PennState
College of Engineering

NucE 497: Reactor Fuel Performance

Lecture 7: Analytical solution of the fuel temperature profile

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Some content taken Professor Motta's book, chapter 9

Today we will begin our discussion of heat transport in the fuel

- Module 1: Fuel basics
- Module 2: Heat transport
  - Intro to heat transport and the heat equation
  - **Analytical solution of the heat equation**
  - Numerical solution of the heat equation
  - 1D solution of the heat equation using Matlab
  - 2D solution of the heat equation using Matlab
  - Coolant temperature change, power generation, and melting
- Module 3: Mechanical behavior
- Module 4: Materials issues in the fuel
- Module 5: Materials issues in the cladding
- Module 6: Accidents, used fuel, and fuel cycle

In order to solve for the temperature profile in the fuel and cladding, we make assumptions

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

- *Assumption 1:* We only care about the steady state solution

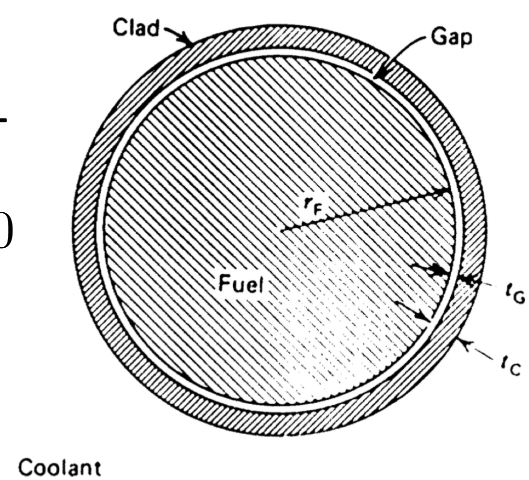
$$\nabla \cdot (k \nabla T) + Q = 0$$

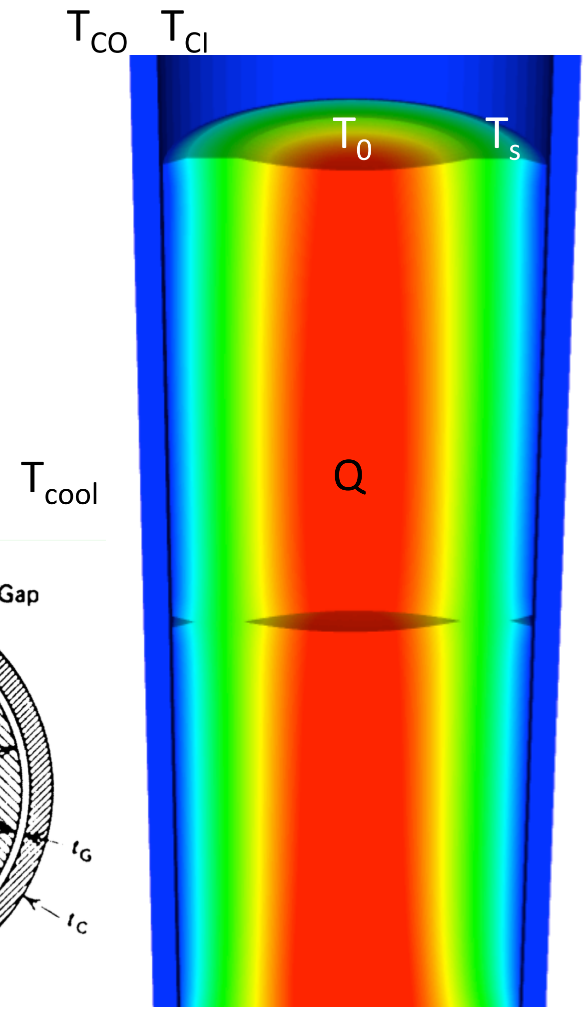
- *Assumption 2:* The behavior is axisymmetric

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r k(T) \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + Q(r,z) = 0$$

- *Assumption 3: T is constant in z*
- *Assumption 4:* The thermal conductivity k is independent of T

