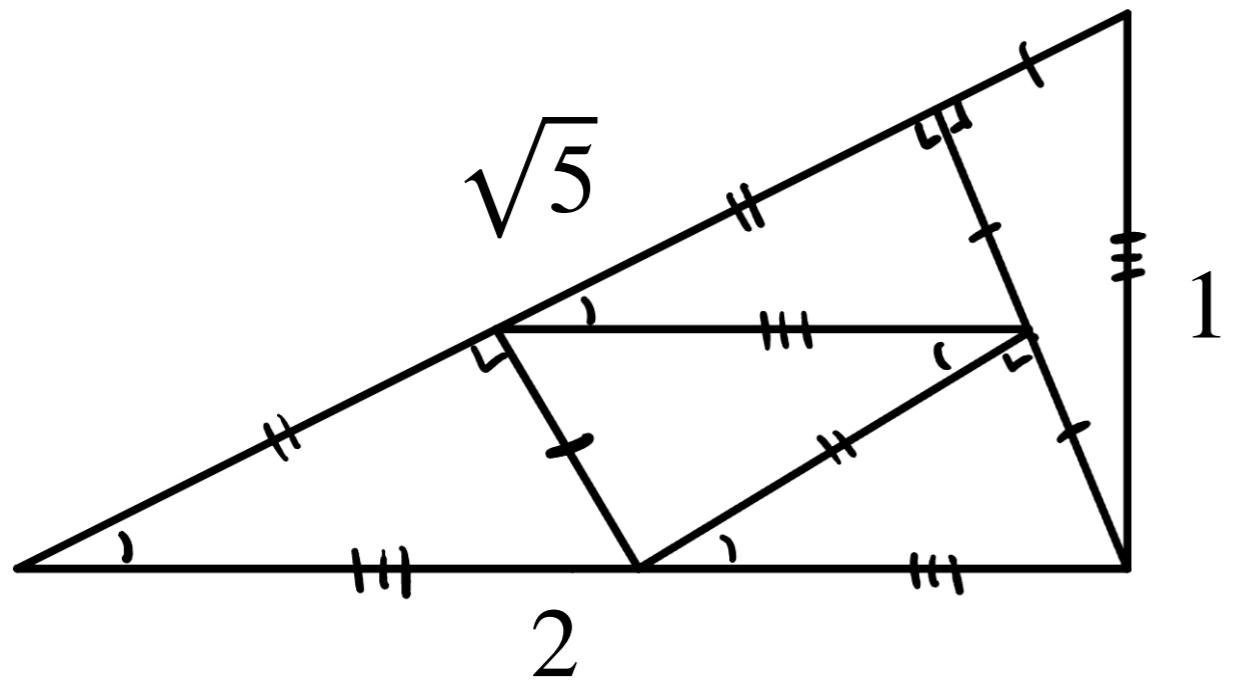


rep-5

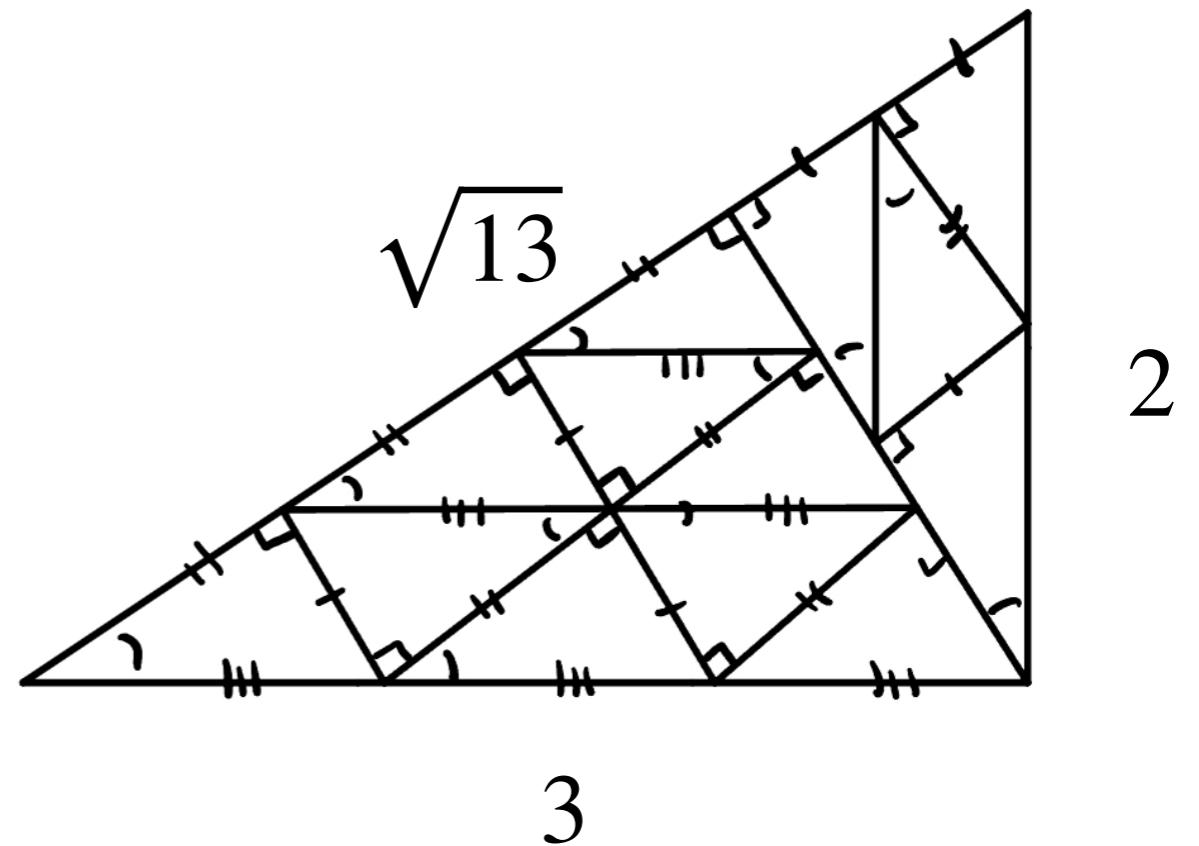
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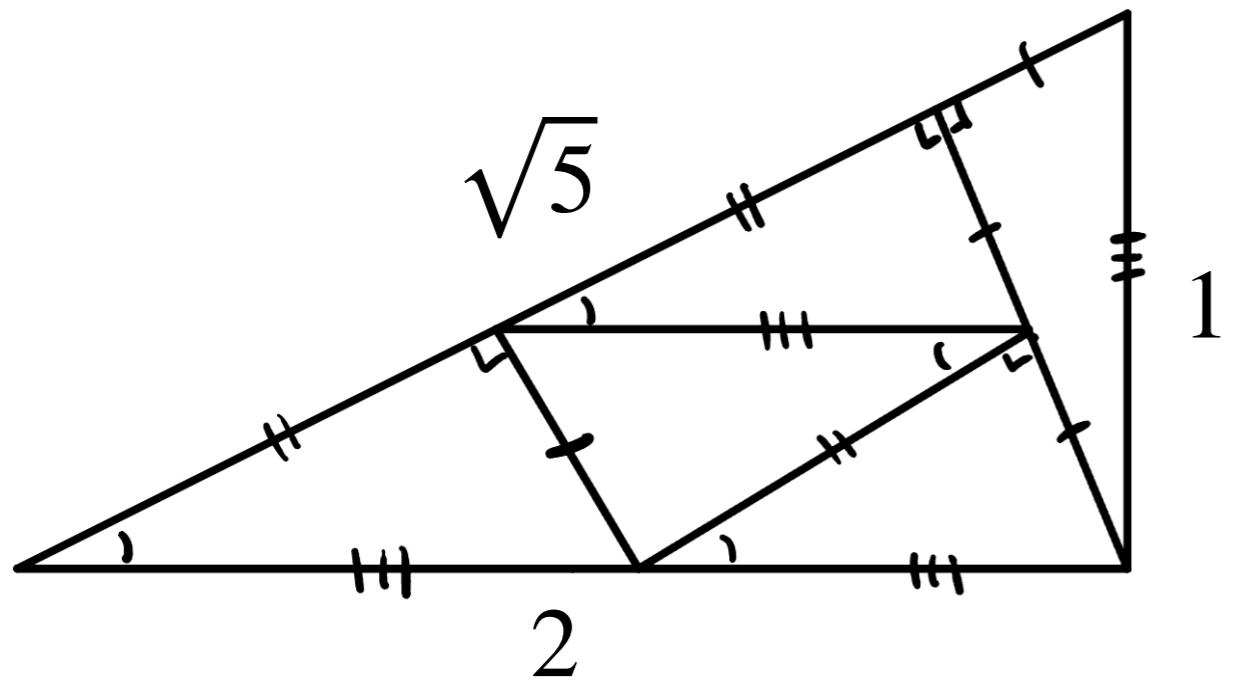


