

downloaded in batches of no more than 50, which can be done with an additional URL argument:

http://www.heliosvoting.org/elections/<ELECTI</pre>

And the next batch can be requested using the after parameter:

http://www.heliosvoting.org/elections/<ELECTI</pre>

The result of an election is available at:

http://www.heliosvoting.org/elections/<ELECTI</pre>

While the proof of this result is available at:

http://www.heliosvoting.org/elections/<ELECTI

For testing purposes, the following election identifier can be used:

agxoZWxpb3N2b3RpbmdyDwsSCEVsZWN0aW9uG0AQDA

which means that its election information is at http://www.heliosvoting.org/elections /agxoZWxpb3N2b3RpbmdyDwsSCEVsZWN0aW9uGOAQDA.

All data for this election was generated using the Helios system, and a verification program, built to the guidelines that follow, should be able to check all of the results.

If one wants to check the audit trail of a ballot, a new ballot audit trail can be generated at any time using the voting booth interface for this election. For convenience, one such ballot audit trail is available here, with choices 2 (Drummond) and 3 (Axel). The ballot fingerprint is 6PkosgVAY011FbVvqHGBeRo1SNs.

# **Data Formats**

We begin with a description of the data types and their representations. All data made available by Helios is in JavaScript Object Notation (JSON) format, with keys in <u>alphabetical order</u> and <u>no extraneous whitespace</u> other than that specified by JSON. These two conditions are particularly important, as hashing data structures will only yield the same hash if the conditions are respected. An example of a JSON data structure with these two conditions is:

Example (not an actual Helios data structure) {"email": ["ben@adida.net", "ben@mit.edu"], "first\_name": "Ben", "last\_name": "Adida"}

#### **Basic Cryptographic Datatypes**

All large integers are represented in decimal form as strings, rather than integers. The reason is that some languages do not support big integers natively, and thus cannot properly parse large integers in JSON integer form. An El-Gamal public-key is then a dictionary including the prime p, the primer-order q of its intended subgroup, the generator g, and the public-key value v (with keys in alphabetical order): <ELGAMAL PUBLIC KEY> {"g": "6811145128679259384514506369165", "p": "16998971978194099593503959", "q": "8499485989097049796751", "y": "7464668703479877164253720373259704"} An El-Gamal ciphertext is a JSON structure containing properties alpha and beta, the two components modulo p. <ELGAMAL CIPHERTEXT> {"alpha": "72348234202340223423", "beta": "123498235234234234324" } In Helios, all ciphertexts are *Exponential ElGamal*, so alpha =  $q^r \mod p$ , and beta =  $q^m \sqrt{r} \mod p$ . Voter A single voter in Helios is represented using a few fields that identify the voter: <VOTER> {"email": "benadida@gmail.com", "name": "Ben Adida", "vote hash": null, "voter id": "agxoZWxpb3N2b3RpbmdyCwsSBVZvdGVyGCcM"} The voter id is unique across all elections hosted at this particular Helios server. The vote hash is the SHA1 hash of the voter's encrypted ballot. Until the voter casts a ballot, vote hash is null. Voters may be identified by OpenID URL rather than email address, in which case their JSON representation is:  $\langle VOTER \rangle$  (2) {"name": "Ben Adida", "openid": "http://benadida.myopenid.com", "vote\_hash": null, "voter\_id": "agxoZWxpb3N2b3RpbmdyCwsSBVZvdGVyGCcM"} In elections where voters are placed in categories (e.g. precincts), an additional category field is present: <VOTER> (3) {"category": "02139", "email": "benadida@gmail.com", "name": "Ben Adida", "vote hash": null, "voter id": "agxoZWxpb3N2b3RpbmdyCwsSBVZvdGVyGCcM"} Once a voter has cast a ballot, their JSON representation is augmented with their encrypted vote: <VOTER WITH VOTE> {"category": "02139", "name": "Ben Adida", "openid": "http://benadida.myopenid.com", "vote" : <VOTE>, "vote hash":

