Exam 1 - NE591

88

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1 $T'(x_0)=0 \quad x_0=0 \quad x_1=X \quad T(x_1)=T_1$

12/12

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+Q=0$$

Assumptions: Steady state
Axisymmetric } Equivalently, constant in y & z for the Cartesian system
Constant in z }
Constant thermal conductivity (w.r.t. Temperature)
i.e. $k \neq k(T)$

$$\int \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+Q\,dx = \int 0\,dx$$

$$k\frac{\partial T}{\partial x}+Qx+C_1=0$$

$$kT'(x)+Qx+C_1=0 \xrightarrow{\text{Apply boundary condition}} \underbrace{kT'(0)}_{=0}-\cancel{Q(0)}+C_1=0$$

$$\Rightarrow C_1=0$$

$$\int kT'(x)+Qx+C_1\,dx=\int 0\,dx$$

$$kT(x)+\frac{Q}{2}x^2+C_1x+C_2=0$$

$$kT(x)+\frac{Q}{2}x^2+C_2=0$$

$$kT(X)+\frac{Q}{2}X^2+C_2=0$$

$$kT_1+\frac{QX^2}{2}=-C_2 \Rightarrow C_2=-kT_1-\frac{QX^2}{2}$$

$$\Rightarrow kT(x)+\frac{Qx^2}{2}-kT_1-\frac{QX^2}{2}=0$$

Final answer on pg 2

$$\boxed{T(x) = \frac{Q}{2k}(X^2 - x^2) + T_i}$$ ✓

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13/18

$\boxed{2}$ Find Centerline temp $(r=0)$ and $T(0.4\text{ cm})$

$T_{cool} = 800\text{ K} \longrightarrow T_{cant} - 800\text{ K} = \frac{Q}{2h_{cooling}} R_{fuel} = \frac{(400 \frac{W}{cm^3})(0.86\text{ cm} - 0.85\text{ cm})}{2(5 \frac{W}{m \cdot K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm})$

$= 24\text{ K}$

$T_{clad} - 824\text{ K} = \frac{Q}{2h_{clad}} R_{fuel} \qquad \Rightarrow T_{cant} = 824\text{ K}$

$T_{clad} - 824\text{ K} = \frac{400 \frac{W}{cm^3}}{2(55 \frac{W}{K\,cm^2})}(0.6\text{ cm}) = 21.82\text{ K}$

Switched these steps

-1

$T_{clad} = 845.82\text{ K}$

$T_{gap} - 845.82\text{ K} = \frac{Q t_{clad}}{2k_{clad}} R_{fuel}$

$T_{gap} - 845.82\text{ K} = \frac{(400 \frac{W}{cm^3})(0.85\text{ cm} - 0.8\text{ cm})}{2(15 \frac{W}{m \cdot K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm}) = 40\text{ K}$

$T_{gap} = 885.82\text{ K}$

0.2 not 0.25
0.8 - 0.6

$T_{fuel} - 885.82\text{ K} = \frac{Q t_{gap}}{2k_{gap}} R_{fuel} = \frac{400 \frac{W}{cm^3}(0.85\text{ cm} - 0.6\text{ cm})}{2(25 \frac{W}{m \cdot K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm}) = 120\text{ K}$

-3

$T_{fuel} = 1005.82\text{ K}$

$T_o - T_{fuel} = \frac{Q}{4k} R_{fuel}^2$

$T_o - 1005.82\text{ K} = \frac{(400 \frac{W}{cm^3})}{4(0.5 \frac{W}{cm \cdot K})}(0.6\text{ cm})^2 = 72\text{ K}$ ✓

wrong T_o gives you wrong $T(r)$

$\boxed{T_o = 1077.82\text{ K}}$ $\qquad T_o - T(0.4\text{ cm}) = \frac{Q}{4k}(0.4\text{ cm})^2 = \frac{(400 \frac{W}{cm^3})}{4(0.5 \frac{W}{cm \cdot K})}(0.4\text{ cm})^2$

-1

$1077.82\text{ K} - T(0.4\text{ cm}) = 32\text{ K} \Rightarrow \boxed{T(0.4\text{ cm}) = 1045.82\text{ K}}$

12/14

3 a) $Q = E_f N_f \sigma_f \phi$

$3\left[235(0.195) + 238(1-0.195)\right] + 2(28) = 769.25 \ \frac{g}{mol}$ ✓

$\left(15.67 \frac{g}{cm^3}\right)\left(\frac{1}{769.25 \ g/mol}\right)\left(6.022\times10^{23} \ \frac{molecules}{mol}\right)(0.195)\left(5 \ \frac{atoms}{molecule}\right) = 7.186\times10^{21} \ \frac{atoms}{cm^3}$ ✓ of U^{235}

$Q = (200\times10^6 \ eV)\left(1.602\times10^{-19} \ \frac{J}{eV}\right)\left(7.186\times10^{21} \ \frac{U^{235} \ atoms}{cm^3}\right)\left(2\times10^{12} \ \frac{n}{cm^2 \ s}\right)\left(570\times10^{-24} \ cm^2\right)$

$\boxed{Q = 262.475 \ \frac{J}{s \ cm^3} = 262.475 \ \frac{W}{cm^3}}$ ✓

b) $7.186\times10^{21} \ \frac{U^{235} \ atoms}{cm^3}$ is the required atom density

Atomic mass $= 235(x) + 238(1-x) + 2(16)$ −2
$= -3x + 238 + 32$
$= -3x + 250$ → math error $238+32 = 270$

