Lecture 1 Differentiation

Benay Tomas

April 5, 2024

1 What is a Derivative?

- Geometric Interpretation
- Physical Interpretation
- The Importance to all measurements

2 How to differentiate any function you want

Geometric Interpretation

By the Geometric interpretation, the derivative is the slope of the tangent line in a point P in any function F(x).

But what is a tangent line?

Objective: Find the tangent line to f(x) at P(x_0, y_0):

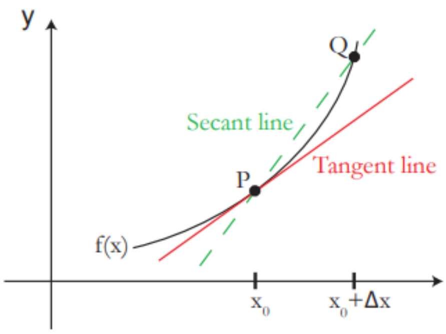

Figure 1: Function with tangent and secant lines

Picture a secant line that crosses through P and another random point Q, now imagine this point Q gets closer and closer to P, the limit of the secant line as the distance between the two points goes to zero IS the tangent line.

Let's consider the distance in the X-axis from P to Q as Δx. So P is on $(x_0, f(x_0))$ and Q is on $(x_0 + \Delta x, f(x_0 + \Delta x))$.

And know to calculate the tangent line we'll use the **the slope equation:** m $= (y - y_0)/(x - x_0)$.

But instead of 'y' it will be Q's Y-coordinate, instead of 'y0' it'll be P y-coordinate, 'x-x0' will be simply Δx as it is the X difference already.

We'll have:

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0)$$

Thus the derivative.

Example 1. $f(x) = \dfrac{1}{x}$

One thing to keep in mind when calculating the derivative, is to never plug Δx = 0, because of the underdetermination of dividing by 0. So, instead, what we do is manipulate the function and its properties to make sure we don't end up dividing something by zero.

$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} = \frac{1}{\Delta x}\left[\frac{x_0 - (x_0 + \Delta x)}{(x_0 + \Delta x)x_0}\right] = \frac{1}{\Delta x}\left[\frac{-\Delta x}{(x_0 + \Delta x)x_0}\right] = \frac{-1}{(x_0 + \Delta x)x_0}$$

Taking the limit as $\Delta x \to 0$,

$$\lim_{\Delta x \to 0} \frac{-1}{(x_0 + \Delta x)x_0} = \frac{-1}{x_0^2}$$

Finding the tangent line

Now that we know the slope of the function is $\dfrac{-1}{x_0^2}$. We just have to use the equation for the tangent line again and plug in the values of the function in x0 and the slope.

And we'll get:

$$y = \frac{-1}{x_0^2}(x - x_0) + \frac{1}{x_0}$$

Simplifying:

$$y = -\frac{x - 2x_0}{x_0^2}$$

3 Notations

The derivative can be expressed as $f'(x)$ or $\frac{d}{dx}f(x)$ or $\frac{dy}{dx}$ or $\frac{df}{dx}$ and $\mathrm{D}f$

4 Properties

This section will have a separated document file just for it, due to having an enourmous amount of Calculus Properties, not only derivatives but limits and integrals too.