rep-13

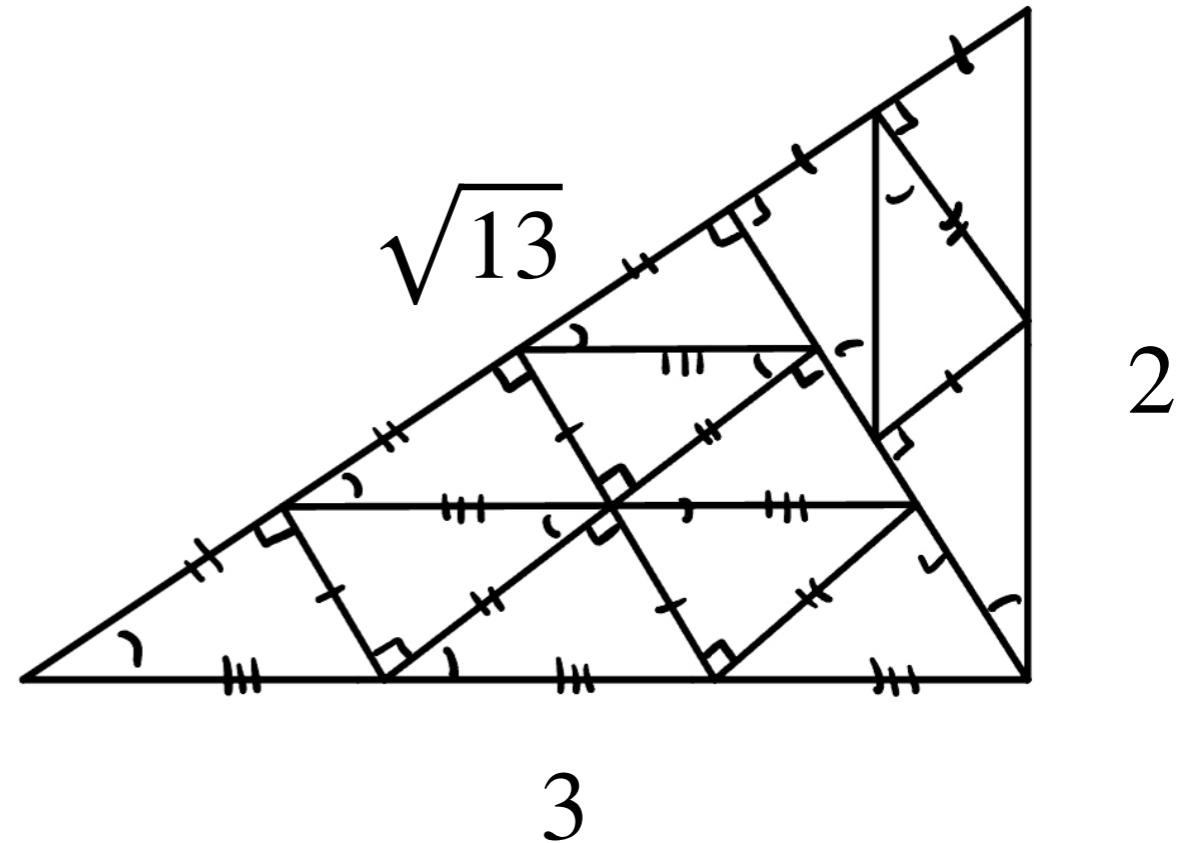
Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?



rep-5



rep-13

$k = a^2 + b^2$ prime: right triangle with sides (a, b, \sqrt{k}) is rep- k .

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

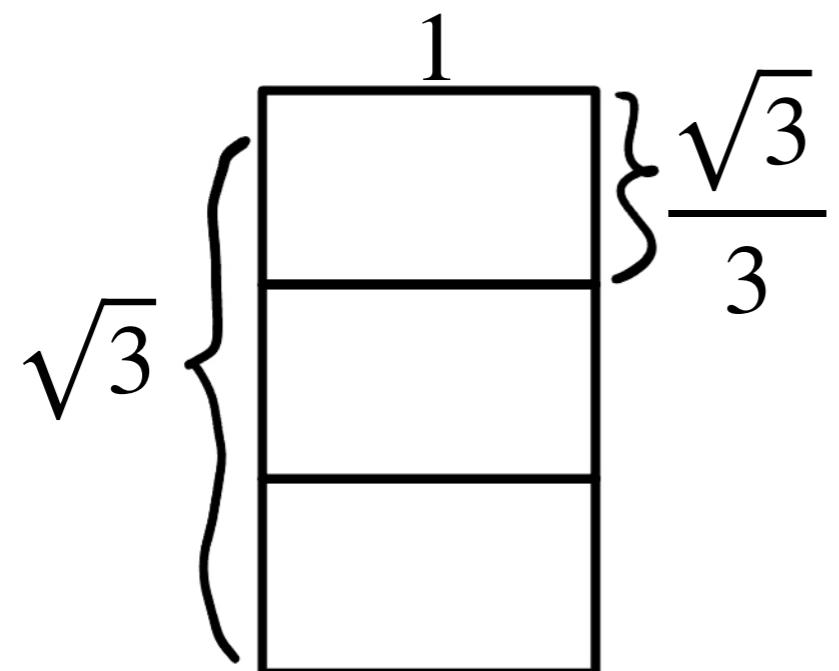
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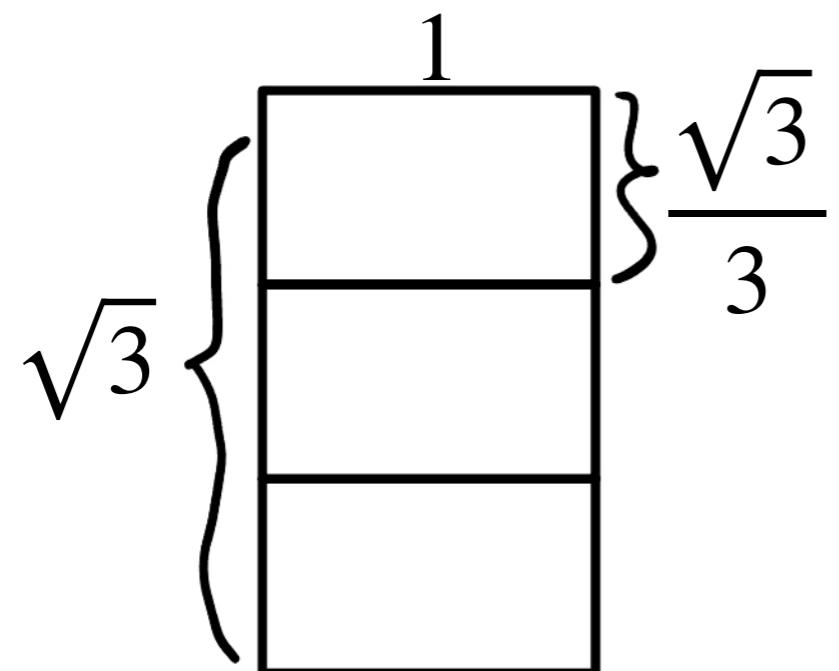


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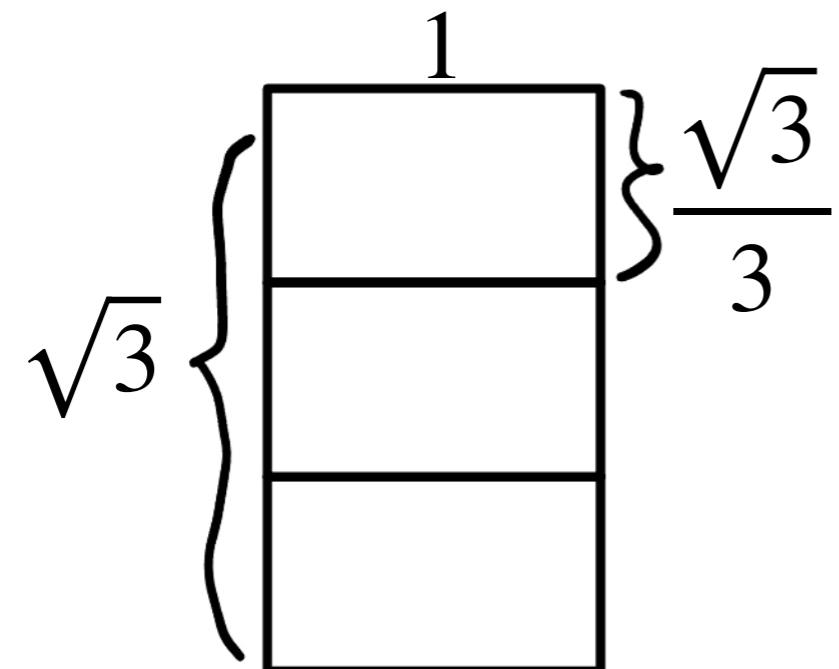
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for big rectangle:

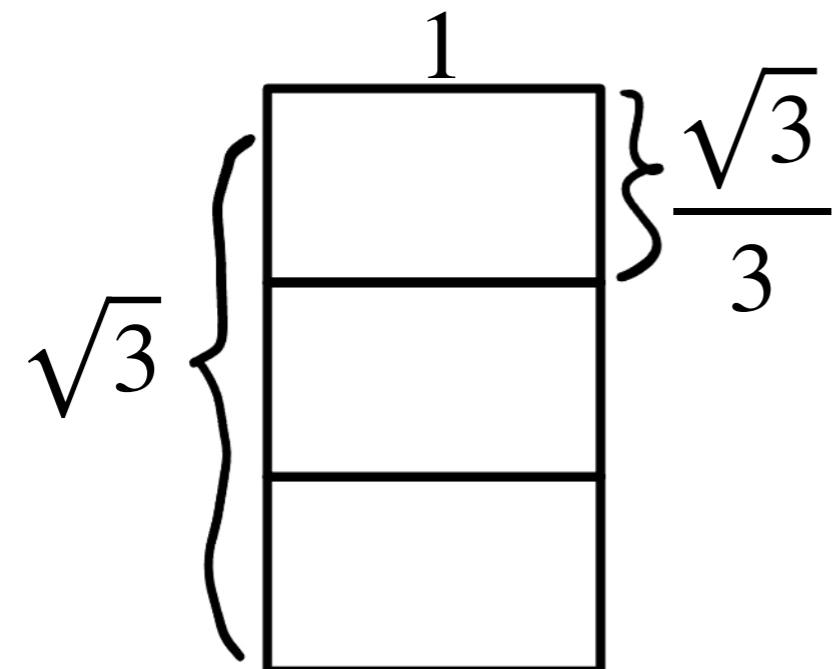
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To be a rep-tile, the big and smaller rectangles need to have the same side ratios.

for big rectangle:

$$\frac{\text{long}}{\text{short}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

for small rectangle:

$$\frac{\text{long}}{\text{short}} = \frac{1}{\sqrt{3}/3} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Paper rep-tiles

A0

841 X 1189 mm

A1

595 X 841mm

A3

297 X 420 mm

A5

148 X 210 mm

A6

105 X 148 mm

A4

297 X 210 mm

A2

595 X 420 mm

Paper rep-tiles

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Why these dimensions?

