Chapter 1, Exercise 2

The answer is $p(A = 1|B = 1) = 0.8$, because to have independence $p(A = 1|B = 1) = p(A = 1|B = 0)$.

Chapter 1, Exercise 4 Inspector Clouseau

Inspector Clouseau arrives at the scene of a crime. The Butler (B) and Maid (M) are his main suspects. The inspector has a prior belief of 0.6 that the Butler is the murderer, and a prior belief of 0.2 that the Maid is the murderer. These probabilities are independent in the sense that $p(B, M) = p(B)p(M)$. (It is possible that both the Butler and the Maid murdered the victim or neither). The inspector's *prior* criminal knowledge can be formulated mathematically as follows:

$$\Omega_b = \Omega_m = \{\mathsf{murderer}, \mathsf{not\ murderer}\}$$

$$\Omega_k = \{\mathsf{knife\ used}, \mathsf{knife\ not\ used}\}$$

$$p(b = \mathsf{murderer}) = 0.6, \quad p(m = \mathsf{murderer}) = 0.2$$

$$\begin{array}{lll}
p(\mathsf{knife\ used}|b = \mathsf{not\ murderer}, & m = \mathsf{not\ murderer}) & = 0.3 \\
p(\mathsf{knife\ used}|b = \mathsf{not\ murderer}, & m = \mathsf{murderer}) & = 0.2 \\
p(\mathsf{knife\ used}|b = \mathsf{murderer}, & m = \mathsf{not\ murderer}) & = 0.6 \\
p(\mathsf{knife\ used}|b = \mathsf{murderer}, & m = \mathsf{murderer}) & = 0.1
\end{array}$$

The victim lies dead in the room and the inspector quickly finds the murder weapon, a Knife (K). What is the probability that the Butler is the murderer? (Remember that it might be that neither is the murderer).

Using b for the two states of B and m for the two states of M,

$$p(B|K) = \sum_m p(B, m|K) = \sum_m \frac{p(B, m, K)}{p(K)} = \frac{p(B) \sum_m p(K|B, m)p(m)}{\sum_b p(b) \sum_m p(K|b, m)p(m)}$$

Plugging in the values we have

$$\begin{aligned}
p(B = \mathsf{murderer}|\mathsf{knife\ used}) &= \frac{\frac{6}{10}(\frac{2}{10} \times \frac{1}{10} + \frac{8}{10} \times \frac{6}{10})}{\frac{6}{10}(\frac{2}{10} \times \frac{1}{10} + \frac{8}{10} \times \frac{6}{10}) + \frac{4}{10}(\frac{2}{10} \times \frac{2}{10} + \frac{8}{10} \times \frac{3}{10})} \\
&= \frac{300}{412} \approx 0.73
\end{aligned}$$

Hence knowing that the knife was the murder weapon strengthens our belief that the butler did it.

Chapter 2, Exercise 2

By Bayes' rule

$$p(class = 0|color = c) = \frac{p(color = c|class = 0)p(class = 0)}{p(color = c)}$$

where

$$p(color = c) = p(color = c|class = 0)p(class = 0) + p(color = c|class = 1)p(class = 1)$$

Therefore, for the first row in the dataset we have: so

$$p(Poisonous|Red) = \frac{p(Red|Poisonous)p(Poisonous)}{p(Red)} = 0.608$$

and, since $0.608 > 0.5$, our prediction is Poisonous. For the second row

$$p(Poisonous|White) = \frac{p(White|Poisonous)p(Poisonous)}{p(White)} = 0.307$$

and so our prediction is Edible.

$$p(Poisonous|Brown) = \frac{p(Brown|Poisonous)p(Poisonous)}{p(Brown)} = 0.143$$

and so our prediction is Edible. The probability $p(Brown|Poisonous)$ has be obtained as:

$$p(Brown|Poisonous) = 1 - p(Red|Poisonous) - p(White|Poisonous)$$

Therefore, our prediction is $[0, 1, 1]$, while the true class is $[0, 0, 1]$. The accuracy is

$$\frac{\text{number of correctly classified instances}}{\text{number instances}} = \frac{2}{3}.$$