Cole Takasugi

1. This assumes 1-D in x, (constant y, z) steady state and constant thermal conductivity

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+Q=0$$

$$k\frac{\partial T}{\partial x}=-Qx+C_1$$

using $\partial T/\partial x\,(x_0)=\partial T/\partial x\,(0)=0$

$$0=C_1$$

$$\Rightarrow \frac{\partial T}{\partial x}=\frac{-Qx}{k}$$

$$T=\frac{Qx^2}{k}+C_2$$

using $T(x_1)=T_1$ and $x_1=\bar{X}$

$$T(\bar{X})=T_1=\frac{-Q\bar{X}}{k}+C_2$$

$$\Rightarrow C_2=T_1+\frac{Q\bar{X}}{k}$$

$$\Rightarrow T(x)=\cancel{\frac{Q(x-\bar{X})}{k}+T_1}$$

$$\frac{Q(\bar{X}-x)}{k}+T_1$$

2 $k_f = 0.5 \frac{W}{cmK}$ $K_{gap} = 25 \frac{W}{mK}$

$K_{clad} = 15 \frac{W}{mK}$ $K_{coat} = 5 \frac{W}{mK}$

$h_{cool} = 5.5 \frac{W}{cm^2K}$ $T_{cool} = 800K$

$Q = 400 W/cm^3$

$$T_{coat} - T_{cool} = \frac{QR_{fuel}}{2h_{cool}}$$

$$= \frac{(400\ W/cm^3)(0.6\ cm)}{2(5.5\ W/cm^2K)} = 21.82\ K$$

$$T_{clad} - T_{coat} = \frac{Q}{2K_{coat}} R_{fuel}\, t_{coat}$$

$$= \frac{(400\frac{W}{cm^3})}{2(0.05\frac{W}{cmK})}(0.6\,cm)\,(0.01\,cm) = 24K$$

$$T_{gap} - T_{clad} = \frac{Q\, t_{clad}}{2K_{clad}} R_{fuel}$$

$$= \frac{(400\frac{W}{cm^3})(0.05\,cm)}{2(0.15\frac{W}{cmK})}(0.6\,cm) = 40K$$

$$T_{fuel} - T_{gap} = \frac{Q}{2h_{gap}} R_{fuel} = \frac{Q\, t_{gap}}{2k_{gap}} R_{fuel}$$

$$= \frac{(400\frac{W}{cm^3})}{2(0.25\frac{W}{cmK})}(0.2\,cm)(0.6\,cm) = 96K$$

$$T_0 - T_{fuel} = \frac{Q}{4K} R_f^2$$

$$= \frac{(400\frac{W}{cm^3})}{4(0.5\frac{W}{cmK})}(0.6\,cm)^2 = 72K$$

$$T_0 = T_{cool} + \delta T_{cool} + \delta T_{clad} + \delta T_{gap} + \delta T_{fuel} + \delta T_{coat}$$

$$\simeq 800 + 21.82 + 24 + 40 + 96 + 72 = 1053.8K$$

at $r = 0.4$ cm

the final term is instead

$$T(0.4) - T_{fuel} = \frac{Q}{4k}(R-r)^2$$

$$= \frac{\left(400\frac{W}{cm^3}\right)(0.6-0.4)^2}{4\left(0.5\frac{W}{cmK}\right)} = 8\text{ K}$$

so $T(0.4) = 989.8$ K

3) $\sigma_f = 570b$ 19.5% enrich

$14.5 \frac{W}{mK}$ $\rho = 15.67 g/cm$

U_3Si_2

a $\phi = 2\times10^{12}$ $n/cm^2\text{-}s$

$Q = E_f N_F \phi \sigma_f$

$m_u =$

$0.805(238) + 0.195(235)$

$= 237.415$ amu

$m_{Si} = 28$

$m_{U_3Si_2} = 3(237.415) + 2(28) = 768.245$ amu

$$N_f = \frac{15.67g}{cm^3} \frac{1 mol}{768.245g} \frac{6.022\times10^{23}}{1 mol} \frac{3U}{U_3Si_2}(0.195)$$

$= 7.18565\times10^{21}$ fuel atoms/cm³

$$Q = (200\times10^6 eV)(1.602\times10^{-19} J/eV)$$

$$\left(2\times10^{12}\frac{n}{cm^2\text{-}s}\right)\left(7.18565\times10^{21}\frac{U235}{cm^3}\right)\left(570\times10^{-24}cm^2\right) = 262.46 \; W/cm^3$$

3'b) need same N_F

$$7.185605 \times 10^{21} \frac{U235}{cm^3} = \frac{6.022 \times 10^{23}}{mol} \frac{10.97\,g}{cm^3} \frac{1U}{UO_2} X_\varepsilon / \left(\left(X_\varepsilon 235 + (1-X_\varepsilon) 238 + 2(16) \right) g/mol \right)$$

$\rightarrow X_\varepsilon = 0.29273$

or 29.273% enriched

4a)

~~150 W/cm~~

$$LHR = 150\ \frac{W}{cm} \cos\left(\frac{\pi}{2 \cdot 1.1}\left(\frac{1.8\,m}{1.5\,m} - 1\right)\right)$$

$$= 143.924\ W/cm$$

b)

$$\frac{dT}{dz} = \frac{LHR}{\dot{m} C_p}$$

so larger temp change is for lower $\dot{m} C_p$

$$4200 \cdot 0.22 = 924$$
$$1404 \cdot 0.12 = 168.48$$

so sodium has largest dT

$y(t_0) = 6$

$t_0 = 1$ $\quad dy/dt = 4t - 3t^2$

5 forward

$$f(t_{0.33}) = f(t_0) + dt \frac{df}{dt_n}$$

$$y(1.33) = 1 + 0.33\left[4(1.33) - 3(1.33)^2\right]$$

$$= 1.33$$

$$y(1.66) = 1.33 + 0.33\left[4(1.33) - 3(1.33)^2\right]$$

$$= 1.334$$

$$y(2) = 1.334 + 0.33\left[4(1.66) - 3(1.66)^2\right]$$

$$= 0.79$$

backward

$$y(1.33) = 1 + 0.33\left[4(1.33) - 3(1.33)^2\right]$$

$$= 1.004$$

$$y(1.66) = 1.004 + .33\left(4(1.66) - 3(1.66)^2\right)$$

$$= 0.467$$

$$y(2) = 0.467 + .33 \ldots$$

$$= -0.853$$

6 fissionable can undergo fission by capture of a high energy neutron, fissile can sustain a fission chain reaction for neutrons of any energy. Fertile cannot undergo fission but can be converted to a fissile nuclide by absorption of neutrons and resulting nuclide conversions

7 - Pure U metal swells dramatically during thermal cycling

- alpha phase has anisotropic expansion and irradiation growth

8 Smear density is the fraction of internal volume of clad filled by fuel. It is important when considering fissile material density, swelling, and thermal effects that result

9 Without enrichment, the density of fissile material is too low to sustain fission in LWRs.
UF_6 gas is used for enrichment
Centrifuges spin the gas, heavier U_{238} will be pushed to the outside and lighter U_{235} to the inside. This is due to centrifugal force, the gas is then drawn from the centrifuge

10 two primary fission
product species
include Mo $A = 96$
Cs $A = 133$

which are largely
produced by fission and
are near peak values
This is approximate regardless
of fissioning material

11 Finite difference,
finite element,
and finite volume

finite element is used because finite volume cannot solve for stress and finite difference does not allow the nonuniformity necessary for discrete pellets and core heterogeneity