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Key property:
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This is ideal for
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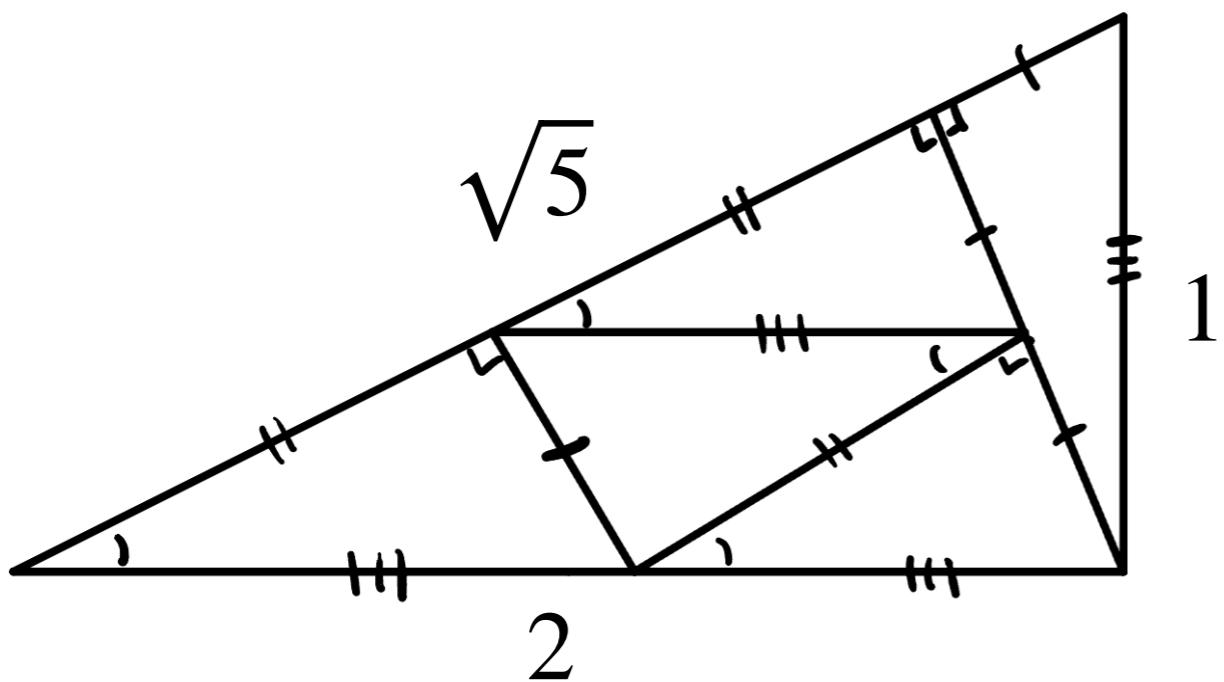
Open Question

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rep-5 reptiles:

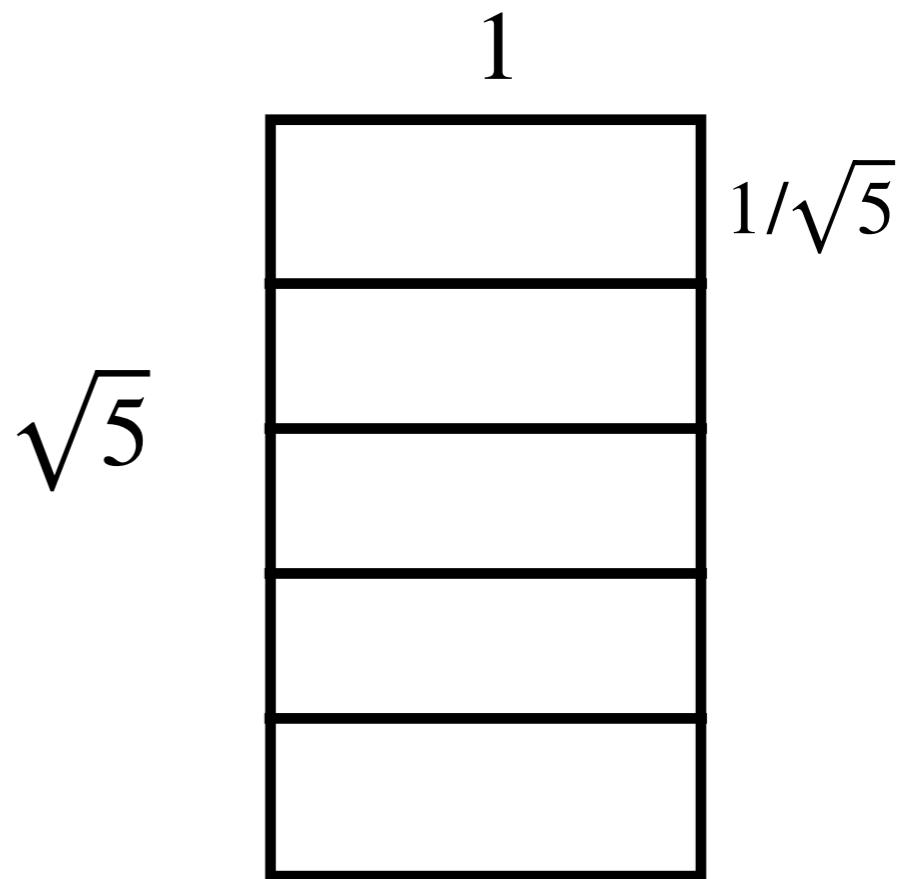
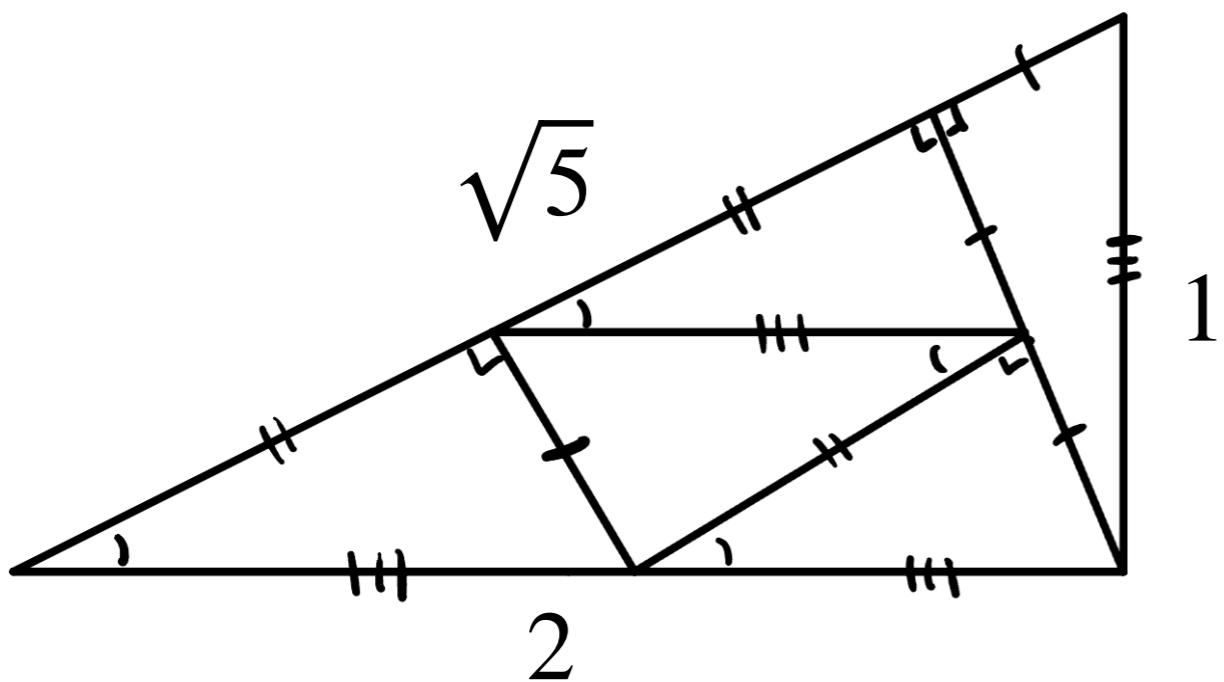
Open Question

