Asymptotic expansion homogenization for multiscale nuclear fuel analysis

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Introduction

- Experiments on irradiated materials provide essential data for predicting the behaviour in general.
- However, the experiments are difficult and expensive to perform due to radioactivity.
- With computational experiments, number of physical experiments can be reduced.
- Asymptotic expansion homogenization (AEH) can couple macro- and microscale together by passing homogenized properties to the engineering scale.

Homogenization

- A method of studying partial differential equations with rapidly varying coefficients.
- Concerned with averages of solutions of equations to be able to obtain macroscopic equations for systems from fine microscopic structure.
- Tries to represent rapidly varying medium by slowly varying medium so that fine-scale structure is averaged out in some way.
- Resembles heat conduction equation.

Homogenization

$$\frac{\partial}{\partial x_i} a_{ij}^{H} \frac{\partial u^{(0)}}{\partial x_j} = f$$

- $u^{(0)}$ is the expression for macroscopic solution.
- Homogenized properties are represented by α_{ij}^{H}.
- χ is the macroscale information.
- f is the source density.

Standard benchmark problem results

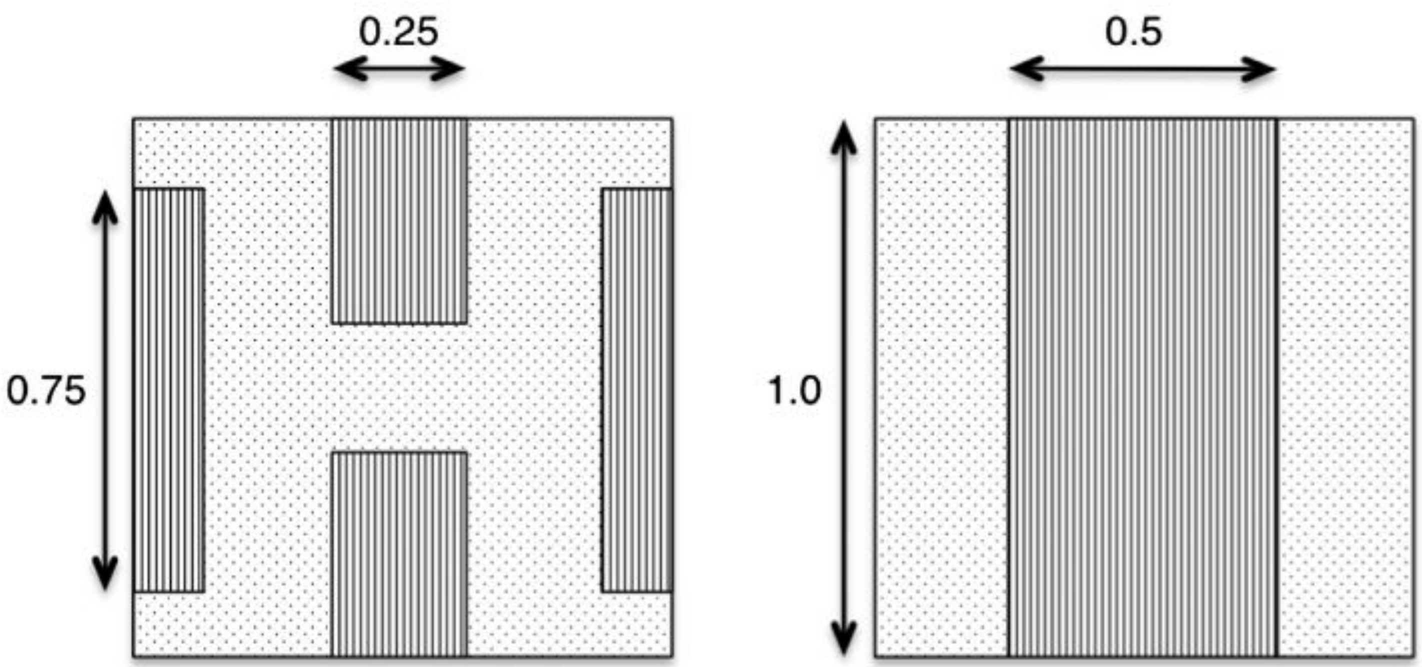


Fig. 1. Short fiber (left) and long fiber (right) geometries. Dark areas are fibers, light areas are matrix. Figure reproduced from [4].

