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Cole Manfred, 1/24/25, Exam 1

1. UN fuel, $\varepsilon = 19\%$, $\rho = 14.3\ g/cm^3$, $\sigma = 587b$

$\phi = 4\times10^{12}\ n/cm^2\cdot s$, $A_N = 14$, $E_F = 200\ MeV$

10/10

$Q = E_F N_F \sigma_F \phi$

$N_F = \rho_{UN} \dfrac{N_A}{A_{UN}} \cdot \text{atom frac.} \cdot \varepsilon$

$A_{UN} = [235\varepsilon + 238(1-\varepsilon)] + 14$ ✓

$= 235(0.19) + 238(1-0.19) + 14$

$= 251.43\ g/mol$ ✓

$N_F = 14.3 \dfrac{g}{cm^3} \cdot \dfrac{6.022\times10^{23}\ \text{atom/mole}}{251.43\ g/mole} \cdot \dfrac{1\ \text{at. U}}{1\ \text{mol. UN}} \cdot 0.19$ ✓

$N_F = 6.507\times10^{21}$ atoms U-235/cm^3 ✓

$Q = 200\times10^6\ eV \cdot \dfrac{1.602\times10^{-19}\ J}{1\ eV} \cdot 6.507\times10^{21} \dfrac{U235}{cm^3}$

$\cdot\ 587\times10^{-24} \dfrac{cm^2}{n} \cdot 4\times10^{12}\ n/cm^2\cdot s$ ✓

$\boxed{Q = 490\ W/cm^3}$

$Q = 490 \rightarrow N_F = 6.507\times10^{21}$ ✓

$6.507\times10^{21} = \left(10.97 \dfrac{g}{cm^3}\right)\left(6.022\times10^{23}\right)\left(\dfrac{1}{1}\right)\left(\dfrac{\varepsilon}{235\varepsilon + 238(1-\varepsilon) + 2(16)}\right)$ ✓

$9.850\times10^{-4} = \dfrac{\varepsilon}{235\varepsilon + 238(1-\varepsilon) + 32}$ ✓

$0.231\varepsilon + 0.234 - 0.234\varepsilon + 0.031 = \varepsilon$ ✓

$0.266 = 1.003\varepsilon \rightarrow \boxed{\varepsilon = 0.265 = 26.5\%}$

17/18

2. $T_o = ?$ $T(r=0.1cm) = ?$ Gap all He

$K_c = 0.18$ W/cmK, $K_f = 0.04$ W/cmK, $h_{cool} = 1.5$ W/cm²K

$Q = 350$ W/cm³, $R_f = 0.4$ cm, $t_g = 30$ um

$t_c = 0.05$ cm, $T_{cool} = 500$ K

$$T_{oc} - T_{cool} = \frac{Q}{2h_{cool}} R_f$$

$$T_{oc} = \frac{350}{2(1.5)}(0.4) + 500 = \underline{546.7\text{ K}}$$

$$T_{ic} - T_{oc} = \frac{Q t_c}{2K_c} R_f$$

$$T_{ic} = \frac{350(0.05)(0.4)}{2(0.18)} + 546.7 = \underline{566.1\text{ K}}$$

$$T_f - T_{ic} = \frac{Q t_g}{2K_g} R_f \quad K_g = 16\times10^{-6}\, T_{ic}^{0.79}$$

$$K_g = 16\times10^{-6}(566.1)^{0.79} = 0.0024 \text{ W/cmK}$$

$$T_f = \frac{350(30\times10^{-4})}{2(0.0024)}(0.4) + 566.1 = \underline{653.9\text{ K}}$$

$$T_o - T_f = \frac{Q}{4K_f} R_f^2 \rightarrow T_o = \frac{350}{4(0.04)}(0.4)^2 + 653.9$$

$$\boxed{T_o = 1004\text{ K}}$$

$$T(r) = \frac{Q(R_f^2 - r^2)}{4K_f} + T_f$$

$$T(r=0.1cm) = \frac{350(0.4^2 - 0.1^2)}{4(0.04)} + 653.9 = \boxed{982\text{ K}}$$

- cladding midpoint T?

$t_{ox} = 60\ \mu m = 0.006\ cm$

$k_{ox} = 0.015\ W/cmK$

$T_{ox} - T_{cool} = \frac{Q}{2h_{cool}} R_f \rightarrow T_{ox} = 546.7K$ ✓

$T_{oc} - T_{ox} = \frac{Q t_{ox}}{2k_{ox}} R_f \rightarrow T_{oc} = 574.7K$ ✓

$T_{ic} - T_{oc} = \frac{Q t_c}{2k_c} R_f \rightarrow T_{ic} = 594.1K$ ✓

$T_f - T_{ic} = \frac{Q t_g}{2k_g} R_f \rightarrow k_g = 0.0025$ ✓

$T_f = 678.6\ K$

$T_o - T_f = \frac{Q}{4k_f} R_f^2 \rightarrow \boxed{T_o = 1028.6}$ ✓

$\Delta T_o = 24.6$ ✓

3.