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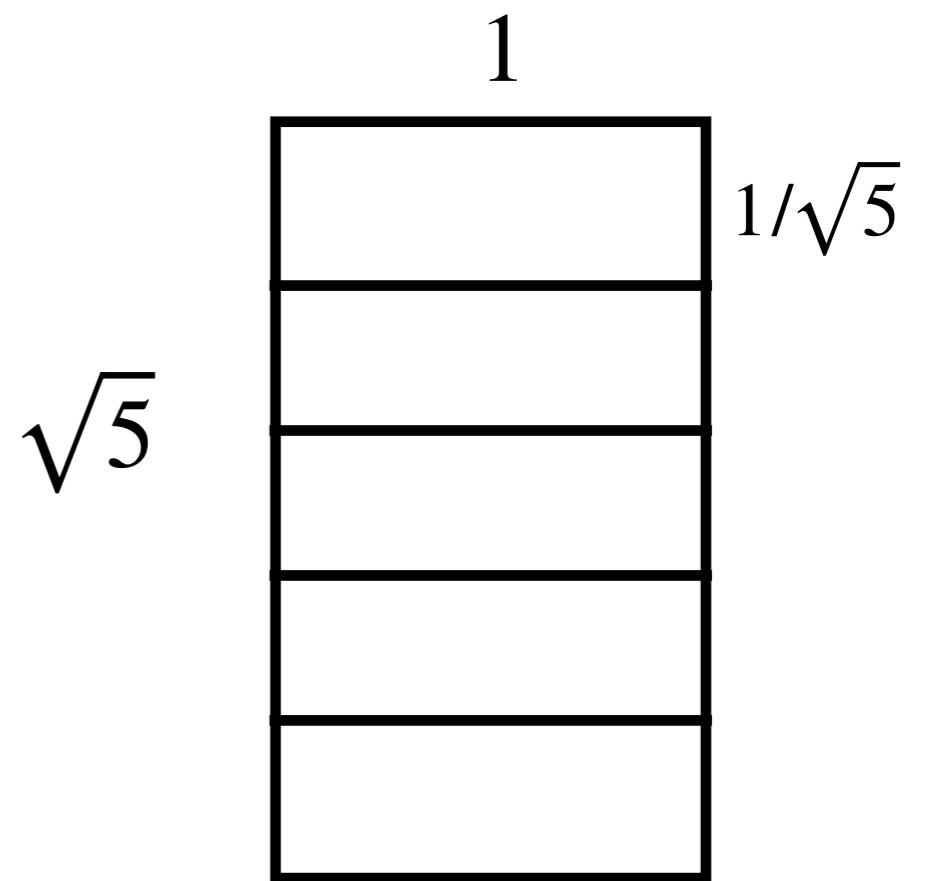
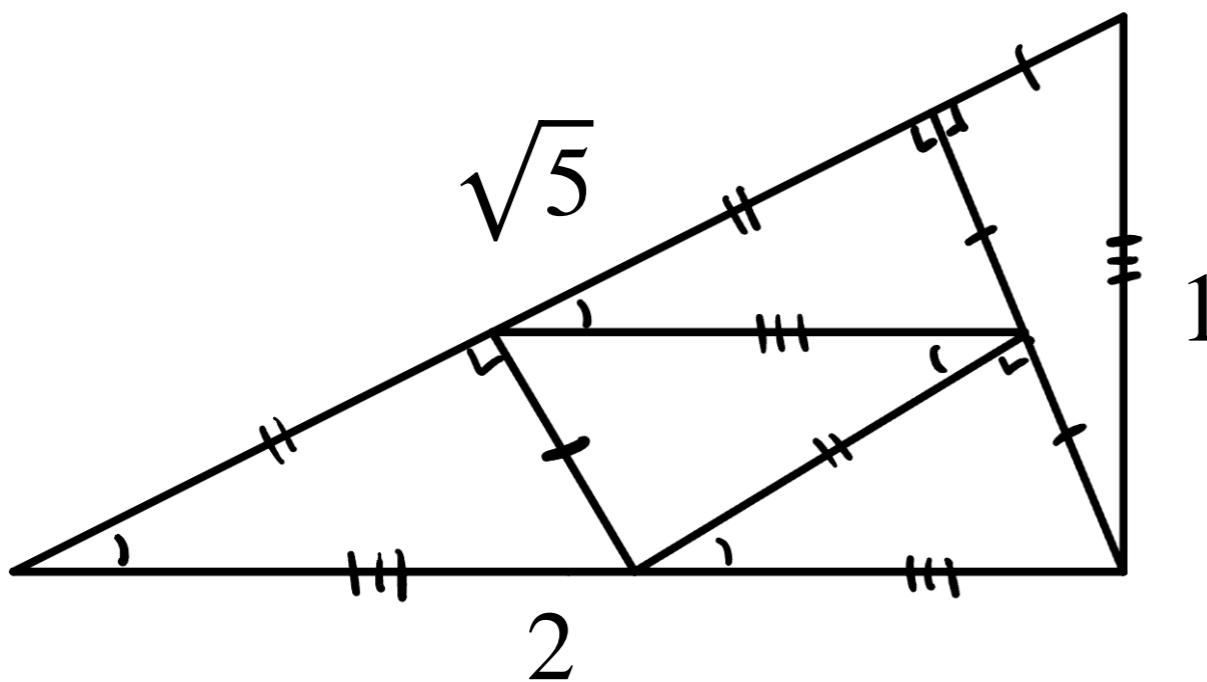
Open Question

rep-5 reptiles:



Open Question

rep-5 reptiles:



Is there any reptile of multiplicity 5 that's not a triangle or a quadrilateral?

Build your own rep-tile

Build your own rep-tile

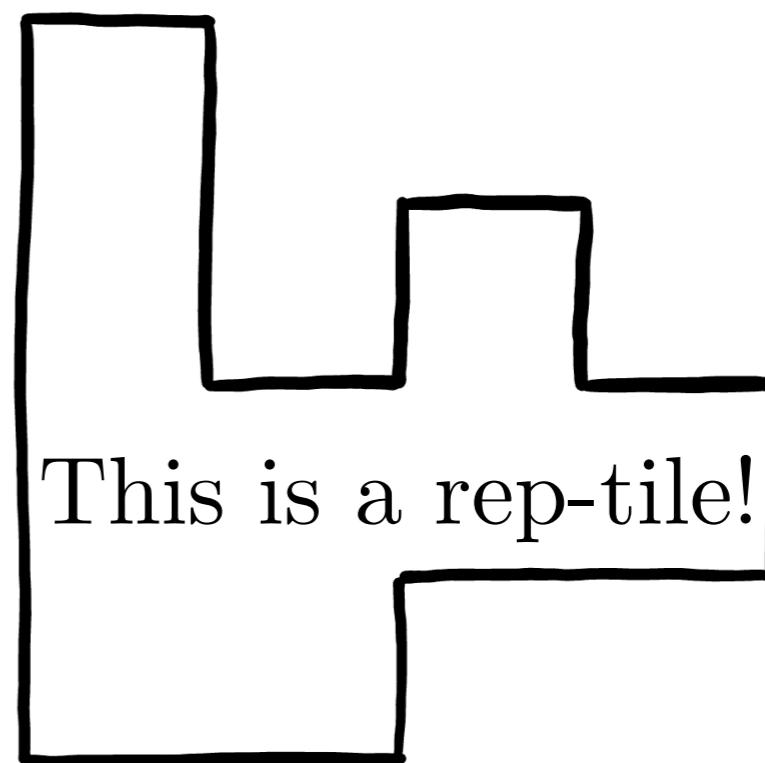
1D	4D	5D	3C	2C	1C
2D	6D	8D	7C	6C	4C
3D	7D	9D	9C	8C	5C
5A	8A	9A	9B	7B	3B
9A	6A	7A	8B	6B	2B
1A	2A	3A	5B	4B	1B

Step 1: Start with a square grid.
Choose one of 1a, 1b, 1c, 1d;
choose one of 2a, 2b, 2c, 2d;
etc.

