

Exam 1

NE 591 - 10

Nuclear Fuel Performance

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## Question 1

Heat conduction equation:

$$\nabla \cdot (k \nabla T) + Q = \rho c_p \frac{\partial T}{\partial t}$$

If we assume the system is in steady state, the equation becomes

$$\nabla \cdot (k \nabla T) + Q = 0$$

Also, if we take only one dimension (namely  $x$ -axis) in cartesian coordinates, then it becomes,

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + Q = 0$$

As we assumed, the system is in steady state, we can replace the partial derivatives with derivatives.

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) + Q = 0$$

$$\Rightarrow \frac{d}{dx} \left( k \frac{dT}{dx} \right) = -Q$$

$$\Rightarrow k \frac{dT}{dx} = -Qx + C_1$$

From  $x = x_0 = 0$  and  $T'(x_0) = 0$ , we get

$$k(0) = -0 + c_1$$

$$\Rightarrow c_1 = 0$$

So,  $k \frac{dT}{dx} = -\phi x$

$$\Rightarrow T = -\frac{\phi x^2}{2k} + c_2$$

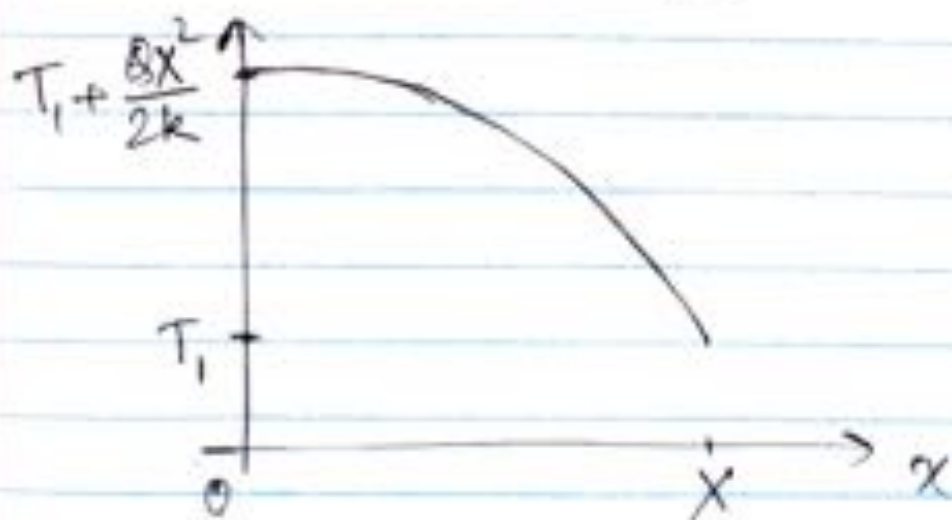
From  $x = x_1 = X$  and  $T(x_1) = T_1$ , we get

$$T_1 = -\frac{\phi X^2}{2k} + c_2$$

$$\Rightarrow c_2 = T_1 + \frac{\phi X^2}{2k}$$

Hence,  $T = -\frac{\phi x^2}{2k} + T_1 + \frac{\phi X^2}{2k}$

$$\Rightarrow T = T_1 + \frac{\phi}{2k} (X^2 - x^2)$$



## Question 2

Given,

$$k_{\text{coat}} = 5 \text{ W/m}\cdot\text{K}$$

$$k_{\text{clad}} = 15 \text{ W/m}\cdot\text{K}$$

$$k_{\text{fuel}} = 0.5 \text{ W/cm}\cdot\text{K}$$

$$k_{\text{gap}} = 25 \text{ W/m}\cdot\text{K}$$

$$h_{\text{cool}} = 5.5 \text{ W/cm}^2\cdot\text{K}$$

$$r_1 = 0.6 \text{ cm}$$

$$r_2 = 0.8 \text{ cm}$$

$$r_3 = 0.85 \text{ cm}$$

$$r_4 = 0.86 \text{ cm}$$

$$T_{\text{cool}} = 800 \text{ K}$$

$$Q = 400 \text{ W/cm}^2$$

Now,

$$\text{LHR} = Q \pi r_1^2$$

$$= 400 \pi \times 0.6^2$$

$$= 452.16 \text{ W/cm}$$

$$T_{\text{cool, i}} - T_{\text{cool}} = \frac{\text{LHR}}{2\pi r_1} \frac{1}{h_{\text{cool}}}$$



$$\Rightarrow T_{\text{coat},o} = \frac{452.4}{2\pi(0.6)} \cdot \frac{1}{5.5} + 800$$

$$= 821.8 \text{ K}$$

$$T_{\text{coat},i} - T_{\text{coat},o} = \frac{\text{LHR}}{2\pi r_i} \cdot \frac{t_{\text{coat}}}{k_{\text{coat}}}$$