L

L

T

3. $L = 3.6m$, $z_0 = 1.8\,m$, $LHR^{\circ} = 400\,W/cm$
$\gamma = 1.25$, $z = 2.1m$

5/8

$$LHR(z) = LHR^{\circ} \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right]$$ ✓

$$LHR(z=2.1) = 400 \cos\left[\frac{\pi}{2(1.25)}\left(\frac{2.1}{1.8} - 1\right)\right]$$ ✓

$$\boxed{LHR = 391\ W/cm}$$ ✓

$C_p = 4200\ J/kg\,K$, $\dot{m} = 0.1\,kg/s$, $T_{cool}^{in} = 500\,K$

$$T_{cool} - T_{cool}^{in} = \frac{2\gamma}{\pi}\,\frac{z_0 \times LHR^{\circ}}{\dot{m}\,c_p}\left\{\sin\left(\frac{\pi}{2\gamma}\right) + \sin\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right]\right\}$$

$$\boxed{T_{cool} = 501.6\ K}$$

doesn't make sense

I don't see steps ... but I assume you left z_0 in m of $z_0 \times LHR$

needs same units

- show steps for more partial credit!

4)

0/8

5. U_3O_8 is found in ore and converted to UF_6 for enrichment. The goal is to increase the lighter U-235 in the gas. Centrifuge spins the heavier U-238 (UF_6) molecules out while collecting the lighter U-235 at the center.

4/6

6. $$DNBR = \frac{\text{Critical Heat Flux}}{\text{Actual Heat Flux}}$$ - in hottest channel

7/8

CHF is the maximum flux before transition boiling begins, and you run the risk of film boiling on your fuel rods. Goal is to always operate below CHF.

- would like a little more depth

7. Smear density is the ratio of volume taken up by pellets/fuel vs. the volume available inside cladding. It is a good metric for determining your fuel system gap and determining how much thermal expansion of the fuel is allowed before straining the cladding. - allows swelling

4/5

- all degradation is from phonon scattering... what causes it?

8. Two sources of thermal conductivity degradation in UO_2 are porosity and phonon scattering.

3/4

9. Two primary fission product species are Xe and Ba (?). Xe is a well known fission product poison and Ba is a guess

3/4

- double hump distribution

10. Cladding protects the fuel and maintains mechanical integrity of system, while protecting from corrosion. Also transports heat to coolant more effectively than fuel.

4/4

11. Fuel system consists of fuel pellet, the fill gas gap, and the cladding.

4/4

12. Fuel performance is determined by:

- Effectiveness of transporting heat
- Length of time it can operate, stability
- Safety (or predictability) in an accident

4/4

13. In finite difference, you can't model anything more complex than a 1.5 D model.

3/4

- wanted more here...

14. Positive: high melting point, cubic structure
Negative: susceptible to corrosion, low thermal conductivity

yes, but not too bad

4/6

15. Explicit time integration - using the values at a given time to determine future time

4/6

Implicit - using values at future time to solve for other future values

Implicit is more stable. Can use for "stiff" equations.

NE 533 Test 1 Study Sheet

$T_0 - T_S = \frac{Q}{4k_f} R_f^2 = \frac{LHR}{4\pi k}$, $T_S - T_{IC} = \frac{Q}{2h_g} R_f = \frac{LHR}{2\pi R_f h_g}$, $h_g = \frac{k_g}{t_g}$

$T_{IC} - T_{OC} = \frac{Q t_c}{2k_c} R_f = \frac{LHR \cdot t_c}{2\pi R_f k_c}$. $T_{OC} - T_{cool} = \frac{Q}{2h_{cool}} R_f = \frac{LHR}{2\pi R_f h_{cool}}$

$1\,eV = 1.602\times10^{-19}\,J$
$\sigma_f = 587\,b\ [235]$
$N_A = 6.022\times10^{23}$
$1\,b = 10^{-24}\,cm^2$

- Ronen fissile rule: For $90 \le Z \le 100$, isotopes with neutrons $(A-Z) = 2Z - N$, $N = 43 \pm 2$
- Smear Density $= \pi r_1^2 h / \pi r_2^2 h = r_1^2 / r_2^2 > 90\%$ typically
- $W_{SWU} = P\cdot V(x_P) + T\cdot V(x_T) - F\cdot V(x_F)$

$V(x) = (2x-1)\ln\left(\frac{x}{1-x}\right)$, $\frac{F}{P} = \frac{x_P - x_T}{x_F - x_T}$, $\frac{T}{P} = \frac{x_P - x_F}{x_F - x_T}$

$\theta = 548.68$
$C_1 = 302.27$
$C_2 = 8.463\times10^{-3}$
$C_3 = 8.741\times10^{7}$
$E_a = 18531.7$

$$k_o = \frac{100}{7.5408 + 17.629T + 3.6142T^2} + \frac{6400}{T^{5/2}}\exp\left[\frac{-16.35}{T}\right]$$

$$C_p = \frac{C_1 \theta^2 e^{\theta/T}}{T^2 (e^{\theta/T} - 1)^2} + 2C_2 T + \frac{C_3 E_a e^{-E_a/T}}{T^2}$$

$E_f = 200\,MeV$
F: Feed
P: Product
T: Tails

$Q = E_f N_f \sigma_f \phi$, $N_f = \rho_x \cdot \frac{N_A}{A_x} \cdot \frac{\text{atoms U}}{\text{molecules } x} \cdot$ enrich.

