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CEE/MAE M20
UID: 005114992 **Homework 4** October 15, 2020

1 Correlation between Competing Lynx and Hare Populations

1. Introduction

   The objective of this problem is to simulate the population trends of two competing populations in a predator-prey relationship. The program computes the population values at each time step and plots the entire data set over time on the same figure.

2. Models and Methods

   The basis of this calculation is the Lotka-Volterra equations given in the assignment.

$$\frac{dx}{dt} = 0.4x - 0.018xy$$

$$\frac{dy}{dt} = -0.8y + 0.023xy$$

   However, to be able for MATLAB to interpret these equations and create the arrays of data, they need to be discretized into the following form:

$$x(k) = x(k-1) + dt * (0.4x(k-1) - 0.018x(k-1)y(k-1))$$

$$y(k) = y(k-1) + dt * (-0.8y + 0.023x(k-1)y(k-1)$$

   To calculate the correlation coefficient, a subsection of the data corresponding to one period starting at t = 5.2 is extracted and the following equation is applied:

$$r_{xy} = \frac{\Sigma_i(x_i - \overline{x})(y_i - \overline{y})}{\Sigma_i(x_i - \overline{x})^2 \Sigma_i(y_i - \overline{y})^2}$$

   The summations in the above equation are taken over the entire subsection of data. The overbars over each variable signify the average value for that respective population over the subsection.

3. Calculations and Results

   With the initial values set by the assignment, the program prints the following to the command window and generates the plot below:

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The correlation coefficient is: -0.00267.
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   The calculated correlation coefficient is around -0.0027. This number is extremely close to zero.

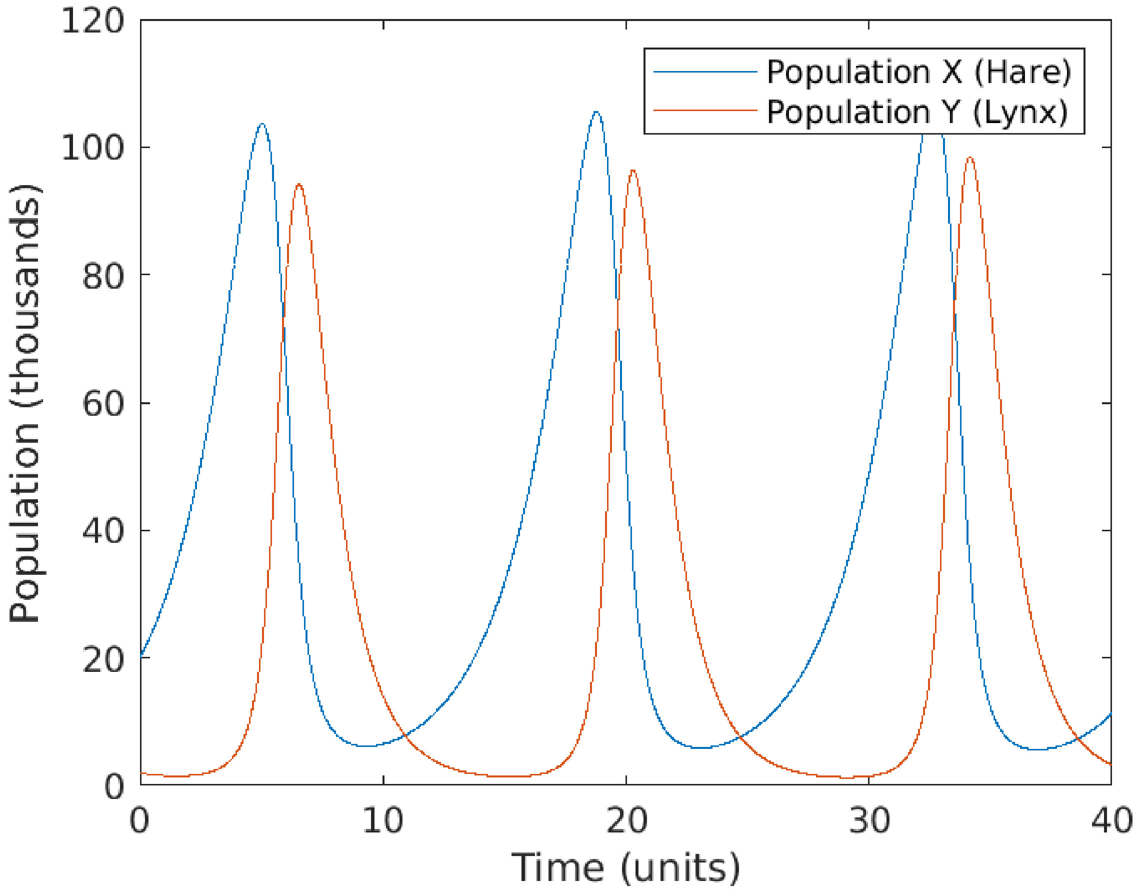

Figure 1: Clearly, the two populations oscillate periodically with the predator peak (lynx, in red) lagging slightly behind the prey peak (hare, in blue) in time. This is expected. Also note that the populations are not decreasing over time, rather they are very slightly increasing. The period, calculated by examining the data points at two peaks of either graph, is around 13.75 units of time or 1375 time steps.