$$\Rightarrow T_{\text{coat},i} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.01}{0.05} + 821.8$$

$$= 845.8 \text{ K}$$

$$T_{\text{coat},i} = T_{\text{clad},o} = 845.8 \text{ K}$$

$$T_{\text{clad},i} - T_{\text{clad},o} = \frac{\text{LHR}}{2\pi r_i} \cdot \frac{t_{\text{clad}}}{k_{\text{clad}}}$$

$$\Rightarrow T_{\text{clad},i} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.05}{0.15} + 845.8$$

$$= 885.8 \text{ K}$$

$$T_{\text{clad},i} = T_{\text{gap}} = 885.8 \text{ K}$$

$$T_{\text{fuel}} - T_{\text{gap}} = \frac{\text{LHR}}{2\pi r_i} \cdot \frac{t_{\text{gap}}}{k_{\text{gap}}}$$

$$\Rightarrow T_{\text{fuel}} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.2}{0.25} + 885.8$$

$$= 981.8 \text{ K}$$

$$\text{Now, } T(r) = \frac{LHR}{4\pi k} \left(1 - \frac{r^2}{R^2}\right) + T_{\text{fuel}}$$

$$\therefore T(r=0) = \frac{452.9}{4\pi (0.5)} + 981.8$$

$$= 1053.8 \text{ K}$$

$$\text{and, } T(r=0.4) = \frac{452.9}{4\pi (0.5)} \left[1 - \left(\frac{0.4}{0.5}\right)^2\right] + 981.8$$

$$= 1007.7 \text{ K}$$

### Question 3

$$\begin{aligned} \text{(a)} \quad N_f &= (15.67 \frac{\text{g}}{\text{cc}}) \times \left(\frac{1 \text{ mol}}{A_g}\right) \times \left(\frac{6.022 \times 10^{23}}{1 \text{ mol}}\right) \\ &\quad \times \left(\frac{3u}{1u_3\text{Si}_2}\right) \times 19.5\% \end{aligned}$$

$$A = (19.5\%) 235 + (80.5\%) 238$$

$$\approx 237.415$$

$$\therefore N_f = 2.33 \times 10^{22}$$

$$\therefore \phi = E_f N_f \sigma_f \phi_{th}$$

$$= (200 \times 10^6 \text{ eV}) \left( 1.6 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right) \\ (2.33 \times 10^{22}) (570 \times 10^{-29} \text{ cm}^2) \\ (2 \times 10^{12} \text{ n/cm}^2\text{-s})$$

$$= 898.23 \frac{\text{W}}{\text{cm}^2}$$

(b)

$$A = x \cdot 235 + A(1-x) \cdot 238$$

$$\therefore A = 238 - 3x$$

$$N_f = (10.97) \left( \frac{1}{238 - 3x} \right) (6.02 \times 10^{23}) (x)$$

$$\therefore \phi = E_f N_f \sigma_f \phi_{th}$$

$$= (200 \times 10^6) (1.6 \times 10^{-19}) N_f (570 \times 10^{-29}) \\ (2 \times 10^{12})$$

$$\Rightarrow N_f = 2.33 \times 10^{22}$$

$$\Rightarrow \frac{x}{238 - 3x} = \frac{2.33 \times 10^{22}}{6.02 \times 10^{23} \times 10.97}$$

$$\Rightarrow x = 0.829 = 82.9\%$$



# Question 9

(a)

$$L = 3 \text{ m}$$

$$\text{LHR}^0 = 150 \text{ W/cm}$$

$$\gamma = 1.1$$

$$\begin{aligned}\text{LHR} &= \text{LHR}^0 \cos \left[ \frac{\pi}{2\gamma} \left( \frac{z}{z_0} - 1 \right) \right] \\ &= 150 \cos \left[ \frac{\pi}{2 \cdot 2} \left( \frac{1.8}{1.5} - 1 \right) \right] \\ &= 143.9 \text{ W/cm}\end{aligned}$$

(b)

$$T_{\text{cool}} - T_{\text{cool}}^{\text{in}} = \frac{1}{1.2} \frac{z_0 \times \text{LHR}^0}{m C_{pw}}$$

$$\left[ m(1.2) + m \left( 1.1 \left( \frac{z}{z_0} - 1 \right) \right) \right]$$

(i)

water

$$\left. \begin{aligned} T_{\text{cool}} - T_{\text{cool}}^{\text{in}} &= \frac{1}{1.2} \frac{150 \times 150}{0.22 \times 200} \left[ 2 m(1.2) \right] \\ &= 37.83 \text{ K} \end{aligned} \right\}$$

(ii)