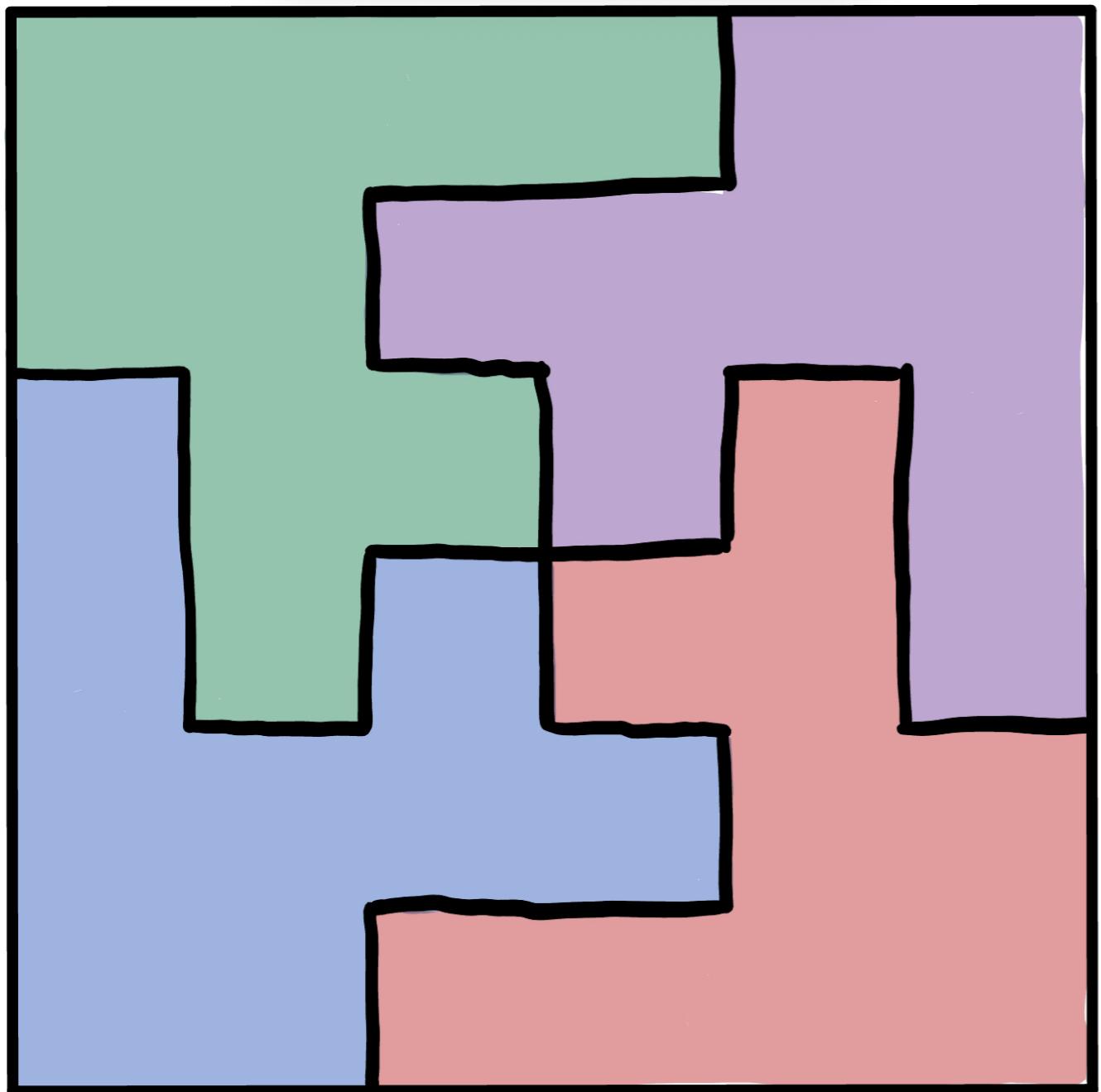
Build your own rep-tile

1D	4D	5D	3C	2C	1C
2D	6D	8D	7C	6C	4C
3D	7D	9D	9C	8C	5C
5A	8A	9A	9B	7B	3B
9A	6A	7A	8B	6B	2B
1A	2A	3A	5B	4B	1B

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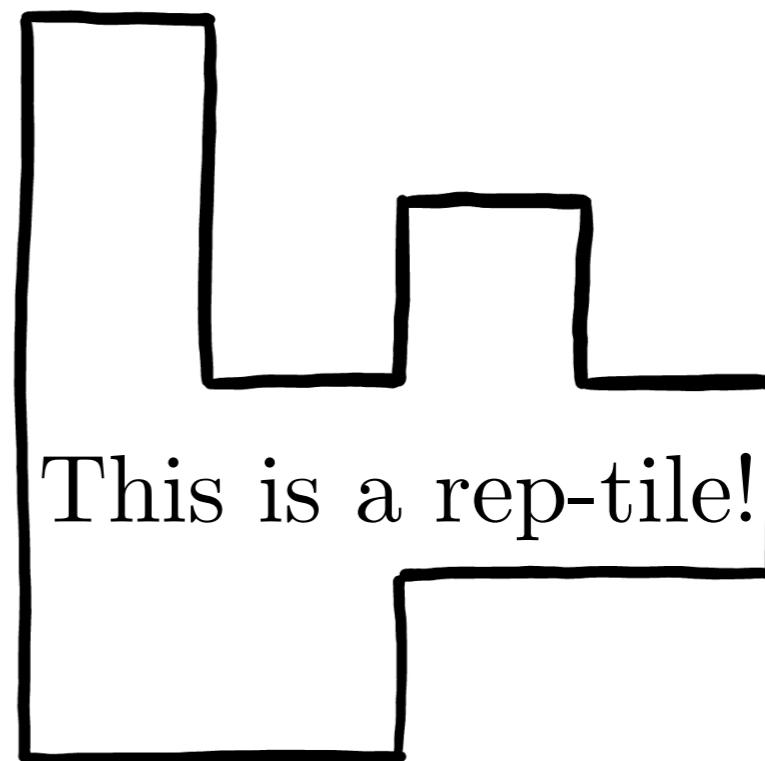
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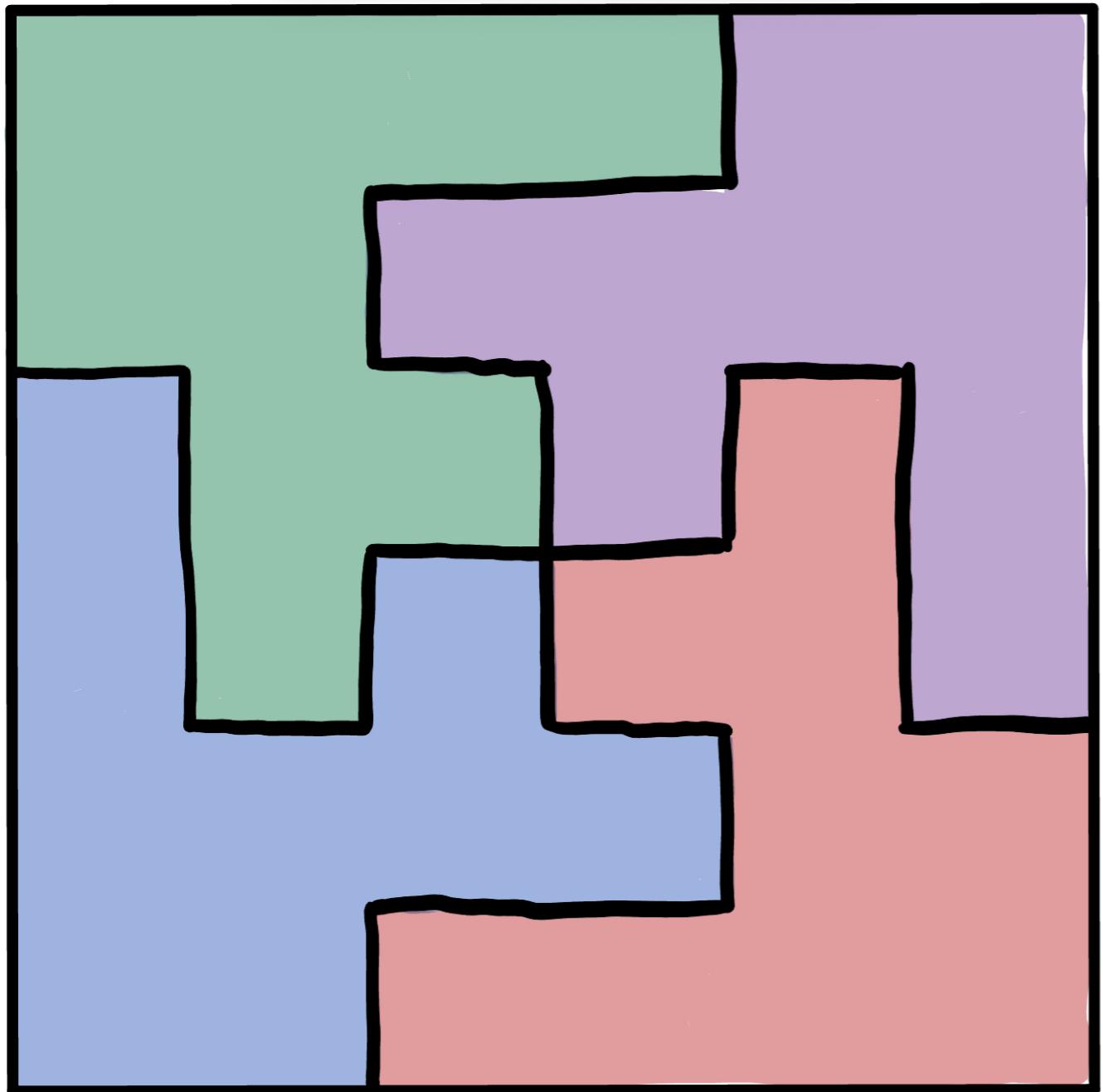
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The resulting shape tiles a square.