$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$, $LHR = \pi R^2 Q_{av}$ — W/m

$\rightarrow \rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r k(T) \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + Q(r,z)$

$T(r) = \frac{Q(R^2 - r^2)}{4k_f} + T_S \rightarrow T_0 - T_S = \frac{LHR}{4\pi k_f}$, $T(x) = T_s - \frac{T_s - T_{IC}}{\delta} x$

$q_x = hA_s(T_s - T_\infty)$
$q''_x = h(T_s - T_\infty)$

$q'_{gap} = -k_{gap}\frac{T_{IC} - T_S}{t_g}$, $q'_g = \frac{LHR}{2\pi R_f}$, $\frac{LHR}{2\pi R_f} = k_{gap}\frac{T_S - T_{IC}}{t_g}$

$h_g = \frac{k_{gap}}{t_{gap}}$, $T_S - T_{IC} = \frac{LHR}{2\pi R_f h_g}$

$q_x = -kA_x \frac{dT}{dx}$
$q''_x = -k\frac{dT}{dx}$
$q''_x = -k\frac{T_2 - T_1}{\Delta x}$

Pure He: $k_g = 16\times10^{-6}\,T^{0.79}$ $(W/cm\cdot K)$, Pure Xe: $k_g = 0.7\times10^{-6}\,T^{0.79}$

Mix Rule: $k_g = k_{He}(1-y) + k_{Xe}\,y$, y = mole/atom frac. of Xe

$q'_c = -k_c \frac{T_{OC} - T_{IC}}{t_c}$, $q'_c = LHR/2\pi R_f$, $T_{IC} - T_{OC} = \frac{LHR}{2\pi R_f}\frac{t_c}{k_c}$

$T_{OC} - T_{cool} = LHR/2\pi R_f h_{cool}$, $\frac{1}{h} = \frac{t_g}{k_g} + \frac{t_c}{k_c} + \frac{1}{h_{cool}}$

$LHR(z/z_0) = LHR^0 \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right] = LHR^0 F(z/z_0)$, $LHR^0 @ z = z_0$, $L = 2z_0$

$\gamma = \frac{z_{ex} + z_0}{z_0} \sim 1.3$, $\pi/2\gamma \sim 1.2$

$\dot m C_{pw}\, \partial T_{cool}/\partial z = LHR(z/z_0)$, $\dot m C_{pw}(T_{cool} - T_{cool}^{in}) = z_0 \int_0^{z/z_0} LHR(z/z_0)\, d(z/z_0)$

$\dot m C_{pw}(T_{cool} - T_{cool}^{in}) = z_0 \times LHR^0 \int_0^{z/z_0} F(z/z_0)\, d(z/z_0)$

$T_{cool} - T_{cool}^{in} = \frac{2\gamma}{\pi}\frac{z_0 \times LHR^0}{\dot m C_{pw}}\left\{\sin\left(\frac{\pi}{2\gamma}\right) + \sin\left[\frac{\pi}{2\gamma}\left(z/z_0 - 1\right)\right]\right\}$, $K_{ox} = \frac{1}{A + BT}$

$\frac{1}{B}\ln\left(\frac{A + BT_0}{A + BT_S}\right) = \frac{LHR}{4\pi}$, FIMA: Fissions per Initial heavy Metal Atom

Forward: $y_{n+1} = y_n + h y'(t_n)$

Backward: $y_{n+1} = y_n + h y'(t_{n+1})$

Improved: $y_{n+1} = y_n + \frac{1}{2}h\left(f(t_n, y_n) + f(t_{n+1}, y_{n+1})\right)$

Plate: $T(x) - T_S = \frac{LHR}{2\pi k}\left(1 - \frac{x^2}{t^2}\right)$

Sphere: $T(r) - T_S = \frac{LHR}{6\pi k}\left(1 - \frac{r^2}{R^2}\right)$

$A = 3.8 + 200\times FIMA$ $(cm\cdot K/W)$, $B = 0.0217$ (cm/W)

$T(r) = \frac{LHR}{4\pi k_f}\left(1 - \frac{r^2}{R_f^2}\right) + T_s$

$\frac{h D_e}{k_{cool}} = 0.023\,Re^{0.8}\,Pr^{0.4}$

$CHF = \max q''$

Nucleate Boil: $q'' = 6(T_{OC} - T_{cool})^4$ W/m^2

$DNBR = \frac{CHF}{q''}$

$\frac{K_{eff}}{K_{ox}} = 1 - P^{2/3}$