Standard benchmark problem results (cont.)

Table 1
Homogenized elasticity constants for the short fiber problem calculated by BISON/MARMOT, shown with results from [4] in the second and third columns for comparison. BISON/MARMOT results are very close to the HOMO2D and Fish and Wagiman solutions.

	BISON/MARMOT	HOMO2D	Fish and Wagiman
E1111 (GPa)	122.457	122.4	122.457
E2222 (GPa)	151.351	151.2	151.351
E1212 (GPa)	42.112	42.10	42.112
E1122 (GPa)	36.191	36.23	36.191

Standard benchmark problem results (cont.)

Table 2
Homogenized elasticity constants for the long fiber problem calculated by BISON/MARMOT, shown with results from [4] in the second and third columns for comparison. BISON/MARMOT results are very close to the HOMO2D and Fish and Wagiman solutions.

	BISON/MARMOT	HOMO2D	Fish and Wagiman
E1111 (GPa)	136.138	136.1	136.147
E2222 (GPa)	245.810	245.8	245.81
E1212 (GPa)	46.8498	46.85	46.85
E1122 (GPa)	36.076	36.08	36.076

Mesh Convergence

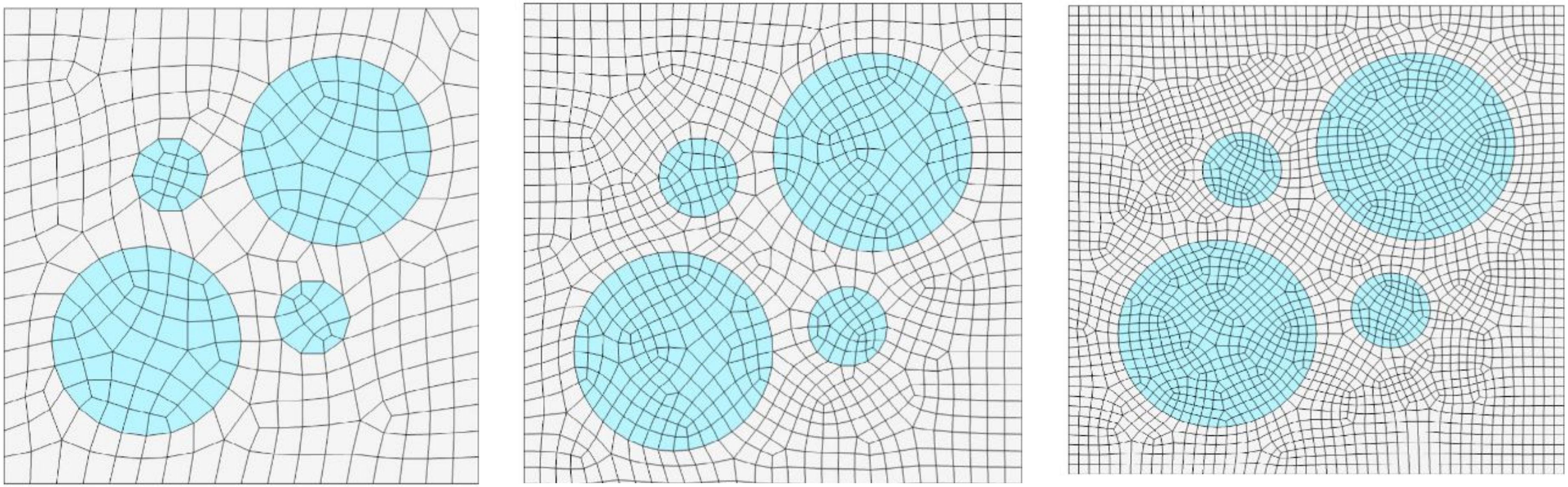


Fig. 2. Lower scale meshes used in the mesh convergence study with 398, 1092, and 2697 nodes.

