PennState
College of Engineering

NucE 497: Reactor Fuel Performance

Lecture 12: Introduction to Solid Mechanics for Fuel and cladding

February 6, 2017

Michael R Tonks
Mechanical and Nuclear Engineering

Today we will discuss solving for the temperature profile in the fuel using 2D transient FEM

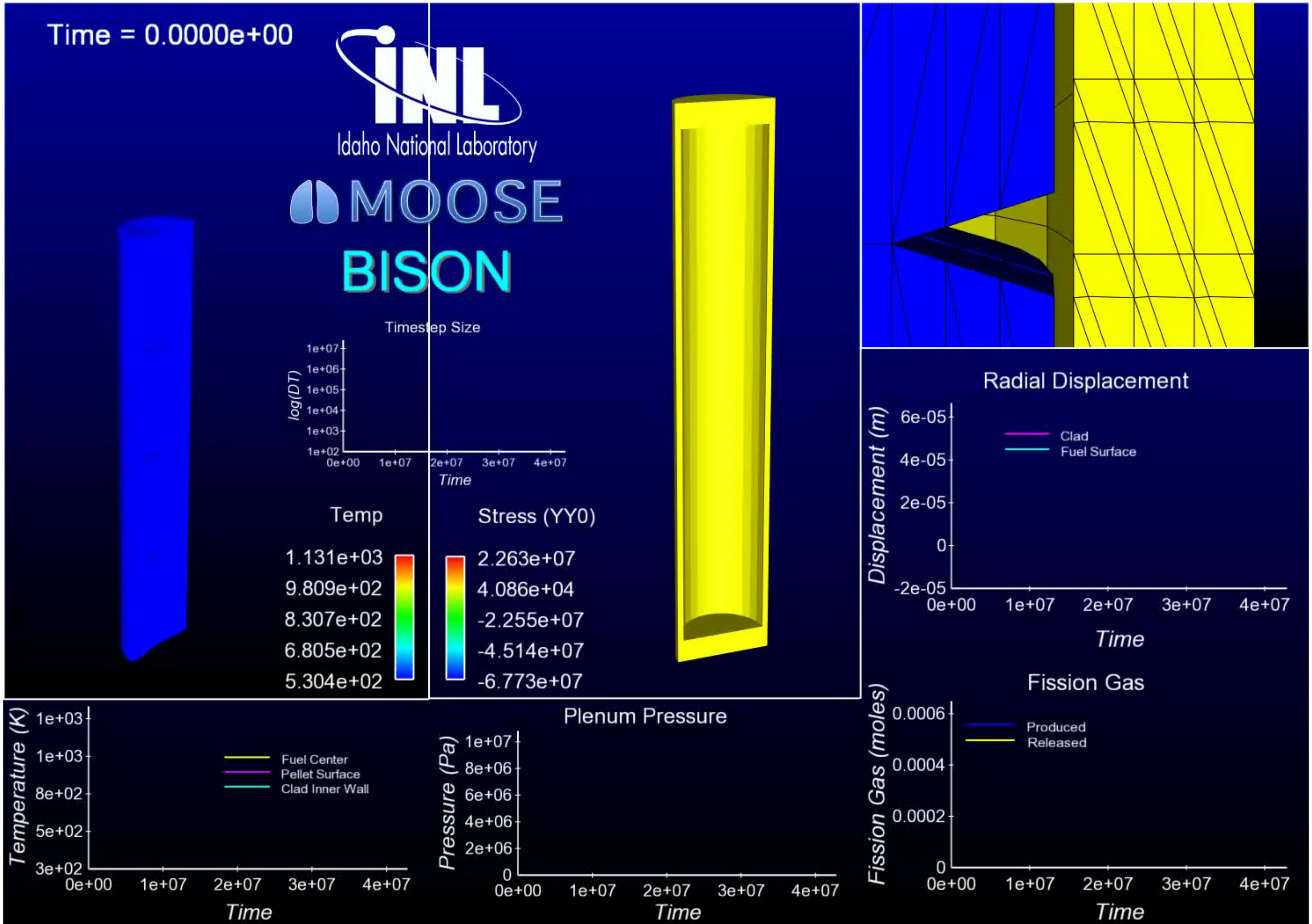
- Module 1: Fuel basics
- Module 2: Heat transport
- Module 3: Mechanical behavior
    - **Introduction to solid mechanics**
    - Thermomechanics, thermal expansion
    - Analytical solutions of the mechanics equations
    - Solving equations in 1D numerically
    - Solving in multiple dimensions with FEM
    - Summary of fuel performance codes
- Module 4: Materials issues in the fuel
- Module 5: Materials issues in the cladding
- Module 6: Accidents, used fuel, and fuel cycle

Here is some review from last time

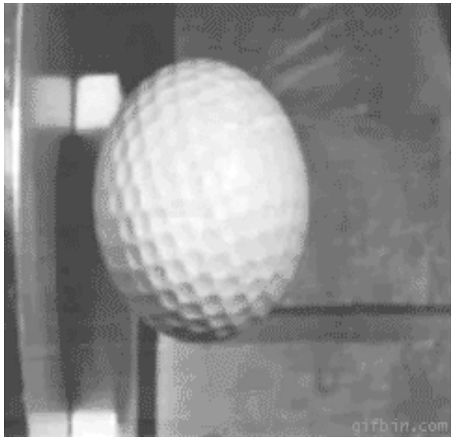
- What reactor types are intrinsically limited on the difference between the inlet and outlet temperatures?
  a) PWR
  b) BWR
  c) VHTR

a) What is NOT a negative side effect of raising the outlet temperature?
   a) Fuel could melt
   b) Materials problems are accelerated
   c) Increased plenum pressure
   d) Less thermal expansion

Though heat transport is the purpose of a reactor, we must also consider the mechanical behavior of the fuel

