1. Introduction

An investigation into referee bias by Price, Remer, and Stone (2012), in their article, *Subperfect Game: Profitable Biases of NBA Referees*, found that NBA referees favor home teams in terms of turnovers and personal fouls. Since referee bias has been a highly debated topic in all professional sports, it's important to know what implications these biases have, whether economic or moral. The moral implications argue that the referee's bias ruins the integrity of the game. For the purposes of this paper, we do not focus on the moral implications of referee bias. The economic implications we focus on within this paper are whether the estimated biases distort the betting markets.

Why is referee bias so important for betting markets, then? There are a few reasons why it's important to understand the implications of referee bias on the betting markets. Bookmakers need and want clean competition. If bookmakers are not considering the biases in their betting lines, this can lead bettors to leave the bookmakers and cause them to lose their businesses. In addition, bettors want all information before placing their bets. If I, as a bettor, am placing a bet on one team, but the referees favor the other, then I will more than likely lose my bet. Lastly, leagues, players, and team owners have reputations and money on the line. If the biases are altering game outcomes counter to what the market expects, then that could lead to the overall consumption of the league to dwindle. There would be no point in betting, creating betting lines, or watching the games because the referees predetermine who is likely to win. With these concerns in mind, we investigate whether the estimated biases toward home teams found in Price, Remer, and Stone (2012) influenced the side-lines betting market in the NBA from 2006-2017.

To test for market inefficiencies caused by referee bias in the side-lines betting market, we take a similar approach to the standard market efficiency tests found in the betting market literature. In the standard market efficiency test, the specification is conducted by recognizing that the points differential and closing side-lines are from the point of view of the favorite in the matchup. Our specification deviates only slightly, as we are conducting the analysis from the point of view of the home team to determine if the biases found favoring the home teams influence the market. Overall, we find that, while there is weak evidence of market inefficiencies in the side-lines market from 2006-2017, the inefficiencies are not caused by the estimated biases toward the home team found in Price, Remer, and Stone (2012).

The paper is organized as follows. Section II briefly reviews the results found in Price, Remer, and Stone (2012). Section III introduces some betting market background literature and the standard market efficiency tests on our data. Section IV reviews the data used and the specification of our model. Section V presents the main results. Section VI concludes.

2. Background on Price, Remer, and Stone (2012)

Price, Remer, and Stone (2012) conducted a series of analyses on referee bias in the NBA from 2002-2023 through the 2007-2008 seasons. Their analysis uses a play-by-play dataset that allows them to divide turnovers and fouls into categories more heavily influenced by referees. They begin by distinguishing turnovers into discretionary (DTOs) and non-discretionary turnovers (NTOs). The implication is that referees have more influence over DTOs than they do on NTOs. Referee influence is more difficult to distinguish in personal fouls; however, they separate them into shooting and non-shooting fouls.

Table I. Results from Table III in Price, Remer, and Stone (2012)				
	Discretionary Turnovers	Nondiscretionary Turnovers	Shooting Fouls	Nonshooting Fouls
Home	-0.0824***	-0.0128	-0.0759***	-0.0449***
	-0.0215	-0.0133	-0.0111	-0.0123

Table I presents the results from Table III in the Price, Remer, and Stone (2012) article on the effect of referee bias against the home team. The results indicate that, on average, referees call 8.24% statistically fewer DTOs against the home team than the away team. In addition, they find that referees, on average, call 7.59% and 4.49% statistically fewer shooting and non-shooting fouls against the home team compared to the away team. The authors conclude that since these coefficients are statistically significant, and by their definitions that referees have more influence over DTOs compared to NTOs, indicates that referees favor the home teams by calling statistically fewer turnovers and fouls against them.

To obtain the predicted impacts on the number of turnovers and fouls, we recognize that the estimations in Table III from Price, Remer, and Stone (2012) are for the game-minute-team observations. There are 48 minutes in an NBA game, so we multiply 48 by the estimated coefficients to find the impacts of these referee biases. This leads to 3.95 fewer discretionary turnovers against the home team, 3.64 fewer shooting fouls against the home team, and 2.15 fewer non-shooting fouls against the home team. Since the data we use is box score data and the turnover/foul type cannot be distinguished, we combined the two foul impacts to get the predicted impact on the total personal fouls, which is 5.79 fewer personal fouls called against the home team. These two estimates, 3.95 and 5.79, will be used in Section 4.2 when we define the games exhibiting heavy referee bias.