4. Discussion

   This program provides an insightful picture into the relationship between predator and prey species using the forward Euler method on Lotka-Volterra equations. With the Pearson's correlation coefficient being virtually zero, there is no linear correlation between the populations. This is an interesting result considering that the population are directly related mathematically (both values are dependent upon a function that includes the other. I admit it is possible that an error occurred during the calculation, but after several hours of debugging I do not believe that is the case.

Another explanation could be that a linear correlation is not the best method to describe the relationship between the two populations and therefore a linear coefficient would essentially be zero.

2 Pendulum Simulation

1. Introduction

   The goal of this problem is to simulate the motion of a pendulum over time. The program calculates the angle and angular velocity of the simulated pendulum using both the explicit Euler and semi-implicit Euler method. The calculation at each time step is stored in a data array and plotted over time.

2. Models and Methods

   The program is founded upon the following Lotka-Volterra equations for the pendulum's physical motion:

$$\frac{d\omega}{dt} = -\frac{g}{L} sin(\theta)$$

$$\frac{d\theta}{dt} = \omega$$

   L is the length of the pendulum (in this case, 10m) and g is the acceleration due to Earth's gravity (10 m/s/s). It then uses their discretized forms in two ways: explicit and semi-implicit Euler method. This is the explicit Euler form:

$$\omega(k) = \omega(k-1) + dt(-\frac{g}{L} sin(\theta(k-1)))$$

$$\theta(k) = \theta(k-1) + dt(\omega(k-1))$$

   This is the semi-implicit Euler form:

$$\omega(k+1) = \omega(k) + dt(-\frac{g}{L} sin(\theta(k))$$

$$\theta(k+1) = \theta(k) + dt(\omega(k+1))$$

   However, these equations do not adequately portray the motion of a true pendulum because they do not account for damping factors. The adjusted Lotka-Volterra equations that include damping are as follows:

$$\omega(k) = \omega(k-1) + dt(-\frac{g}{L} sin(\theta(k-1)) - \omega(k-1)Ld)$$

$$\theta(k) = \theta(k-1) + dt(\omega(k-1))$$

   for the explicit Euler method and

$$\omega(k+1) = \omega(k) + dt(\frac{-\frac{g}{L} sin(\theta(k)}{1 + dtLd})$$

$$\theta(k+1) = \theta(k) + dt(\omega(k+1))$$

for the semi-implicit Euler method. These equations include the damping factor d, which for the purposes of the problem is initialized to 0.01 m*s. However, as will be later discussed, there is a plot generated for this assignment that uses variable values for the damping factor.

3. Calculations and Results

For part a of the assignment, the following plot is generated as an example of what the pendulum looks like without damping in the system.

