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Exam 1
NE 591 (010)

1) $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + Q = 0$

$\Rightarrow \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -Q \Rightarrow k\frac{\partial T}{\partial x} = -Qx + c_1$

at $x = 0 \Rightarrow \quad 0 = +c_1$

$\frac{\partial T}{\partial x} = -\frac{Q}{k}x \Rightarrow T(x) = -\frac{Q}{2}x^2 + c_2$

at $x_1 = X \Rightarrow T_1 = -\frac{Q}{2}X^2 + c_2 \Rightarrow c_2 = T_1 + \frac{Q}{2}X^2$

$T(x) = T_1 + \frac{Q}{2}X^2 - \frac{Q}{2}x^2 = \underline{T_1 + \frac{Q}{2}(X^2 - x^2)}$

The assumption used are:
- Steady state
- Constant in z and y
- Constant thermal conductivity.

2) $LHR = Q\pi R_1^2 = 400 \times \pi \times 0.6^2 = 452 \frac{W}{cm}$

~~$T_{r_0} = T_{cool} =$~~

$$T_{r_4} - T_{cool} = \frac{LHR}{2\pi R_1 h_{cool}} = \frac{452}{2\times\pi\times 0.6\times 5.5} = 21.8\ K$$

$$\underline{T_{r_4}} = T_{cool} + 21.8 = 800 + 21.8 = \underline{821.8\ K}$$

$$T_{r_3} - T_{r_4} = \frac{LHR}{2\pi R_1 h_{coating}} = \frac{452}{2\pi\times 0.6} \times \frac{r_4 - r_3}{\underset{\uparrow\, h_{coating}}{0.05}} = \frac{452}{2\pi\times 0.6}\ \frac{0.86 - 0.85}{0.05}$$

$$= 2\cancel{3}4.0\ K$$

$$\underline{T_{r_3}} = T_{r_4} + 24.0\ K = 821.8 + 24 = \underline{845.8\ K}$$

$$T_{r_2} - T_{r_3} = \frac{LHR}{2\pi R_1 h_{clad}} = \frac{LHR}{2\pi R_1} \times \frac{r_3 - r_2}{k_{clad}}$$

$$= \frac{452}{2\pi\times 0.6} \times \frac{0.85 - 0.8}{0.15}$$

$$= \cancel{37}.40.0\ K$$

$$\underline{T_{r_2}} = T_{r_3} + 40.0 = 845.8 + 40.0 = \underline{885.8\ K}$$

$$T_{r_1} - T_{r_2} = \frac{LHR}{2\pi R_1 h_{gap}} = \frac{LHR}{2\pi R_1} \times \frac{r_2 - r_1}{k_{gap}}$$

$$= \frac{452}{2\pi\times 0.6} \times \frac{0.8 - 0.6}{0.25} = 95.9\ K$$

$\underline{T_{r_4}} = T_{r_3} + 95.9\ K = 885.8 + 95.9 = \underline{981.7}\ K$

$\underset{\substack{\uparrow \\ \text{centerline}}}{T_0} - T_{r_4} = \frac{LHR}{4\pi k_f} = \frac{436}{4\times\pi\times0.5} = 71.9\ K$

$\underline{T_0} = T_{r_4} + 71.9 = 981.7 + 71.9 = \underline{1053.6}\ K$

↑
centerline temperature

$T(r=0.4) = \frac{Q}{4k}(R_f^2 - r^2) + T_{r_4}$

$= \frac{400}{4\times0.5}(0.6^2 - 0.4^2) + 981.7$

$\underline{T(r=0.4) = 1021.7\ K}$

3) a) $Q = E_f \times N_f \times \sigma_f \times \phi$

$E_f = 200\ MeV$ $\qquad \phi = 2\times10^{13}\ \frac{n}{cm^2 \cdot s}$

$\sigma_f = 570\times10^{-24}\ cm^2$

~~Nfiss At mass U = 0.195×235 =~~

mass U : $0.195\times235 + (1-0.195)\times238 = 237.4$ amu

$M_{U_3Si_2} = 3\times237.4 + 2\times28 = 768.2$ amu density ↓ avogadro number ↓ $\frac{3U}{U_3Si_2}$ ↓

$$N_f = 15.67 \times \frac{1}{768.2} \times 6.02\times10^{23} \times 3 \times 0.195$$

↑ $M_{U_3Si_2}$

$$= 7.18 \times 10^{21} \ \frac{U^{235}}{cm^3}$$

$$Q = 200\times10^{6} \times 1.602\times10^{-19} \times 7.18\times10^{21} \times 570\times10^{-24} \times 2\times10^{12}$$

$$\underline{Q} = \cancel{256}\ 262.1 \ \frac{W}{cm^3}$$

b) x = enrichment

mass U : $235 \times x + (1-x)\,238$.

