Übung 10 #2 | Möglichkeit ohne Imaginärteil-betrachtungen

$$\tilde{V}(q) = -V_0 \int 2\pi \, d\cos\bar{\theta} \, r^2 dr \, e^{-iqr\cos\bar{\theta} - r/R_0}$$

$$= -V_0 \, 2\pi \int r \, dr \left(\frac{1}{-iq}\right)\left(e^{-r\left(\frac{1}{R_0}+iq\right)} - e^{-r\left(\frac{1}{R_0}-iq\right)}\right)$$

$\alpha$-Trick:

$$\left[\int_0^\infty dr \, r \, e^{-\alpha r} = \int_0^\infty dr \left(-\frac{d}{d\alpha}\right) e^{-\alpha r} = \frac{1}{\alpha^2}\right]$$

$$= -V_0 \, 2\pi \, \frac{1}{-iq}\left(\frac{1}{\left(\frac{1}{R_0}+iq\right)^2} - \frac{1}{\left(\frac{1}{R_0}-iq\right)^2}\right)$$

$$= -V_0 \, 2\pi \, \frac{1}{-iq} \, \frac{-4iq\frac{1}{R_0}}{\left(\frac{1}{R_0}+iq\right)^2\left(\frac{1}{R_0}-iq\right)^2}$$

$$= -V_0 \frac{8\pi}{R_0} \, \frac{1}{\left(\frac{1}{R_0^2}+q^2\right)^2}$$

$$= -V_0 \, 8\pi R_0^3 \, \frac{1}{\left(1+R_0^2 q^2\right)^2}$$