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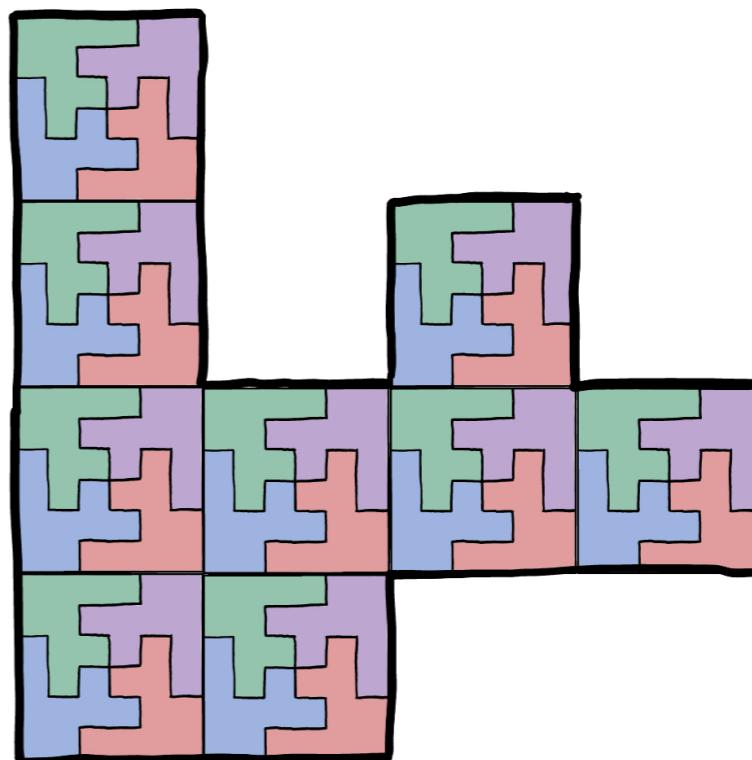
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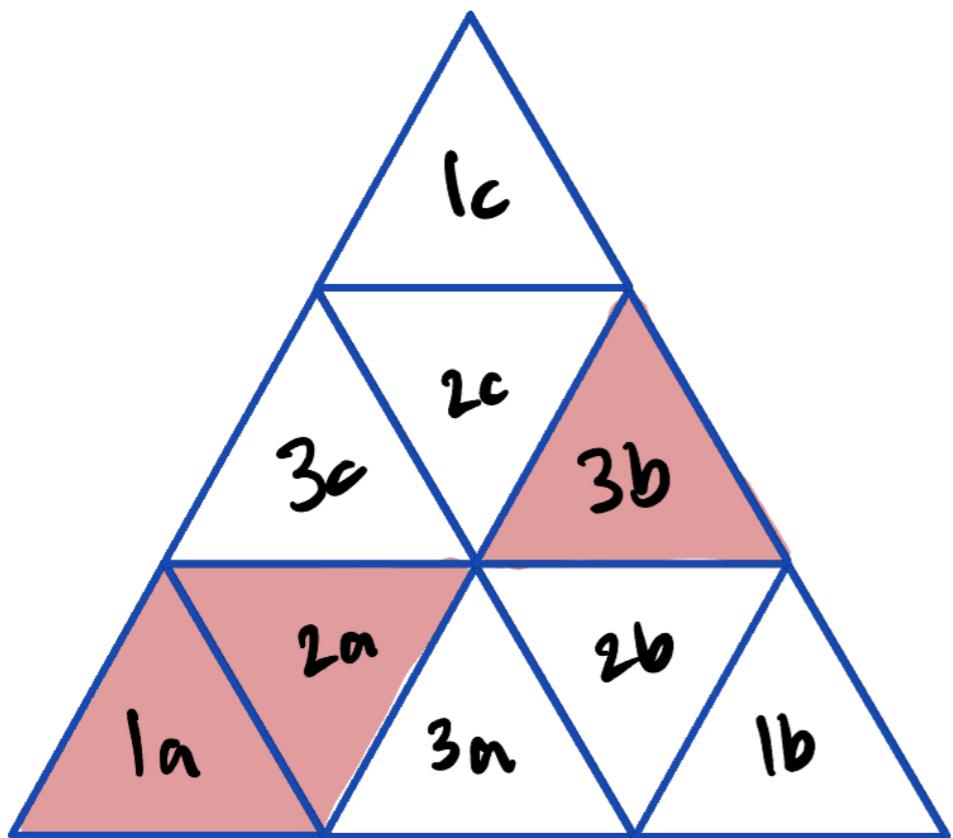
Step 3:

The square tiles the shape.



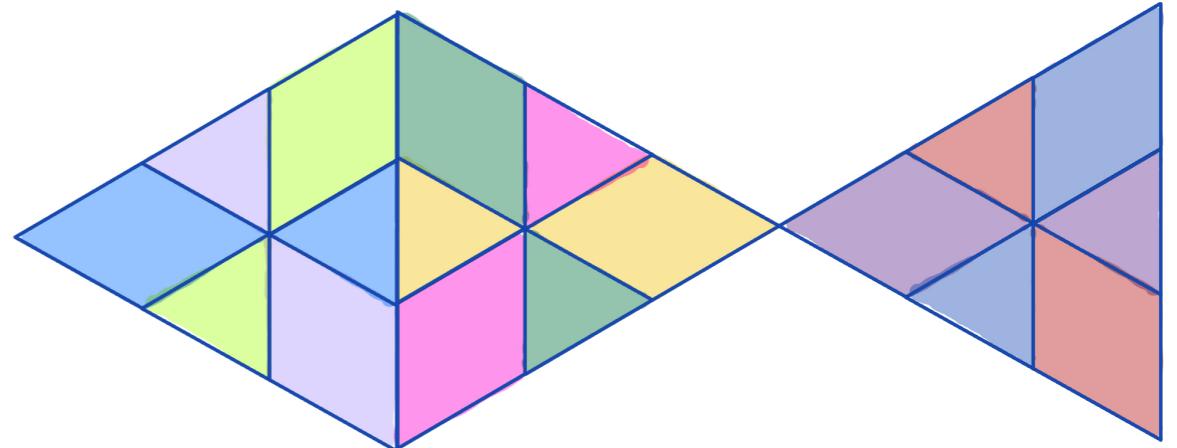
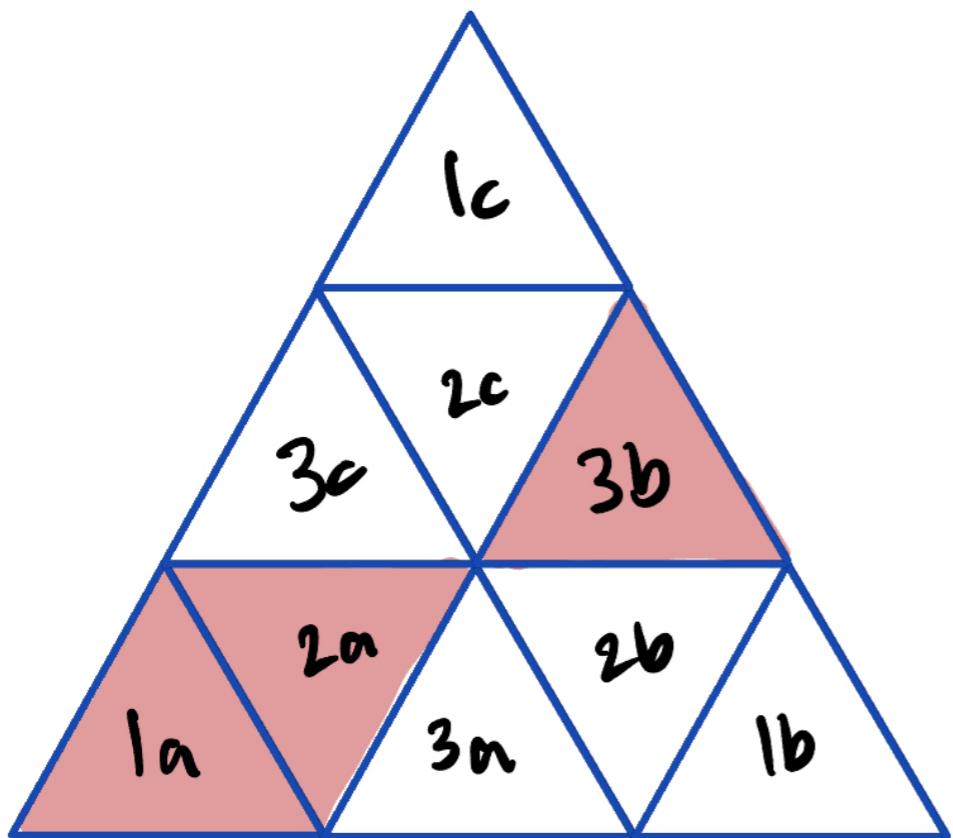
The fish, revisited

Remark: The same procedure recovers the fish!



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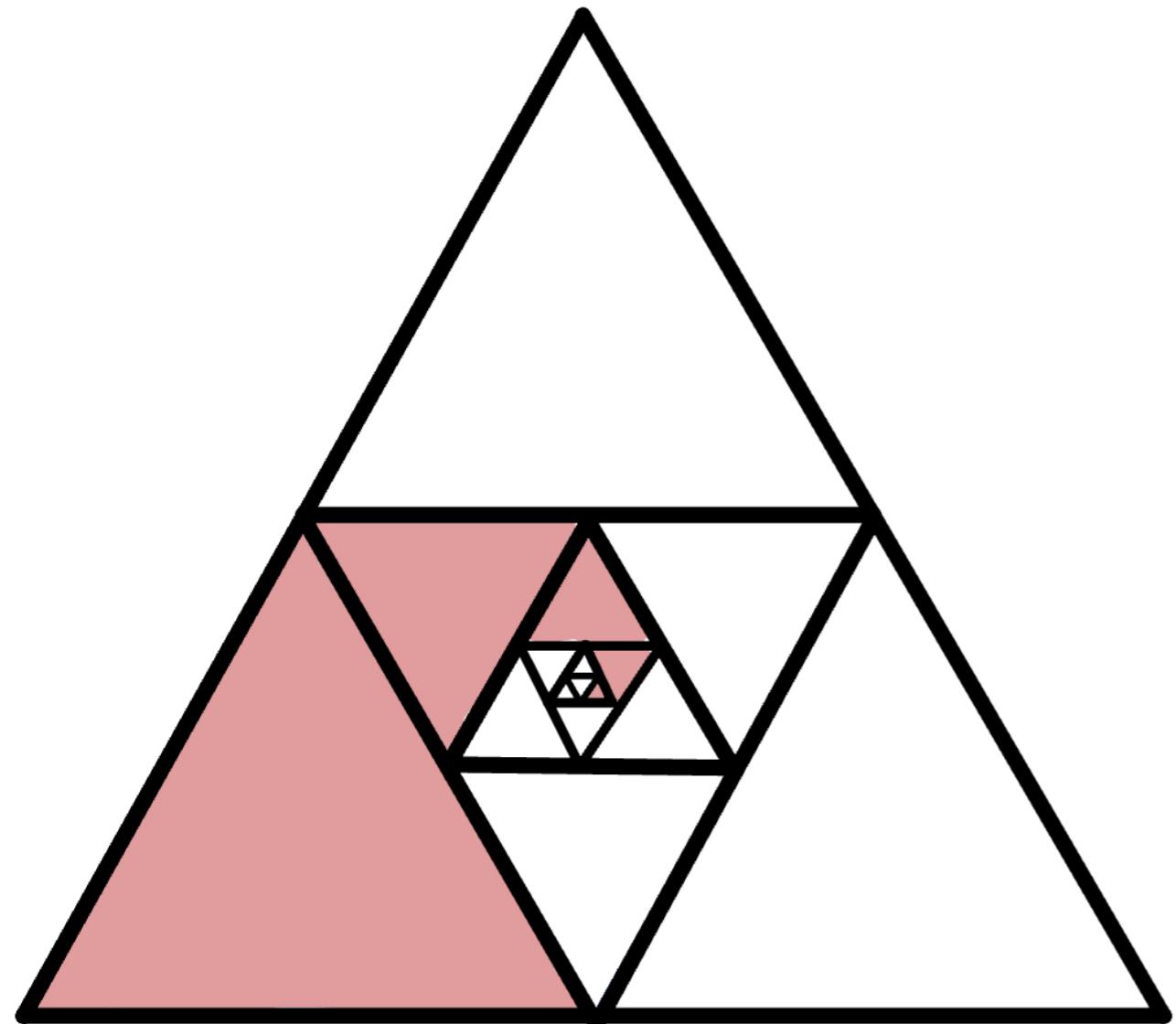


The elephant, revisited

A similar procedure recovers the elephant!

