1604 Grading 90/100 Evan Simpson

(1) $K = 12.5 \frac{W}{m \cdot K}$ $\rho = \frac{7g}{cm^3}$ -3, 27/30

a.) uranium-235 at 0.7% for natural uranium

b.) $Q = E_f N_f \sigma_f \phi_{th}$

$q = 3\%$ $\phi_{th} = 3.2e^{13} \frac{n}{cm^2 \cdot s}$ $\sigma_f = 550 \times 10^{-24} cm^2$

$E_f = 3 \times 10^{-11} \frac{J}{fission}$

$$N_f = q \cdot \frac{\rho_u NA}{M} = (0.03)\left[\frac{11.31 \frac{g}{cm^3} \cdot 6.022 \times 10^{23} \, atoms/mol}{238 \, g/mol}\right]$$

$$N_f = 8.585 \times 10^{20} \frac{atoms}{cm^3}$$

$$Q(U_3Si_2) = (3 \times 10^{-11})(3.2 \times 10^{13})(550 \times 10^{-24})(8.585 \times 10^{20}) = 453.288 \frac{W}{cm^3}$$

$$N_f = q\left[\frac{7 \cdot 6.022 \times 10^{23}}{238}\right] = 1.771 \times 10^{22} q$$

$$453.288 \frac{W}{cm^3} = q \cdot 1.771 \times 10^{22} \frac{atoms}{cm^3} (3 \times 10^{-11} J)(3.2 \times 10^{13} \frac{n}{cm^2 \cdot s})(550 \times 10^{-24} cm^2)$$

$$q = .0485 \Rightarrow \boxed{q \simeq 4.85\%}$$

-3, Just use density of U

c.) $12.5 \frac{W}{m \cdot K} \cdot \frac{1m}{100cm} = .125 \frac{W}{cm \cdot K}$

U_3Si_5 would make a worse fuel than U_3Si_2 because a higher enrichment is need to get the same energy rate per volume & the thermal conductivity is lower than U_3Si_2 ($.23 \frac{W}{cm \cdot K}$)

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-0, 35/35

(2) $R = .45\,cm \quad t_c = .06\,cm \quad t_{gap} = 40\times10^{-4}\,cm$

$LHR = 250\ W/cm \quad T_{cool} = 580\,K \quad c_{Xe} = .05$

$h_{cool} = 2.5 \frac{W}{cm^2\cdot K}$

a) $T_s = ?$

$$T_{co} = T_\infty + \frac{LHR}{2\pi R_F h_{cool}} = \frac{250 \frac{W}{cm^2\cdot K}}{2\pi(.45cm)(2.5\frac{W}{cm^2\cdot K})} + 580K$$

$T_{co} = 615.37\,K \qquad \Delta T = 35.37\,K$

$$T_{ci} = T_{co} + \frac{LHR}{2\pi R_F K_c}(.06) = 615.37K + \frac{250}{2\pi(.45)(.17\frac{W}{cm\cdot K})}(.06)$$

$T_{ci} = 646.58K \qquad \Delta T = 31.21$

$K_{He} = 16\times10^{-6}(646.58K)^{.79} = .00266 \frac{W}{cm\cdot K}$

$K_{Xe} = .7\times10^{-6}(646.58K)^{.79} = .000116\ W/cm\cdot K$

$K_{gap} = K_{He}^{(1-c)}\cdot K_{Xe}^{(c)} = (.00266)^{(1-.05)}\cdot .000116^{(.05)} = .00227 \frac{W}{cm\cdot K}$

$h_{gap} = K_{gap}/t_{gap} = .00227\ W/cm\cdot K \,/\, 40\times10^{-4}\,cm$

$h_{gap} = .284 \frac{W}{cm^2\cdot K}$

$$T_s = T_{ci} + \frac{LHR}{2\pi R_F h_{gap}} = 646.58K + \frac{250}{2\pi(.45)(.284)}$$

$$\boxed{T_s = 957.92\,K}$$

b) $\eta = r/R_F \quad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)} \quad \sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2)$

$$T_0 = T_s + \frac{LHR}{4\pi k} = 957.92K + \frac{250}{4\pi(.2)}$$

$T_0 = 1057.31K$

$\longrightarrow$

② cont

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$$\sigma^* = \frac{(7.5\times10^{-6}\ 1/K)(246.7\times10^{9}\ Pa)[1057.39K - 957.92K]}{4(1-.25)}$$

$$\sigma^* = 61.35\ MPa$$

$$\sigma_{\theta\theta}(\eta=1) = -61.35\,MPa\,(1-3(1)^2)$$

$$\boxed{\sigma_{\theta\theta} = 122.7\ MPa}$$

c.) Stress would be higher in a UO_2 pellet since the temperature gradient would be larger. This leads to a proportional increase in $\sigma_{\theta\theta}$.

