

# Quantentheorie II Übung 11

Besprechung: 2021WE27 (KW27)

SS 2021

Prof. Dominik Stöckinger (IKTP)

## 1. Questions

- (a) What is the "optical theorem" and its interpretation?
- (b) What are the so-called "unitarity bounds" on the partial cross sections  $\sigma_l$ ?
- (c) Which partial wave is dominant in case of the  $\delta$ -function potential? (In other words, which  $\delta_l$  are largest/non-zero?)
- (d) Which of the spherical harmonics  $Y_{lm}$  represents best the temperature distribution of the earth's surface?

## 2. Green function and Fourier transformation: compute the integral

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{x}}}{k^2 - q^2 - i\varepsilon}$$

in the limit  $\varepsilon \rightarrow 0_+$  (the sign of the imaginary term is opposite to the lecture).

## 3. S-wave scattering: the differential cross section of a pure s-wave scattering is given as

$$\frac{d\sigma}{d\Omega} = a, \text{ where } a > 0.$$

What is the complex scattering amplitude  $f(\theta)$ ?

## 4. Scattering on sphere: we consider s-wave scattering on a homogeneous sphere potential

$$V(r) = \begin{cases} V_0 > 0 & \text{for } r < a, \\ 0 & \text{for } r \geq a. \end{cases}$$

- (a) Take first  $V_0 \rightarrow \infty$  and solve the radial part of the Schrödinger equation exactly for  $l = 0$ , i.e. obtain the radial function  $u_0(r)$  and the scattering phase  $\delta_0$  exactly.
- (b) What is the equation for the scattering phase  $\delta_0$  when  $E < V_0$  (now  $V_0 < \infty$ )?
- (c) Estimate the scattering phase  $\delta_0$  and the partial cross section  $\sigma_0$  when the energy of the incoming particle is small,  $k \rightarrow 0$ .

## 5. Scattering on exponential potential (again): a particle with mass $m$ is scattered on the potential

$$V(r) = -V_0 e^{-\frac{r}{R_0}}, \quad V_0 > 0.$$

- (a) Compute the scattering phase of s-wave scattering  $\delta_0$ . Use the scattering amplitude  $f(\theta, \phi)$  obtained in the exercise 10.
- (b) What is the equation to determine the phase of the p-wave scattering  $\delta_1$ ?
- (c) The Born approximation is valid when  $|\psi^{(1)}(\vec{r})| \ll |\phi_k(\vec{r})| = 1$ , or equivalently

$$\frac{m}{2\pi} \left| \int d^3r' V(r') \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} e^{ikz'} \right| \ll 1 \quad \xrightarrow{r \rightarrow 0} \quad \left| \int_0^\infty dr V(r) (e^{2ikr} - 1) \right| \ll \frac{k}{m}, \quad (1)$$

What are the conditions to satisfy Eq. (1) for  $kR_0 \gg 1$  and  $kR_0 \ll 1$  respectively?