Lecture 1 Mathematics for Computer Science

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1 Proof

Proof is a method for ascertaining the truth.
These are some forms of proving:

- Experimentation, Observation
- Sampling, Counter Exampling
- Common Sense

A mathematical proof is a verification of a propositions by a chain of logical deductions from a set of axioms.

1.1 Proposition

A proposition is a statement that can be true or false.
Examples.

2+3=5 is True. The sky is yellow is False.

$\forall n \in N$ [$n^2 + n + 41$ is a prime number]

$\exists a, b, c, d \in N_+$ [$a^4 + b^4 + c^4 = d^4$]

$313(x^3+y^3) = z^3$ [has no positive integer solutions]

Goldbach's proposition: Every even positive integer but 2 is the sum of 2 primes.

$\forall n \in Z_+$ [$n \geq 2 \implies n^2 \geq 4$]

The part inside the brackets is called predicate, and it's a proposition where to be truth it depends on a variable value.
And the arrow is called an implication.

1.1.1 Implication

An implication is a statement in which one proposition implies another one.

For example, let P and Q be two different propositions,
P $\implies$ Q reads as P implies Q.

In this case, we have some possibilities for P and Q and for the statement as whole. These possibilities are ordely written in what's called a Truth Table:

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

There is also the double implication or biconditional, in which is exactly like the implication but Q also implies P.

The notation is:

$P \iff Q$

And the Truth Table for this one would be:

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

1.2 Axioms

Now an axiom is a proposition in which we assume to be True. We say assume because most of it is so fundamental and basic that we don't have ways to prove it, so we assume to be True by common sense and use it to prove others that we actually have to prove.

Examples.

If a = b, and b = c, then a = c.

If A $*$ B = 0, then A=0 or B=0 or A=B=0.

A general rule about axioms is that they must be consistent and non-redundant.

Being consistent means it should be impossible for an axiom to be True and False at the same time. And non-redundant means they must mean something and must be useful to proof.