1 Ein-Teilchen

Lorentzgruppe

Generatoren

$$k_x = i\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \qquad k_y = i\begin{pmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix} \qquad k_z = i\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}$$

$$l_x = i\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix} \qquad l_y = i\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{pmatrix} \qquad l_z = i\begin{pmatrix}0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0\end{pmatrix}$$

$$[l_i, l_j] = \epsilon_{ijk} l_k \qquad [k_x, k_j] = -i\epsilon_{ijk} l_k \qquad [l_i, k_j] = i\epsilon_{ijk} k_k$$

Lorentztransform

$$\Lambda^\mu_\nu = \delta^\mu_\nu + \omega^\mu_\nu$$
$$\text{or } S(\Lambda) := \mathbb{1} - \frac{i}{2}\omega^{\mu\nu} L_{\mu\nu})$$
$$\omega \ldots \text{antisymmetrisch}$$

Spinoren

$$\text{Drehung von Spinorlsg: } S(R_i(\theta)) = \mathrm{e}^{-i\theta S_i} \tag{1}$$

$$S_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu] \qquad S^\dagger_{\mu\nu} = \gamma^0 S_{\mu\nu}\gamma^0 \qquad S^1(\Lambda) = \gamma^0 S^\dagger(\Lambda)\gamma^0 = 1 + \frac{i}{2}\omega^{\mu\nu} S_{\mu\nu}$$

Spinoridentitäten unter Lorentztransform

$$\begin{aligned}
\Psi &\mapsto S(\Lambda)\Psi \\
\overline{\Psi} &\mapsto \overline{\Psi} S^{-1}(\Lambda) \\
\overline{\Psi}\Psi &\mapsto \overline{\Psi}\Psi \\
\overline{\Psi}\gamma^\mu\Psi &\mapsto \Lambda^\mu_\nu \overline{\Psi}\gamma^\Psi \\
S^{-1}(\Lambda)\gamma^\mu S(\Lambda) &\mapsto \Lambda^\mu_\nu \gamma^\nu
\end{aligned}$$

Matrixidentitäten

Gamma	$\gamma^{\mu *} = \gamma^2\gamma^\mu\gamma^2$	$\gamma^{i\dagger} = \gamma^0\gamma^i\gamma^0 i$	
	$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}$	$\gamma^i\gamma^i = -\mathbb{1}$	with $i \in \{1,2,3\}$
Sigma	$\sigma^i\sigma^j = \delta_{i,j}\mathbb{1} + i\sum_{k=1}^{3}\epsilon_{ijk}\sigma^k$	$[\sigma_j, \sigma_k] = 2i\epsilon_{jkl}\sigma_l$	$\{\sigma_j, \sigma_k\} = 2\delta_{jk}\mathbb{1}$

Relativistisch

Klein-Gordon-Gleichung: $\Box\Phi(x) + m^2\Phi(x) = 0$

4-Stromdichte: $j^\mu = \frac{i}{2m}[\Psi^*\partial^\mu\Psi - \Psi\partial^\mu\Psi^*] \qquad j^0 = \rho \qquad j^i = \vec{j}$

Dirac-Gleichung: $(i\partial_\mu\gamma^\mu - m)\psi = 0 = (i\not\partial - m)\Psi = (p - m)\Psi$

Ansatz: $\Psi(x) = \omega(p)\mathrm{e}^{\mp ip_\mu x^\mu} \to (\pm\not p - m)\omega p = 0$

$$+\not p - m1 \qquad = E\gamma^0 - p_x\gamma^1 - p_y\gamma^2 - p_z\gamma^3 = \begin{pmatrix} E-m & 0 & -p_z & -p_x + ip_y \\ 0 & E-m & p_x - ip_y & p_z \\ p_z & p_x - ip_y & -E-m & 0 \\ p_x + ip_y & -p_z & 0 & -E-m \end{pmatrix}$$

$$\text{mit} \begin{cases} \text{Teilchenspinoren} u \\ \text{Antiteilchenspinoren} v \end{cases}$$

LSG: $u_1 = N\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} \qquad u_2 = N\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad \text{mit } N = \frac{1}{\sqrt{(E+m)} \to u\overline{u} = 2E}$

$v_1 = N\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix} \qquad v_2 = N\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}$

Dirac-Hamiltonian: (γ^0v. links multiplizieren)

$i\partial_t\psi = (-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi = \hat{H}_D\psi$

Electronmagn. Eichinvarianz:

$A^\mu(x) \to A^\mu(x) + \partial^\mu\theta(x) \qquad \psi(x) \to \mathrm{e}^i e\theta(x)\psi(x)$

$D^\mu\psi := (\partial^\mu - ieA^\mu)\psi \qquad P^\mu\psi \to \mathrm{e}^i e\theta(x)D\mu\psi$

Viel-Teilchen

Symetrisierungsoperator

$S_N^\pm := \frac{1}{N!}\sum_{\mathcal{P}}(\pm)^p\mathcal{P}$ mit $\mathcal{P} = \Pi\mathcal{P}$ Alle Permutationen

$hermitisch$ $\qquad S_N^\pm S_N^\pm = S_N^\pm$

$[P_{ij}, S_N] = 0$ $\qquad P_{ij}S_N^\pm = \pm S_N^\pm$

$[P_{ij}, \hat{A}_N] = 0 \quad \forall sinnvolle\hat{A}_N$

Fock-Raum:

Bosonen:$\mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2^\pm \oplus \mathcal{H}_3^\pm \ldots$ Fermionen:$\mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2^\mp \oplus \mathcal{H}_3^\mp \ldots$