Mesh Convergence (cont.)

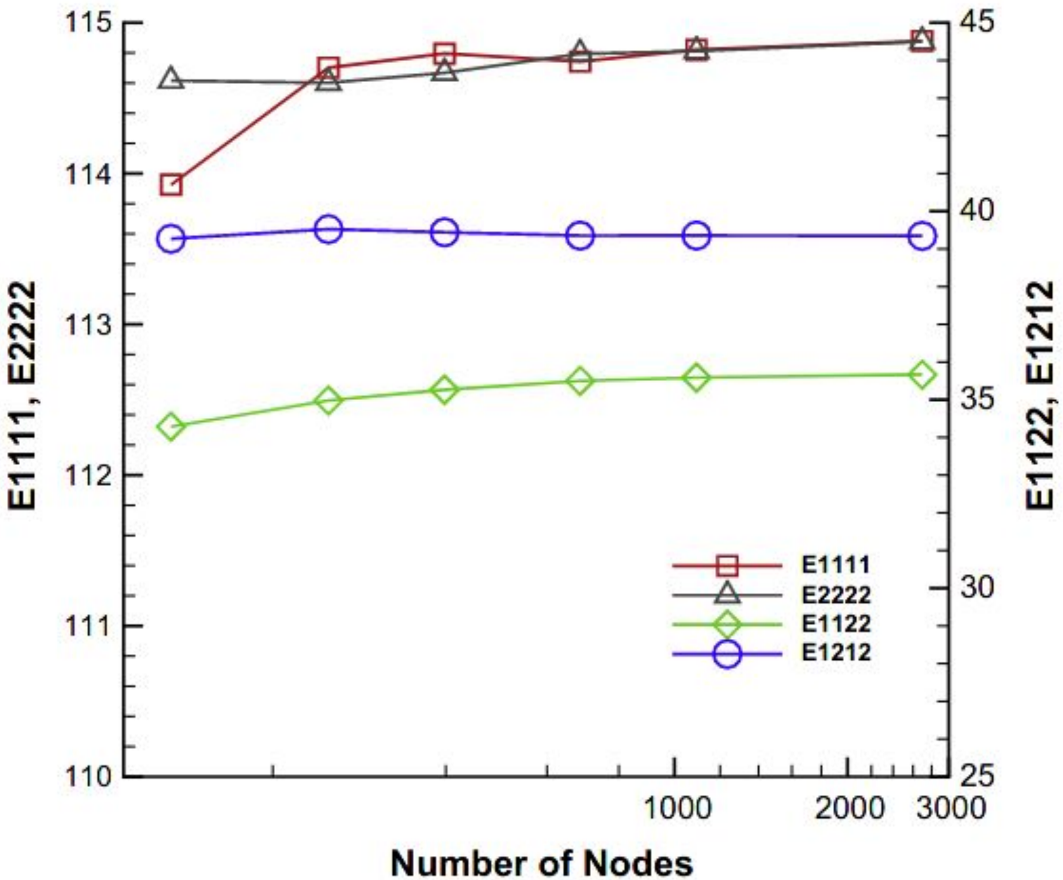
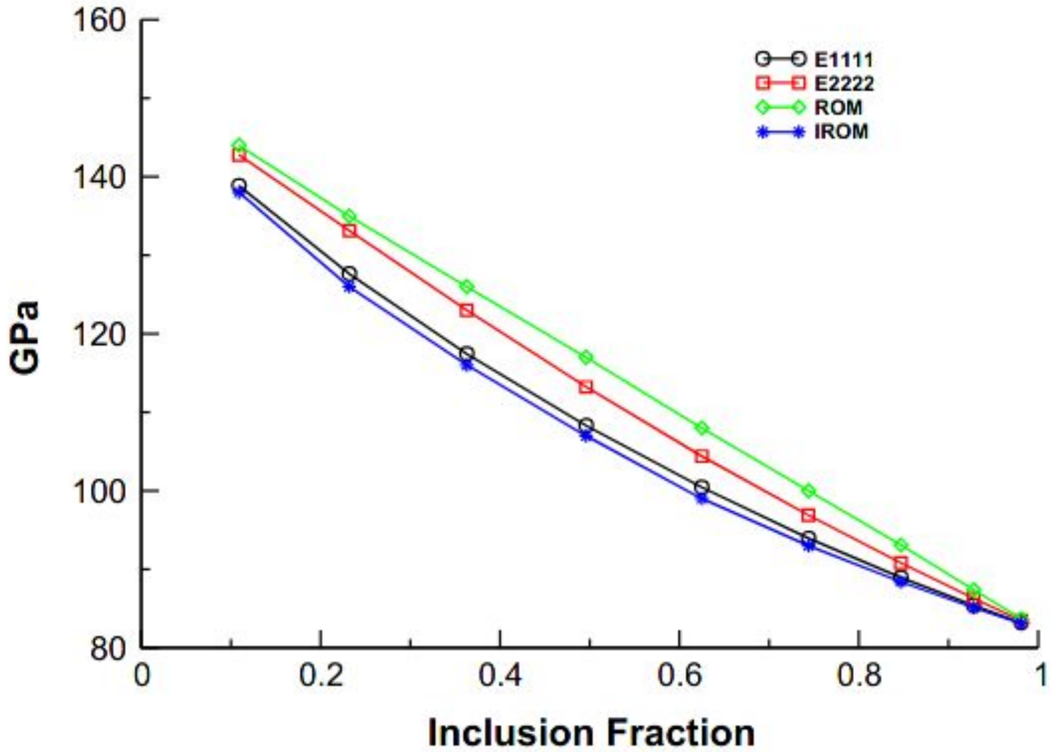


