Fuel Performance

NE 591

Last Time

- Developed analytical solutions for temperature profile
- This time, we move from the analytical into the numerical framework

- $T_0 - T_{fuel} = \frac{Q}{4k} R_{fuel}^2$ $\quad T_0 - T_{fuel} = \frac{LHR}{4\pi k}$
- $T_{fuel} - T_{gap} = \frac{Q}{2h_{gap}} R_{fuel}$ $\quad T_{fuel} - T_{gap} = \frac{LHR}{2\pi R_{fuel} h_{gap}}$ $\quad h_{gap} = \frac{k_{gap}}{t_{gap}}$
- $T_{gap} - T_{clad} = \frac{Q t_{clad}}{2k_{clad}} R_{fuel}$ $\quad T_{gap} - T_{clad} = \frac{LHR\, t_{clad}}{2\pi R_{fuel} k_{clad}}$
- $T_{clad} - T_{cool} = \frac{Q}{2h_{cool}} R_{fuel}$ $\quad T_{clad} - T_{cool} = \frac{LHR}{2\pi R_{fuel} h_{cool}}$

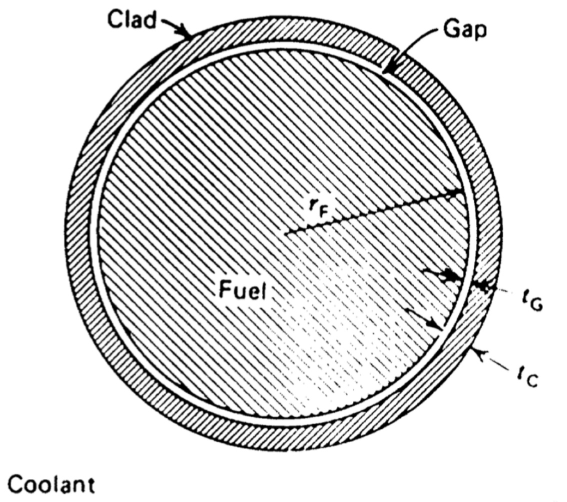
Review of Assumptions

- Analytical solution requires:
  - Steady-state solution
  - Temperature is axisymmetric
  - T is constant in Z
  - Thermal conductivity is independent of temperature
  - Temperature profile in the fuel is parabolic, linear profiles in gap, clad and coolant

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + Q(r,z)$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T}{\partial r}\right) + Q(r)$$



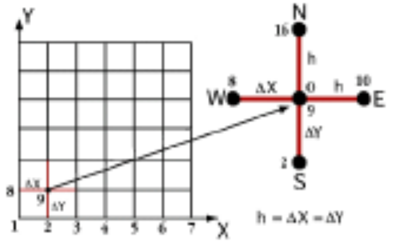
Solution of Heat Equation

- Parabolic profile of temperature in fuel
- Cosine profile of LHR as a function of z on fuel rod

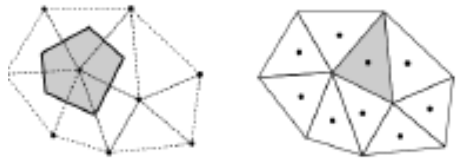
- The heat equation can be solved using numerical methods
- Numerical solution methods are needed for time derivative and gradients.
- Time derivative solution methods march through time in steps and can be
  - Explicit: utilizes current state to make prediction about future state
  - Implicit: utilizes current state and future state to make prediction about future state

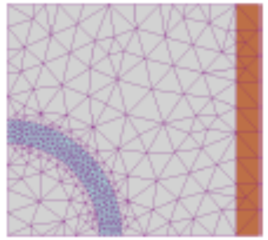
SPATIAL DISCRETIZATION

Spatial resolution



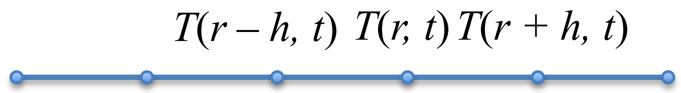
- To numerically solve in space, we need to discretize
  - Finite difference
    - convert differential equations into a system of equations that can be solved by matrix algebra techniques
  - Finite volume
    - volume integrals in a partial differential equation that contain a divergence term are converted to surface integrals, using the divergence theorem. These terms are then evaluated as fluxes at the surfaces of each finite volume
  - Finite element
    - subdivides a large system into smaller, simpler parts that are called finite elements, the equations that model these finite elements are then assembled into a larger system of equations that models the entire problem