$7.186\times10^{21} \ \frac{U^{235} \ atoms}{cm^3} = (x)\left(6.022\times10^{23} \ \frac{molecules}{mol}\right)\left(10.96 \ g/cm^3\right)\left(\frac{1}{-3x+250 \ g/mol}\right)$

$0.0011 = \frac{x}{-3x+250}$ $\quad -0.0033x + 0.272 = x$

$0.272 = 1.0033x$
$x = 0.271$ → $\boxed{27.1\% \text{ enrichment needed}}$

~ should be 0.959 $\quad$ 6/12

$\boxed{4}$ a) $LHR\left(\frac{z}{z_0}\right) = 150\,\frac{W}{cm^2}\cos\left[\frac{\pi}{2.2}\left(\frac{z}{z_0}-1\right)\right]$

$LHR\left(\frac{z}{z_0}\right)\Big|_{z=1.5m} = 150\,\frac{W}{cm^2}(0.99999) = \boxed{149.999\,\frac{W}{cm^2}}$ ×

b) $T_{cool} - T_{cool}^{in} = \frac{1}{1.2}\,\frac{z_0\,LHR^0}{\dot{m}C_{pw}}\left\{\sin(1.2) + \sin\left[1.2\left(\frac{z}{z_0}-1\right)\right]\right\}$

Evaluate at $z = L = 2z_0$ ✓

$\Delta T_{cool} = \frac{1}{1.2}\,\frac{z_0\,LHR^*}{\dot{m}C_{pw}}\left\{\sin(1.2) + \sin\left[1.2(1)\right]\right\}$

$\left(\frac{\pi}{2\gamma}\right) \rightarrow$ here 1.43

As $\dot{m}C_{pw}$ increases, ΔT_{cool} decreases

$\therefore$ higher $\dot{m}C_{pw}$ indicates lower ΔT_{cool}

$\Rightarrow$ $\boxed{\text{Water sees a higher } \Delta T}$ ×

$\dot{m}C_p(H_2O) > \dot{m}C_p(Na)$

$\rightarrow$ Na has higher ΔT

[5] $\frac{dy}{dt} = 4t - 3t^2$ $\quad dt = 0.33 \quad y(1) = 6$ 16/16

Backwards

$t_1 = 1.33 \quad f(t_1) = 6 + 0.33(4(1.33) - 3(1.33)^2) = 6.004$

$t_2 = 1.66 \quad f(t_2) = 6.004 + \underbrace{0.33(4(1.66) - 3(1.66)^2)}_{= -0.5368} = 5.467$

$t_3 = 1.99 \approx 2 \quad f(t_3) = 5.467 + \underbrace{0.33(4(2) - 3(2^2))}_{= -1.32} = 4.147$

Forwards

$t_1 = 1.33 \quad f(t_1) = 6 + \underbrace{0.33(4(1) - 3(1)^2)}_{= 0.33} = 6.33$

$t_2 = 1.66 \quad f(t_2) = 6.33 + \underbrace{0.33(4(1.33) - 3(1.33)^2)}_{= 0.0041} = 6.334$

$t_3 = 1.99 \approx 2 \quad f(t_3) = 6.334 + \underbrace{0.33(4(1.66) - 3(1.66)^2)}_{= -0.5368} = 5.797$

[6] fertile - can be converted into a fissile nuclide 5/5
fissile - capable of sustaining chain reaction w/ nt of any energy
fissionable - capable of undergoing fission w/ high energy nt's

[7] Pure U experiences a very large amount of swelling, and 4/4
the α phase experiences anisotropic irradiation growth. In general,
too many phases on phase diagram that could be present in-core

swelling

[8] Smear density is the ratio of fuel volume to total volume of a fuel element. 2/4
It allows for the accounting of material pores, gaps, etc in the fuel rods.

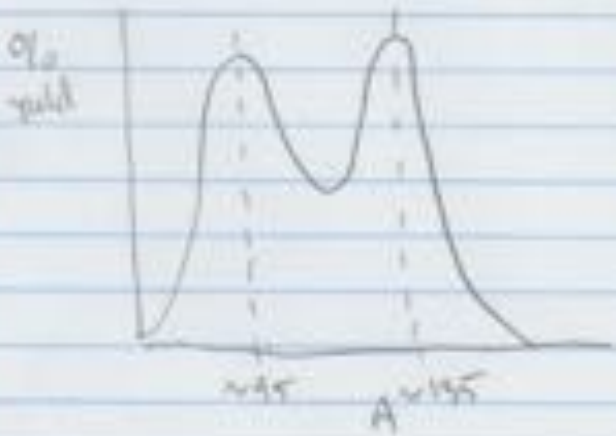
9 We need to enrich U due to the primary component, U-238, not being fissile. We need a higher density of U-235 to be able to reliably sustain a fuel cycle that is acceptably long, as there won't be enough excess reactivity at beginning of life otherwise

6/8

Uranium is converted from "yellow-cake" into UF_6 gas in order to be enriched. The gas is centrifuged and tapped from the center
This works since U-235 is physically heavier than U-238, so it isn't thrown as far towards the outside of the centrifuge, and is able to move slightly faster than U-238

opposite

10 2 primary fission product species are Molybdenum & Cesium,
Typically the 2 fission products aren't equal weights, rather 1 light & 1 heavy. The typical curve is shown below



Mo has A=95
Cs has A=133

4/4

8/8

11 Finite Difference
Finite Volume
$\boxed{\text{Finite Element}}$ → Can model any system & any BC
geometry
→ continuous representation
→ allows for heterogeneous properties in materials

utilized
~ S.O.A