3. Introduction to Betting Markets

There have been several studies on the efficiency of betting markets in professional sports. Bower et al. (1985) conducted two efficiency tests of the gambling market in the NFL and found weak evidence to establish efficiency. However, they find profitable betting opportunities, indicating there are possible inefficiencies in the market. Gandar et al. (1988) find statistical evidence that the NFL gambling market behaves rationally, though they find economic evidence that there are profitable betting behaviors, indicating the market is not behaving rationally. Gander et al. (1998) finds that informed trade betting leads to significantly improved accuracy of betting lines forecasts of game outcomes.

The standard gambling market efficiency test is regressing the points differential at the end of the game against the closing side-line. This is typically done from the viewpoint of the favorite in the matchup, so point differentials and closing side-lines will be from the viewpoint of the favorites. The statistical tests for market efficiency is that the coefficient on the closing side-line is one ($\beta_1 = 1$), the constant is equal to zero ($\beta_0 = 0$), and the joint test that $\beta_1 = 1$ and $\beta_0 = 0$. If we reject the null in any test, we conclude that there is a market inefficiency present. The model's specification is:

$$favdiff_i = \beta_0 + \beta_1 slfav_i + e_i$$

Conducting the standard market efficiency tests on our dataset in Table II finds evidence in both sample periods there is a market inefficiency present. Since we reject the null for $\beta_0 = 0$ in both sample periods and the joint test that $\beta_1 = 1$ and $\beta_0 = 0$ in the full sample, we conclude there is a market inefficiency.

Table II. Standard Gambling Market Efficiency Results		
	favdiff (2006-2017)	**favdiff** (2006-2007)
Favorite Side-Lines	1.06***	0.9959***
	(0.0264)	(0.1910)
Constant	0.9959***	1.5084***
	(0.1910)	(0.4746)
F-Stat. β_0=0	27.18	10.1
F-Stat. β_1=1	5.15	3.65
F-Stat. β_0=0 & β_1=1	23.22	6.45

Based on our results of the standard market efficiency tests, we find that there is a market inefficiency present. Do we think, however, that the predicted referee biases from Price, Remer, and Stone (2012) are contributing to these inefficiencies? The bias could be enough to change game outcomes counter to market expectations. However, the NBA and bookmakers don't seem worried about referee bias because there have been no news stories, scandals, or blue-ribbon commissions to investigate them. So, either the market knows the bias exists and builds it into the existing betting lines, or it doesn't matter enough to alter market expectations. This paper will contribute to the existing literature by determining if the concerns over the referee bias estimated by Price, Remer, and Stone (2012) are purely aesthetic/moral or is the source of inefficiency in the side-lines betting market.

4. Data and Empirical Strategy

4.1 Data

Our data consists of betting market data and box score data for all NBA games from 2006-2017.[1] The data includes the date of individual games, what teams were home and away, and in-game statistics, such as turnovers, personal fouls, and points scored by each team. Also included in the dataset are the closing side-lines and final point differentials for each team. Betting lines were obtained from multiple online sports books and then the average side-line was obtained for each game. We look at two periods to determine whether the referee bias estimated in Price, Remer, and Stone (2012) influenced the side-lines betting market. The first period is from 2006-2007 since these two years are the only overlapping years in the dataset used by Price, Remer, and Stone (2012). The next period is for the full sample from 2006-2017. In our estimations, we have roughly 14,900 observations in the full sample and 2,500 observations in the smaller sample.

Table III displays summary statistics of the full sample for the four main variables used in our analysis. The variables are the total number of personal fouls and turnovers committed by the home team in a single game, along with the point differential at the end of the game from the viewpoint of the home team and the closing side-lines before tip-off from the viewpoint of the home team. Based on the summary statistics, the home team committed, on average, 13.49 turnovers and 20.19 personal fouls per game. The home team scored, on average, 3.01 more points than the away team, and the home team was favored by 3.41 points over the away team before each game began.

[1]Missing exact location of where data was obtained from.

