NE 533 - Spring 2023

Exam 1 Solutions

1) UN $x_e = 0.195$ $\rho = 12.3\ g/cc$ $\sigma_f = 587\ b$ $\phi = 5\times10^{13}\ n/cm^2\text{-}s$

a) $Q = E_f N_f \phi_f \sigma_f$ =?

$m_u = 0.195 \times 235 + 0.805 \times 238 = 237.415$

$m_{UN} = 237.415 + 14 = 251.415$

$N_f = 12.3\ g/cc \ \frac{1 mol}{251.4 g} \ \frac{6.022\times10^{23}}{1 mol} \ \frac{1 U}{1 UN} \times 0.195 = 5.74\times10^{21}\ U^{235}/cc$

$Q = (200\times10^{6})(1.602\times10^{-19})(5.74\times10^{21})(587\times10^{-24})(5\times10^{13})$

$\boxed{Q = 540\ W/cc}$

b) $x_E (UO_2)$ =?

$N_f(UN) = N_f(UO_2)$

$N_f = 5.74\times10^{21}\ U^{235}/cc$ $\qquad$ $N_f = \frac{\rho N_A}{M(UO_2)} x_E$

$M(UO_2) = 32 + x_E 235 + (1-x_E) 238 = 270 - 3x_E$

$5.74\times10^{21} = \frac{(10.97)(6.022\times10^{23})}{270 - 3x_E} x_E$

$1.55\times10^{24} - 1.72\times10^{22} x_E = 6.606\times10^{24} x_E$

$\boxed{x_E = 0.234}$

2) $\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0$ $\frac{dT}{dx}(x_0) = 0$ $x_0 = 0$

$x_1 = X$ $T(x_1) = T_1$

$$\left(\int \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = \int -Q\right) dx$$

$$\underset{0}{\underset{\Downarrow}{k \frac{\partial T}{\partial x}}} = \underset{0}{\underset{\Downarrow}{-Qx}} + \underset{0}{\underset{\Downarrow}{C}}$$

$\frac{\partial T}{\partial x}(x_0) = 0$

$x_0 = 0$

$$k \frac{dT}{\partial x} = -Qx \rightarrow \left(\int k \frac{\partial T}{\partial x} = \int -Qx\right) dx$$

$$T(x) = \frac{-Q}{2k} x^2 + C$$

$$T_1 = \frac{-Q}{2k} X^2 + C \qquad C = T_1 + \frac{Q}{2k} X^2$$

$$T(x) = \frac{-Q}{2k} x^2 + T_1 + \frac{Q}{2k} X^2$$

$$T(x) = T_1 + \frac{Q}{2k}\left(X^2 - x^2\right)$$

- 1-D (constant y-z)
- constant K
- steady-state

3) $Q = 350 \ W/cc$ $\qquad R_f = 0.6$ cm $\qquad k_f = 0.03 \ W/cm\text{-}K$

$T_{cool} = 550$ K $\qquad t_g = 0.003$ cm $\qquad k_c = 0.15 \ W/cm\text{-}K$

$t_c = 0.05$ cm $\qquad h_{cool} = 2.5 \ W/cm^2\text{-}K$

$$LHR = Q \pi R_f^2$$

$$LHR = (350)(\pi)(0.6)^2 = 396 \ W/cm \qquad \frac{LHR}{2\pi R_f} = 105 \ W/cm^2$$

$$T_{co} - T_{cool} = \frac{LHR}{2\pi R_f} \frac{1}{h_{cool}} = 105\left(\frac{1}{2.5}\right) = 42K \qquad \underline{T_{co} = 592 \ K}$$

$$T_{ci} - T_{co} = \frac{LHR}{2\pi R_f} \frac{t_c}{k_c} = 105 \left(\frac{0.05}{0.15}\right) = 35 \ K \qquad \underline{T_{ci} = 627 \ K}$$

$$T_S - T_{ci} = \frac{LHR}{2\pi R_f} \frac{t_g}{k_g} \qquad k_g(He) = 16 \times 10^{-6} \ T^{0.79}$$