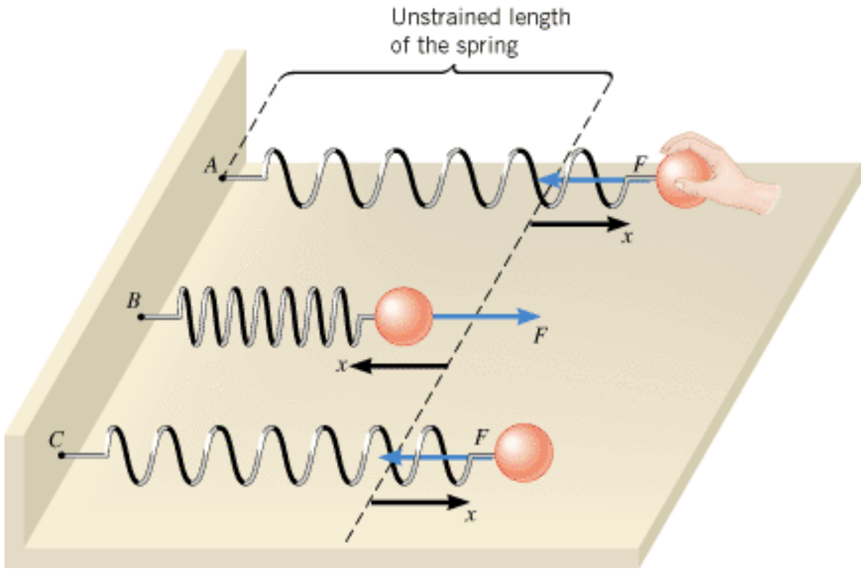


When a load is applied to a body, it changes shape and perhaps size



- Motions throughout a body are called **displacements**
  - **u**(**r**, t)
- Rigid body displacements do not change the shape and/or size
- Changes in shape and/or size are call **deformations**
- The objective of **Solid Mechanics** is to relate loads to deformation

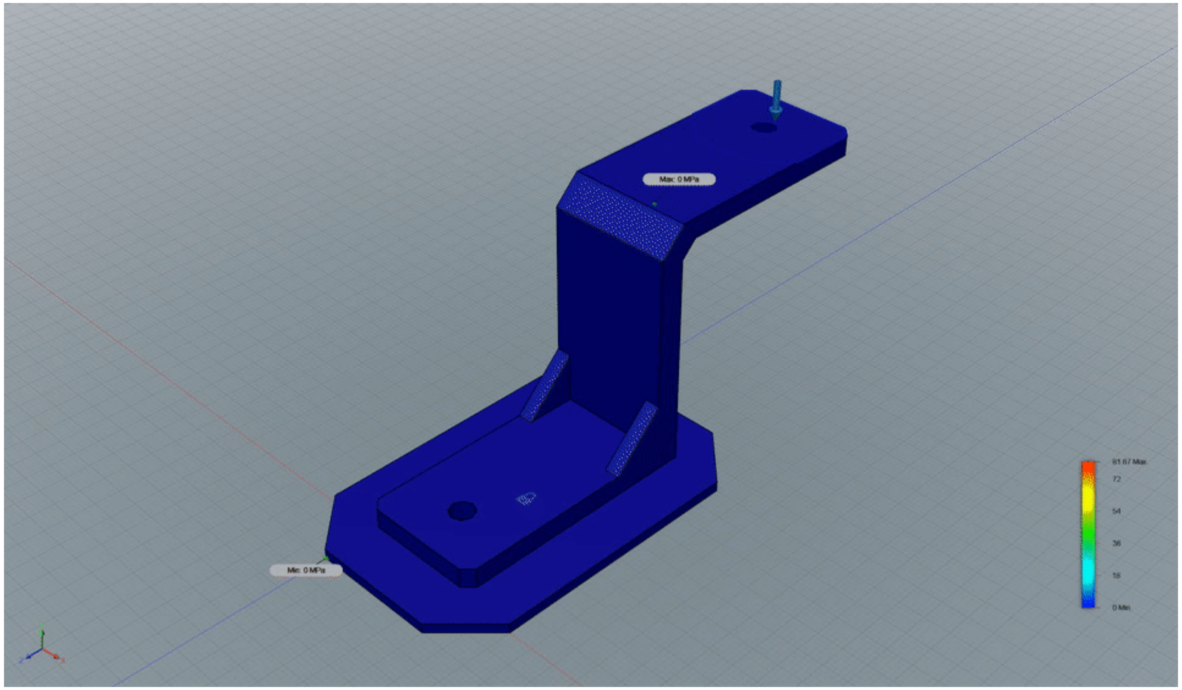
Before we get into the details of stress and strains, it helps to review an ideal spring



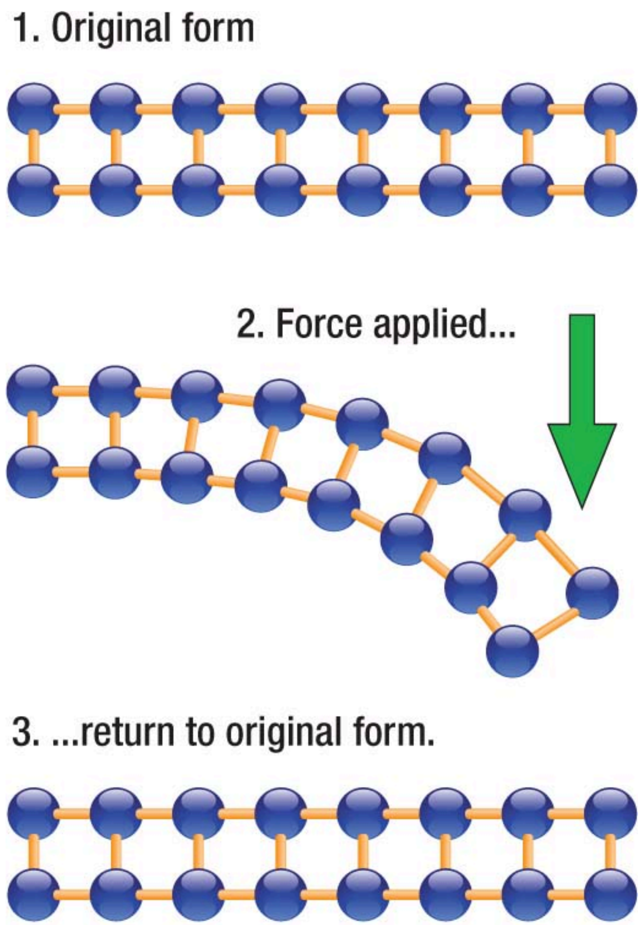
- When we apply some force F, we get some displacement x
  - $F = k\,x$
- When the spring is displaced by x, there is force that responds in the opposite direction equal to kx
- Due to the displacement, there is a stored energy $E = ½\,k\,x^2$

Solid mechanics is similar to the behavior of an ideal spring but throughout a body

- An applied load results results in deformation.
- The internal **strain** is like the displacements x
- The internal **stress** is like the internal force F