Table III. Summary Statistics 2006-2017				
	Mean	Std. Dev.	Min	Max
Home Turnovers	13.49	3.81	2	29
Home Personal Fouls	20.19	4.29	7	41
Home Point Differentials	-3.01	13.31	-61	58
Home Closing Side-lines	-3.41	6.38	-23	17.5

4.2 Identifying Games with Heavy Home Bias

To determine whether a game has a heavy bias by the referee's, we take a relatively simple approach. In Section 2 of the paper, we found that referees called 3.95 and 5.79 statistically fewer turnovers and personal fouls per game (48 minutes) against the home team, respectively. These predicted impacts are similar to the standard deviations found for our variables, home turnovers and home personal fouls (3.95 and 5.79 predicted impacts compared to 3.81 and 4.29 standard deviations). By recognizing this similarity, we create two indicator variables that indicate whether a game experiences "heavy" referee bias in favor of the home team. The first indicator variable is equal to one when the home team's turnovers were fewer than 3.95 times 1.5 standard deviation, less the sample mean (heavyhtov). The second indicator variable is equal to one when the home team's personal fouls were fewer than 5.79 times 1.5 standard deviations, less the sample mean (heavyhpf). The specification for heavily biased games is:

$$heavyhtov = 1 \; if \; htov_i < 13.49 - (1.5 * 3.95)$$

$$heavyhpf = 1 \; if \; hpf_i < 20.19 - (1.5 * 5.79)$$

It's important to note that we choose 1.5 standard deviations here arbitrarily.[2] The specification can be changed from 1.5, though, as we make our way closer to two standard deviations, we do not have many games that fit the heavy bias criteria. We then interact the indicator variables with the spread from the viewpoint of the home team.

Figure 1.1

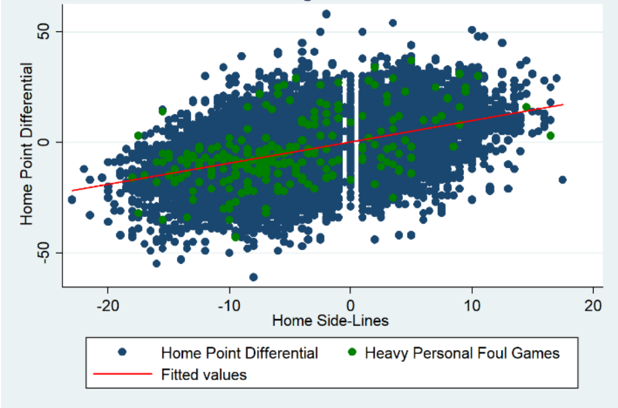
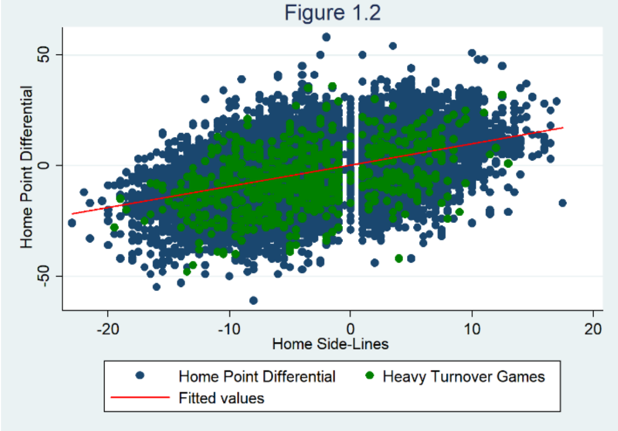


Figure 1.2



Figures 1.1 and 1.2 display two scatter plots showcasing the home closing side-lines (blue dots) and potentially heavily biased games for turnovers and personal fouls (green dots). Figure 1.1 showcases the heavily biased games for personal fouls, and Figure 1.2 showcases the heavily biased games for turnovers. The purpose of these graphs is to show that the heavily

[2] We find similar results using 1 standard deviations, or just the predicted impact of turnovers and fouls less the sample mean.

biased games do not all come from one side of the side-line distribution or the other. Essentially, the heavily biased games are not coming from the sample when the home team is heavily favored to win (large negative home side-line) or the home team is a heavy underdog (large positive home side-line). The red fitted lines in both figures indicate a positive relationship between the home side-line and home points differential.

4.3 Empirical Model

Earlier in this paper, we discussed that the standard market efficiency test for the betting market is to regress the point differential at the end of the game from the viewpoint of the favorite against the closing side-line from the viewpoint of the favorite. Since the objective of this paper is to estimate whether the predicted biases toward the home teams influence the side-lines betting market, we must adjust the specification of the model to be from the viewpoint of the home team rather than from the viewpoint of the favorite. The adjustment is to regress the point differential from the viewpoint of the home team against the closing side-lines from the viewpoint of the home team and introduce the two interaction terms, which will allow us to test for whether the referee bias is influencing the market. The new specification of the model is:

$$homediff_i = \beta_0 + \beta_1 slhome_i + \beta_2 slhome_i * heavyhtov_i + \beta_3 slhome_i * heavyhpf_i + e_i$$

In addition to adjusting our model, we must adjust the efficiency tests to account for the referee bias. Our first test of efficiency is to determine whether the coefficient on the closing side-line from the viewpoint of the home team is equal to one ($\beta_1 = 1$). If we reject the null in this case, we conclude that the side-lines betting market is inefficient. Our second efficiency test is to determine whether the coefficients β_1, β_2, and β_3 are jointly equal to one

$(\beta_1 + \beta_2 + \beta_3 = 1)$. If the null is rejected in this case, we conclude that the biases toward the home teams are causing inefficiencies in the side-lines betting market.