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r k(T) \frac{\partial T}{\partial r}\right) + Q(r) = 0$$





We will start by solving for the temperature profile in the fuel

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + Q = 0$$

- Consider the radius r = 0 in the center and r = R_f on the outer edge
- Use the boundary conditions: T'(0) = 0, T(R_f) = T_s
- Solve for the temperature T(r)

$$\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) = -Q\,r$$

$$rk\frac{\partial T}{\partial r} = -\frac{Q\,r^2}{2} + C_1$$

$$0 = -\frac{Q\,0^2}{2} + C_1$$

$$\frac{\partial T}{\partial r} = -\frac{Q\,r}{2k}$$

$$T(r) = -\frac{Q\,r^2}{4k} + C_2$$

$$C_2 = \frac{Q\,R_f^2}{4k} + T_s$$

$$T(r) = \frac{Q\,(R_f^2 - r^2)}{4k} + T_s$$

$$T_0 - T_s = \frac{Q\,R_f^2}{4k}$$

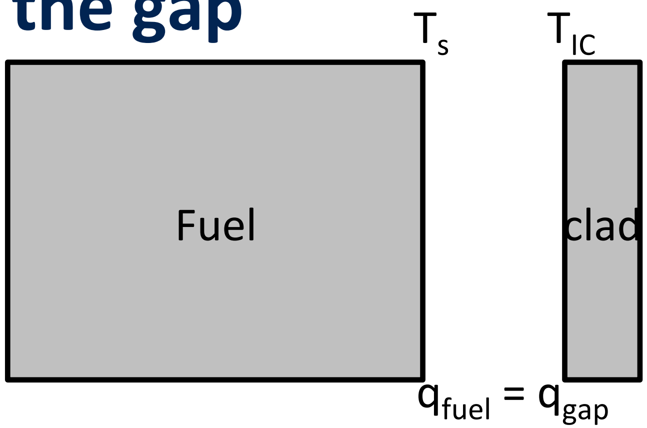
The linear heat rate is the heat rate delivered per unit length of fuel

- LHR = $\pi R_f^2 Q_{av}$ (W/cm), where Q_{av} is the radially averaged heat generation rate
- If we substitue LHR into our temperature equations, we get

$$T(r) = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_f^2}\right) + T_s$$

$$T_0 - T_s = \frac{LHR}{4\pi k}$$

Next, we need to determine the heat transport through the gap



$$q_{gap} = k_{gap} \frac{T_s - T_{IC}}{t_G}$$

$$q_{fuel} = \frac{LHR}{2\pi R_f}$$

- Derive an expression to solve for $T_s - T_{IC}$

$$k_{gap} \frac{T_s - T_{IC}}{t_G} = \frac{LHR}{2\pi R_f}$$

$$T_s - T_{IC} = \frac{LHR}{2\pi R_f} \frac{t_G}{k_{gap}}$$

- We can define the gap conductance

$$h_{gap} = \frac{k_{gap}}{t_G}$$

- So,

$$T_s - T_{IC} = \frac{LHR}{2\pi R_f h_{gap}}$$

- We assume a linear temperature profile across the gap

The thermal conductivity of the gap depends on the gas that is filling it

- The gas can is filled with He at the beginning of life, but begins to fill with Xe due to fission gas release.

$$k_{gas} = A \times 10^{-6} T^{0.79} \quad W / cm - K$$

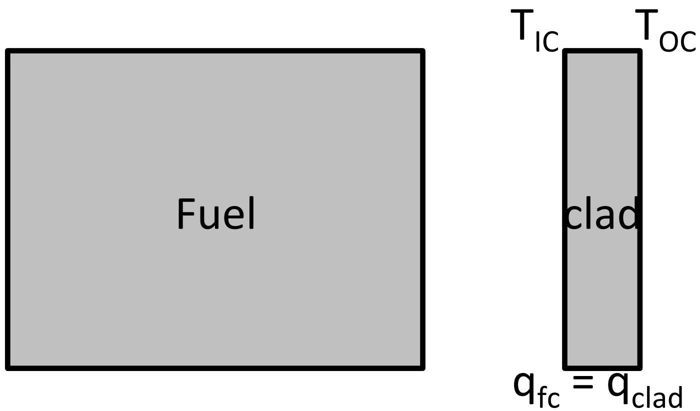
- A = 16 for He and 0.7 for Xe
- The fraction of Xe in the gas mixture, y, increases with time
- The thermal conductivity of the mixture is determined with