Finite Difference

- The finite difference method solves on a grid and uses numerical derivatives
- Derivatives are approximated by differences
- Boundary conditions must have either a fixed T or dT/dr
- Typically restricted to handle rectangular shapes
- Once you compute the time derivative, you can use either forward or backward Euler to march through time



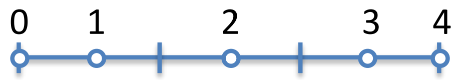
$$\dot{T}(r,T) = \frac{1}{\rho\, c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T(r,T)}{\partial r}\right) + \frac{1}{\rho\, c_p}Q(r)$$

$$q = rk(T)\frac{\partial T(r,t)}{\partial r} = \frac{rk(T(r,t))}{2h}\left(T(r+h,t) - T(r-h,t)\right)$$

$$\dot{T}(r,t) = \frac{1}{\rho c_p}\frac{1}{r}\frac{\partial q}{\partial r} + \frac{1}{\rho c_p}Q(r) = \frac{1}{\rho c_p}\frac{1}{2h\, r}\left(q(r+h,t) - q(r-h,t)\right) + \frac{1}{\rho c_p}Q(r)$$

Finite Volume

- Discretize the domain by subdomains
  - Domain size h
  - We place points in the subdomain centers and on either boundary
- The finite volume method balances fluxes across the boundaries of your divided subdomains
- Integrate our PDE across the subdomain
- Evaluate the integral using a linear approximation of the variable
- Restricted to flux boundary conditions, often used in flow-type problems



$$\frac{d}{dx}k\frac{dT}{dx} + q = 0$$

$$\int_a^{a+h} \frac{d}{dx}k\frac{dT}{dx} + q \; dx = 0$$

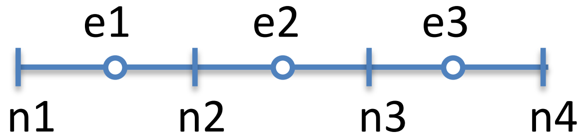
$$k\frac{dT}{dx}\bigg|_{a+h} - k\frac{dT}{dx}\bigg|_{a} + qh = 0$$

$$\frac{T_{i+1} - T_i}{h_2} - \frac{T_i - T_{i-1}}{h_1} + q\frac{h_2}{k} = 0$$

$$T_i = \frac{h_1 h_2}{h_1 + h_2}\left(\frac{T_{i+1}}{h_2} + \frac{T_{i-1}}{h_1} + q\frac{h_2}{k}\right)$$

Finite Element

- In the finite element method, we interpolate the variable using nodal values and integrate over elements
- Systematically recombine all sets of element equations into a global system of equations for the final calculation
- Write the strong form of the equation, rearrange to get zero on the right-hand side, multiply by the test function, integrate over the domain, yielding weak form
- The **strong form** states conditions that must be met at every material point, whereas **weak form** states conditions that must be met only in an average sense
- Finite element works for any geometry and any boundary condition



$$0 = \rho c_p \dot{T}(r,t) - \frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T(r,t)}{r}\right) - Q(r)$$

$$-\nabla \cdot \frac{\mathbf{K}}{\mu}\nabla p = 0 \in \Omega$$

$$(\nabla\psi, \frac{\mathbf{K}}{\mu}\nabla p) - \langle\psi, \frac{\mathbf{K}}{\mu}\nabla p \cdot \hat{n}\rangle = 0$$

