Quantentheorie II Übung 10

Besprechung: 2021WE26 (KW26)

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1. **Questions**
   (a) In the lecture simple classical scattering situations were discussed (football against concrete wall, football rolls on/off curb, etc). Map these cases to the cases of quantum mechanical scattering at Yukawa/Coulomb/δ-function potential and on the potential of a Gaussian charge distribution. Answer in particular:
      i. What is the $\cos\theta$-dependence of scattering at the δ-potential — explain in simple terms!
      ii. What is the approximate $\cos\theta$-dependence of scattering at the Yukawa potential for very small k^2 — explain in simple terms!
      iii. Which case(s) are similar to rolling over a curb — consider in particular the possibility of large scattering angles.

   (b) Remind yourself of the key observation in Rutherford scattering — how does Rutherford scattering prove the existence of a hard and small nucleus as opposed to the earlier "plum pudding" model of atoms? Which of our situations is similar to this comparison?

2. **Scattering:** a particle of mass m scatters at a potential

$$V(r) = -V_0 e^{-r/R_0}, \text{ where } V_0 > 0, \text{ and } r \equiv |\vec{x}|.$$

   (a) Compute the scattering amplitude $f(\theta, \phi)$ in first Born approximation.
   (b) Compare with the results for the Yukawa potential at small/large momentum transfer $\vec{q} = \vec{k}' - \vec{k}$, small/large angles and small/large r.

3. **Scattering by sphere:** a particle of mass m scatters at a potential

$$V(r) = \begin{cases} -V_0 & \text{for } r < R_0 \\ 0 & \text{for } r \geq R_0 \end{cases} \quad \text{where } , V_0 > 0 \text{ and } r \equiv |\vec{x}|.$$

   (a) Compute the scattering amplitude $f(\theta, \phi)$ in first Born approximation.
   (b) Compare with the results for the Yukawa potential and δ-function potential at small/large momentum transfer $\vec{q} = \vec{k}' - \vec{k}$, small/large angles and small/large r.
   (c) Determine the differential cross section $d\sigma/d\Omega$ and discuss for small particle energies $kR_0 \ll 1$.

4. **Green function for spherical waves:** look up the expressions for Δ and $\vec{\nabla}$ in spherical coordinates and use the known result $\Delta \frac{-1}{4\pi|\vec{x}|} = \delta^{(3)}(\vec{x})$ to show

$$(\Delta + k^2) \frac{-e^{ik|\vec{x}|}}{4\pi|\vec{x}|} = \delta^{(3)}(\vec{x}). \tag{1}$$

5. **Fourier transformation:** compute the Fourier transform of the Yukawa potential by explicit integration!