"f1d2d2f924e986ac86fdf7b36c94bcdf32beec15", "voter id": "agxoZWxpb3N2b3RpbmdyCwsSBVZvdGVyGCcM"} We describe the details of the <V0TE> data structure later in this document, once we have described all of the required components. It may be confusing to note that vote hash and vote are both present within the data structure, when the hash is clearly derived from the vote. The reason for the presence of both is that, when voters are listed in a <V0TER LIST>, only the vote hash is listed, not the complete vote, for efficiency purposes. Election An election is represented as: <ELECTION> {"election id": "agxoZWxpb3N2b3RpbmdyDgsSCEVsZWN0aW9uGAYM", "name": "foo234234", "pk": <ELGAMAL PUBLIC KEY>, "questions": <QUESTION\_LIST>, "voters\_hash": "G6ySVdAZm5hKnCn5cRgBGdw3yGo", "voting ends at": null, "voting\_starts\_at": null} election id is a unique identifier for the election, and name is the election's name. <ELGAMAL PUBLIC KEY> is, as detailed earlier, the JSON data structure that represents an El-Gamal public key. <QUESTION LIST> is a data structure that represents the list of questions and available answers to those questions. <QUESTION LIST> [<QUESTION>, <QUESTION>, ...] and a single question is a JSON object: <QUESTION> {"answer urls": ["http://example.com/alice", null], "answers": ["alice", "bob"], "max": 1, "question": "Who Should be President?", "short\_name": "President"} which, in this case, contains two possible answers (alice and bob), URLs that describe these answers in greater detail, the text of the question, and a short name for the question. The parameter max indicates the maximum number of options that a voter can select, most often 1. Note how, given that this max value should be a small integer, it is in fact serialized as an integer, not as a string. voters hash is the base64 encoding of the SHA1 hash of the list of voters for the election. The list of voters is a JSON array, where each voter is represented without a cast ballot and without the vote hash, of course (otherwise voters hash

changes as voters cast their ballot). For example, a list of voters might be:

<VOTER\_LIST> (example) [{"email": "ben@adida.net", "name": "Ben Adida", "voter\_id": "453"}, {"name": "Ella", "openid": "http://ella.example.com", "voter\_id": "834"}]

**Note:** we strictly follow the JSON specification, so the forwardslash character "/" is escaped as "\/". Not all JSON toolkits do this correctly, given that escaping the forward-slash is not necessary given JavaScript specifications, but for interoperability, we choose the strict interpretation. Python's simplej son does the right thing.

### **Open Registration**

Helios supports "open registration elections", when the election administrator so desires. In those elections, the voter list is not set ahead of time. In that case, an election data structure does not contain voters\_hash, but does contain a new field, open reg:

<ELECTION> {"election\_id": "agxoZWxpb3N2b3RpbmdyDgsSCEVsZWN0aW9uGAYM", "name": "foo234234", "openreg": True, "pk": <ELGAMAL\_PUBLIC\_KEY>, "questions": <QUESTION\_LIST>, "voting\_ends\_at": null, "voting\_starts\_at": null}

## **Election Fingerprint**

Once an election is ready to be used for voting, the administrator *freezes* the election, at which point Helios prevents changing any of the question parameters and voter registration settings: an open election remains an open election, and a closed election remains closed with a fixed voter list.

Such a frozen election can be designated by its **Helios Election Fingerprint**, which is the base-64-string-encoded SHA1 of the election data structure serialized as JSON (with properly alphabetized field names). Note how this fingerprint depends on the list of voters if the election registration status is closed, but not if it is open. In any case, this fingerprint does *not* depend on any cast vote or cast-vote hash.

## Vote

A vote contains a list of encrypted answers, and a reference to the election, both by ID (for convenience) and by hash (for integrity.) The hash is the election fingerprint just described.

<VOTE> {"answers": [<ENCRYPTED\_ANSWER>, <ENCRYPTED\_ANSWER>, ...], "election\_hash": "Nz1fWLvVLH3eY3Ox7u5hxfLZPdw", "election\_id": "agxoZWxpb3N2b3RpbmdyDgsSCEVsZWN0aW9uGAYM"} Each "encrypted answer" corresponds to one election question: it contains a list of ciphertexts (one for each possible choice for that question), a list of corresponding proofs that the ciphertext is correctly formed, and an overall proof that all of the ciphertexts for that election question, taken together, are correctly formed.