$$k_{gap} = k_{He}^{1-y} k_{Xe}^{y}$$

- Then, compute the conductance

$$h_{gap} = \frac{k_{gap}}{t_G}$$

Now we find the temperature change in the cladding



$$q_{clad} = k_C \frac{T_{IC} - T_{OC}}{t_C}$$

$$q_{fc} = \frac{LHR}{2\pi(R_f + t_C/2)}$$

$$\approx \frac{LHR}{2\pi R_f}$$

- The expression for $T_{IC} - T_{OC}$ is

$$T_{IC} - T_{OC} = \frac{LHR\, t_C}{2\pi R_f k_C}$$

The last step is to compute the temperature change through the coolant

- Heat conduction through the coolant is due to convection

$$T_{CO} - T_{cool} = \frac{LHR}{2\pi R_F h_{cool}}$$

- h_{cool} is the convective heat transfer coefficient

So, to summarize the analytical temperature equations:

- Note, $t_C = \delta_c$

Inc terms of Q

Fuel: $$T_m - T_S = \frac{Q}{4k} R_f^2$$

Gap: $$T_S - T_{CI} = \frac{Q}{2h_{gap}} R_f$$

Cladding: $$T_{CI} - T_{CO} = \frac{Q}{2k_c} R_f \delta_c$$

Coolant: $$T_{CO} - T_{cool} = \frac{Q}{2h_{cool}} R_f$$

In terms of LHR

$$T_0 - T_s = \frac{LHR}{4\pi k}$$

$$T_s - T_{IC} = \frac{LHR}{2\pi R_f h_{gap}}$$

$$h_{gap} = \frac{k_{gap}}{t_G}$$

$$T_{IC} - T_{OC} = \frac{LHR\, t_C}{2\pi R_f k_C}$$

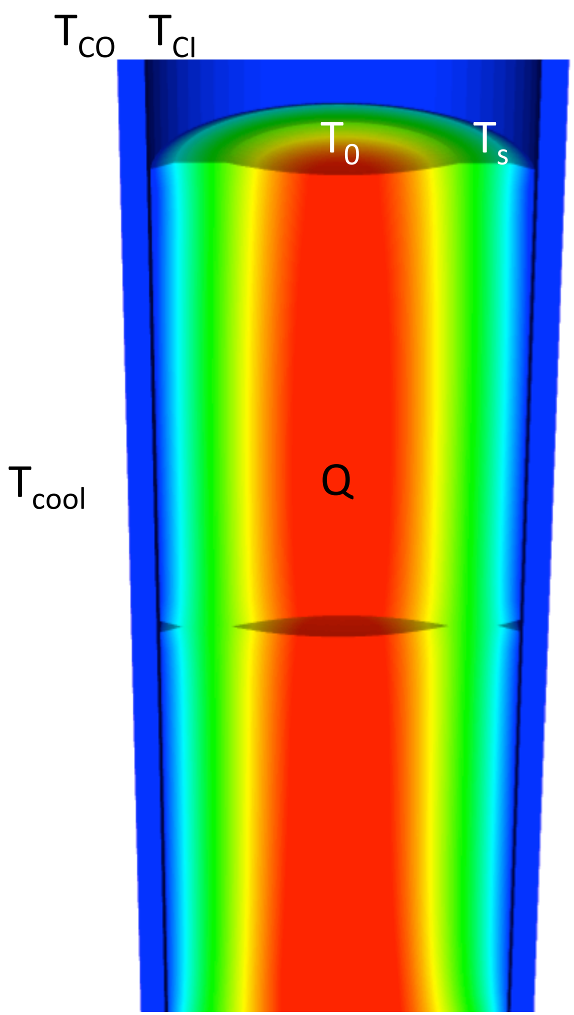
$$T_{CO} - T_{cool} = \frac{LHR}{2\pi R_F h_{cool}}$$