Fig. 3. Elasticity constants with mesh refinement. The constants vary little when the mes contains more than ~1000 nodes.

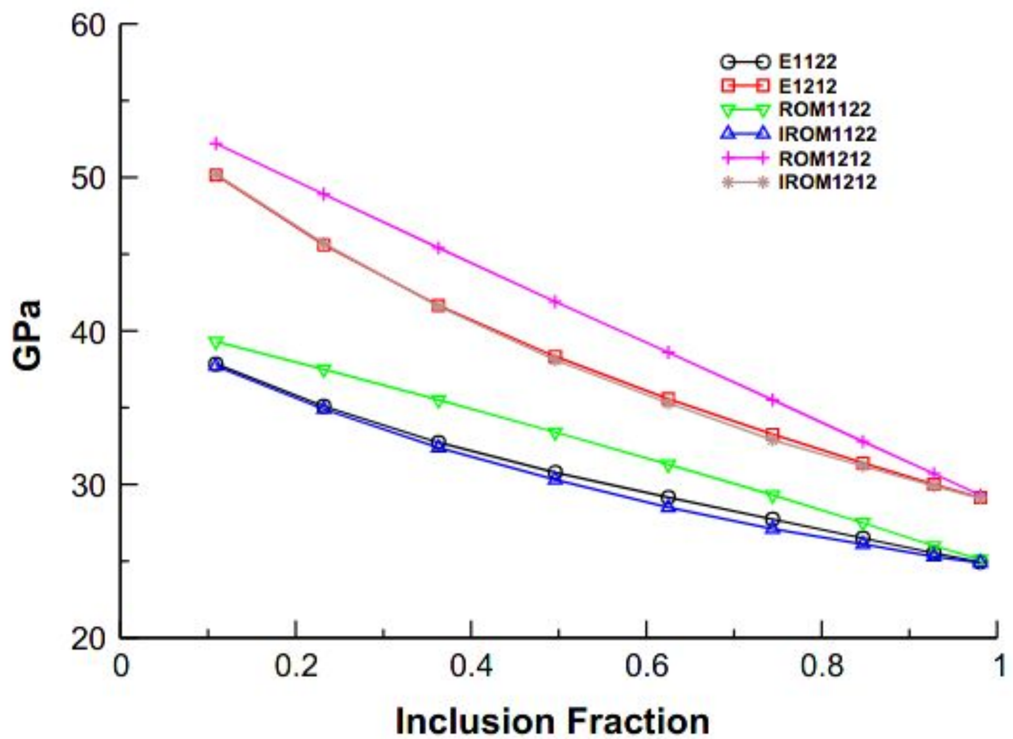
Application to nuclear fuel simulations



(a)

Fig. 4. In (a) and (b), AEH, rule of mixtures (ROM), and inverse rule of mixtures (IROM) elasticity tensor values. In (c), ratios of AEH to ROM and IROM values.

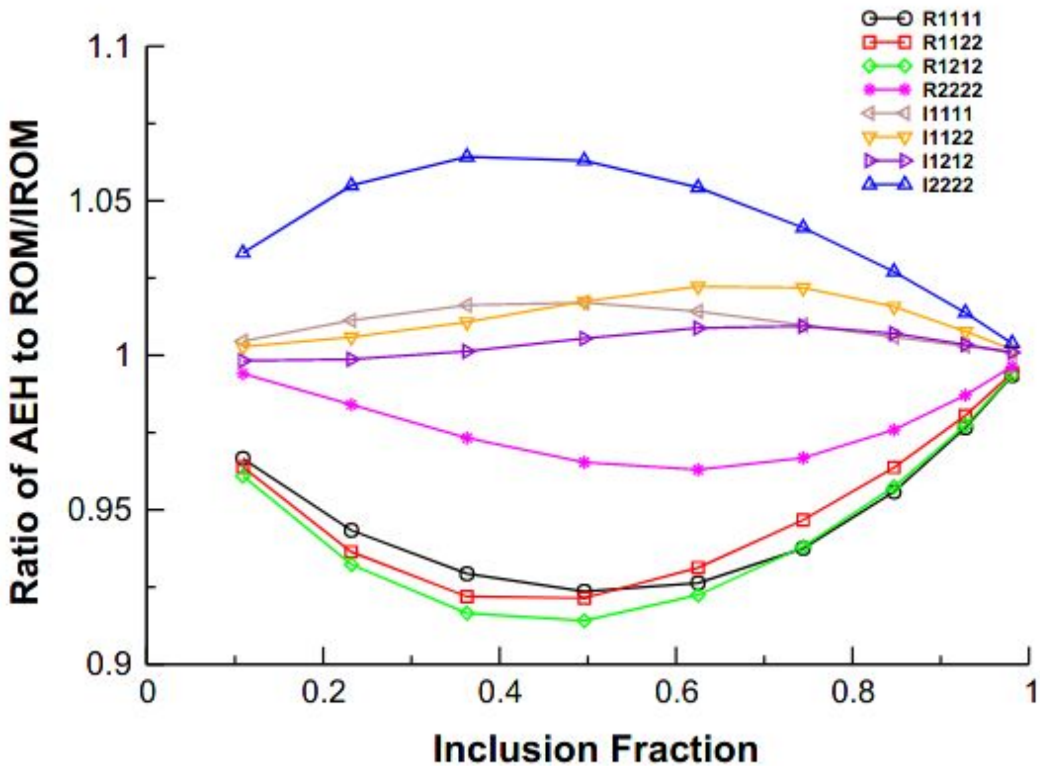
Application to nuclear fuel simulations (cont.)



(b)

Fig. 4. In (a) and (b), AEH, rule of mixtures (ROM), and inverse rule of mixtures (IROM) elasticity tensor values. In (c), ratios of AEH to ROM and IROM values.

Application to nuclear fuel simulations (cont.)



(c)

Fig. 4. In (a) and (b), AEH, rule of mixtures (ROM), and inverse rule of mixtures (IROM) elasticity tensor values. In (c), ratios of AEH to ROM and IROM values.

