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Solution of Equations in $\mathbf{R}^n$ (Part 2)

NORTH-HOLLAND

Blagovest Sendov, Andrey Andreev, Nikolay Kjurkchiev

NUMERICAL SOLUTION OF POLYNOMIAL EQUATIONS

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