Discrete Math 2 HW 1

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Problem 9.1.6.

c.

Reflexive proof:
1. $x - x = 0$ where x is a real number (algebra)
2. 0 can be represented as $\frac{0}{1}$ and is therefore a rational number (defintion of rational)
3. Therefore for all real x, (x, x) is in the relation

Symmetric proof:
1. Assume $x - y = \frac{a}{b}$ where x, y are real numbers and a, b are integers (definition of rational)
2. $-\frac{a}{b} = y - x$ (algebra)
3. $-\frac{a}{b} = 0 - \frac{a}{b} = \frac{0}{b} - \frac{a}{b} = \frac{0-a}{b}$ (algebra)
4. $k = 0 - a$ where k is some integer (closure)
5. $\frac{k}{b} = y - x$ (algebra)
6. Therefore, if (x, y) is in the relation, (y, x) must also be in the relation

Not anti-symmetric because (10, 5) and (5, 10) are in the set and $10 \neq 5$

Transitive proof:
1. Assume, $x - y = \frac{a}{b}$ where x, y are real numbers and a, b are integers (definition of rational)
2. Assume, $y - z = \frac{c}{d}$ where y, z are real numbers and c, d are integers (definition of rational)
3. $x = \frac{a}{b} + y$ (algebra)
4. $-z = \frac{c}{d} - y$ (algebra)
5. $(\frac{a}{b} + y) + (\frac{c}{d} - y) = x - z$ (algebra)
6. $\frac{a}{b} + \frac{c}{d} = x - z$ (algebra)
7. $x - z = \frac{a}{b} + \frac{c}{d} = \frac{a*d}{b*d} + \frac{c*b}{b*d} = \frac{a*d+c*b}{b*d} = \frac{k}{h}$ for some integer k, h (algebra, closure)
8. Since k, h are integers $\frac{k}{h}$ is rational (definition of rational)
9. Therefore if (x, y) and (y, z) are in the relation, (x, z) is in the relation making the relation transitive

d.

Not reflexive because (5, 5) is not in the set because $5 \neq 2 * (5)$

Not symmetric because (10, 5) is in the set $10 = 2 * (5)$, but (5, 10) is not in the set $5 \neq 2 * (10)$

Anti-symmetric proof:

1. Assume, $x = 2y$ (definition (x, y))
2. Assume, $y = 2x$ (definition (y, x))
3. $y \neq 4y$ (algebra, contradiction)
4. The first part of the implication is always false therefore the whole thing is always true

Not transitive because (4, 2) is in the set $4 = 2 * (2)$ and (2, 1) is in the set $2 = 2 * (1)$, but (4, 1) is not in the set $4 \neq 2 * (1)$

Problem 9.1.32.

$R \circ S$: {(3, 2), (3, 3), (4, 4), (2, 3), (4, 3), (2, 2), (3, 4)}
$S \circ R$: {(1, 2), (1, 1), (2, 1), (2, 2)}

Problem 9.1.36.

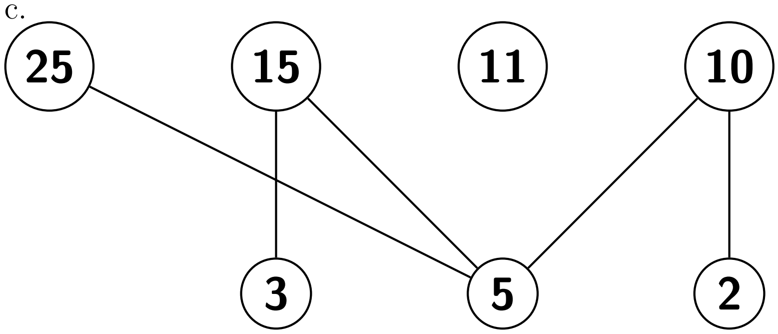
c. $\{(a, b) \in R^2\}$
e. $\{(a, b) \in R^2 | a > b\}$

Problem 9.6.10.

Not partial order because it is not transitive. c → d, d → b, but c does not go to b

Problem 9.6.22.

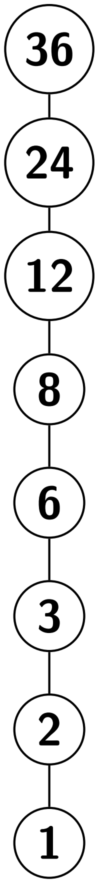
c.

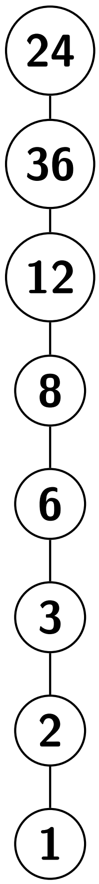


Problem 9.6.26.

{(a,a), (b,b), (c,c), (d,d), (e,e), (a, e), (a, d), (a, b), (a, c), (b, e), (b, d), (c, d)}

Problem 9.6.62.

1. 



2.

Problem Consider the poset....

a. No maximal
b. (0, 0)
c. No greatest
d. (0, 0)
e. (3, 5)
f. (1, 3)
Reflexive proof:
1. Let (a, b) where a and b are arbitrary natural numbers
2. $(a, b) \preccurlyeq (a, b)$ because $a \leq a$ and $b \leq b$
Not Symmetric: $(1, 1) \preccurlyeq (2, 2)$ $1 \leq 2$ and $1 \leq 2$, but $(2, 2) \not\preccurlyeq (1, 1)$ $2 \nleq 1$ and $2 \nleq 1$
Anti-Symmetric proof:
1. Assume $(a1, a2) \preccurlyeq (b1, b2)$ and $(b1, b2) \preccurlyeq (a1, a2)$ where a1, a2, b1, and b2 are arbitrary natural numbers
2. $a1 \leq b1$ and $a2 \leq b2$ (definition of relation)
3. $b1 \leq a1$ and $b2 \leq a2$ (definition of relation)
4. $a1 = b1$ and $a2 = b2$ (algebra)
5. Therefore if $(a1, a2) \preccurlyeq (b1, b2)$ and $(b1, b2) \preccurlyeq (a1, a2)$, $a1 = b1$ and $a2 = b2$
Transitive proof:
1. Assume $(a1, a2) \preccurlyeq (b1, b2)$ and $(b1, b2) \preccurlyeq (c1, c2)$ where a1, a2, b1, b2, c1, and c2 are arbitrary natural numbers

2. $a1 \leq b1$ and $a2 \leq b2$ (definition of relation)
3. $b1 \leq c1$ and $b2 \leq c2$ (definition of relation)
4. $a1 \leq c1$ and $a2 \leq c2$ (algebra)
5. Therefore $(a1, a2) \preccurlyeq (c1, c2)$, and the relation is transitive

Problem Let S be the set of strings....

a. The set of all permutations for the string "abcdefghijklmnopqrstuvwxyz"
b. "" (empty string)
c. No greatest
d. "" (empty string)
e. No least upper bound. Shortest upper bound: "piondk"
f. "thumbrown"
e. No least upper bound. Longest lower bound: "brn"
h. "rin"