Def. $|1,1\rangle := |\uparrow\uparrow\rangle$

$|1,0\rangle := \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$

$|1,-1\rangle := |\downarrow\downarrow\rangle$

$|0,0\rangle := \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

S M

$[S^2, S_z]$

$S(S+1)$ M

$$S^2 = (S_1 + S_2)^2$$
$$= S_1^2 + S_2^2 + 2S_1S_2$$
$$= S_1^2 + S_2^2 + 2S_{z_1}S_{z_2} +$$

$S_\pm = S_x \pm i S_y$

$$\to S_\pm S_\mp = S_x^2 + S_y^2 \mp i[S_x, S_y] = S_x^2 + S_y^2 \pm S_z$$
$$= (S_x \pm i S_y)(S_x \mp i S_y) = \pm i(S_yS_x - S_xS_y)$$
$$= S^2 - S_z^2 \pm S_z$$
$$\Rightarrow S^2 = S_\pm S_\mp + S_z^2 \mp S_z$$

$$S_{\pm 1}S_{\mp 2} = S_{x_1}S_{x_2} + S_{y_1}S_{y_2} \pm \dots$$
$$S_{\mp 1}S_{\pm 2} = S_{x_1}S_{x_2} + S_{y_1}S_{y_2} \mp \dots$$

$$2S_1S_2 = 2S_{x_1}S_{x_2} + 2S_{y_1}S_{y_2} + 2S_{z_1}S_{z_2}$$
$$= \underbrace{S_{\pm 1}S_{\mp 2} + S_{\mp 1}S_{\pm 2} + 2S_{z_1}S_{z_2}}$$

$$S^2 = S_1^2 + S_2^2 +$$

z.z. $S(S+1)|SM\rangle = \hat{S}^2|SM\rangle$

$$\hat{S}^2|SM\rangle = \left(S_1^2 + S_2^2 + \underbrace{S_{+1}S_{-2} + S_{-1}S_{+2}} + 2S_{z_1}S_{z_2}\right)\left(\frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{4} + \frac{3}{4} + \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} - \frac{1}{2}\right)|\dots\rangle = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix}|SM\rangle$$

$$\left(\frac{1}{4}+\frac{1}{4}+\{-1\}\right) = -\frac{1}{2}$$

$$S_1^2|\uparrow\downarrow\rangle = \frac{1}{2}\left(\frac{1}{2}+1\right)|\uparrow\downarrow\rangle = \frac{3}{4}|\uparrow\downarrow\rangle$$

$$S_{+1}S_{-2}\left(\frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}\right) = \pm\frac{|\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$S_{\pm}|sm\rangle = \sqrt{s(s+1)-m(m\pm1)}\,|s,m\pm1\rangle$$

$$S_{-1}S_{+2}\left(\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\right) = \frac{|\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$S_+|\uparrow\rangle = 0$$

$$S_+|\downarrow\rangle = \sqrt{\frac{3}{4}+\frac{1}{4}}\,|\uparrow\rangle = |\uparrow\rangle$$

$$S_-|\uparrow\rangle = |\downarrow\rangle$$

$$S_-|\downarrow\rangle = 0$$

$\oplus$: $\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$

$\ominus$: $\frac{|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle}{\sqrt{2}} = -1\cdot\frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$

$$2\,S_{z_1}S_{z_2}\left(\frac{|\uparrow\downarrow\rangle}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}}\left(S_z|\uparrow\rangle \otimes S_z|\downarrow\rangle\right) = -\frac{2}{\sqrt{2}}\cdot\frac{1}{2}\cdot\frac{1}{2}\underbrace{(|\uparrow\rangle\otimes|\downarrow\rangle)}_{|\uparrow\downarrow\rangle}$$

$$= \underline{\underline{-\frac{1}{2}}}\cdot\frac{|\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$S^2|1,0\rangle = 2|1,0\rangle = \left\{ S(S+1) \right\}|SM\rangle$$

$$S^2|0,0\rangle = 0|0,0\rangle =$$

EZ. von $S_z = \frac{1}{2}\sigma_z$

1. Teilchen : $\underbrace{|\uparrow\rangle \quad m=\frac{1}{2} \qquad |\downarrow\rangle \quad m=-\frac{1}{2}}_{s=1/2}$

$$\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \rightarrow |\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$