Antimony

$$\left. \begin{aligned} T_{\text{cool}} - T_{\text{cool}}^{\text{in}} &= \frac{1}{1.2} \frac{150 \times 150}{0.12 \times 1924} \left[ 2 m(1.2) \right] \\ &= 207.45 \text{ K} \end{aligned} \right\}$$



Question 5

$$\Delta t = 0.33$$

$$\frac{dy}{dt} = 4t - 3t^2$$

$$t_0 = 1 \Rightarrow y(t_0) = 6$$

$$y(t_i) = y(t_{i-1}) + \frac{dy}{dt} \Delta t$$

Forward

$$y(t_1) = y(t_0) + \left. \frac{dy}{dt} \right|_{t_0} \Delta t$$

$$= 6 + (0.33)$$

$$= 6.33$$

$$t_1 = t_0 + \Delta t$$

$$= 1.33$$

$$y(t_2) = y(t_1) + \left. \frac{dy}{dt} \right|_{t_1} \Delta t$$

$$= 6.33 + (0.0133)(0.33)$$

$$= 6.33 + 3.89$$

$$t_2 = 1.66$$

$$y(t_3) = y(t_2) + \left. \frac{dy}{dt} \right|_{t_2} \Delta t$$

$$= 6.33 + 3.89 + (-1.6268)(0.33)$$

$$= 5.7975$$

$$t_3 = 2$$

~~1/4~~

Backward

$$y(t_1) = y(t_0) + \left. \frac{dy}{dt} \right|_{t_1} \Delta t$$

$$\begin{aligned} t_1 = 1.33 &= 6 + 0.4389 \\ &= 6.4389 \end{aligned}$$

$$y(t_2) = y(t_1) + \left. \frac{dy}{dt} \right|_{t_2} \Delta t$$

$$\begin{aligned} t_2 = 1.66 &= 6.4389 + (-1.6468)(0.33) \\ &= 5.902 \end{aligned}$$

$$y(t_3) = y(t_2) + \left. \frac{dy}{dt} \right|_{t_3} \Delta t$$

$$\begin{aligned} t_3 = 2 &= 5.902 + (-4)(0.33) \\ &= 4.582 \end{aligned}$$

## Question 6

- Fissionable nuclide is capable of undergoing fission after capturing a high energy neutron
- Fissile nuclide is capable of sustaining fission chain reaction with neutrons of any energy.
- Fertile material is not fissionable, but it can be converted into a fissile nuclide by neutron absorption.

## Question 7

- During thermal cycling pure U swells dramatically
- $\alpha$ -U has both anisotropic expansion and  $\alpha$  irradiation growth.



### Question 8

Linear density

= Ratio of fuel volume  
to total internal  
volume

$$= \frac{\pi r_i^3 h}{\pi r_o^3 h}$$

$$= \frac{r_i^3}{r_o^3}$$

$$= \left(\frac{r_i}{r_o}\right)^3$$

→ It's necessary for thermal  
hydraulic and efficiency  
measurements.

### Question 9

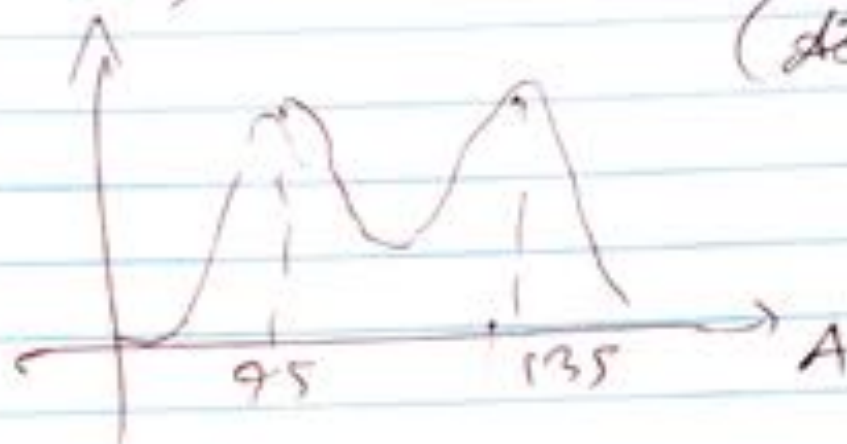
Oxide  $\rightarrow$   $UF_6$   $\rightarrow$   $UO_2$   $\rightarrow$  pellet  $\rightarrow$  rods

→ Because in a centrifuge U-238  
gets further than U-235.

### Question 10

One around  $A = 95$   
Another around  $A = 135$

(detonides)



### Question 11

1. Finite difference
2. Finite volume
3. Finite element

Finite element is used in  
SOA fuel performance simulations  
as it can model any geometry.