Discrete Math 2 HW 1

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Problem 9.1.6.

c.

Reflexive proof:
1. $x - x = 0$ where x is a real number (algebra)
2. 0 can be represented as $\frac{0}{1}$ and is therefore a rational number (defintion of rational)
3. Therefore for all real x, (x, x) is in the relation

Symmetric proof:
1. Let $x - y = \frac{a}{b}$ where x, y are real numbers and a, b are integers (definition of rational)
2. $-\frac{a}{b} = y - x$ (algebra)
3. $-\frac{a}{b} = 0 - \frac{a}{b} = \frac{0}{b} - \frac{a}{b} = \frac{0-a}{b}$ (algebra)
4. $k = 0 - a$ where k is some integer (closure)
5. $\frac{k}{b} = y - x$ (algebra)
6. Therefore, if (x, y) is in the relation, (y, x) must also be in the relation

Not anti-symmetric because (10, 5) and (5, 10) are in the set and $10 \neq 5$

Transitive proof:
1. Let $x - y = \frac{a}{b}$ where x, y are real numbers and a, b are integers (definition of rational)
2. Let $y - z = \frac{c}{d}$ where y, z are real numbers and c, d are integers (definition of rational)
3. $x = \frac{a}{b} + y$ (algebra)
4. $-z = \frac{c}{d} - y$ (algebra)
5. $(\frac{a}{b} + y) + (\frac{c}{d} - y) = x - z$ (algebra)
6. $\frac{a}{b} + \frac{c}{d} = x - z$ (algebra)
7. $x - z = \frac{a}{b} + \frac{c}{d} = \frac{a*d}{b*d} + \frac{c*b}{b*d} = \frac{a*d+c*b}{b*d} = \frac{k}{h}$ for some integer k, h (algebra, closure)
8. Since k, h are integers $\frac{k}{h}$ is rational (definition of rational)
8. Therefore for (x, z) are in the relation and the relation is transitive

d.

Not reflexive because (5, 5) is not in the set because $5 \neq 2 * (5)$

Not symmetric because (10, 5) is in the set $10 = 2 * (5)$, but (5, 10) is not in the set $5 \neq 2 * (10)$

Anti-symmetric proof:
1. $x = 2y$ (definition (x, y))
2. $y = 2x$ (definition (y, x))
3. $y \neq 4y$ (algebra)

4. The first part of the implication is always false therefore the whole thing is always true

Not transitive because (4, 2) is in the set $4 = 2 * (2)$ and (2, 1) is in the set $2 = 2 * (1)$, but (4, 1) is not in the set $4 \neq 2 * (1)$