Fuel and Cladding Thermal Properties

Material	**Density (g/cm^3)**	**Heat Capacity Cp (J/g-K)**	**Thermal Conductivity k (W/cm-K)**	**Thermal Expansion Coefficient a (K^{-1})**
UO_2	10.98	0.33	0.03	1.2×10^{-5}
Zr	6.5	0.35	0.17	1.0×10^{-5}
Stainless steel	8.0	0.5	0.17	9.6×10^{-6}

To solve for the temperature profile across the radius, you first solve for the transition temperatures

- You first solve for the transition temperatures.
- Start from the coolant and work inward
- Then, assume a linear profile everywhere except in in the fuel
- Finally, solve for the temperature profile throughout the fuel

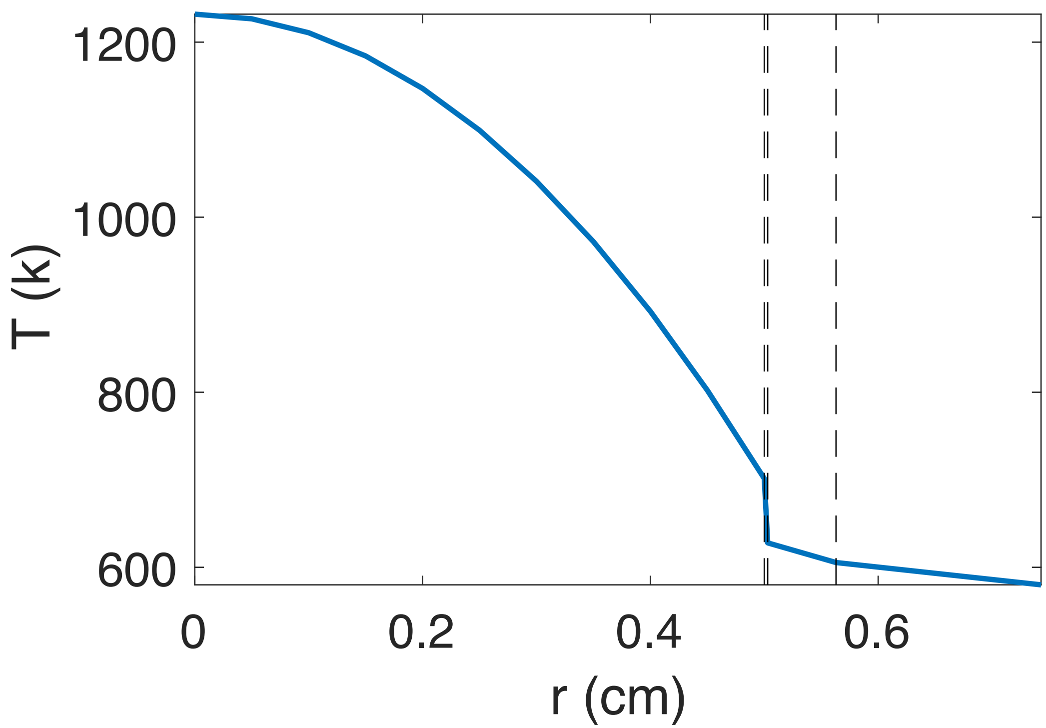


Example: Calculate the temperature profile for a fuel rod using the following data:

