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NE 533 Exam 1 Mahmoud Hawary

Problem 1

12/12

$* \; T'(x_0) = 0$

$x_0 = 0$

$* \; x_1 = X$

$T(x_1) = T_1$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0 \Rightarrow \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = -Q$$

By integration: $\boxed{k \frac{\partial T}{\partial x} = -Q x + C_1}$

To find $C_1 \Rightarrow$ $T'(x_0) = 0$ where $x_0 = 0$

$$\therefore k T'(x_0) = -Q x_0 + C_1 = -Q(0) + C_1 = 0$$

$$\boxed{\therefore C_1 = 0}$$

$$k \frac{\partial T}{\partial x} = -Q x \Rightarrow \frac{\partial T}{\partial x} = -\frac{Q}{k} x$$

By integration:

$$T(x) = \frac{-Q}{2k} x^2 + C_2$$

To find $C_2 \Rightarrow$

$$T(x_1) = \frac{-Q}{2k} X^2 + C_2 = T_1 \Rightarrow \boxed{C_2 = T_1 + \frac{Q}{2k} X^2}$$

$$\therefore T(x) = \frac{-Q}{2k} x^2 + T_1 + \frac{Q}{2k} X^2 = T_1 + \frac{Q}{2k}\left[X^2 - x^2\right]$$

$$\boxed{\therefore T(x) - T_1 = \frac{Q}{2k}\left[X^2 - x^2\right]}$$

~~Cont'd~~ Proplom $\boxed{1}$ 'Cont'

The assumptions made are:

1. steady-state condition
2. axisymmetric
3. temperature is constant in Z-direction
4. thermal conductivity is constant (doesn't depend on T).

Preplem ②

$T_o = ?$

Coating $\Rightarrow$ $k = 0.015$ w/cm.k

cladding $\Rightarrow$ $k = 0.15$ w/cm.k

fuel $\Rightarrow$ $k = 0.05$ w/cm.k

gap $\Rightarrow$ $k = 0.004$ w/cm.k

$T_{coat} = 600k$

$Q = 250$ w/cm^3

$R_f = 0.6$ cm

$t_g = 0.005$ cm

$t_{clad} = 0.05$ cm

$t_{coat} = 0.01$ cm.

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$\Rightarrow$ with Coating,

$$* \; T_{co} - T_{coat} = \frac{Q * t_{coat}}{2 k_{coat}} * R_f = \frac{250 \frac{w}{cm^3} * 0.01 cm}{2 * 0.015 \; w/cm.k} * 0.6 = 50 K$$

✓

$T_{co} = 650 k$

$$* \; T_{ci} - T_{co} = \frac{Q \, t_{clad} * R_f}{2 k_{clad}} = \frac{250 * 0.05 * 0.6}{2 * 0.15} = 25 K$$

✓

$T_{ci} = 675 K$

✓

$$* \; T_f - T_{ci} = \frac{250 * 0.005 * 0.6}{2 * 0.004} = 93.75 \Rightarrow T_f = 768.75 K$$

$$* \; T_o - T_f = \frac{Q}{4 k_f} R_f^2 = \frac{250 * 0.6^2}{4 * 0.05} = 750 K \Rightarrow \boxed{T_o = 1518.75 K}$$

- missed the R_f^2

⇒ without Coating :

✓

$T_{ci} = 25\,K + 600 = 625\ K$

$T_f = 93.75 + 625 = 718.75\ K$

$T_o = 750 + 718.75 = 1468.75\ K$

Problem $\boxed{3}$:

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UN $\Rightarrow$ enrich = 19.5 %

$\rho = 12.3\ g/cm^3$

$\sigma_f = 570$ barns

$Z_N = 14$

ⓐ heat generation rate for $\phi = \cancel{2\times10}$ $5 \times 10^{12}\ n/cm^2.sec$

$$Q = E_f N_f \sigma_f \phi_f$$

$UN = 0.5U + 0.5N$

To find N_f:

$M_u = 235 * 0.195 + 238 * 0.805 = 237.415$ a.m.u

252.036 $M_{UN} = 0.77 * 237.415 + 0.23 * 14 = 252.036\ g/mol$

- slightly high

$N_{U-235} = 12.3 \frac{g}{cm^3} * \frac{1}{252.036} \frac{mol}{gm} * 6.022 * 10^{23} * \frac{0.77}{1} * 0.195$

$= 4.412 * 10^{21}$ atom/cm3

$Q = 200 * 10^6 * 1.602 * 10^{-19} * 4.412 * 10^{21} * 570 * 10^{-24} * 5 * 10^{12}$

$= 0.00402\ w/cm^3$?? to small

- calculator error?