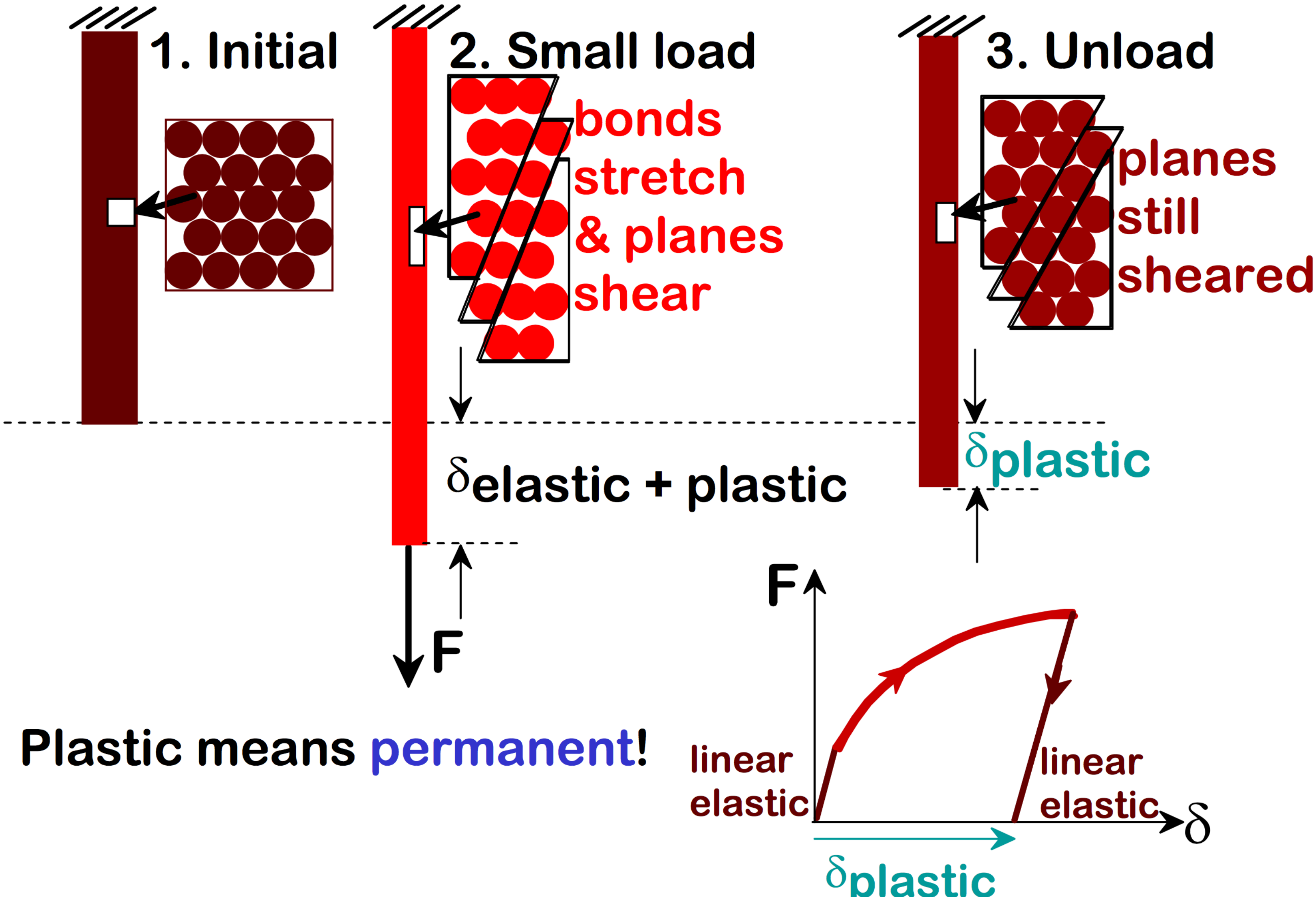


Elastic deformations are small and they are recoverable (they spring back)



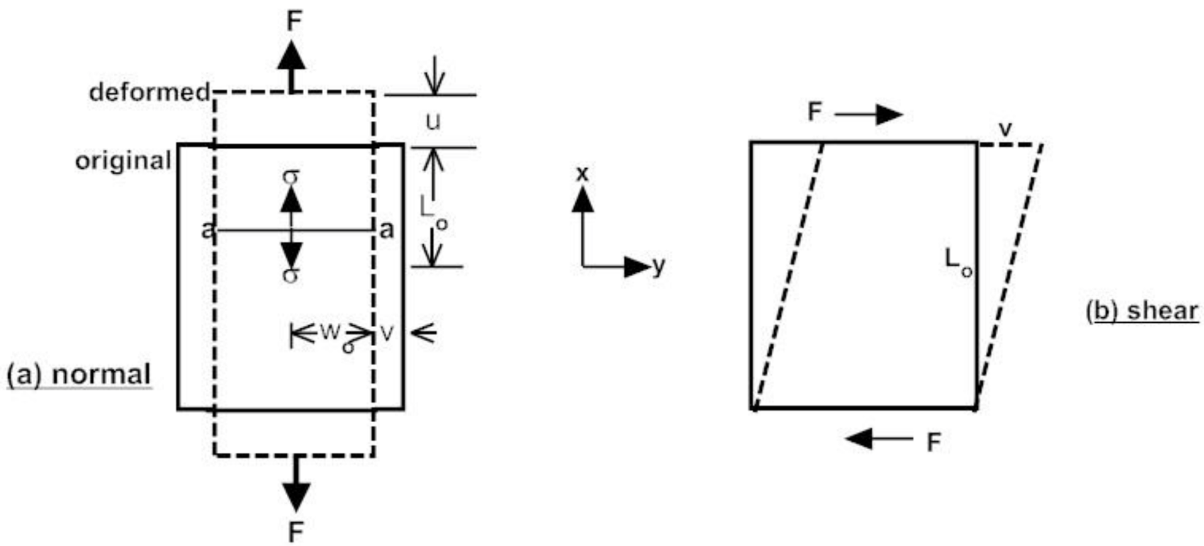
- In elastic deformation, we are stretching the atomic bonds.
- The more we stretch the bonds, the more force it takes to stretch.
- When we release the load, the atoms spring back into their lattice sites.

