Midterm 1

Math 2420

Due Friday, March 7 by 5pm

This midterm is an opportunity for you to show what you've learned in this course so far. Please justify your answers. The exam is not meant to have any tricks or surprises. Please read everything carefully, including the instructions.

Instructions:

- Solutions must be submitted on Gradescope by Friday at 5pm. **Solve as many problems as you can.** Show your work. Partial solutions will earn partial credit.
- **You may use the book and your lecture notes. You may not use any other resources. You must work by yourself.**
- You may either submit handwritten or typed solutions. If you type your solutions, please make your own tex file. If you write by hand, please try to be neat.
- **To receive credit, you must include a page with your name, and the following academic honesty statement:** "I affirm that I have completed this exam without using any outside resources and that the work I am submitting is my own."
- If you have any questions, please ask via email. Good luck!

Problem	Points	Possible
1		15
2		15
3		5
4		15
Total		50

Problem 1. *Which of the following manifolds is $\mathbb{Z}$-orientable? Explain your answer.*

(a) $\mathbb{R}P^2 \# \mathbb{R}P^2$

(b) $\mathbb{R}P^3$

(c) $\mathbb{R}P^2 \times \mathbb{R}P^2$

Problem 2. *True or false. Be sure to justify your answer.*

(a) Every map $S^2 \times S^2 \to \mathbb{C}P^2$ has even degree.

(b) For a closed 4-manifold, if X is simply connected, then $H^*(X;\mathbb{Z})$ contains no torsion.

(c) Every manifold with fundamental group $\mathbb{Z}/3\mathbb{Z}$ is $\mathbb{Z}$-orientable.

Problem 3. *Give an explicit example of a cocycle representing a nonzero class in the image of the map $\mathrm{Ext}(H_{k-1}(X),\mathbb{Z}) \to H^k(X;\mathbb{Z})$ arising in the universal coefficient theorem, for some X, k.*

Problem 4. *Give an example or explain why no example exists.*

(a) A space X and $k \geq 0$ such that $H^k(X;\mathbb{Z}) = 0$ and $H_k(X) \neq 0$.

(b) A compact 3-manifold whose boundary is the Klein bottle.

(c) A manifold that is covered by $\mathbb{C}P^2$, other than $\mathbb{C}P^2$ itself.