$M_{UO_2} = 235x + (1-x)\,238 + 2\times16$

$= 270 - 3x$

$$Q = 200\times10^{6} \times 1.602\times10^{-19} \times N_f \times 570\times10^{-24} \times 2\times10^{12}$$

$$= 3.65\times10^{-20} \times N_f$$

$$N_f = \frac{262.1}{3.65\times10^{-20}} = 7.18\times10^{21} \ \frac{U^{235}}{cm^3}$$

$$7.18 \times 10^{24} = 10.97 \times \frac{1}{270.03} \times 6.02 \times 10^{23} \times 1 \times x$$

$$1.94 \times 10^{24} - 2.15 \times 10^{22} x = 6.60 \times 10^{24} x$$

$x = 0.294$

It would require an enrichment of ~~29.4~~ 20.4%

4) a) $LHR\left(\frac{z}{z_0}\right) = LHR^0 \cos\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right)$

$z_0 = 1.5\,m$

$\gamma = 1.1$

At $z = 1.8\,m$

$$LHR = 150 \times \cos\left(\frac{\pi}{2 \times 1.1}\left(\frac{1.8}{1.5} - 1\right)\right)$$

~~LHR~~ $\underline{LHR = 143.9}\ \frac{W}{cm}$ at $z = 1.8\,m$.

b) $$T_{cool}^{out} - T_{cool}^{in} = \frac{1}{\frac{\pi}{2\gamma}} \times \frac{z_0 \times LHR_0}{\dot{m}\, c_{pw}}\left(\sin\left(\frac{\pi}{2\gamma}\right) - \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right)\right)$$

For coolant i): water

$$T_{cool}^{out} - T_{cool}^{in} = \frac{1}{\frac{\pi}{2 \times 1.1}} \times \frac{1.5 \times 150}{0.22 \times 4200} \times \left(\sin\left(\frac{\pi}{2 \times 1.1}\right) - \sin\left(\frac{\pi}{2 \times 1.1} \times \left(\frac{3}{1.5} - 1\right)\right)\right)$$

$= 0.338\ K/rod$

sodium

For coolant ii)

$$T_{cool}^{out} - T_{cool}^{in} = \frac{1}{\frac{\pi}{2 \times 1.1}} \times \frac{1.5 \times 150}{0.22 \times 1406} \times \left(2 \sin\left(\frac{\pi}{2 \times 1.1}\right)\right) = 1.85\ K/rod$$

The coolant with sodium has the largest change in temperature.

6)

<u>Forward</u>:

$dt = 0.33$

$t_0 = 1$

~~y~~ $\underline{t_1 = 1.33}$

$$y(t_1) = y(t_0) + dt \left.\frac{\partial y}{\partial t}\right]_{t_0} = 6 + 0.33 \times (4 \times 1 - 3 \times 1^2)$$

$$\underline{y(t_1) = 6.33}$$

$\underline{t_2 = 1.66}$

$$y(t_2) = y(t_1) + dt \left.\frac{\partial y}{\partial t}\right]_{t_1} = 6.33 + 0.33 \times (4 \times 1.33 - 3 \times 1.33^2)$$

$$\underline{y(t_2) = 6.3344}$$

$t_3 = 2$

$$y(t_3) = y(t_2) + dt \left.\frac{\partial y}{\partial t}\right]_{t_2} = \cancel{6.3344 + 0.33 \times (4 \times 2 - 3 \times 2^2)}$$

$\cancel{y(t_3) = 5.0}$

$$y(t_3) = 6.3344 + 0.33 \times (4 \times 1.66 - 3 \times 1.66^2)$$

$$\underline{y(t_3) = 5.7976}$$

Backward:

$t_1 = 1.33$

$$y(t_1) = y(t_0) + dt \left.\frac{\partial y}{\partial t}\right]_{y\,t_1}$$

$$= 6 + 0.33 \times (4 \times 1.33 - 3 \times 1.33^2)$$

$y(t_1) = 6.0044$

$t_2 = 1.66$

$$y(t_2) = y(t_1) + dt \left.\frac{\partial y}{\partial t}\right]_{t_2}$$

$$= \cancel{6+0}\ 6.0044 + 0.33 \times (4 \times 1.66 - 3 \times 1.66^2)$$

$y(t_2) = 5.4676$

$t_3 = 2$

$$y(t_3) = y(t_2) + dt \left.\frac{\partial y}{\partial t}\right]_{t_3}$$

$$= 5.4676 + 0.33 \times (4 \times 2 - 3 \times 2^2)$$

$y(t_3) = 4.1476$.

6) fissionable: a nuclide capable to undergo fission with thermal or fast neutron

fissile: a nuclide capable to undergo fission with thermal neutron

fertile: a nuclide not fissionable but capable of ~~bei~~ being converted into a fissionable nuclide.

7) - Pure uranium has swells dramatically during thermal cycle
- One of the phase (α phase) has anisotropic thermal expansion and anisotropic irradiation growth.

8) Smear density: ratio of the volume occupy by the fuel element to the total ~~available~~ available volume inside the cladding.
It is important because all fuel ~~de~~ material ~~swell~~ swell and so need more space in order to not increase the stress on the cladding.

9) We enriched uranium because ^{235}U natural enrichment is 0.7%.

During enrichment, the following compounds are used:
- U_3O_8
- UF_6
- UO_2

Centrifuge based enrichment:
gaseous UF6 is injected at the center of the centrifuge. Because $^{238}UF_6$ is heavier than $^{235}UF_6$, it ~~travels~~ velo faster toward the edge of the centrifudge leaving a higher enrichment close to the center.

10) $^{96}_{42}Mo$ and $^{137}_{55}Cs$

The fission ~~yield~~ product yield ~~are~~ has 2 peaks centered at A=95 and A=135

11) There are:

- finite difference
- finite volume
- finite element.

Finite element is used by state of the art software because it can be used for any geometry, BCs, and can determine the stress.