- T_{cool}=580 K; LHR = 200 W/cm; h_{cool} =2.5 W/cm^2- K
- Fuel pellet radius R_F =0.5 cm, Cladding thickness δ_C= 0.06 cm; Gap width δ_{gap}= 30 μm
- First, we calculate T_{CO} = LHR/(2 π R_f h_{cool}) + T_{cool}
  T_{CO} = 200/(2 π*0.5*2.5) + 580 = 25.5 + 580 = 605.5 K;
- Then, T_{CI} = LHR δ_C/(2 π R_f k_c) + $T_{CO,}$ where k_c = 0.17 W/cm-K (from table)
  T_{CI} = 200*0.06/(2 π*0.5*0.17) + 605.5 = 22.5 + 605.5 = 628.0 K ;
- Next, T_s = LHR /(2 π R_f h_{gap}) + $T_{CI,}$ where k_{He} = 16-e6 $T_{ci}^{0.79}$ = 0.0026 W/cm-K
  h_{gap} = k_{He}/δ_{gap} = 0.0026/30e-4 = 0.87 W/cm^2-K
  T_s = 200/(2 π*0.5*0.87) + 628.0 = 73.5 + 628.0 = 701.5 K ;
- Finally, T_0 = LHR/(2 π k) + $T_{s,}$ where k = 0.03 W/cm-K (from table)
  T_{CI} = 200/(4 π*0.03) + 701.5 = 530.5 + 701.5 = 1232.0 K ;

We finish, by calculating the temperature profile throughout the fuel

$$T(r) = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_f^2}\right) + T_s$$



In class problem: Calculate the fuel centerline temperature if 30% of the gas in the gap is Xe

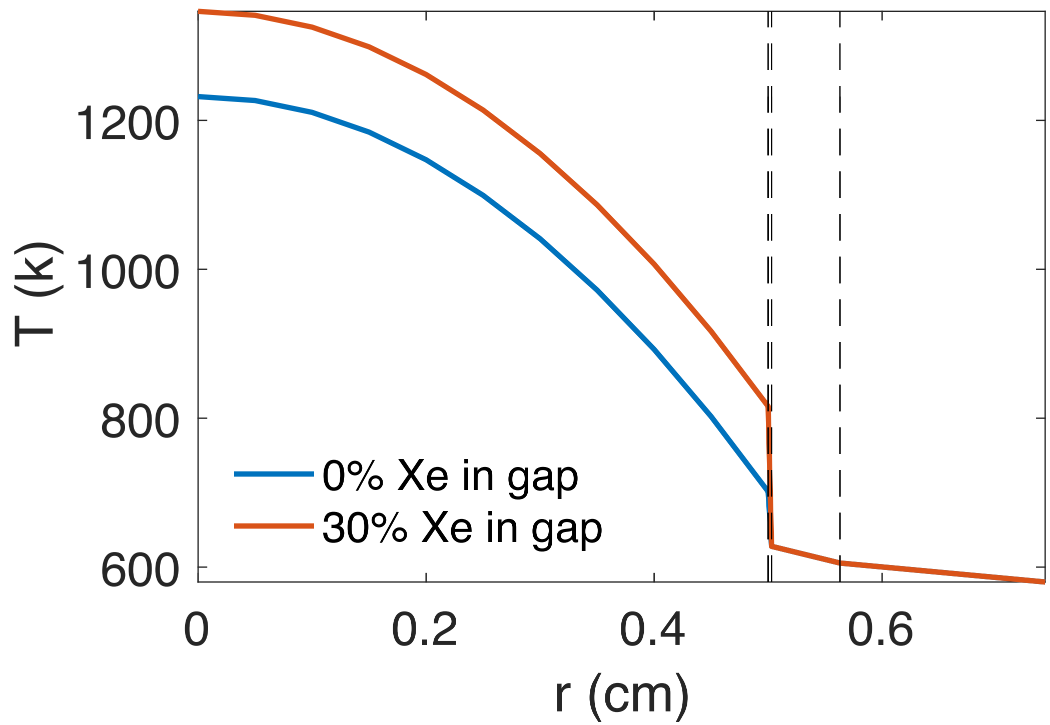
- T_{cool}=580 K; LHR = 200 W/cm; h_{cool} =2.5 W/cm^2- K
- Fuel pellet radius R_F =0.5 cm, Cladding thickness δ_C= 0.06 cm; Gap width δ_{gap}= 30 μm
- T_{OC} = 605.5 K; T_{IC} = 628.0 K; $T_0 - T_{IC}$ = 530.5 K
- k_{He} = 0.0026 W/cm-K, k_{Xe} = 0.7e-6 $T_{CI}^{0.79}$, $k_{gap} = k_{He}^{1-y} k_{Xe}^{y}$