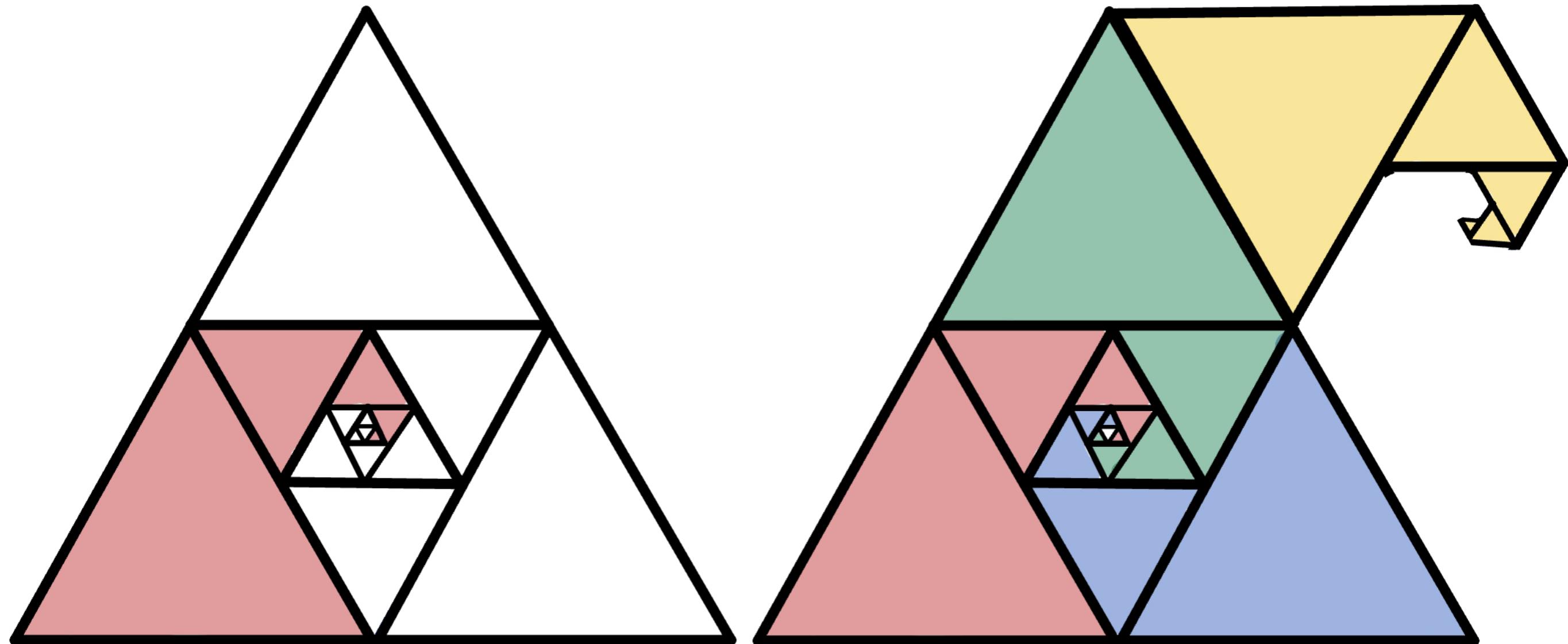
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The elephant, revisited

A similar procedure recovers the elephant!



Siphonaptera

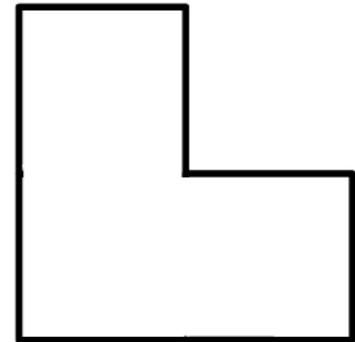
by Augustus De Morgan (1872)

Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

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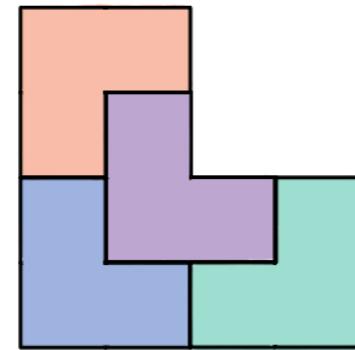


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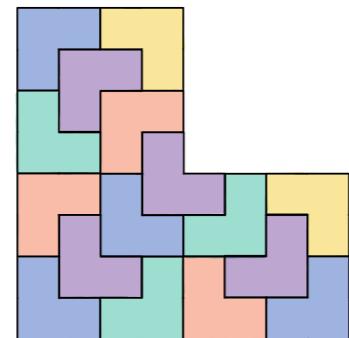


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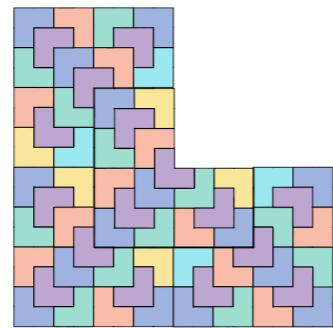


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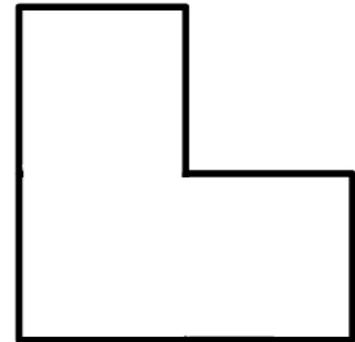


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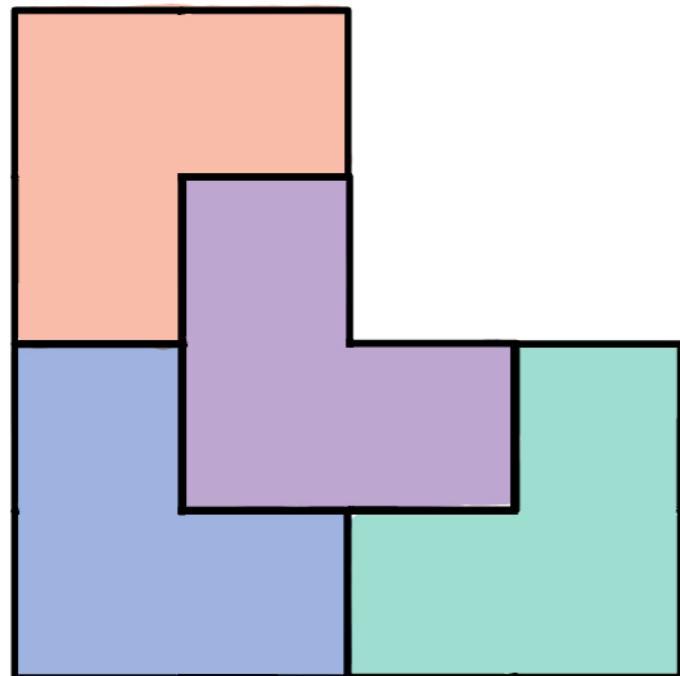