<ENCRYPTED\_ANSWER> {"choices": [<ELGAMAL\_CIPHERTEXT>, <ELGAMAL\_CIPHERTEXT>, ...], "individual\_proofs": [<ZK\_PROOF\_0..1>, <ZK\_PROOF\_0..1>, ...], "overall\_proof": <ZK\_PROOF\_0..max>}

The value of max in overall\_proof matches the value of max in the election's question definition.

When a voter generates a ballot, Helios provides the ballot fingerprint, which is the base64-encoding of the SHA1 hash of the <V0TE> data structure defined above.

## Proofs

A zero-knowledge proof, denoted <ZK\_PROOF\_0..max>, is a transcript of a non-interactive proof that the corresponding ciphertext encodes an integer value between 0 and max. For the overall proof, the ciphertext whose value is being proven to be between 0 and max is the homomorphic sum (element-wise product) of the choices ciphertexts.

In Helios, all 0..max proofs are disjunctive proofs (CDS & CP), meaning that the transcript includes max+1 proofs, one for each possible value of the plaintext, 0 through max. The max+1 individual challenges must sum up to the single actual protocol challenge, which ensures that one of the proofs is real (while the others are simulated.)

<ZK\_PROOF\_0..max> [<ZK\_PROOF(0)>, <ZK\_PROOF(1)>, ..., <ZK\_PROOF(max)>]

A single ZK proof is then composed of three messages: the commitment, the challenge, and the response. Since the proof is a Chaum-Pedersen proof of a DDH tuple, the commitment is composed of two values, A and B. Thus, a ZK proof is:

<ZK\_PROOF(plaintext)> {"challenge": "2342342", "commitment": {"A": "28838", "B": "9823723"}, "response": "970234234"}

## Ballot Audit Trail

When a voter chooses to audit their ballot, each encrypted answer contains additional information concerning the actual selected choice and the randomness used to encrypt each choice's ciphertext. Specifically, the JSON structure for <VOTE\_WITH\_PLAINTEXTS> is as follows.

<VOTE\_WITH\_PLAINTEXTS>

{"answers": [<ENCRYPTED\_ANSWER\_WITH\_PLAINTEXT>, <ENCRYPTED\_ANSWER\_WITH\_PLAINTEXT>, ...], "election\_hash": <B64\_HASH>, "election\_id": <ELECTION\_ID>} And the contained <ENCRYPTED\_ANSWER\_WITH\_PLAINTEXT> is as follows.

<ENCRYPTED\_ANSWER\_WITH\_PLAINTEXT> {"answer": 1, "choices": [<ELGAMAL\_CIPHERTEXT>, <ELGAMAL\_CIPHERTEXT>, ...], "individual\_proofs": [<ZK\_PROOF\_0..1>, <ZK\_PROOF\_0..1>, ...], "overall\_proof": <ZK\_PROOF\_0..max>, "randomness": [<BIGINT>, <BIGINT>, <BIGINT>]}

## Result

The result of an election is represented using two structures: <RESULT> and <RESULT\_PROOF>. The result simply displays the count of votes for each candidate within each question, in an array of arrays format.

<RESULT> [[<QUESTION\_1\_CANDIDATE\_1\_COUNT>, <QUESTION\_1\_CANDIDATE\_2\_COUNT>, <QUESTION\_1\_CANDIDATE\_3\_COUNT>], [<QUESTION\_2\_CANDIDATE\_1\_COUNT>, <QUESTION\_2\_CANDIDATE\_2\_COUNT>]]

The <RESULT\_PROOF> data structure is formatted exactly the same way, with a Chaum-Pedersen proof of proper decryption for each candidate within each question:

<RESULT\_PROOF> [[<QUESTION\_1\_CANDIDATE\_1\_PROOF>, <QUESTION\_1\_CANDIDATE\_2\_PROOF>, <QUESTION\_1\_CANDIDATE\_3\_PROOF>], [<QUESTION\_2\_CANDIDATE\_1\_PROOF>, <QUESTION\_2\_CANDIDATE\_2\_PROOF>]]

# A Note on the Source Code in this Specification

In the rest of this document, we show how to verify various aspects of a Helios election using Python code for concreteness and legibility. We assume that certain data structures have been defined: election, vote, proof, disjunctive\_proof, and a few others, all of which correspond to collections of fields that directly map to the JSON data structures described above. However, we note that a verification program need not necessarily parse these JSON strings into custom Python objects. It is perfectly acceptable to extract individual fields when necessary.

In particular, in a number of cases, our sample code will call election.toJSON(), or vote.toJSON() in order to reconvert the data structure to JSON so that it can be hashed and checked for integrity. A verification program that handles JSON strings directly without de-serializing them to Python objects would obviously not need to re-serialize to JSON, either. The original JSON provided by the Helios server hashes appropriately to the intended values.

# Verifying a Single Ballot

Recall the Chaum-Pedersen proof that a ciphertext (alpha, beta) under public key (y, (g,p,q)) is proven to encode the value m by proving knowledge of r, the randomness used to create the ciphertext, specifically that g, y, alpha, beta/g^m is a DDH tuple, noting that alpha = g^r and beta/g^m = y^r.

- Prover sends A = g<sup>w</sup> mod p and B = y<sup>w</sup> mod p for a random w.
- Verifier sends challenge, a random challenge mod q.
- Prover sends response = w + challenge \* r.
- Verifier checks that:
  - o g^response = A \* alpha^challenge

verify\_proof(ciphertext, plaintext, proof, public\_key): if pow(public\_key.g, proof.response, public\_key.p) != ((proof.commitment.A \* pow(ciphertext.alpha, proof.challenge, public\_key.p)) % public\_key.p): return False beta\_over\_m = modinverse(pow(public\_key.g, plaintext, public\_key.p), public\_key.p) \* ciphertext.beta beta\_over\_m\_mod\_p = beta\_over\_m % public\_key.p if pow(public\_key.y, proof.response, public\_key.p) != ((proof.commitment.B \* pow(beta\_over\_m\_mod\_p, proof.challenge, public\_key.p)) % public\_key.p): return False return True

In a disjunctive proof that the ciphertext is the encryption of one value between 0 and max, all max+1 proof transcripts are checked, and the sum of the challenges is checked against the expected challenge value. Since we use this proof in non-interactive Fiat-Shamir form, we generate the expected challenge value as SHA1(A0 + "," + B0 + "," + A1 + "," + B1 + ... + "Amax" + "," + Bmax) with A0, B0, A1, B1, ..., Amax, Bmax in decimal form. (Ai and Bi are the components of the commitment for the i'th proof.)

Thus, to verify a <ZK\_PROOF\_0..max> on a <ELGAMAL\_CIPHERTEXT>, the following steps are taken.

verify\_disjunctive\_0..max\_proof(ciphertext, max, disjunctive\_proof, public\_key): for i in range(max+1): # the proof for plaintext "i" if not verify\_proof(ciphertext, i, disjunctive\_proof[i], public\_key): return False # the overall challenge computed\_challenge = sum([proof.challenge for proof in disjunctive\_proof]) % public\_key.q # concatenate the arrays of A,B values list\_of\_values\_to\_hash = sum([[p.commitment.A, p.commitment.B] for p in disjunctive\_proof], []) # concatenate as strings str\_to\_hash = ",".join(list\_of\_values\_to\_hash) # hash expected\_challenge = int\_sha(str\_to\_hash) # last check return computed\_challenge == expected\_challenge

Thus, given <ELECTION> and a <V0TE>, the verification steps are as follows:

verify\_vote(election, vote):