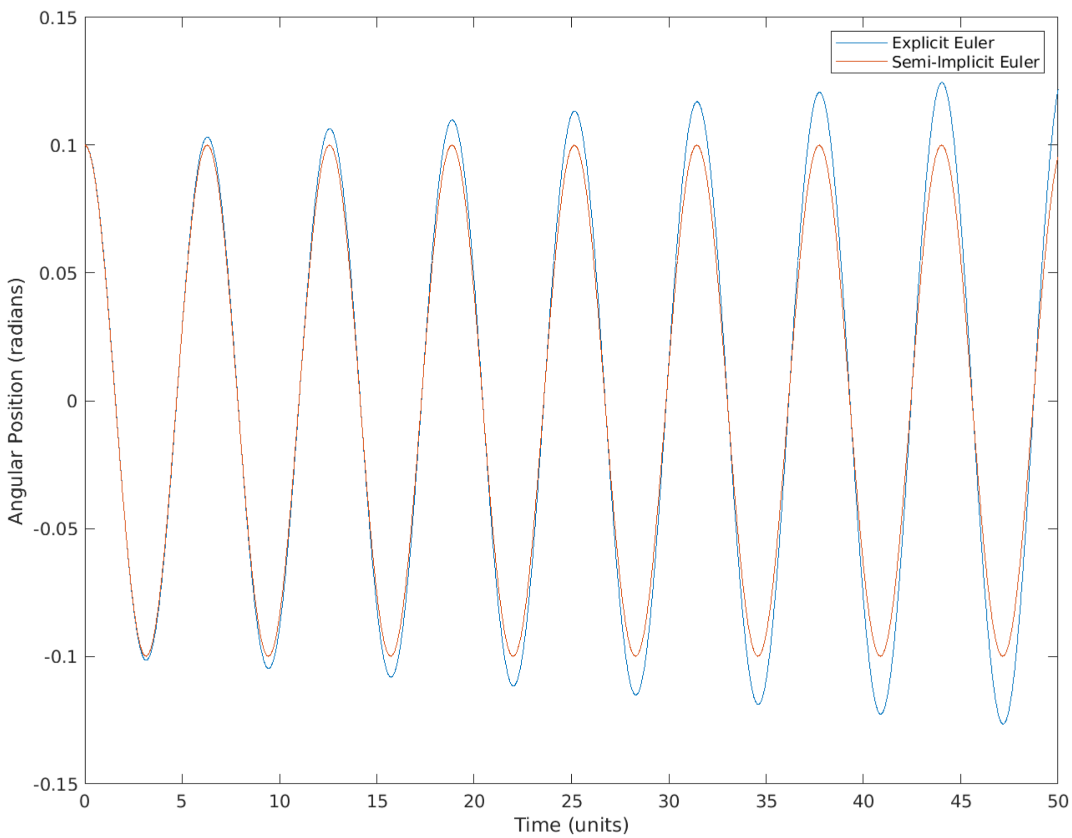

Figure 2: This plot describes the motion of the pendulum without damping. Note that the explicit Euler graph increases in amplitude over time, whereas the semi-implicit Euler graph maintains a constant sinusoidal amplitude. The more physically accurate model is the semi-implicit method because it does not appear to introduce energy into the system from a unknown source.

The actual program submitted for this problem simulates the pendulum in the presence of damping. The plot for this simulation is below.

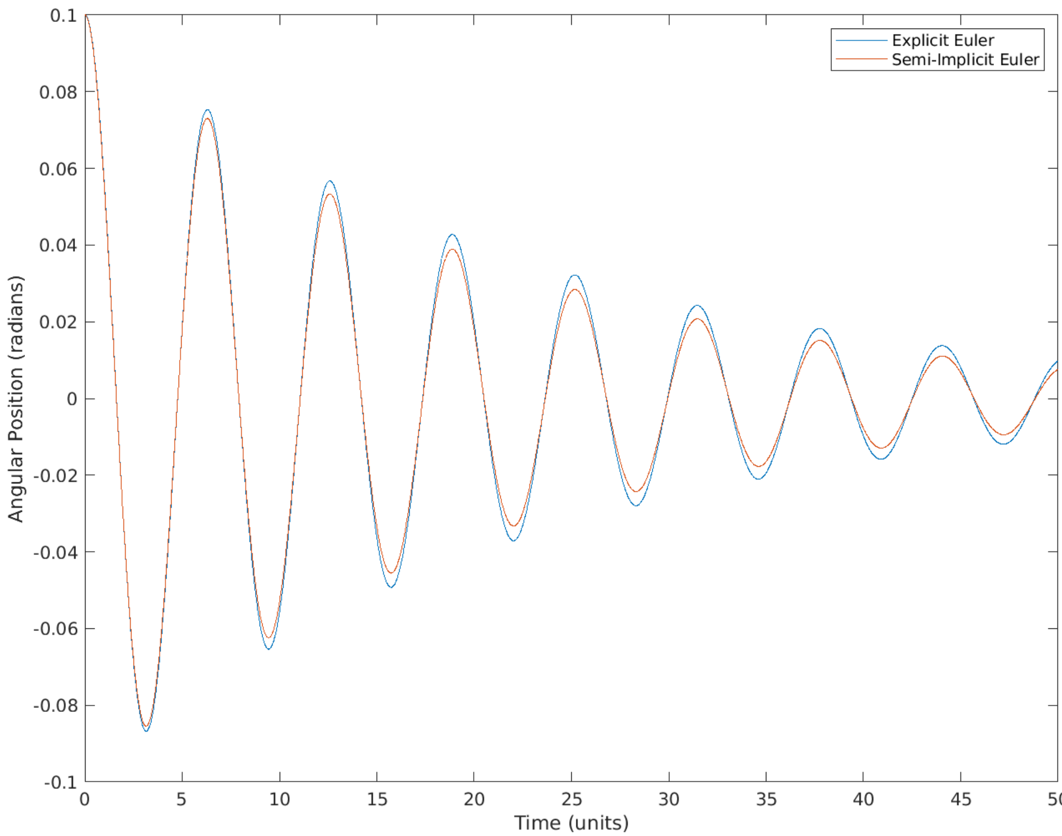

Figure 3: This plot clearly demonstrates the visible effect that damping has on the pendulum, decreasing its maximum amplitude significantly with each period.