$$T_s - T_{IC} = \frac{LHR}{2\pi R_f h_{gap}}$$

$$h_{gap} = \frac{k_{gap}}{t_G}$$

- k_{Xe} = 0.7e-6 $628^{0.79}$ = 1.36e-4 W/cm-K
- k_{gap} = $0.0026^{(1-0.3)}$*$(1.36\text{e-}4)^{0.3}$ = 0.001 W/cm-K
- h_{gap} = 0.001/30e-4 = 0.3386 W/cm^2-K
- T_s = 200/(2π0.5*0.3386) + 628.0 = 188.0 + 628.0 = 816.0 K
- T_0 = 816.0 + 530.5 = 1346.5 K

Even 30% Xe in the gap has a large impact on the fuel centerline temperature

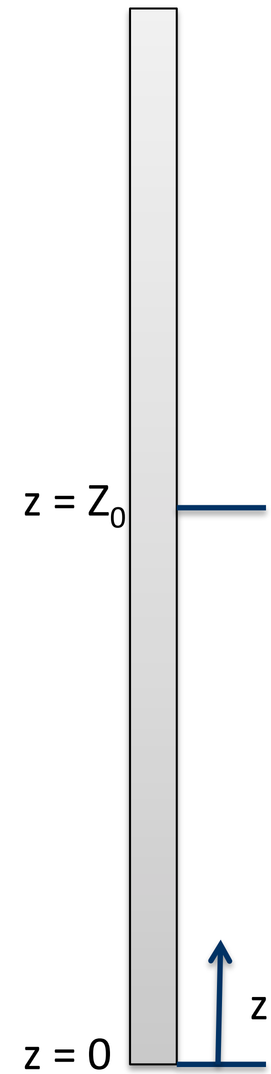


The heat generation rate varies axially, so the LHR does as well

- Consider a fuel rod of length $2Z_0$,

$$LHR\left(\frac{z}{Z_o}\right) = LHR^o \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{Z_o} - 1\right)\right] = LHR^o\, F\left(\frac{z}{Z_o}\right)$$

- Where
  - LHR^0 is the centerline linear heat rate (z = Z0)
  - $\gamma = (Z_{ex} + Z_o)/Z_o$ where Z_{ex} is the extrapolation distance
  - $\gamma \approx 1.3$

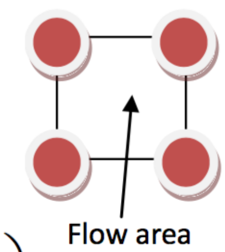


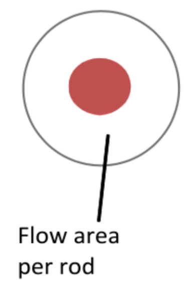
The coolant temperature actually varies with axial position along the rod

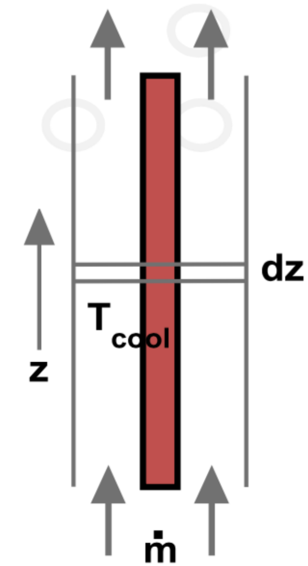
$$\dot{m}C_{PW}\frac{dT_{cool}}{dz} = LHR\left(\frac{z}{Z_o}\right)$$

Integrating from the core entry (z = 0) to height z:

$$\dot{m}C_{PW}(T_{cool} - T_{cool}^{in}) = Z_o \int_0^{z/Z_o} LHR\left(\frac{z}{Z_o}\right) d\left(\frac{z}{Z_o}\right)$$