Plastic deformations are larger and are permanent



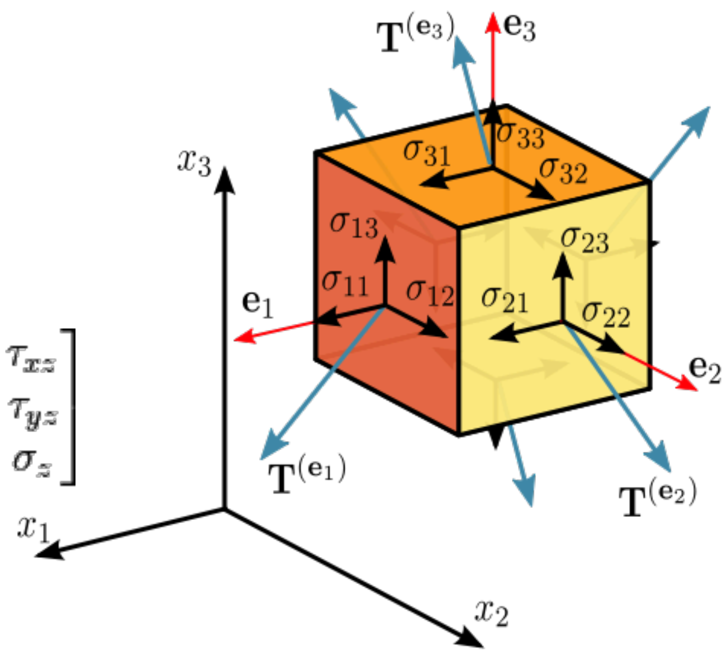
Plastic means permanent!

A stress is a force per unit area



$\boldsymbol{\sigma_{ij} = F_{ij}/A_i}$

- i is the face the force is applied to
- j is the direction it is applied

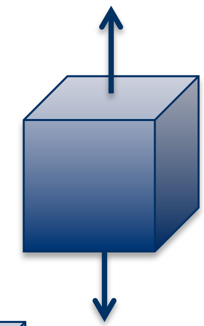


- It has SI units of Pa = N/m^2
- The stress is a 2nd order tensor (a 3 by 3 matrix)

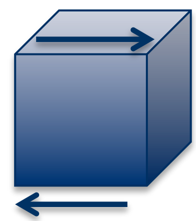
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$$

The stress in a material is the RESPONSE to an applied load (force) or an applied displacement

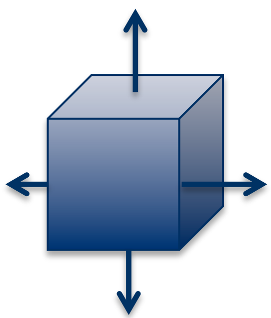
- Uniaxial (simple) tension or compression
  - Only one non-zero stress: σ_{ii} $(\sigma_{11}, \sigma_{22}, \sigma_{33})$
  - Tension means positive stress, compression negative
  - Examples: Cables, tension tests



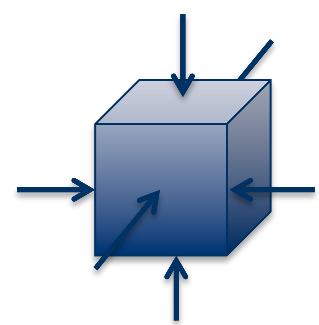
- Pure shear
  - Only one non-zero stress: $(\sigma_{12}, \sigma_{13}, \sigma_{23})$
  - Examples: drive shaft



- Biaxial tension
  - Two non-zero stress (e.g. $\sigma_{11} = 1, \sigma_{22} = 2)$
  - Examples: Pressure cylinder or vessel



- Hydrostatic compression (pressure)
  - $\sigma_{11} = \sigma_{22} = \sigma_{33} =$ -p
  - Anything underwater



We describe material deformation in terms of strain

- The **strain** in a tensile test is the deformation divided by a representative length $e = \frac{\Delta L}{L} = \frac{\ell - L}{L}$
- Strain is a second order tensor, like the stress and is computed using gradients
  - Let **u** be a vector of the displacements
  - The small strain tensor is

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$$

- The common strain states are the same as the stress (uniaxial tension, etc)

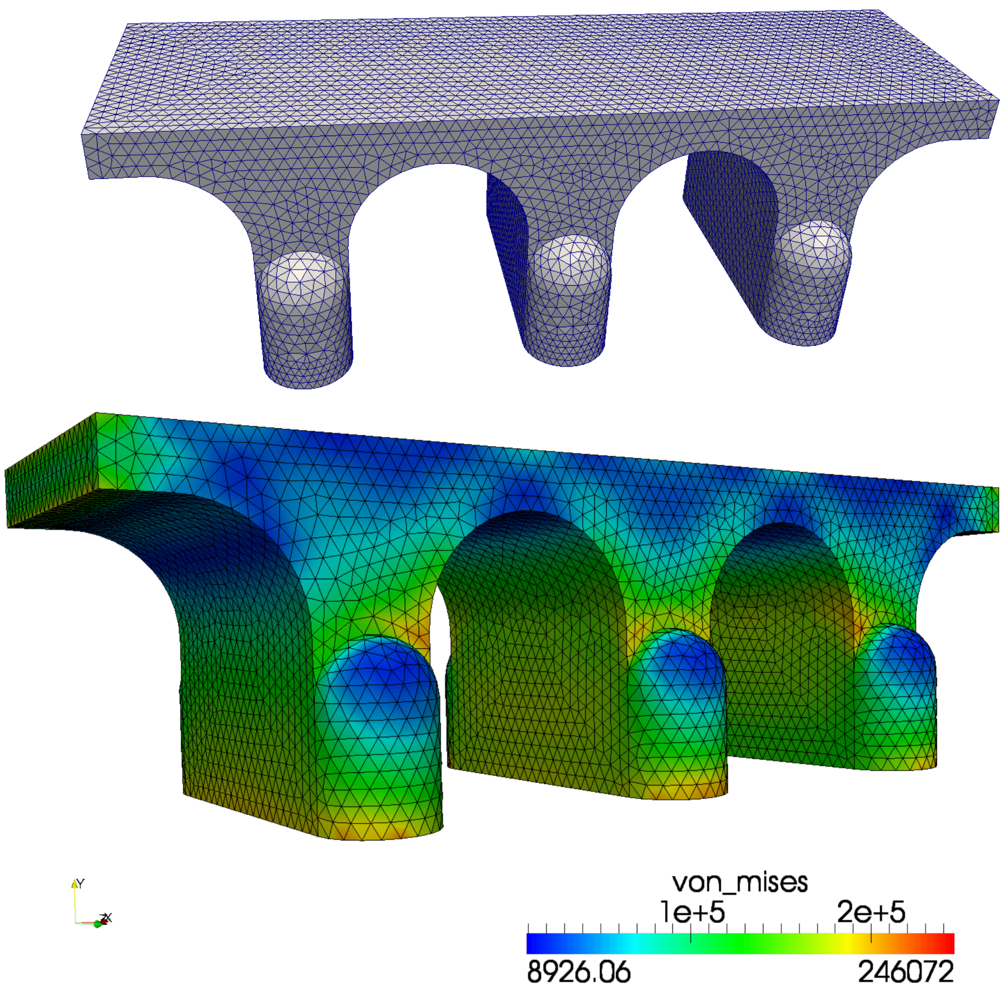
A deformation (strain) results in stress within a material