True

# check hash (remove the last character which is a useless '=')
computed\_hash =

base64.b64encode(hash.new(election.toJSON()).digest())[:-1] if computed hash != vote.election hash: return False # go through each encrypted answer by index, because we need the index # into the question array, too for figuring out election information for question num in range(len(vote.answers)): encrypted answer = vote.answers[question num] question = election.guestions[guestion num] # initialize homomorphic sum (assume operator overload on add with 0 special case.) homomorphic sum = 0 # go through each choice for the question (loop by integer because two arrays) for choice num in range(len(encrypted answer.choices)): ciphertext = encrypted\_answer.choices[choice\_num] disjunctive\_proof = encrypted answer.individual proofs[choice num] # check the individual proof (disjunctive max is 1) if not verify disjunctive 0..max proof(ciphertext, 1, disjunctive proof, election.public key): return False # keep track of homomorphic sum homomorphic sum = ciphertext + homomorphic sum # check the overall proof if not verify\_disjunctive\_0..max\_proof(homomorphic\_sum, guestion.max, encrypted answer.overall proof, election.public key): return False # done, we succeeded return

# Auditing/Spoiling a Single Ballot

Given a <V0TE\_WITH\_PLAINTEXTS> and a claimed vote fingerprint, verification entails checking the fingerprint, checking all of the proofs to make sure the ballot is well-formed, and finally ensuring that the ballot actually encodes the claimed choices.

verify\_ballot\_audit(vote\_with\_plaintexts, election, vote\_fingerprint) # check the proofs if not verify\_vote(election, vote\_with\_plaintexts): return False # check the proper encryption of each choice within each question # go through each encrypted answer for encrypted\_answer in vote\_with\_plaintexts.answers: # loop through each choice by integer (multiple arrays) for choice\_num in range(len(encrypted\_answer.choices)): # the ciphertext and randomness used to encrypt it ciphertext = encrypted\_answer.choices[choice\_num] randomness = encrypted\_answer.randomness[choice\_num] # the plaintext we expect, g^1 if selected, or g^0 if not selected if choice\_num == encrypted\_answer.answer: plaintext = public key.g else: plaintext = 1 # check alpha if pow(public\_key.g, randomness, public\_key.p) != ciphertext.alpha: return False # check beta expected\_beta = (pow(public\_key.y, randomness, public\_key.p) \* plaintext) % public\_key.p if expected\_beta != ciphertext.beta: return False # check the fingerprint vote\_without\_plaintexts = vote\_with\_plaintexts.remove\_plaintexts() computed\_fingerprint = base64.b64encode(hash.new(vote\_without\_plaintexts.toJSON()).d return computed\_fingerprint == vote\_fingerprint

# Verifying a Complete Election Tally

To verify a complete election tally, one should:

- display the computed election fingerprint.
- ensure that the list of voters matches the election voterlist hash.
- display the fingerprint of each cast ballot.
- check that each cast ballot is correctly formed by verifying the proofs.
- homomorphically compute the encrypted tallies and verify, using the result proof, that they correctly decrypt to the claimed results. Display these results.

In other words, the complete results of a verified election includes: the election fingerprint, the list of ballot fingerprints, and the actual count. Any party who verifies the election should re-publish all of these items, as they are meaningless without one another. This is effectively a "re-tally".

Part of this re-tally requires checking a decryption proof, which is almost the same, but not quite the same, as checking an encryption proof with given randomness. First, we document the verification of a decryption proof.

verify decryption proof(ciphertext, plaintext, proof, public key): # Here, we prove that (g, y, alpha, beta/g^m) is a DDH tuple. # Before we were working with (g, alpha, y, beta/g^m) if pow(public\_key.g, proof.response, public\_key.p) != ((proof.commitment.A \* pow(public\_key.y, proof.challenge, public key.p)) % public key.p): return False beta over m = modinverse(pow(public\_key.g, plaintext, public\_key.p), public key.p) \* ciphertext.beta beta over m mod p = beta\_over\_m % public\_key.p if pow(ciphertext.alpha, proof.response, public key.p) != ((proof.commitment.B \* pow(beta\_over\_m\_mod\_p, proof.challenge, public\_key.p)) % public\_key.p): return False # compute the challenge generation, Fiat-Shamir style str to hash = str(proof.commitment.A) + "," + str(proof.commitment.B) computed challenge = int sha(str to hash) # check that the challenge matches return computed\_challenge == proof.challenge

