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8:28 pm

91/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-0, 30/30

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

Fissile isotope = ^{235}U.

Natural enrichment = 0.7%

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

U_3Si_2: $11.31 \frac{g\,U}{cm^3}$

$\frac{\rho_2}{M} N_A \gamma_2 = 8.5884 \times 10^{20} \frac{atom}{cm^3} = N_U$

U_3Si_5: $7.5 \frac{g\,U}{cm^3}$

$\frac{\rho_2 N_A \gamma_2}{M} = N_U$

$M = 0.03 \cdot 235 + 0.97 \cdot 238$

$= 237.91 \frac{g}{mol}$

$\frac{\rho_5}{M} N_A \gamma_5 = N_u$

$\gamma_5 =$

$\frac{\rho_5 N_A \gamma_5}{M} = N_U$

$\frac{\rho_2 \gamma_2}{\rho_5 \gamma_5} = 1$

$\gamma_5 = \frac{\rho_2}{\rho_5} \gamma_2$

$\boxed{\gamma_2 = 4.5\ \%}$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

$12.5 \frac{W}{mK} \frac{m}{100}$
$= 0.125 \frac{W}{cmK}$

I would rank U_3Si_5 as a poor fuel compared to U_3Si_2. It has a lower thermal conductivity and a lower Uranium density. The other properties such as thermal stability and temperature are unknown.

$k_c = 0.17 \frac{W}{cm\,K}$

$0.008\,cm$

Question 2: $0.45\,cm$ -1, 34/35

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K). $0.06\,cm$

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{CO} = T_{cool} + \frac{LHR}{2\pi R_F h_{cool}} \quad 580\,K + \frac{250\,W/cm}{2\pi (0.45\,cm)\, 2.5\,W/cm^2K} = 615.37\,K$$

$$T_{CI} = T_{CO} + \frac{LHR\, t_c}{2\pi R_F k_c} = 615.37\,K + \frac{250\,W/cm\ 0.06\,cm}{2\pi (0.45\,cm)\, 0.17\,\frac{W}{cm\,K}} = 646.57\,K$$

$$T_S = T_{CI} + \frac{LHR\, t_g}{2\pi R_F k_g} = 646.57 + \frac{250\,\frac{W}{cm} \times 0.008\,cm}{2\pi (0.45\,cm)\, 0.002227\,\frac{W}{cm\,K}} = \boxed{957.80\,K}$$

$$T_0 = T_s + \frac{LHR}{4\pi k} = 957.80\,K + \frac{250\,W/cm}{4\pi (0.2\,W/cm\,K)} = 1057.27\,K$$

$k_{gap} = k_{He}(T_{CI})^{1-\gamma}\, k_{Xe}(T_{CI})^{\gamma}$

$\gamma = 0.05$

$k_{He}(T_{CI}) = 16\times10^{-6}\, T_{CI}^{0.79} = 0.00265$

$k_{Xe}(T_{CI}) = 0.7\times10^{-6}\, T_{CI}^{0.79} = 1.163\times10^{-4}$

$k_{gap} = 0.002227\,\frac{W}{cm\,K}$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$$\sigma_{\theta\theta} = -\sigma^*(1-3\eta^2) \quad \sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)} \quad \eta = \frac{r}{R_F}$$

$$\sigma^* = \frac{7.5\times10^{-6}\,\frac{1}{K}\ 246.7\,GPa\ (1057.27 - 957.80)}{4(1-0.25)} = 0.06135\,GPa = 61.35\,MPa$$

$\sigma_{\theta\theta} = -61.35\,MPa\left(1 - 3\left(\frac{r}{R_F}\right)^2\right)$ max at $r = R_F$

$= \boxed{122.7\,MPa = \text{max stress of pellet}}$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

UO_2: $E = 200\,GPa$

$\alpha = 11\times10^{-6}\,\frac{1}{K}$

$\sigma^* = 461.8\,MPa$

The value of σ^* is much higher for UO_2, so I would expect the stress to be higher in UO_2 than in UN.

-1, Why is sigma* so much higher?

d) What assumptions were made in your calculations for a) and b)? (5 points)

Assumptions:

* Ends of fuel rod have no effects
* steady state
* smeared pellets
* axissymetric
* Isotropic material response

Question 3: -8, 27/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

* Thin walls behave as straight walls
* static body
* Gravity is negligible
* Axisymmetric

-3, Stress is constant through thickness

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\sigma_\theta = \frac{PR}{\delta} = \frac{6\,MPa \cdot 5.6\,mm}{0.6\,mm} = \boxed{56\,MPa = \sigma_\theta}$$

$$\sigma_z = \frac{PR}{2\delta} = \boxed{28\,MPa = \sigma_z}$$

$$\sigma_r = -\frac{1}{2}P = \boxed{-3\,MPa = \sigma_r}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

$$\sigma_{\theta\theta} = P\frac{(R_o/r)^2+1}{(R_o/R_i)^2-1} \qquad R_o = 6.2\,mm \quad R_i = 5.6\,mm \qquad \text{max stress at } r = R_i$$

$$\sigma_{\theta\theta} = 6\,MPa\,\frac{(6.2/5.6)^2+1}{(6.2/5.6)^2-1} = 59.15\,MPa$$

-4, Calculate stress at two radii and compare

The thin-walled approximation underrepresents the stress by about 5%. It is not a conservative estimate.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \quad \text{No shear so} \quad \underline{\underline{\sigma}} = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{pmatrix} = \begin{bmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\theta & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$$\boxed{\underline{\underline{\sigma}} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 56 & 0 \\ 0 & 0 & 28 \end{bmatrix} MPa}$$

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{1}{E}\sigma = \frac{\sigma}{70{,}000\,MPa}$$

$$\begin{bmatrix} -3 \\ 56 \\ 28 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & & & \\ -\nu & 1 & -\nu & & 0 & \\ -\nu & -\nu & 1 & & & \\ & 0 & & & (2+2\nu)I & \end{bmatrix}\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \varepsilon_{r\theta} \\ \varepsilon_{rz} \\ \varepsilon_{\theta z} \end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \cancel{\begin{bmatrix} 9.01 & 0 & 0 \\ 0 & 11.94 & 0 \\ 0 & 0 & 10.55 \end{bmatrix} GPa^2}$$

Units/values make no sense

-1, Math error

$$\boxed{\underline{\underline{\varepsilon}} = \begin{bmatrix} -4.29\times10^{-5} & 0 & 0 \\ 0 & 8\times10^{-4} & 0 \\ 0 & 0 & 4\times10^{-4} \end{bmatrix}}$$