David Henry
DMH5708

79/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

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Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

$\delta_{U_3Si_5} = 7.5 \frac{g}{cm^3}$

$k = 12.5 \ W/mK$

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

$N = 2(Z) - (43 \pm 2) = (2)(92) - (43 \pm 2) = 139, \cancel{140}, 141, \cancel{142}, 143$

$\Rightarrow 231, 23\cancel{3}, 235$

Naturally occurs @ 0.7%

$\boxed{\text{U-235 is the Fissile isotope in } U_3Si_5 \text{ \& Naturally occurs @ an enrichment of } 0.7\%}$

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$\gamma = 0.03$

$\phi_{th} = 3.2 \times 10^{13}$

$$Q = E_F N_F \sigma_F \phi_{th} \rightarrow \cancel{E_F} N_{U_3Si_5} \cancel{\sigma_{235}} \cancel{\phi_{th}} = \cancel{E_F} N_{U_3Si_5} \cancel{\sigma_{235}} \cancel{\phi_{th}} \Rightarrow N_{U_3Si_5} = N_{U_3Si_5} \ldots (1)$$

$$N_F = \frac{\gamma N_a \delta}{M} \Rightarrow \frac{\gamma_{U_3Si_5} \cancel{N_a} \delta_{U_3Si_5}}{M_{U_3Si_5}} = \frac{\gamma_{U_3Si_2} \cancel{N_a} \delta_{U_3Si_2}}{M_{U_3Si_2}} \Rightarrow \gamma_{U_3Si_5} = \frac{(854 \frac{g}{mol})(0.03)(12.2 \frac{g}{cm^3})}{(7.5 \frac{g}{cm^3})(770 \frac{g}{mol})}$$

$\Rightarrow M_{U_3Si_5} = (3)(238) + (5)(28) = 854 \frac{g}{mol}$

$\Rightarrow M_{U_3Si_2} = (3)(238) + (2)(28) = 770 \frac{g}{mol}$

$\delta_{U_3Si_2} = 12.2 \ g/cm^3$

(From Lect. 3)

$\rightarrow \gamma_{U_3Si_5} = 0.0541$

$\boxed{\text{enrichment of } 5.41\%}$

-3, Just use density of U

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

U_3Si_5 would be Ranked lower b/c:

- Have to be enriched more ∴ Costs more
- Thermal conductivity is less than U_3Si_2
  - $\hookrightarrow$ 0.125 W/cm·K < 0.23 W/cm·K

(From table in Lect. 2)

David Henry
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$h_{cool} = 2.5\ W/cm^2 \cdot K$
$R_F = 4.5mm = 0.45cm$
$\delta_{gap} = 80\mu m = 80\times10^{-4} cm$
$\delta_c = 0.6mm = 0.06cm$
$LHR = 250\ W/cm$
$T_{cool} = 580K$
$Y = 0.05$

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{co} = \frac{LHR}{2\pi R_F h_{cool}} + T_{cool} = \frac{250}{(2\pi)(0.45)(2.5)} + 580 = 615.368\ K$$

$$T_{ci} = \frac{\delta_c LHR}{2\pi R_F k_c} + T_{co} = \frac{(0.06)(250)}{(2\pi)(0.45)(0.17)} + 615.368 = 646.575K$$

$k_c = 0.17\ \frac{W}{cmK}$ (from table)

$$T_S = \frac{LHR}{2\pi R_F h_{gap}} + T_{ci} = \frac{250}{(2\pi)(0.45)(0.278)} + 646.575 \Rightarrow \boxed{T_S = 964.36K}$$

$T_I = 2$

-1, 0.0227

$$h_{gap} = \frac{k_{gap}}{\delta_{gap}} = \frac{\boxed{0.0223}}{80\times10^{-4}} = 0.278\ W/cm^2 \cdot K$$

$$\Rightarrow k_{gap} = k_{He}^{1-Y} k_{Xe}^{Y} = (0.0026)^{0.95}(1.16\times10^{-4})^{0.05} = 0.00223$$

$$\Rightarrow k_{Xe} = (0.7\times10^{-6})T_{ci}^{0.79}$$

0.0026 (From Lect. 6) $= (0.7\times10^{-6})(646.575)^{0.79} = 1.16\times10^{-4}\ W/cm \cdot K$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