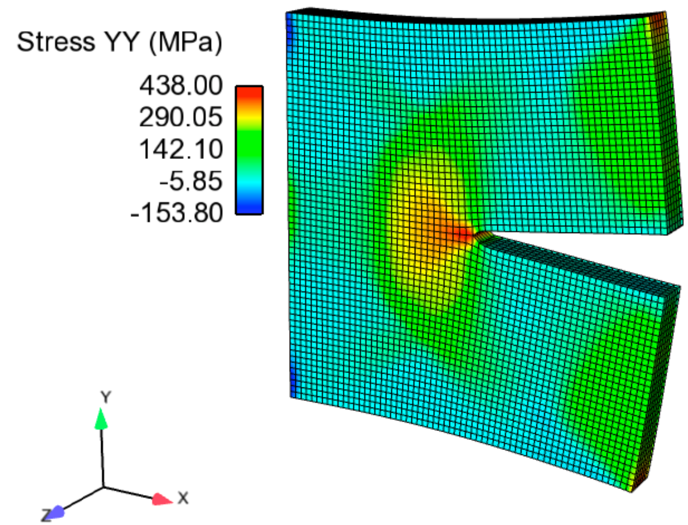
- $\boldsymbol{\sigma} = F(\boldsymbol{\epsilon})$
- For small strains, the stress is elastic and is a linear function of the strain
  - $\boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}(\boldsymbol{\epsilon})$
- For larger deformation, some of the strain is elastic and increases the stress. The rest is plastic and does NOT contribute to the stress.
  - $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_e + \boldsymbol{\epsilon}_p$

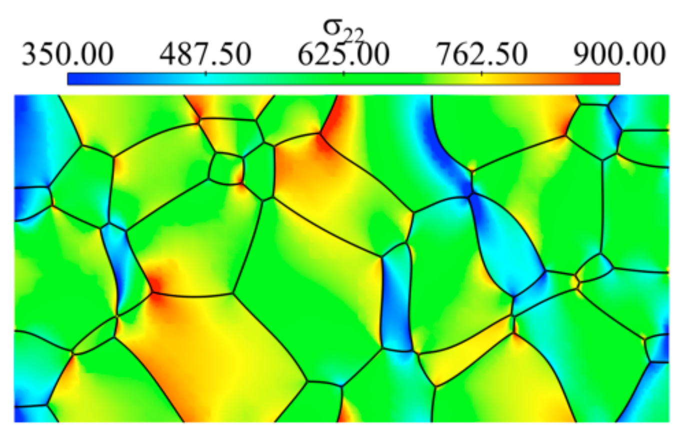
    $\boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}\boldsymbol{\epsilon}_e$
- The elastic energy density in a material is a scalar quantity equal to

$$E_{el} = \frac{1}{2}\boldsymbol{\epsilon}_e \cdot \boldsymbol{\sigma}$$

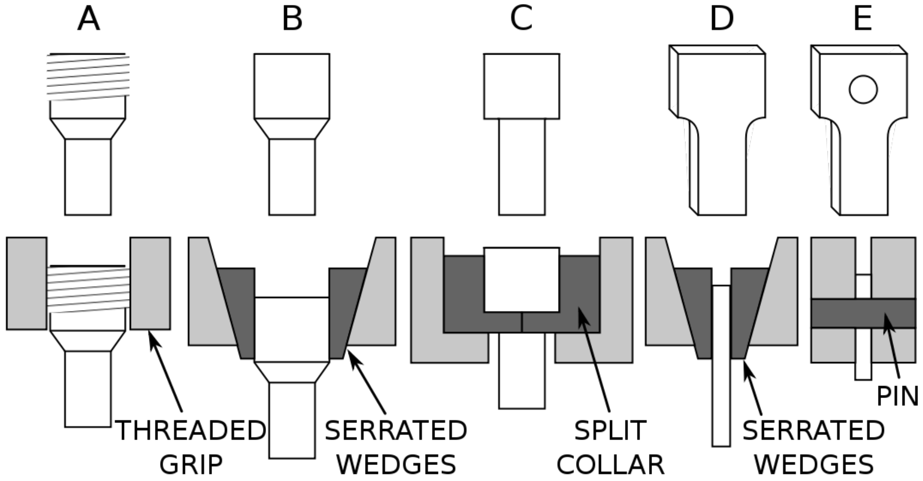
In actual materials, the stress and the strain change throughout the material

