GARCH Models and Applications

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1 Abstract

This paper will investigate both the theoretical underlying of what a GARCH model is trying to measure as well as potential real applications of the family of models. It will also focus on determining which version of the standard GARCH model works the best in predicting the future volatility of various different asset classes. This will involve first understanding the components of the GARCH model (formulation, properties etc.), researching its potential applications in predicting volatility clustering in specific markets, then implementing it into code with data to test it on. Finally, statistical testing will be used to compare the different GARCH models between both each other as well as other volatility models (EWMA).

2 Definitions

An ARCH model uses past returns along with a constant long term volatility term to model future volatility. It is described in the form ARCH(p) where p is the number of lags and can be written as

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 ... + \alpha_q \epsilon_{t-q}^2$$

A GARCH model is known as the "generalized ARCH model" and accounts for past variance in addition to past returns (like ARCH) and a constant to estimate future volatility. It can be written in the form of GARCH(p,q) which is expressed with

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \alpha_q \epsilon_{t-q}^2 + \beta_p \sigma_{t-p}^2$$

where ϵ_t is previous return at time t, σ_t is the previous variance at time t, and ω, α_i and β_i are coefficients > 0

These coefficients are commonly found using the statistical concept of maximum likelihood estimate (MLE), which is the value of an arbitrary parameter θ

that maximizes the probability of getting the observed data (Penn State, n.d.). It can be defined as

$$L(\theta) = P(X_1 = x_1, ...X_n = x_n) = f(x_1;\theta) \cdot ...f(x_n;\theta) = \Pi_{i=1}^{n} f(x_i;\theta)$$

where $f(x_i;\theta)$ is the probability density function of each X_i.

What both of these models are essentially trying to do is predict a future volatility by using past returns and a constant long term volatility term (or in the case of GARCH, past returns, past volatility, and a constant long term volatility term). The GARCH model will be the main focus of this paper due to its unique ability to capture long term volatility patterns as well as model clustering, characteristics observed in financial markets (Bollerslev, 1986). It will also be compared to the Exponentially Weighted Moving Average model, another common method of volatility forecasting. It is fair in some ways to think of the $EWMA(\lambda)$ model as a GARCH(1,1) with $\omega = 0, \alpha = 1 - \lambda, \beta = \lambda$ (Hull, 2021). It can be modeled as

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1 - \lambda)\epsilon_{t-1}^2$$

where λ is the exponential smoothing parameter and $0 < \lambda < 1$

3 Setup

The coding portion of this research project will be explained on a high level, however the Github repo (`https://github.com/bena777/GARCH-ResearchProject`) with the actual code used for testing is linked. The Python programming language was used to impliment this project along with the libraries "yfinance" to access equity data, "arch" to fit the models, as well as "matplotlib" and "statsmodels" to display results.

I chose to analyze three different assets for this project, each in a different asset class. The S&P 500 to represent equities, WTI Crude Oil to represent commodities, and the EUR/USD currency pair for foreign exchange markets. I chose these three assets due to them being some of the most liquid in their respective asset classes. In particular, I chose oil due to its unique phenomenon such as extreme volatility and negative prices. The time frame used in this analysis will be from 1/01/2006 to 1/01/2024. The period of 1/01/2006-12/31/2019 will serve as the training partition while 1/01/2020-11/01/2024 will be the testing.

4 Process

The base program of this project starts by fitting a GARCH(1,1) model to the asset it is modeling with all the data from the training phase. Thankfully, Python's "arch" module takes care of the dirty work of parameter estimation using the aforementioned MLE. After the model is properly fit, its parameters are plugged into the "garch_generalized" function for whatever model is currently being tested. It predicts the volatility for the duration of the testing phase. The predicted results are then compared to the actual results during the testing phase and summed up in the form of a mean squared error statistic. This process is then repeated for different GARCH models of various lengths. We also are able to describe the predicted volatility model's 95% confidence interval by assuming a normal distribution with a mean return of 0 and multiplying each predicted value by the appropriate z score ($z = 1.96$).