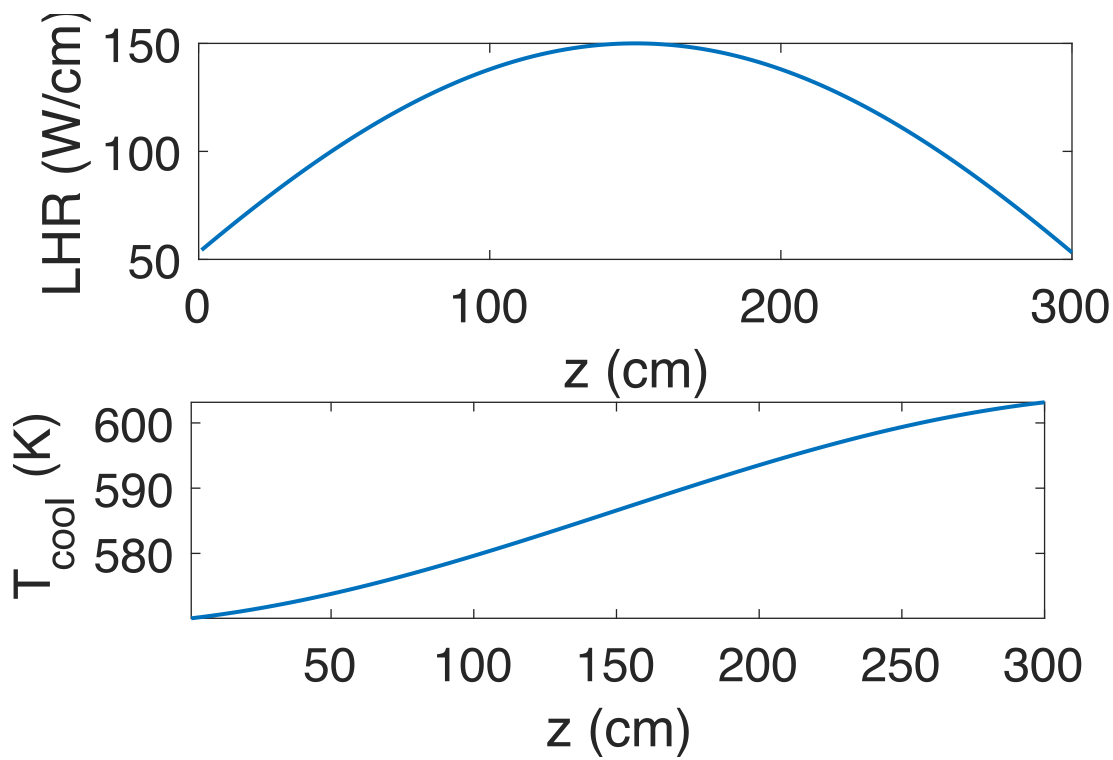
The axial power profile results in axial variation of LHR

$$\dot{m}C_{PW}(T_{cool} - T_{cool}^{in}) = Z_o \times LHR^o \int_0^{z/Z_o} F\left(\frac{z}{Z_o}\right) d\left(\frac{z}{Z_o}\right)$$

$$T_{cool} - T_{cool}^{in} = \frac{2\gamma}{\pi}\frac{Z_0 LHR^0}{\dot{m}C_{pw}}\left(\sin\left(\frac{\pi}{2\gamma}\right) + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{Z_0} - 1\right)\right)\right)$$

Example: We will calculate the LHR and T_{cool} with axial variation

- mdot = 0.25 kg/s-rod; Z_0 = 150 cm; LHR^0 = 150 W/cm; C_{PW} = 4200 J/kg-K; T_{in} = 570 K
- Plugging these values into the equations gives



Summary

- We can derive analytical expressions for the temperature profile within a fuel rod by making four assumptions
  - Steady state solution
  - Temperature is axisymmetric
  - T is constant in z
  - The thermal conductivity is independent of temperature
- The temperature profile in the fuel is parabolic.
- We assume the temperature profiles in the gap, cladding, and coolant are linear