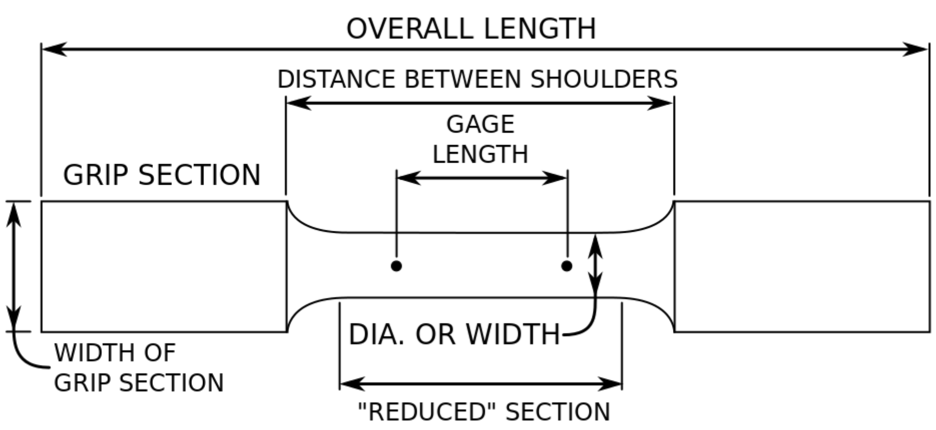


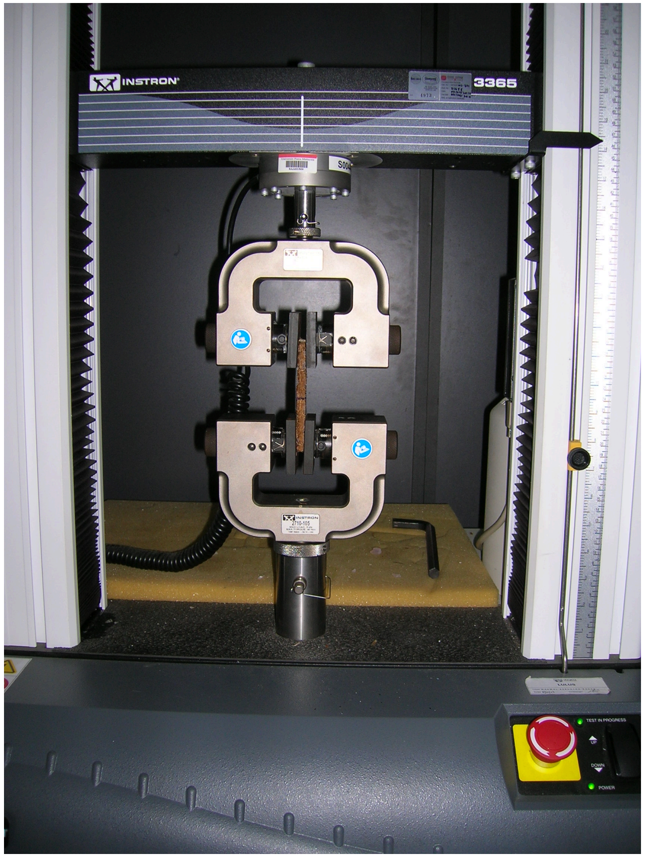




The most common means of determining mechanical properties is a uniaxial tension test