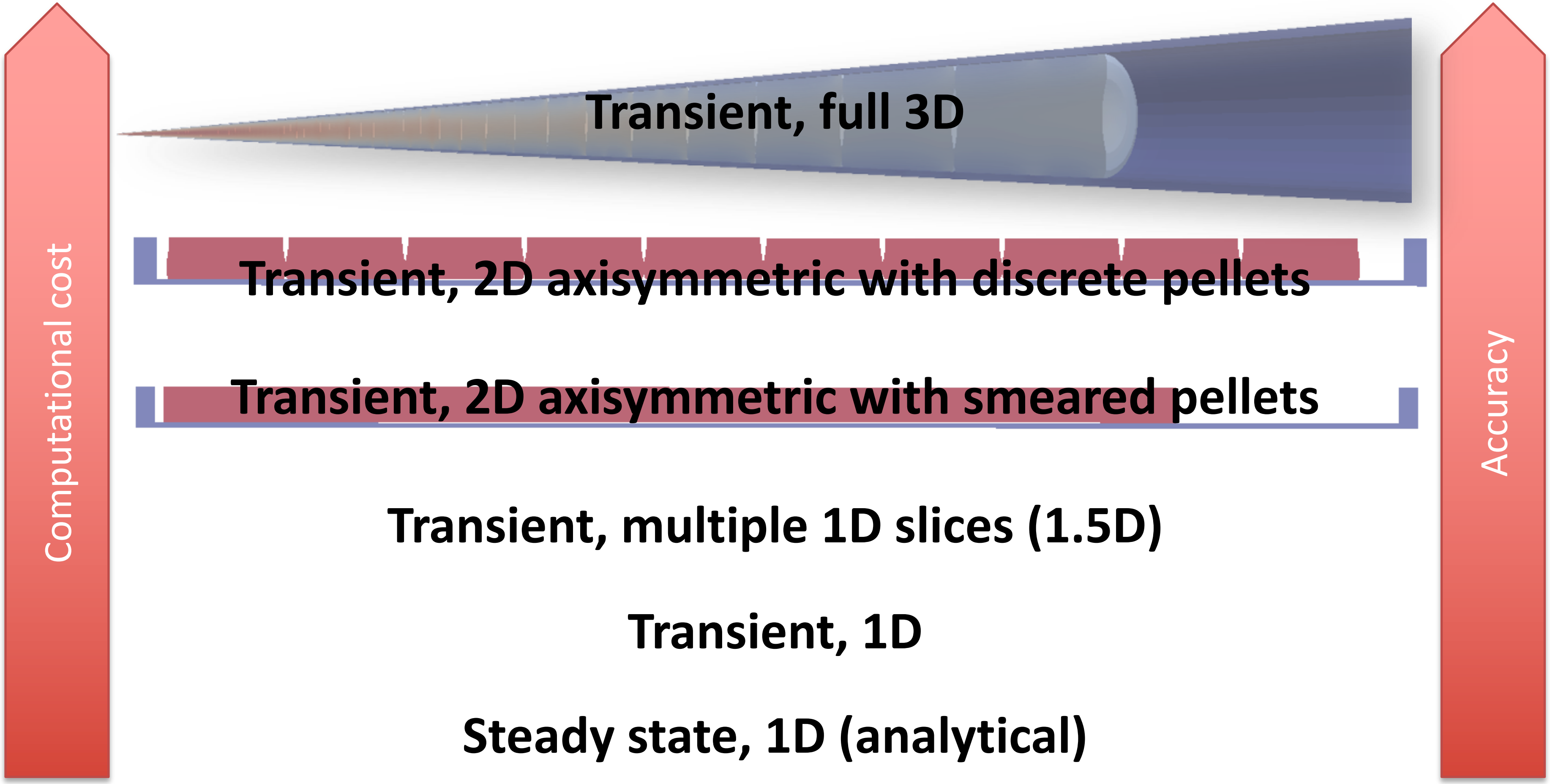
Spatial resolution

- Finite difference
  - Advantages
    - Simple
    - Easy to code
    - Fast
  - Disadvantages
    - Difficult to model complex geometries
    - Difficult to model complex BCs
    - Only represents solution at points
    - Difficult to represent heterogeneous properties

- Finite Volume
  - Advantages
    - Can model any geometry
    - Naturally conservative
    - Heterogeneous properties
  - Disadvantages
    - Boundary conditions add complexity
    - More complicated than finite difference

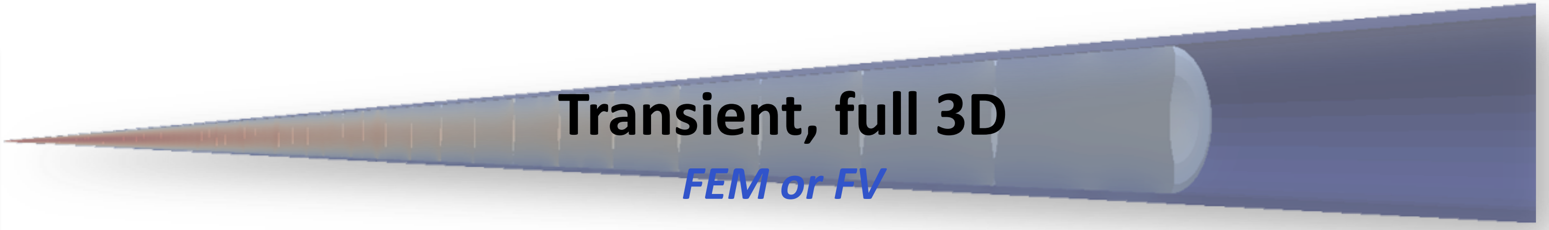
- Finite Element
  - Advantages
    - Can model any geometry
    - Can model any BC
    - Continuous representation
    - Heterogeneous properties
  - Disadvantages
    - Complicated
    - Somewhat more expensive