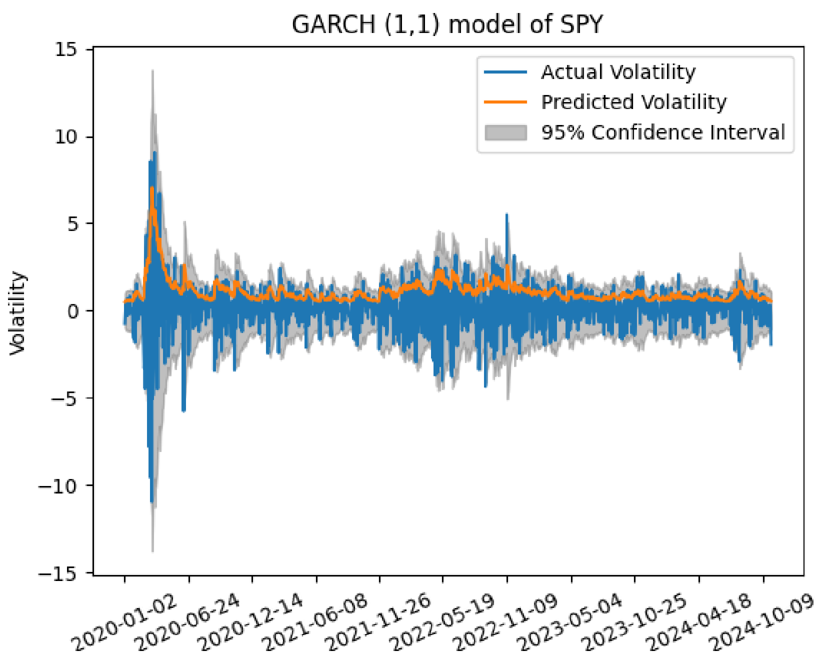

Figure 1: A visual representation of the predicted volatility and confidence interval of a GARCH(1,1) model

5 Results

The first thing that was abruptly clear was that the order of magnitude of the GARCH model does not really change the accuracy of the prediction all that

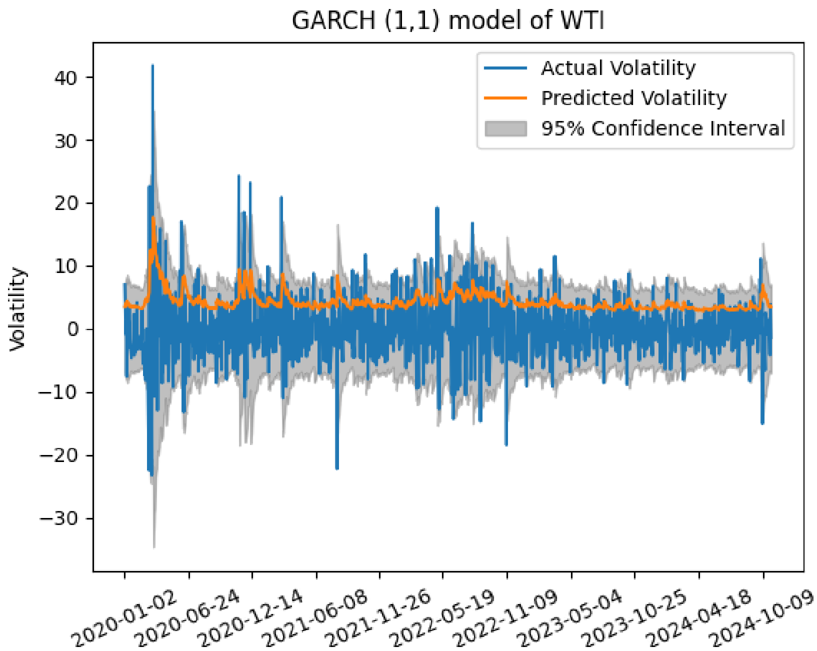

Figure 2: GARCH(1,1) of WTI

much. The mean squared error of different models (figure 3) for the same underlying asset were extremely similar with some minor fluctuations. Because of this, in the majority of cases it is practical to simply use the GARCH(1,1) model. This further confirms the academic consensus of it being the most widely used variation of the classic GARCH model (Jafari et al., 2007). Models can all be looked at from the trade off of complexity vs accuracy, meaning that in this case, the small improvement in accuracy is not worth the exponential increase in complexity.