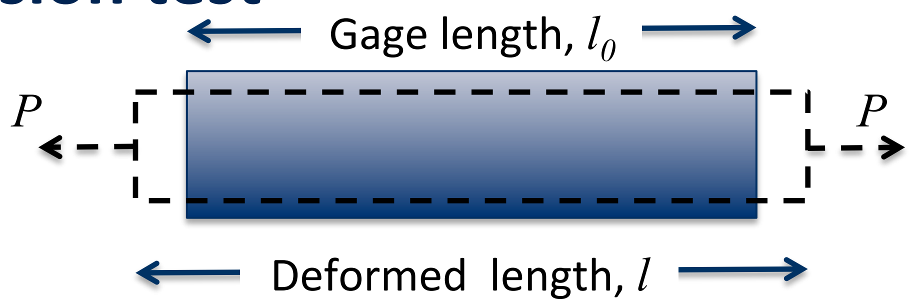




There are standard ways to determine the stress and strain in a tension test

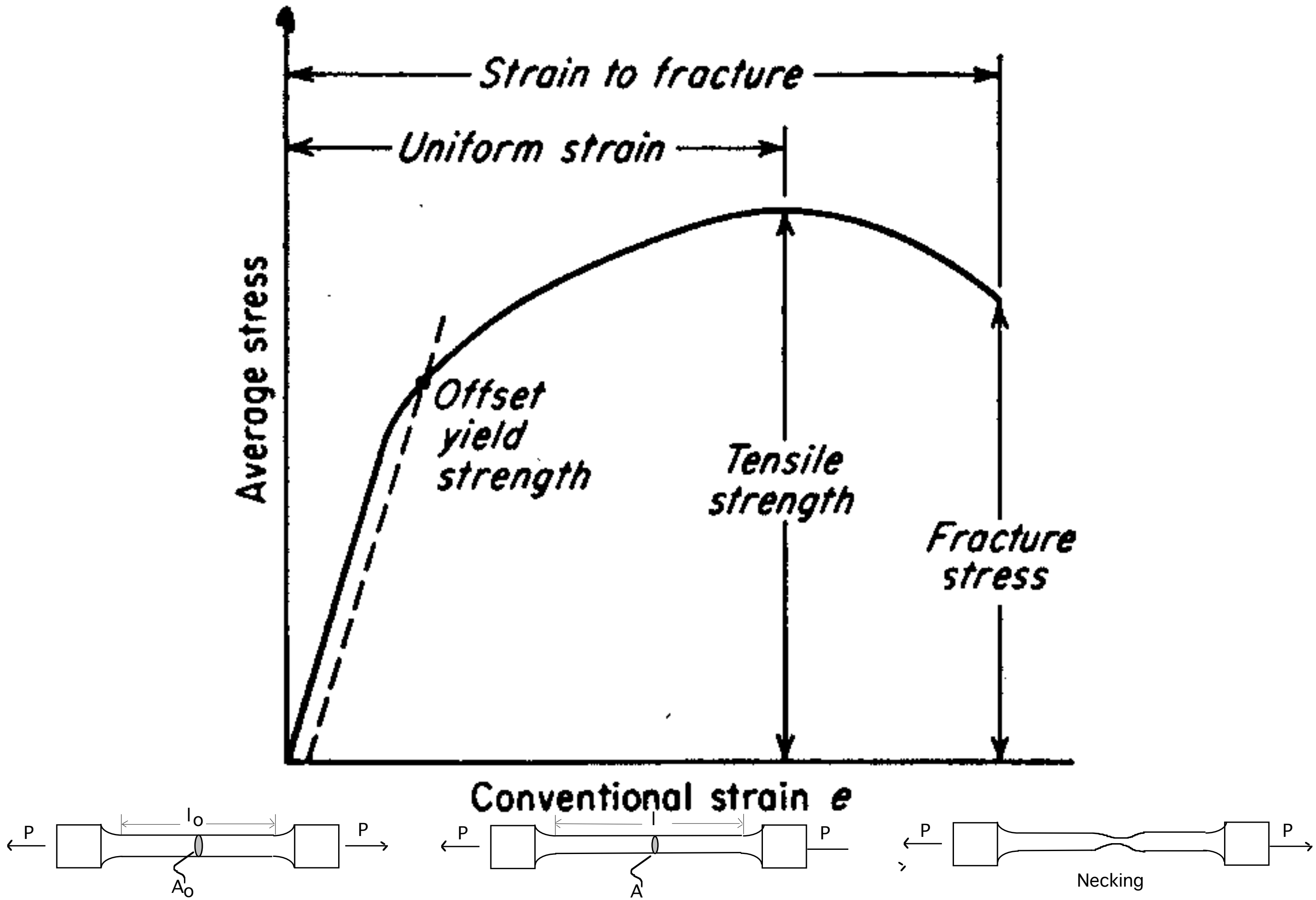
A_0 = Initial cross section area

A = deformed cross section area

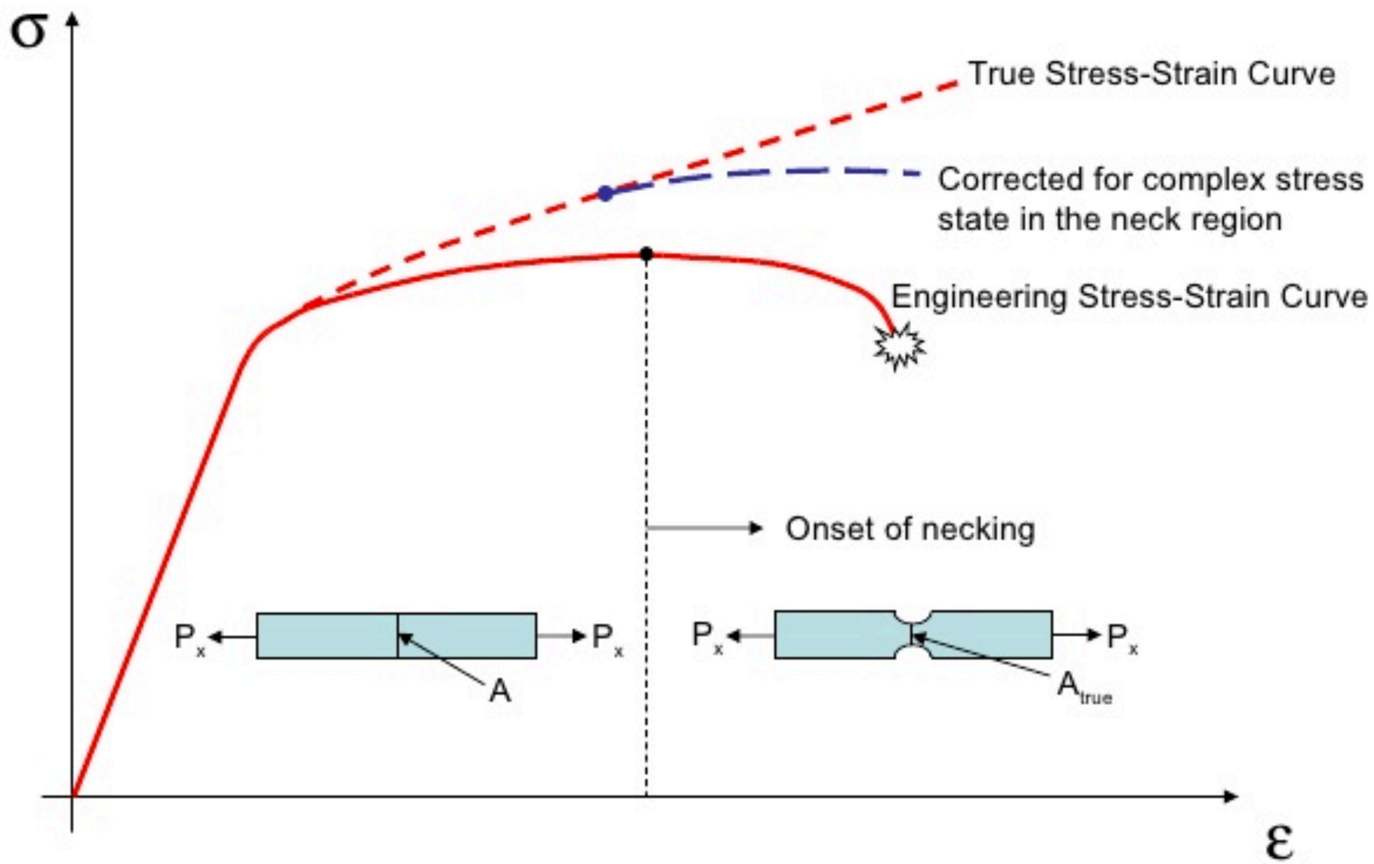


	Engineering	True
stress σ	$\dfrac{P}{A_o}$	$\dfrac{P}{A}$
strain ε	$\dfrac{l-l_o}{l_o}$	$\int_{l_o}^{l}\dfrac{dl}{l} = ln\left(\dfrac{l}{l_o}\right)$

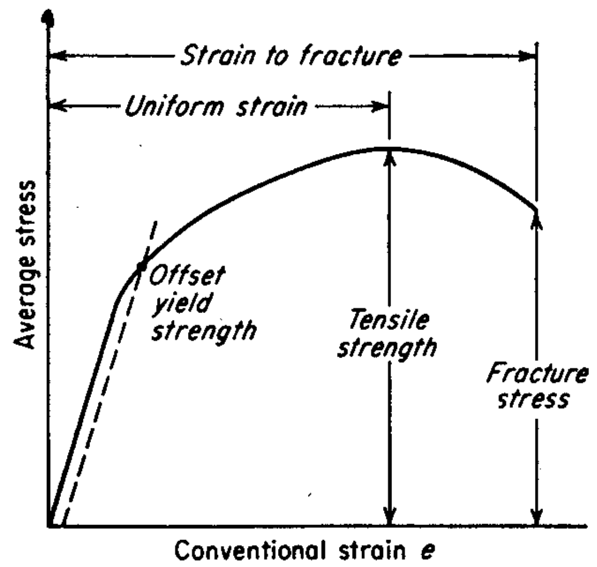
Strain strain curves of ductile materials have specific regions of different behavior



Engineering stress and strain give a different curve than true stress and strain



In the elastic portion of the stress-strain curve, the stress varies linearly with strain