UN

$E = 246.7\ GPa$
$\nu = 0.25$
$\alpha = 7.5\times10^{-6}\ \frac{1}{K}$

hoop stress is greatest $\therefore\ \sigma_{\theta\theta} = -\sigma^*(1-3\eta^2) \Rightarrow \eta = \frac{r}{R_F}$ max @ $\eta = 1$ so $r = R_F$

$$\Rightarrow \sigma^* = \frac{\alpha E(T_o - T_s)}{4(1-\nu)}$$

$$\Rightarrow \sigma^* = \frac{(7.5\times10^{-6})(246.7)(1163.31-964.36)}{(4)(1-0.25)}$$

$$\Rightarrow \sigma^* = 0.122699\ GPa$$

$$\Rightarrow T_o = \frac{LHR}{\boxed{2\pi k_{UN}}} + T_s = \frac{(250)}{(2\pi)(0.2)} + 964.36$$

-2, Should be 4 pi k

$k_{UN} = 0.2$ (Table Lect. 2)

$$\Rightarrow T_o = 1163.31K$$

$$\Rightarrow \sigma_{\theta\theta} = -(0.123)\underbrace{(1-3(1)^2)}_{-2} = 0.245397\ GPa$$

$$\Rightarrow \boxed{\sigma_{\theta\theta} = 245.4\ MPa}$$

c) Would you expect this stress to be higher or lower if the pellet was UO₂? Why? (5 points)

-5, lower thermal conductivity means higher Delta T and higher stress

I would expect this stress to be lower if the pellet was UO_2.

- the stress is lower b/c the thermal conductivity of UO_2 is much smaller
- Smaller Thermal conductivity means smaller temp. gradient & smaller stresses

d) What assumptions were made in your calculations for a) and b)? (5 points)

Assumptions for (a):

1) Steady State Solutions
2) behavior is axisymmetric
3) T is constant in the z-direction
4) Thermal conductivity is independent of T

Assumptions for b):

1) Small Strains
2) Isotropic material Response

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Henry
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Question 3: -10, 25/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

1) Stress is constant through the wall of the cylinder
2) Impact of gravity is negligable
3) Static body
4) Problem is axisymmetric

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$P = 6$ MPa
$R = 5.6\text{mm} = 0.56\text{cm}$
$\delta_c = 0.6\text{mm} = 0.06\text{cm}$

$$\bar{\sigma}_\theta = \frac{PR}{\delta} = \frac{(6)(0.56)}{(0.06)} \Rightarrow \boxed{\bar{\sigma}_\theta = 56\text{ MPa}}$$

$$\bar{\sigma}_z = \frac{PR}{2\delta} = \frac{\bar{\sigma}_\theta}{2} = \frac{56}{2} \Rightarrow \boxed{\bar{\sigma}_z = 28\text{ MPa}}$$

$$\bar{\sigma}_r = -\tfrac{1}{2}P = -(\tfrac{1}{2})(6) \Rightarrow \boxed{\bar{\sigma}_r = -3\text{ MPa}}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

Avg $R = 5.6$mm
$\delta_c = 0.6$mm
$\therefore R_o = 0.59\text{cm}$ & $R_i = 0.53\text{cm}$
hoop stress largest @ $r = 0.53\text{cm}$

$$\sigma_{\theta\theta}(r) = P\frac{(R_o/r)^2+1}{(R_o/R_i)^2-1} = (6)\left[\frac{(0.59/0.53)^2+1}{(0.59/0.53)^2-1}\right] = (6)\left(\frac{2.239}{0.239}\right) = 56.161\text{ MPa}$$

-4, Calculate stress at two radii and compare

$$\left(\frac{|56.161-56|}{56.161}\right)(100) = 0.286\%$$ so thin wall approximation is accurate

The Thin wall approximation would not be conservative b/c the actual value is greater than the approximation.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$E = 70$ GPa
$\nu = 0.41$

$$\epsilon_{rr} = \frac{2r}{5} = \frac{2(0.56)}{5} = 0.224$$
$$\epsilon_{\theta\theta} = \frac{r}{5} = \frac{0.56}{5} = 0.112$$
$$\boxed{\epsilon = \begin{bmatrix} 0.224 & 0 \\ 0 & 0.112 \end{bmatrix}}$$

-2, Strain and strain missing zz component
-4, Calculate strain from stresses from part b

$$C_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{(70)(1-0.41)}{(1.41)(1-0.82)} = 162.727$$

$$C_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{(70)(0.41)}{(1.41)(1-0.82)} = 113.081$$

$$\Rightarrow \sigma_{rr} = (0.224)(162.727) + (0.112)(113.081) = 49.116\text{ GPa}$$

$$\Rightarrow \sigma_{\theta\theta} = (0.112)(162.727) + (0.224)(113.081) = 43.556\text{ GPa}$$

$$\Rightarrow \boxed{\sigma = \begin{bmatrix} 49.116 & 0 \\ 0 & 43.556 \end{bmatrix}}$$