Different Fuel Performance Problems



Numerical Approaches to Different Fuel Performance Problems

Computational cost



Transient, full 3D

FEM or FV

Transient, 2D axisymmetric with discrete pellets

FEM or FV

Transient, 2D axisymmetric with smeared pellets

FEM, FV, or FD

Transient, multiple 1D slices (1.5D)

FEM, FV, or FD

Transient, 1D

FEM, FV, or FD

Steady state, 1D (analytical)

Accuracy

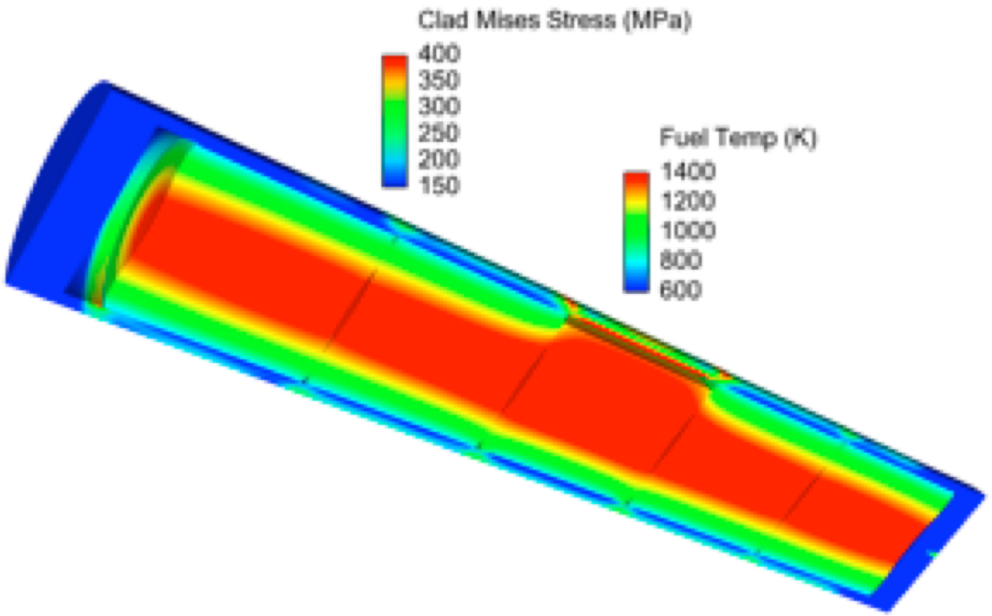
Heat equation solution approach summary

Approach	Solution	Assumptions
1D steady state	Analytical	Steady state, axisymmetric, no axial variation, constant k
1D transient	FEM, FD, FV	Axisymmetric, no axial variation
1.5D transient	FEM, FD, FV with multiple slices	Axisymmetric, no axial variation
2D transient, smeared pellets	FEM, FD, FV	Axisymmetric, fuel pellets act as one body, fuel pellets are perfect cylinders
2D transient, discrete pellets	FEM, FV	Axisymmetric
3D transient	FEM, FV	You have a big computer

Each numerical solution can be solved explicitly or implicitly

Solving with fuel performance codes

- Fuel performance codes primarily use either finite difference or finite element
- The earliest fuel performance codes solved the heat equation in 1.5D using finite difference (with multiple axial slices)
- More modern codes have switched to finite element, due to more flexibility with geometry and boundary conditions
- Finite volume is not used because it can't solve for the stress



Summary

- The heat equation can be solved using numerical methods.
- Spatial derivative solution methods divide the domain up into smaller pieces
  - Finite difference
  - Finite volume
  - Finite element
- Each discretization has strengths/weaknesses
- Finite element is primary method for high fidelity fuel performance codes

Notes

- Exam on Feb. 11
- Will cover all classes up to and including today
- Wellness day next Tuesday, Feb. 9
- Will contain both conceptual and work-through problems
- Major topics covered:
  - Fuel types
  - Heat generation
  - Reactor Systems
  - Fuel fabrication
  - Heat transfer
  - Analytical solution to heat transfer
  - Numerical solution to heat transfer
- Total of 1.5 hours to complete the exam (class period plus 15 minute buffer)
- Will be available for a problem session during office hours next week