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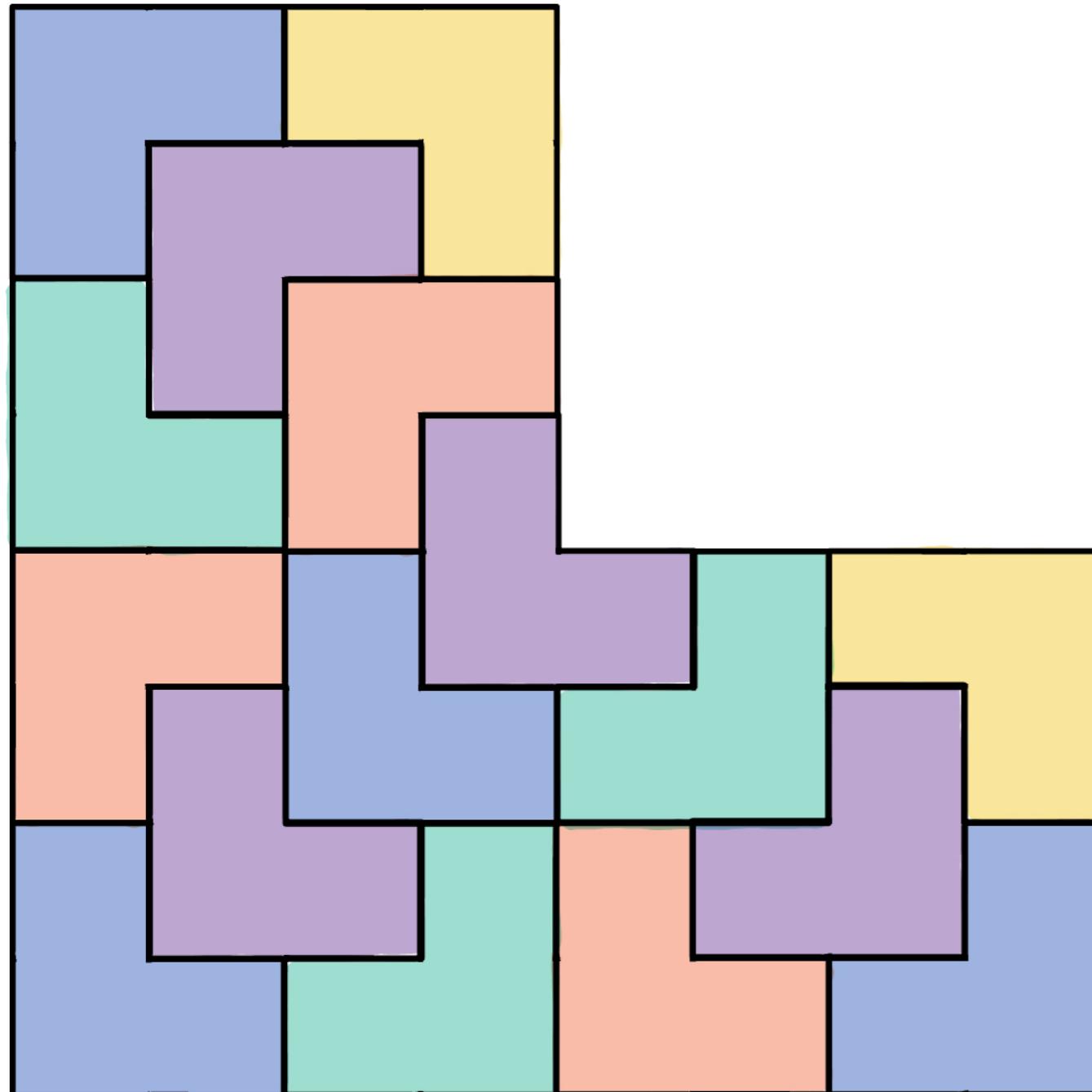


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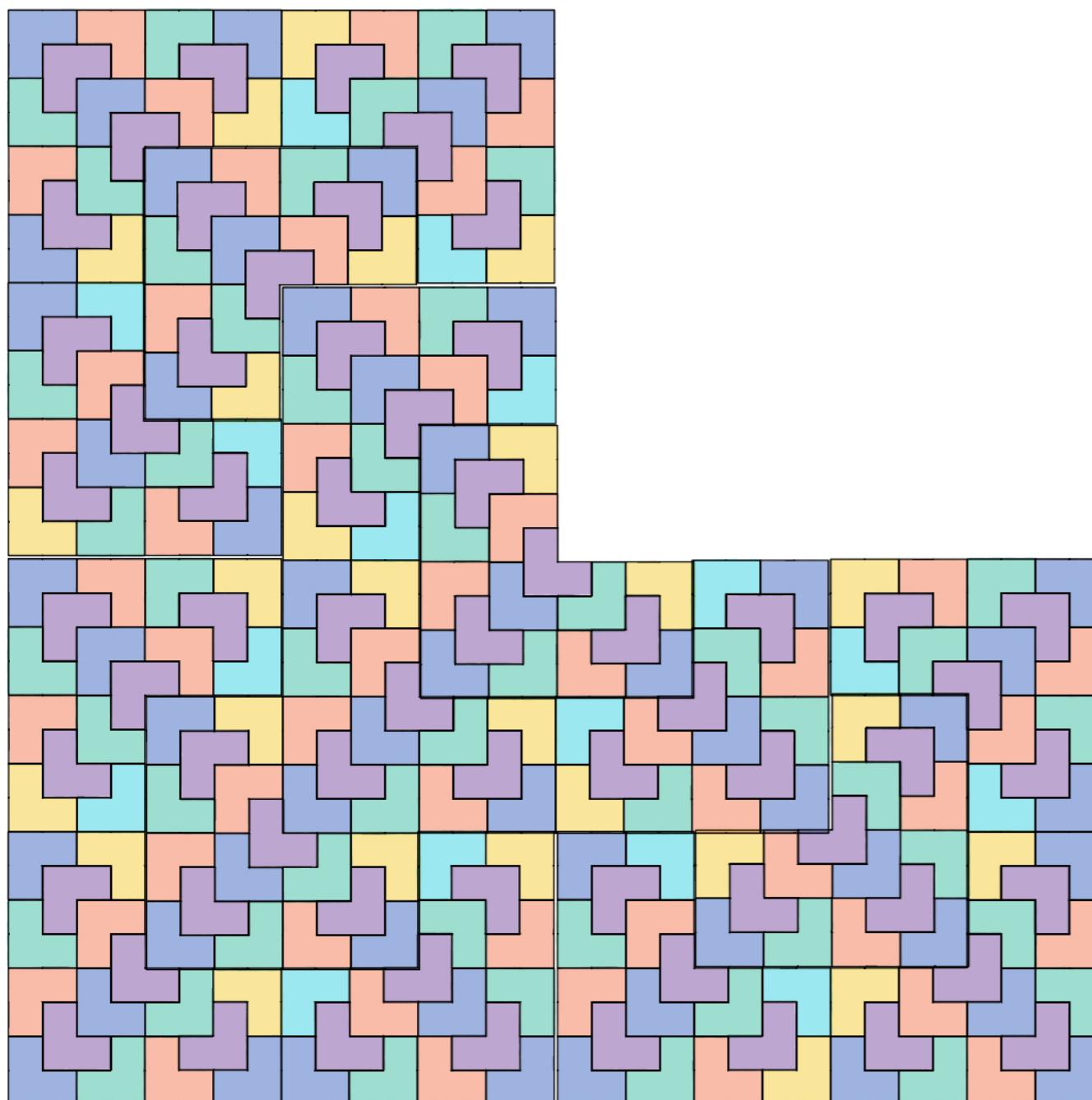


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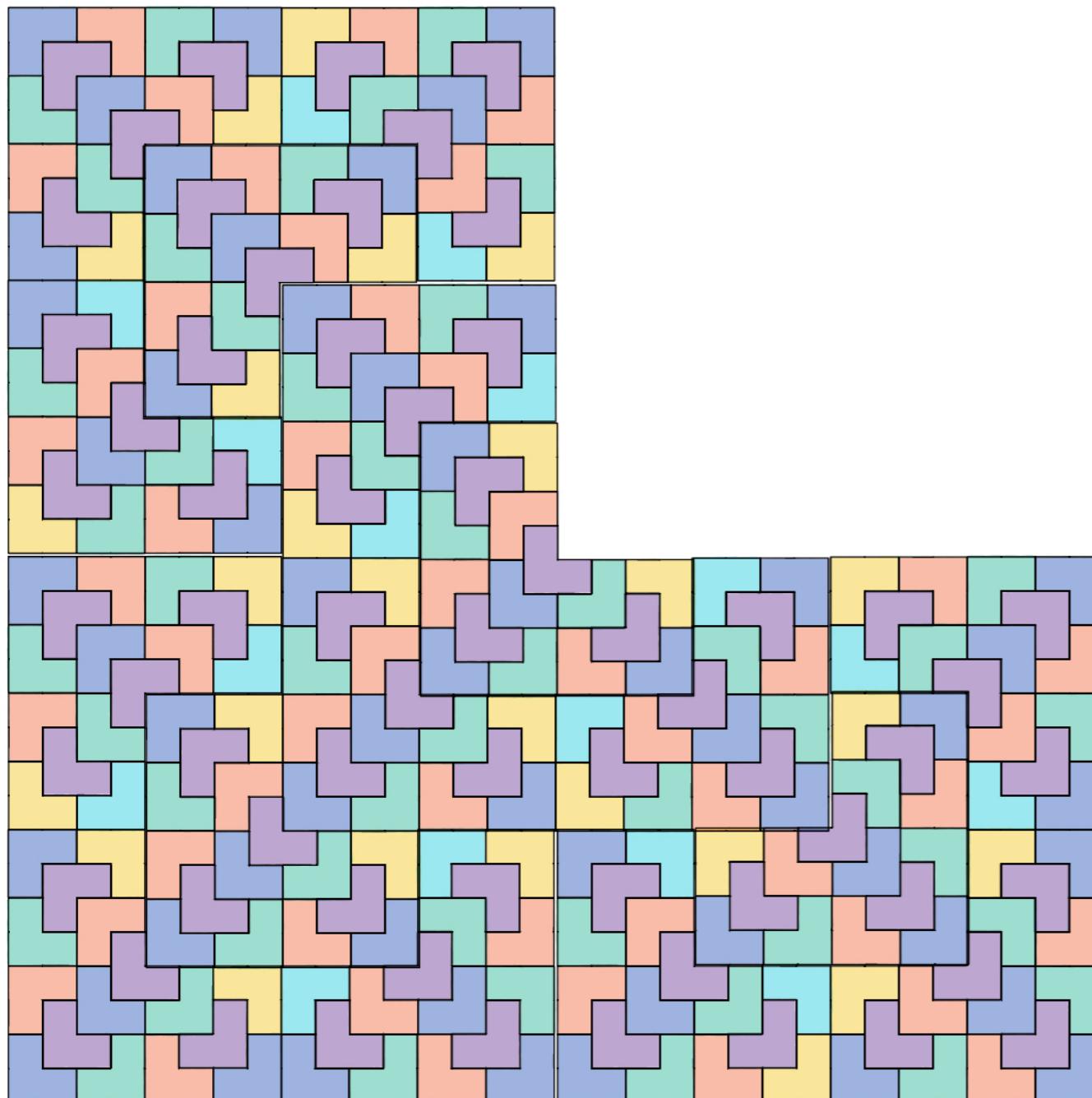


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Thank you