NE 591 Exam 1

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1. This equation is made by the assumption "steady state". $\left(\frac{\partial T}{\partial t} = 0 \right)$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + Q = 0$$

$$\Rightarrow \quad k\frac{\partial T}{\partial x} = -Qx + C_1 \quad \text{——} ①$$

use boundary condition $T'(x=x_0=0) = 0$ into ①

$$0 = 0 + C_1$$

$$\therefore C_1 = 0$$

Therefore, $k\frac{\partial T}{\partial x} = -Qx$

$$\Rightarrow \int \frac{\partial T}{\partial x}\,dx = -\frac{Q}{k}\int x\,dx$$

$$\Rightarrow \int dT = -\frac{Q}{k}\frac{1}{2}x^2 + C_2$$

$$\Rightarrow \quad T = -\frac{Q}{k}\frac{1}{2}x^2 + C_2 \quad \text{———} ②$$

use another B.C. $T(x=x_1) = T_1$ into ②

$$T(x=x_1=X) = -\frac{Q}{k}\frac{1}{2}X^2 + C_2 = T_1$$

$$\therefore C_2 = T_1 + \frac{Q}{k}\frac{1}{2}X^2$$ Insert this into ② again

$$\Rightarrow \quad T(x) = -\frac{Q}{k}\frac{1}{2}x^2 + T_1 + \frac{Q}{k}\frac{1}{2}X^2$$

$$T(x) = \frac{Q}{2k}\left(X^2 - x^2\right) + T_1$$

2.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial T}{\partial r}\right)+Q=0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=-Qr^2$$

$$r^2\frac{\partial T}{\partial r}=\int -Qr^2\,dr \Rightarrow \frac{-Q}{3}r^3+C_1$$

$T(r_1)=T_1$

$T'(r_0)=0,\ r_0=0,\ r_1=R$

$0=0+C_1$

$$r^2\frac{\partial T}{\partial r}=-\frac{Q}{3}r^3 \rightarrow \frac{\partial T}{\partial r}=-\frac{Q}{3}r$$

$$T(r)=\int -\frac{Q}{3}r\,dr \rightarrow \frac{-Q}{6}r^2+C_2$$

$r_1=R,\ T(r)=T_1$

$$T(R)=T_1=-\frac{QR^2}{6}+C_2 \Rightarrow C_2=T_1+\frac{QR^2}{6}$$

$$T(r)=\frac{-Q}{6}r^2+T_1+\frac{QR^2}{6}$$

$$T(r)-T_1=\frac{Q}{6k}(R^2-r^2) \rightarrow \text{spherical 1-D steady state}$$

3.

```
(*heat generation rate by U3Si2*)

In[1]:= sigma = 570 (*barns*);
d = 15.67 (*g/cm^3*);
en = 19.5 (*%*);
phi = 2 * 10^12 (*n/cm^2-s*);
Ef = 200 (*MeV*);
M = 3 * (en * 0.01 * 235 + (1 - en * 0.01) * 238) + 2 * 28;
Nf = (d * 6.02 * 10^23 / M) * 3 * en * 0.01;
Q = Ef * (10^6 * 1.6 * 10^(-19)) * Nf * phi * (sigma * 10^-24)

Out[1]= 262.045
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(*heat generation rate by UO2*)

In[4]:= sigma = 587 (*barns*);
d = 10.97 (*g/cm^3*);
phi = 2 * 10^12 (*n/cm^2-s*);
Ef = 200 (*MeV*);
M = (x * 0.01 * 235 + (1 - x * 0.01) * 238) + 2 * 16;
Nf = (d * 6.02 * 10^23 / M) * x * 0.01;
Solve[262.045 == Ef * (10^6 * 1.6 * 10^(-19)) * Nf * phi * (sigma * 10^-24), x]

Out[4]= {{x → 28.4279}}
```

4.

```
In[ ]:= (*Calculate LHR considering variability in Z*)

In[13]:= Z = 180 (*cm*);
         Z0 = 150 (*cm*);
         L0 = 150 (*W/cm*);
         r = 1.1;
         LHR = L0 * Cos[Pi / (2 r) (Z / Z0 - 1)]

Out[13]= 143.924

         (*Calculate Tcool for water considering variablity in Z*)

         Z = 300 (*cm*);
         Z0 = 150 (*cm*);
         L0 = 150 (*W/cm*);
         r = 1.1;
         m = 0.22 (*kg/s*);
         Cp = 4200 (*J/kg-K*);
         delta T = 2 * r / Pi (Z0 * L0 / (m Cp)) (Sin[Pi / (2 r)] + Sin[Pi / (2 r) (Z / Z0 - 1)])

Out[17]= 33.7575

         (*Calculate Tcool for sodium considering variablity in Z*)

In[19]:= Z = 300 (*cm*);
         Z0 = 150 (*cm*);
         L0 = 150 (*W/cm*);
         r = 1.1;
         m = 0.12 (*kg/s*);
         Cp = 1404 (*J/kg-K*);
         delta T = 2 * r / Pi (Z0 * L0 / (m Cp)) (Sin[Pi / (2 r)] + Sin[Pi / (2 r) (Z / Z0 - 1)])

Out[19]= 185.137
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5.

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(*Backward Euler*)

In[11]:= dt = 0.33;
n = 6;
Do[n = n + dt * (4 (k + dt) - 3 (k + dt) ^2);
 Print[n], {k, 1, 2 - dt, dt}]

6.00439

5.46755

4.17385

(*Forward Euler*)

In[10]:= dt = 0.33; n = 6; Do[n = n + dt * (4 k - 3 k^2); Print[n], {k, 1, 2 - dt, dt}]

6.33

6.33439

5.79755
```

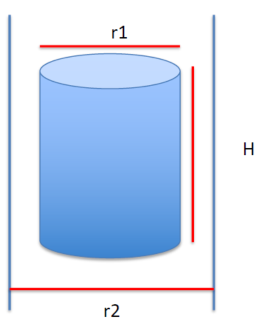
6.

1) A fertile material is a material that, although not itself fissionable, can be converted into a fissile nuclide by neutron absorption and subsequent nuclei conversions.
2) A fissile nuclide is capable of sustaining a nuclear fission chain reaction with neutrons of any energy.
3) A fissionable nuclide is capable of undergoing fission (even with a low probability) after capturing a high energy neutron.

7.

We don't use pure metallic U as a fuel form because pure uranium dramatically swells during thermal cycling.
In addition, pure uranium has three phases, including α-phase is orthorhombic, β-phase is tetragonal, ϒ-phase is body-centered cubic. Abd alpha U has both anisotropic thermal expansion and anisotropic irradiation growth.

8.
Smear density is the ratio of fuel volume to internal volume of the fuel element.



$$\text{Smear density} = \pi r_1^2 h / \pi r_2^2 h$$

From: Professor Beeler's notes

It is important for metal fuel designs to have a low enough smear density to properly allow for fuel swelling over the course of fuel life.

9. Because water has higher neutron absorption. UF6 and UO2. Use different mass to separate U-238 and U235.

10. One broad peak centered around A=95, the other around A=135.

11. Back Euler and Forward Euler.