Application to nuclear fuel simulations (cont.)

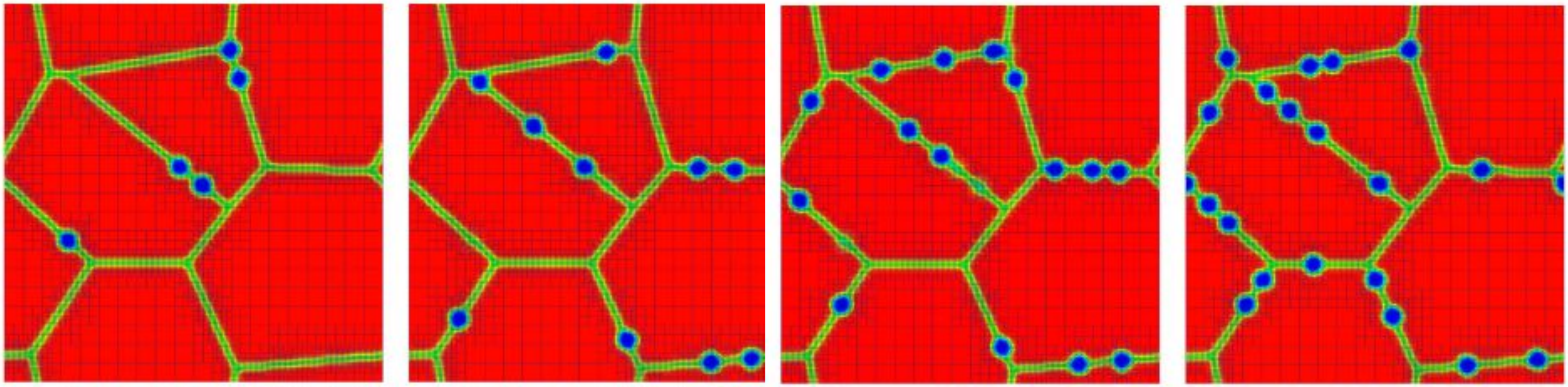


Fig. 5. Five, ten, fifteen, and twenty bubbles along the grain boundary for a 2D model in the xy plane.

Application to nuclear fuel simulations (cont.)