d.) Axisymmetric, isotropic, neglect gravity, static body, temperature is independent of axial position

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-7, 28/35

(3) $P = 6\,MPa$ $\quad \bar{R}_c = .56\,cm$ $\quad t_c = .06\,cm$

a.) The thickness of the cladding is much smaller than the average radius

-3, Stress is constant across thickness

There is no shear stress

b.) $\sigma_\theta = \frac{PR}{t} = \frac{(6\,MPa)(.56\,cm)}{.06\,cm} \Rightarrow \boxed{\sigma_\theta = 56\,MPa}$

$\sigma_z = \frac{PR}{2t} = \frac{56\,MPa}{2} \Rightarrow \boxed{\sigma_z = 28\,MPa}$

$\sigma_r = -\frac{1}{2}P = -\frac{1}{2}(6\,MPa) \Rightarrow \boxed{\sigma_r = -3\,MPa}$

c.) $R_i = \bar{R}_c - \frac{1}{2}(t) = .56\,cm - \frac{1}{2}(.06\,cm)$

$R_i = .53\,cm$

-4, Calculate stress at two radii to see if it is constant

$R_o = \bar{R}_c + \frac{1}{2}(t) = .56\,cm + \frac{1}{2}(.06\,cm)$

$R_o = .59\,cm$

$\sigma_{\theta\theta}(R_i) = p\frac{(R_o/R_i)^2+1}{(R_o/R_i)^2-1} = 6\,MPa\frac{(.59/.53)^2+1}{(.59/.53)^2-1}$

$6\left[\frac{2.239}{.239}\right] \Rightarrow \boxed{\sigma_{\theta\theta} = 56.21\,MPa}$

$\sigma_{rr}(R_i) = -6\,MPa\left[\frac{(R_o/R_i)^2-1}{(R_o/R_i)^2-1}\right] = -6\left[\frac{(.59/.53)^2-1}{(.59/.53)^2-1}\right]$

$\boxed{\sigma_{rr} = -6\,MPa}$

$\sigma_{zz}(R_i) = P\frac{1}{(R_o/R_i)^2-1} = 6\,MPa\frac{1}{(.59/.53)^2-1}$

$\boxed{\sigma_{zz} = 25.1\,MPa}$

$\frac{|56-56.21|}{56.21} \times 100 = .374\%$ error for $\sigma_{\theta\theta}$

$\frac{|6-3|}{6} \times 100 = 50\%$ error for σ_{rr}

$\frac{|25.1-28|}{25.1} \times 100 = 11.55\%$ error for σ_{zz}

The thin-walled assumption would not be accurate enough to determine if a material would fail, as seen by the error calculated on the previous pages.

d.) $E = 70 \times 10^9\ Pa \qquad \nu = .41$

$$\varepsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right)$$

$$= \frac{1}{70\times10^9\,Pa}\left[-3\times10^3\,Pa - .41(56\times10^6\,Pa + 28\times10^6\,Pa)\right]$$

$$\varepsilon_{rr} = -4.92\times10^{-04}$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right)$$

$$\varepsilon_{\theta\theta} = \frac{1}{70\times10^9\,Pa}\left[56\times10^6 - .41(-3\times10^3 + 28\times10^6)\right]$$

$$\varepsilon_{\theta\theta} = 6.36\times10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{rr} + \sigma_{\theta\theta})\right)$$

$$= \frac{1}{70\times10^9}\left(28\times10^6 - .41(-3\times10^3 + 56\times10^6)\right)$$

$$\varepsilon_{zz} = 7.2\times10^{-5}$$

$$\sigma = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 56 & 0 \\ 0 & 0 & 28 \end{bmatrix} MPa$$

$$\varepsilon = \begin{bmatrix} -4.92\times10^{-4} & 0 & 0 \\ 0 & 6.36\times10^{-4} & 0 \\ 0 & 0 & 7.2\times10^{-5} \end{bmatrix}$$