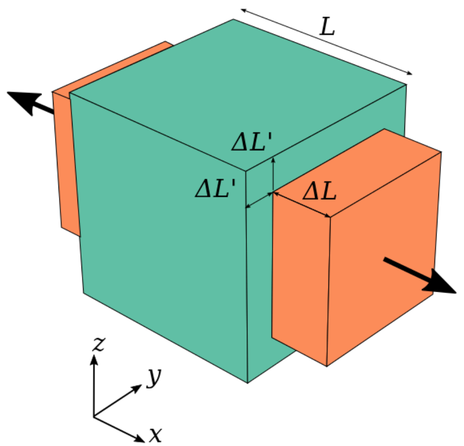
- The slope of the line is Young's Modulus, E

$$\sigma = E\,\varepsilon$$

- Young's modulus is an elastic constant
- Another elastic constant is Poisson's ratio, ν
- Poisson's ratio is the ratio of the shrinkage in cross section due to the extension in the pulling direction

$$\nu = -\frac{d\varepsilon_{\mathrm{trans}}}{d\varepsilon_{\mathrm{axial}}} = -\frac{d\varepsilon_{\mathrm{y}}}{d\varepsilon_{\mathrm{x}}} = -\frac{d\varepsilon_{\mathrm{z}}}{d\varepsilon_{\mathrm{x}}}$$

$$\nu \approx \frac{\Delta L'}{\Delta L}.$$



- The shear modulus, G, defines the stress to strain ratio in shear $\sigma_{12} = G\epsilon_{12}$
- For isotropic materials, $G = E / (2(1 + \nu))$
- To see all the elastic constants, go to https://en.wikipedia.org/wiki/Hooke's_law

Some materials deform the same way in any direction, others don't

- **Isotropic** materials deform the same way no matter in what direction you deform them.
  - They have 2 unique elastic constants
    $\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\epsilon})\mathbf{I} + 2\mu\boldsymbol{\epsilon}$
- **Anistropic** materials behave differently in different directions
  - The elasticity tensor **C** can have 21 unique components defining anisotropy
  - Cubic structured materials have 3 unique elastic constants (UO_2)
  - Hexagonal structured materials have 5 unique elastic constants
- Polycrystalline anisotropic materials can behave as isotropic, because the various grains average out.

The stress and stress relationship can also be written in matrix form

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

The matrix elasticity tensor form depends on the symmetry

Isotropic

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Cubic

$$\begin{bmatrix} 11 & 12 & 12 & 0 & 0 & 0 \\ . & 11 & 12 & 0 & 0 & 0 \\ & . & 11 & 0 & 0 & 0 \\ & . & . & 44 & 0 & 0 \\ & . & . & . & 44 & 0 \\ & . & . & . & . & 44 \end{bmatrix}$$

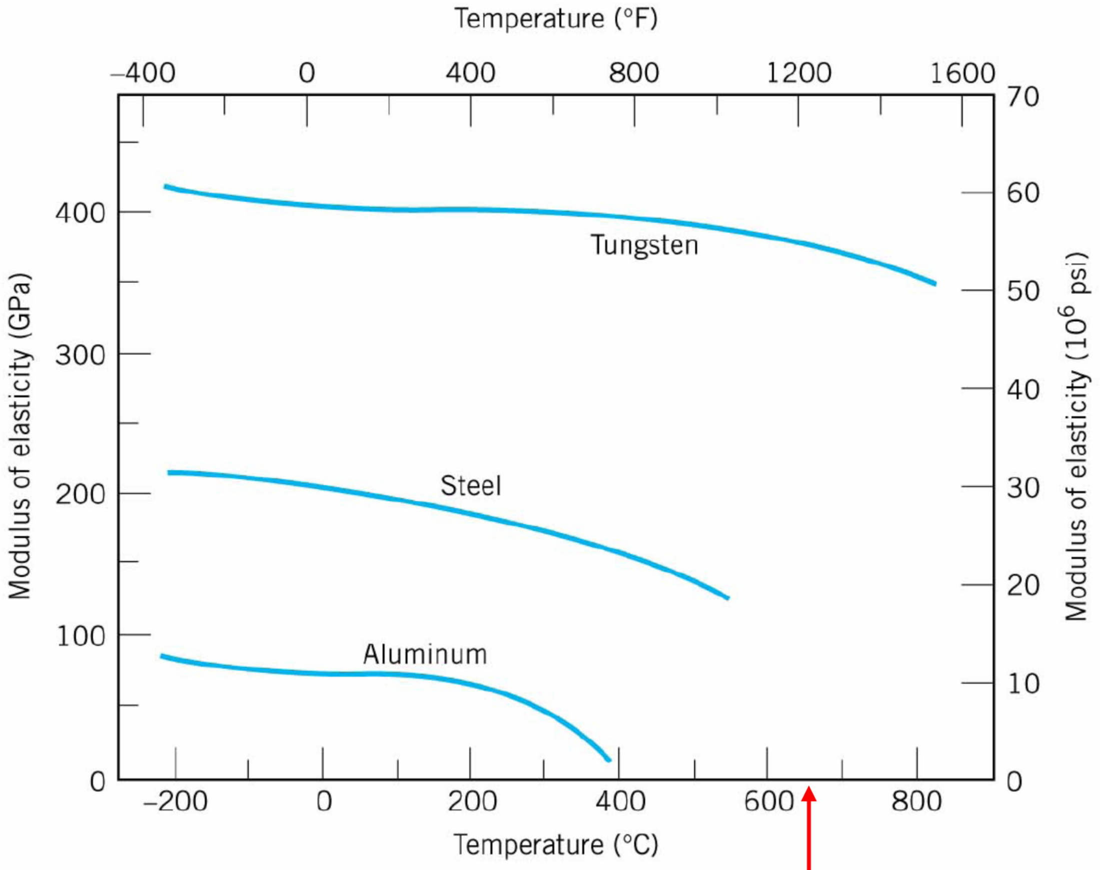
Hexagonal

$$\begin{bmatrix} 11 & 12 & 13 & 0 & 0 & 0 \\ . & 11 & 13 & 0 & 0 & 0 \\ . & . & 33 & 0 & 0 & 0 \\ . & . & . & 44 & 0 & 0 \\ . & . & . & . & 44 & x \\ & & & & & \end{bmatrix}$$

Here are isotropic elastic constants for some materials

Material	E (Gpa)	ν
Aluminum	70.3	.345
Gold	78.0	.44
Iron	211.4	.293
Nickel	199.5	.312
Tungsten	411.0	.28
Zircaloy	80.0	0.41
UO_2	200.0	0.345

Young’s modulus is typically a function of temperature



Melting Point of Al

Summary

- Solid mechanics predicts the deformation of a body from its applied load
- The strain defines the deformation
- The stress defines the material's internal response to the strain
- Materials can have recoverable and permanent deformation
  - Elastic deformation is recoverable and occurs due to the stretching of the atomic lattice
  - Plastic deformation is permanent and results form the breaking of bonds