ⓑ $\rho_{UO_2} = 10.97\ g/cm^3$

$M_{UO_2} = 235x + 238(1-x) + 2 * 28 =$ 16 ✓

$N_{UO_2} = 10.97 * \frac{1}{M_{UO_2}} * 6.022 * 10^{23} * x =$ ✓

then we can find x. ✓

- yes, this is how we would do this

Problem $\boxed{4}$ 8/12

$L = 3.5 \text{ m} = 2Z_0 \Rightarrow Z_0 = 1.75 \text{ m}$

$\mathcal{Z}HR^0 = 350 \text{ W/cm}$

$\gamma = 1.3$

radians, not degrees

(a) $\mathcal{Z}HR(z = 1.4 \text{ m}) = ?$

$$\mathcal{Z}HR\left(\frac{z}{z_0}\right) = \mathcal{Z}HR^0 \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right]$$

$$\therefore \mathcal{Z}HR(z = 1.4) = 350 \frac{W}{cm} \cos\left[\frac{\pi}{2(1.3)}\left(\frac{1.4}{1.75} - 1\right)\right]$$

$$= 349.9 \text{ W/cm}$$

(b) $\Delta T = ?$ $C_p = 4200 \text{ J/kg.K}$, $\dot{m} = 0.22 \text{ kg/sec.rod}$

$$T_{cool} - T_{cool}^{in} = \frac{2\gamma}{\pi} \frac{Z_0 \mathcal{Z}HR^0}{\dot{m} C_p}\left[\sin\left(\frac{\pi}{2\gamma}\right) + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right)\right]$$

$$\therefore \Delta T_{cool} = \frac{2*1.3}{\pi} \frac{175 \text{ cm} * 350 \text{ W/cm}}{0.22 \frac{kg}{sec} * 4200 \text{ J/kg.K}}\left[\sin\left(\frac{\pi}{2(1.3)}\right) + \sin\left(\frac{\pi}{2(1.3)}\left(\frac{3.5}{1.75} - 1\right)\right)\right]$$

$$\boxed{\Delta T_{cool} = 2.3137 \text{ K}}$$

- right equations, but the answers don't make sense

Problem $\boxed{5}$

12/12

$dt = 0.5$, $t_0 = 0$, $t_n = 1.5$, $y_0 = 4$, $\frac{dy}{dt} = t * exp(-2t)$

for Backward

$$y_{n+1} = y_n + dt\, y'_{n+1}$$

$t_1 = 0.5 \Rightarrow y_1 = y_0 + dt\, y'_1 = 4 + 0.5[0.5 * exp(-2*0.5)]$

$y_1 = 4 + 0.919$

$y_1 = 4.091$

$t_2 = 1 \Rightarrow y_2 = y_1 + dt\, y'_2 = 4.091 + 0.5[(1)\, exp(-2*1)]$

$y_2 = 4.159$

$t_3 = 1.5 \Rightarrow y_3 = y_2 + dt\, y'_3 = 4.159 + 0.5[(1.5)\, exp(-2*1.5)]$

$y_3 = 4.1969$

6 4/4

* fertile: a material is not itself fissionable but can converted into a fissile material by neutron absorption
* fissile: capable of sustaining a nuclear fission chain reaction with neutrons of any energy
* fissionable: capable of undergoing fission (even with low probability) after capturing a high energy neutrons.

7 4/4 Pure-U has:

* Anisotropic thermal expansion and irradiation growth
* swells extremily
* low melting point.

8 Smear density: the ratio of fuel volume to the total volume of the fuel element $= \frac{r_1^2}{r_2^2}$

* It's important be cause it allows us to control the fuel swelling under irradiation

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* The natural uranium has low concenteration of fissile nuclides. so we need ~~to~~ enrichment to increase the no of the fissile nuclide in the fuel to generate sufficient heat (energy) so the npps. would be economic.

* The uranium is enriched in the form of UF_6.

* The UF_6 gas is inserted into centrifuge cylinders which rotate at very high speed. Due to the centrifuge force, the UF_6 is separated based on the mass. The heavier nuclides will more outward the center, while the lighter will be closer to the center. There we can absorb the UF_6 with different ~~mass~~ masses based on the position

10 ① Finite difference ② finite volume ③ Finite element

6/6 FEM is the most common because it has the following advantages
① model any geometry
② " any BCs
③ Continuous representation
④ simulate hetragenous properties
Its disadvantages include:
① complicated
② expensive.

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* DNBR: the ratio of the heat flux that causes dryout (the critical heat flux) to the actual heat flux.

* Critical heat flux: heat flux that causes dryout

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TRISO layers:

1. Fuel Kernel
2. Buffer
3. Inner Pyrolitic Carbon (IPyC)
4. SiC
5. outer Pyrolitic Carbon (OPyC).

⇒ Reactors use TRISO:

1. High temp. gas reactor (HTGRs)
2. Molten salt reactors (MSRs).