Model	SPY	WTI	EUR/USD
GARCH(1,1)	3.3495	43.5812	0.4351
GARCH(1,2)	3.3493	43.4785	0.4345
GARCH(2,1)	3.4182	42.4949	0.4360
GARCH(2,2)	3.3525	43.5926	0.4345
GARCH(3,3)	3.3531	43.4687	0.4346
EWMA(0.95)	13.8809	1050.764	0.2746

Figure 3: Mean squared error results for volatility forecasts of SPY, WTI, and EUR/USD using different models.

Another interesting result that was observed was the differing levels of accuracy of the model depending on which asset it was being used on. As shown in figures 2 and 3, the GARCH model does not nearly do as good of a job of predicting oil volatility as it does with the S&P 500. On the other hand, it tends to do a better job predicting the EUR/USD futures volatility. This shows the possibility that the GARCH model is pretty accurate in regards to predicting assets with low-mild volatility, but it breaks down more when high volatility assets are thrown into the mix.

The EWMA model grossly overestimated volatility during extreme periods while performing moderately well during "normal" periods (figure 4). It seems that the model is very sensitive to large price movements which leads it to create unrealistic estimates at times. This was problematic when it was modeling assets that often have crazy jumps like WTI. On the other hand, it performed even better than the GARCH models when dealing with the lower volatility forex trade.

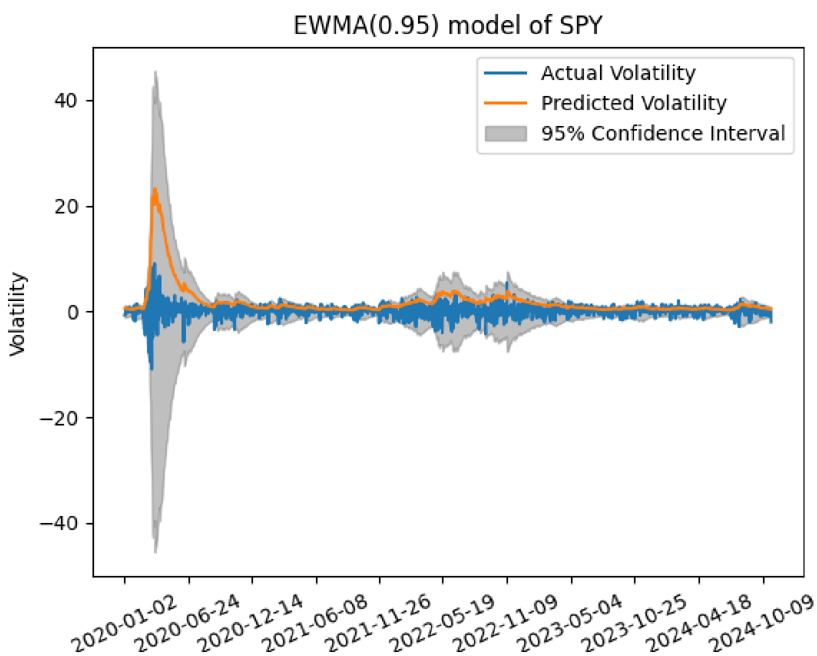

Figure 4: EWMA(0.95) model of SPY

6 Conclusion

The GARCH model can be a simple and effective tool to get a rough estimate of future volatility. It by itself is not accurate enough to be used as a measure of future volatility in a trading strategy. Despite this, it is still better than many other models that try to predict volatility including EWMA, which was shown earlier. The properties of GARCH itself (mean reverting and clustering) form an extremely solid foundation for a volatility model and with the proper steps, a better predictor could be constructed. Another conclusion that can be made is that in the majority of situations, there is minimal performance gain from using any another GARCH(p,q) model over the base GARCH(1,1). Finally, the GARCH model works best when measuring assets with low to mild volatility, however it is not terrible at measuring ones with high volatility either.

7 Future Steps

This research is only the tip of the iceberg when it comes to GARCH models. There are literally hundreds of different models in the GARCH family each with their own unique properties. In the future, I would particularly like to explore the applications of the EGARCH model which takes into account asymmetric returns in its volatility predictions.

8 References

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