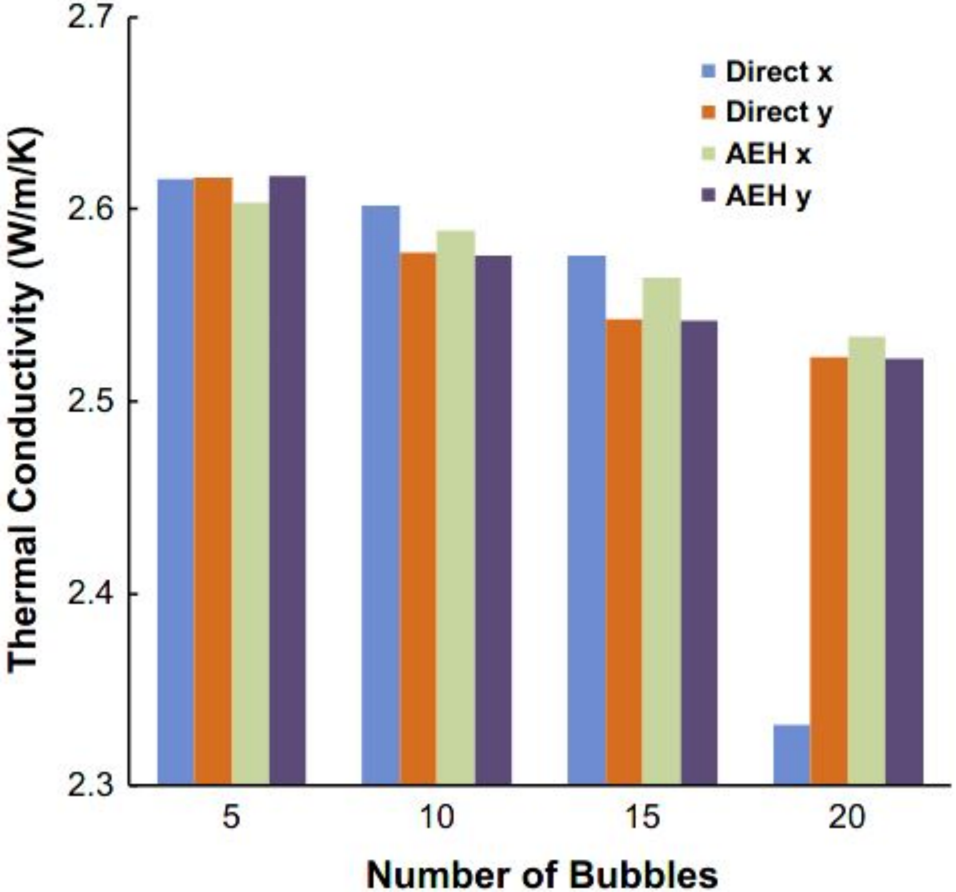


Fig. 6. Homogenized thermal conductivity for the direct and AEH methods in the x and y directions. The direct method gives a significantly lower thermal conductivity in the x direction for the case of 20 bubbles due to a bubble on the domain boundary.

Demonstration

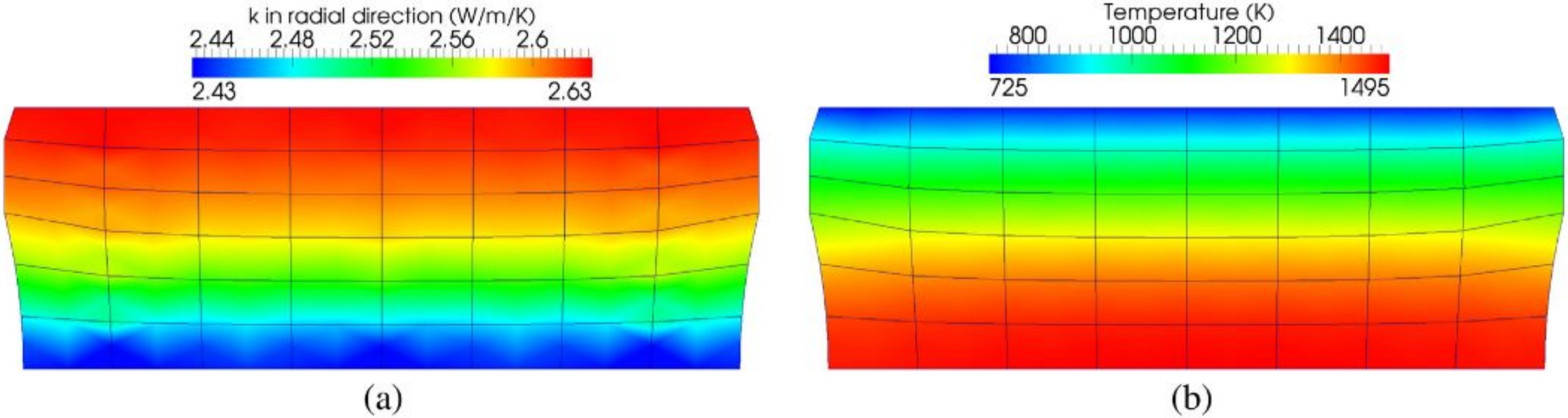


Fig. 7. Thermal conductivity in the radial direction in (a). Temperature distribution in (b).

Conclusions

- Strong proven history in other fields.
- Applicable for homogenization of thermal conductivity, elastic constants, and other quantities.
- Low computational expense.
- Solves benchmark problems accurately.
- Provides good results without overly fine meshes.
- Possible future applications include estimating mechanical response of cladding tubes and computing thermal conductivity in fuels with grain boundary gas bubbles.

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Thank you