$$T = T_{ci}$$

$$= 16 \times 10^{-6} (627)^{0.79} = 0.0026 \ W/cm\text{-}K$$

$$T_S - T_c = 105\left(\frac{0.003}{0.0026}\right) = 121K \qquad \underline{T_S = 748 \ K}$$

$$T_0 - T_S = \frac{LHR}{4\pi k_f} = \frac{396}{4\pi(0.03)} = 1050K \qquad \underline{T_0 = 1798 \ K}$$

$$\frac{1}{B} \ln\left(\frac{A + BT_0}{A + BT_S}\right) = \frac{LHR}{4\pi}$$

$A = 3.8 + 200 \times FIMA \ cm\text{-}K/W$

$B = 0.0217 \ cm/W$

$A = 3.8 \qquad FIMA = 0$

$$\frac{A + BT_0}{A + BT_S} = \exp\left(\frac{B \times LHR}{4\pi}\right)$$

$$3.8 + 0.0217 \, T_0 = (3.8 + 0.0217(748)) \exp\left(\frac{0.0217(396)}{4\pi}\right)$$

$$3.8 + 0.0217 \, T_0 = 39.69$$

$$\underline{T_0 = 1654K} \qquad \text{drop of} \sim 140 \ K$$

4) $L = 3.2\ m$ $\quad LHR^{o} = 300\ W/cm$

$\Downarrow$ $\quad \gamma = 1.1$

$Z_0 = 1.6\ m$

a) $LHR(z = 0.9\,m)$?

$$LHR(z) = LHR^{o} \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right]$$

$$= 300 \cos\left[\frac{\pi}{2 \cdot 1.1}\left(\frac{0.9}{1.6} - 1\right)\right] = \underline{226.7\ W/cm}$$

b) $T_{cool}(z = 0.8)$ $\quad T_{cool}^{in} = 500\ K$

$$T(z) - T_{cool}^{in} = \frac{2\gamma}{\pi}\,\frac{z_0\, LHR^{o}}{\dot{m}\, c_p}\left[\sin\left(\frac{\pi}{2\gamma}\right) + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right)\right]$$

$$= \frac{2.2}{\pi}\,\frac{(160)(300)}{(0.3)(4200)}\left[\sin\left(\frac{\pi}{2.2}\right) + \sin\left(\frac{\pi}{2.2}\left(\frac{0.9}{1.6} - 1\right)\right)\right]$$

$$= (26.6)(0.335) = 9\ K$$

$$T(0.8\,m) = \underline{509\ K}$$

5) we enrich to increase N_F (U-235) to sustain chain reaction and to have a high heat generation rate.
UF_6 is the compound that is enriched.
Gas is inserted into the rapidly spinning canister. Due to the mass difference in U-235 to U-238, heavier U-238 concentrates on the outside of the canister. Enriched flow of U-235 is removed from the center of the cannister. Process repeats.

6) Departure from nucleate boiling is the ratio of the critical heat flux to the actual heat flux in the hottest channel. This is a safety metric which governs heat transfer modes to the coolant. The CHF is the heat flux at which dryout will occur. Reaching the CHF results in a large temperature spike.

7) fertile - can absorb neutrons to become fissile
fissile - can undergo fission w/ neutron of any energy
fissionable - can undergo fission w/ high energy neutron

8) low melting point, anisotropic swelling, FCCI, poor oxidation

9) A reduction in the volume of the fuel compared to the volume available inside the cladding. Allows for fuel swelling.

10) fuel kernel, buffer, IPyC, SiC, OPyC

11) U_3Si_2, UN, Cr-doped UO_2, FeCrAl or SiC cladding

12) Mo, Xe, Cs, Zr, etc.

Double hump distribution of fission products w/ maxima around A: 90 and A: 135

13) Primary barrier between fuel & coolant. Holds fuel in place + maintains fuel shape; Retain fission products.

14) Fuel, gap, cladding, coolant

15) heat transport to coolant, behavior under accidents, ability ~~to~~ operate w/o stoppages