The final part of this problem involves a graphical comparison of three different d values corresponding to varying levels of damping being present in the pendulum system. The following graph plots the data for all three d values over the same time period (t= 0 to t=100).

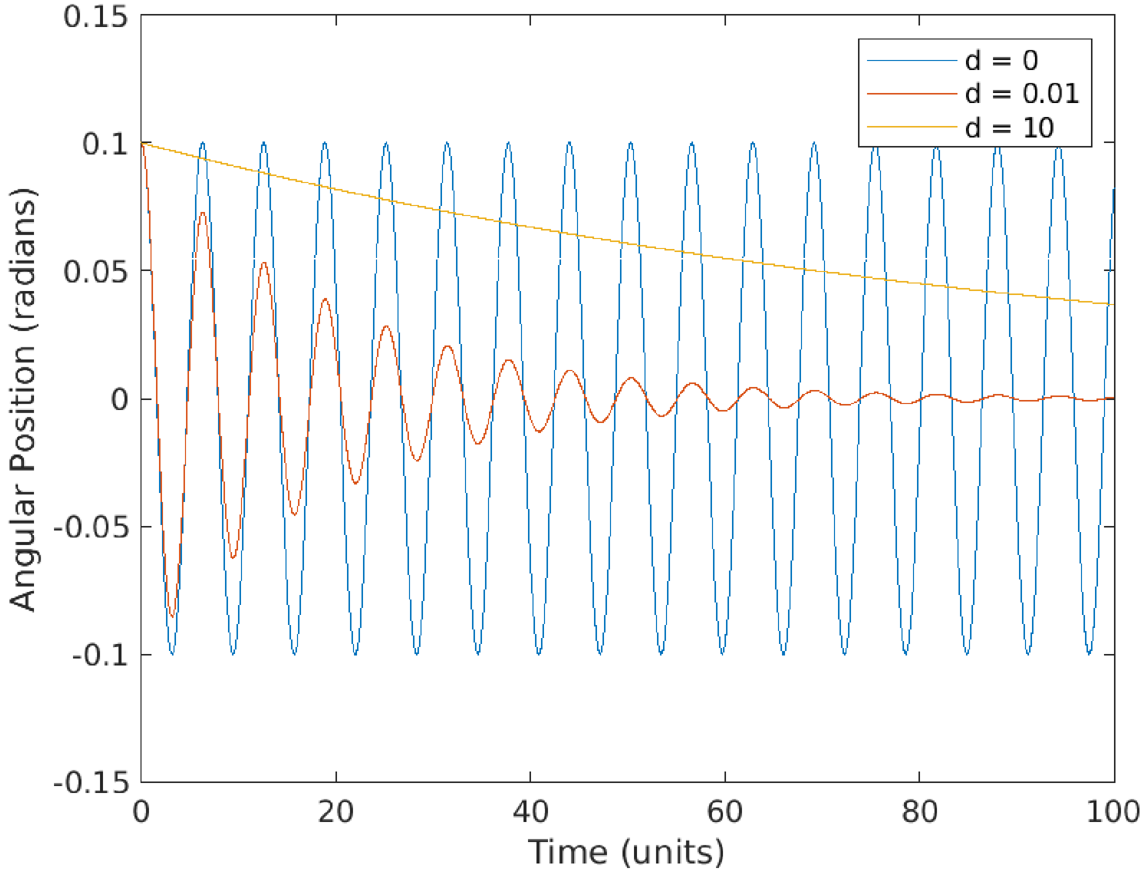

Figure 4: The plot for d = 0 is the same as the the semi-implicit Euler graph from Figure 2, which is to be expected because both graphs represent the pendulum without damping. The other two graphs visibly show the effect that damping has on the amplitude of the pendulum over time, decreasing it significantly for d = 0.01 and completely eliminated the sinusoidal motion for d = 10.

4. Discussion

   This program demonstrates that MATLAB is exceptionally useful for simulating physical phenomena and systems, such as a simple pendulum like this. Being able to account for damping (or not) provides a level of realism that most physical analysis is not able to achieve. Trying to achieve consistency in collecting motion data for a pendulum in a lab setting is difficult and is unlikely to produce a graph that is as easily readable and interpretable as this simulation. The semi-implicit Euler method especially provides a realistic, energy-conserving display of the pendulum's oscillatory motion. The next plot demonstrates an example of damping, most likely underdamped motion, that slowly reduces the amplitude of the pendulum over time without eliminating its sinusoidal motion. The final plot in Figure 4 provides useful insight into how damping truly affects a harmonic system like a pendulum. The graphs for d = 0 and d = 0.01 have already discussed to be undamped and underdamped, respectively. The d = 10 graph shows an overdamped pendulum in which the amplitude of the pendulum decreases extremely slowly but never oscillates. In fact, an overdamped system will take a relatively large amount of time to reach zero (a totally vertical equilibrium).