5. Main Results

Table IV displays the results of our OLS regressions and the corresponding efficiency tests for both periods to determine whether referee bias toward the home teams is influencing the side-lines betting market. The first column displays the results for the full sample (2006-2017), and the second column displays the results for the smaller sample (2006-2007). The two OLS regressions are estimated using robust standard errors.

Table IV. Side-lines Betting Market Regression Results

	(1) **homediff** (2006-2017)	(2) **homediff** (2006-2007)
Home Side-lines	0.9488*** (0.0152)	0.9378*** (0.0385)
*Home Side-lines*Heavy Biased Turnover Games*	0.1808*** (0.0602)	0.421*** (0.1431)
*Home Side-lines*Heavy Biased Personal Foul Games*	0.053 (0.0879)	0.022 (0.1682)
Constant	0.2667** (0.1108)	0.2268 (0.2828)
F-Stat. Hypothesis 1	11.28	2.61
F-Stat. Hypothesis 2	3.15	3.11

*,**,*** denote 10%, 5%, and 1% significance. Columns 1 and 2 are OLS regressions using robust standard errors.

In the full sample, we see that both interaction terms are positive and statistically significant at the 5% level. This suggests when games are experiencing a heavy bias in turnover and fouls in favor of the home team, the market is underpredicting the home team's performance. We find similar results in the smaller sample. The interaction term for heavy bias in turnovers is positive and statistically significant at the 5% level. However, our interaction term that includes heavy bias for personal fouls is now statistically insignificant at the 5% level. This suggests when games are experiencing only a heavy bias in turnovers in favor of the home team, the market underpredicts the home team's performance.

To determine whether the referee's bias toward the home team influences the side-lines betting market, we must look at the two hypotheses and the corresponding F-statistics. In our full sample, we find weak evidence to indicate that we reject the null ($\beta_1 = 1$), and, therefore, concludes that the side-lines betting market is inefficient. However, when we look at the hypothesis test of whether the coefficients β_1, β_2, and β_3 are jointly equal to one, we fail to reject the null. This result suggests that, even though there is a market inefficiency present from 2006-2017, the referee bias estimated by Price, Remer, and Stone (2012) towards the home team is not causing this inefficiency. In the smaller sample, we fail to reject the null for both hypotheses. These results suggest that there is no market inefficiency, whether it be caused by referee bias or not, present from 2006-2007.

6. Conclusion

We have presented evidence that the estimated biases towards the home team in Price, Remer, and Stone (2012) do not influence the side-lines betting market. While we find weak evidence of market inefficiency from 2006-2017, it is not due to referee bias. It now begs the question of whether the implications of these biases are strictly aesthetic or moral. This is a question for future work concerning the implications of referee bias in professional sports. In addition, two other betting markets, the money line (which team wins) and the totals line (over/under total points scored), can be investigated to see if referee bias is an issue.

As noted before, we have taken a simplistic approach to defining games that exhibit heavy referee bias. In this specification, the assumption is that every team has the same offensive and defensive abilities, which is never the case. Teams vary in their abilities, meaning some teams commit more fouls per game than others or they commit less turnovers per game than other teams. To account for this, we need a new specification that can accurately describe a team's accumulated in-season performance coming into the game. The new specification can be described as follows. We first accumulate all performance stats from the first game through the last game of the season. Then generate per-game performance stats for each game moving through the season. Next, compute pre-game averages for in-game stats for both the home and away team. Then use the pre-game values to predict the number of home team turnovers and personal fouls per game. Lastly, compare the actual number of turnovers and fouls to the predicted. If the actual turnovers are less than 3.95 and actual fouls are less than 5.79, then identify the game as having heavy bias. Once we have our games that are identified as heavily biased, we then run our market efficiency tests to see if the results hold.[3]

References.

Gandar, John M., et al. "Informed traders and price variations in the betting market for professional basketball games." The Journal of Finance 53.1 (1998): 385-401.

[3] This specification is still being worked on and is why it is not used within this paper.

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