Then, the re-tally proceeds as follows.

retally\_election(election, voters, result, result\_proof):
# compute the election fingerprint election\_fingerprint =
b64\_sha(election.toJSON()) # compute the voter list hash on

just the voter identities voters\_without\_votes = votes.remove votes() voters hash = b64 sha(voters without votes.toJSON()) # verify, no need to continue if we fail here if voters hash != election.voters hash: return False # keep track of voter fingerprints vote fingerprints = [] # keep track of running tallies, initialize at 0 # again, assuming operator overloading for homomorphic addition tallies = [[0 for a in question.answers] for question in election.questions] # go through each voter, check it for voter in voters: if not verify vote(election, voter.vote): return False # compute fingerprint vote fingerprints.append(b64 sha(voter.vote.toJSON())) # update tallies, looping through questions and answers within them for question num in range(len(election.questions)): for choice num in range(len(election.guestions[guestion num].answers)): tallies[question num][choice num] = voter.vote.answers[guestion num].choices[choice num] + tallies[question num][choice num] # now we have tallied everything in ciphertexts, we must verify proofs for question\_num in range(len(election.questions)): for choice\_num in range(len(election.guestions[guestion\_num].answers)): # verify the tally for that choice within that question # check that it decrypts to the claimed result with the claimed proof if not verify decryption proof(tallies[question num][choice num], pow(election.public key.g, result[question num][choice num], election.public key.p), result proof[question num][choice num], election.public key): return False # return the complete tally, now that it is confirmed return { 'election fingerprint': election fingerprint, 'vote fingerprints' : vote fingerprints, 'verified tally' : result }

# **Election with Multiple Trustees**

A Helios election can be configured to have multiple trustees, each of which holds a share of the election secret key.

Before the trustees have submitted their public key shares, the election's public key is null, e.g.:

Election with Trustees, before shares {"election id":

"agxoZWxpb3N2b3RpbmdyDgsSCEVsZWN0aW9uGAUM", "name": "test-trustees", "pk": null, "questions": [], "voters\_hash": "I9Fw4VUO7kr8CvBlt4zaMCqXZ0w", "voting\_ends\_at": null, "voting\_starts\_at": null}

The list of trustees for an election can be obtained at

http://www.heliosvoting.org/elections/<ELECTI</pre>

which returns the <TRUSTEE\_LIST> data structure as follows:

<TRUSTEE\_LIST> [<TRUSTEE>, <TRUSTEE>, ..., <TRUSTEE>]

where a single <TRUSTEE> data structure is:



decryption factor, much like <RESULT\_PR00F> is an elementwise decryption proof of <RESULT>.

<DECRYPTION\_PROOFS>
[[<Q1\_CANDIDATE\_1\_PROOF>,
<Q1\_CANDIDATE\_2\_PROOF>,
<Q1\_CANDIDATE\_3\_PROOF>],
[<Q2\_CANDIDATE\_1\_PROOF>,
<Q2\_CANDIDATE\_2\_PROOF>]]

Each of these proofs is a DH-tuple proof, just like the original result decryption proof, with the fourth element of the DH tuple the corresponding decryption factor. So, for example,

<Q1\_CANDIDATE\_1\_PR00F> is a transcript of the proof that g, Q1\_C1\_TALLY.alpha, y,

<Q1\_CANDIDATE\_1\_DEC\_FACTOR> is a proper DH tuple. The homomorphic tally, prior to decryption, is computed exactly as it was without trustees.

## **Putting it All Together**

At verification time, the steps for a trustee election are only slightly different:

- Each trustee's public-key share is verified against the corresponding proof of knowledge.
- The election's single public key is indeed the product of the key shares.
- Each individual ballot is verified just as before.
- The encrypted tally for each candidate to each question is homomorphically computed, *just as before*.
- Each partial decryption for each candidate to each question is verified.
- The final tally for a given candidate is obtained by multiplying the partial decryption factors, and dividing it out of the corresponding encrypted tally's beta.

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