81/100

*NucE 497 Fuel Performance Exam 1 covering modules 1 - 3*

**Question 1:** -6, 24/30

$U_3Si_5$ is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

a) What is the fissile isotope in $U_3Si_5$? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

$\boxed{U\text{-}235}$

Natural Uranium: 99.3% = U-238 $\boxed{.7\% = U\text{-}235}$

b) What enrichment would be required for $U_3Si_5$ to have the same energy release rate of $U_3Si_2$ enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that $U_{235}$ has a negligible impact on the total molar mass of U in the fuel (15 points)

$\gamma = .03$

$\phi = 3.2 \times 10^{13}$

$E_f = 3 \times 10^{-11}$ J/s

$\sigma = 550 b$

$\rho_{U_3Si_2} = 12.2 \frac{g}{cm^3}$

$A = (238)3 + (28.1)2 = 770.2$

$\rho_{U_3Si_5} = 9.1 \frac{g}{cm^3}$

$A = (238)(3) + (28.1)(5)$

$$Q(U_3Si_2)\Big|_{\gamma=.03} = Q(U_3Si_5)\Big|_{\gamma=x}$$

$$\cancel{E_f}\,\cancel{\sigma}\,N_{U235}\,\cancel{\phi} = \cancel{E_f}\,\cancel{\sigma}\,N_{U235}\,\cancel{\phi}$$

$$\frac{\rho N_a \gamma}{A} = \frac{\rho N_a \gamma}{A}$$

$$\frac{3(12.2)(6.02\times10^{23})(.03)}{770.2} = \frac{3(9.1)(6.02\times10^{23})(x)}{854.5}$$

-3, Use U density provided

$$8.58\times10^{20}\ \frac{atoms}{cm^3} = (1.92\times10^{22})(x) \longrightarrow \boxed{x = .04 \Rightarrow 4\%}$$

c) How would you rank $U_3Si_5$ as a potential fuel compared to $U_3Si_2$? Why? (8 points)

For same heat generation rate, more U-235 is needed when using $U_3Si_5$. Therefore, $U_3Si_2$ is better because you need less U-235. It is hard to judge how safe it is using Q values alone. -3, thermal conductivity?

(melting temps are unknown for this problem).

-0, 35/35

**Question 2:**

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

does this mean pellet or clad? which surface of clad? inside or outside?

$R_f = .45$ cm

$t_{gap} = .008$ cm

$t_{clad} = .06$ cm

$LHR = 250 \frac{W}{cm}$

$T_{cool} = 580$ K

$h_{cool} = 2.5 \frac{W}{cm^2 K}$

$k_{zr} = .17$

$T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250}{2\pi(.45)(2.5)} + 580 = \boxed{615.4\text{K}}$ outer surface of clad

$T_{ic} = \frac{LHR\, t_c}{2\pi R_f k_c} + T_{co} = \boxed{646.6\text{ K}}$ Inner surface of clad

$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ic}$

$h_{gap} = k_{gas}/t_{gap}$

$k_{gas} = k_{He}^{.95}\, k_{Xe}^{.05} = (16\times10^{-6}(646.6)^{.79})^{.95}(.7\times10^{-6}(646.6)^{.79})^{.05} = .002$

$\boxed{T_s = 957.9\text{k}}$ surface temp of fuel

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$k_{UN} = .2$

$T_o = \frac{LHR}{4\pi k} + T_s = \frac{250}{4\pi(.2)} + 957.9 = 1057.4$ k

$\sigma_{\theta\theta} = -\sigma^*(1-3\eta^2)$

$\sigma^* = \alpha E(T_o - T_s)/4(1-\nu) = (7.5\times10^{-6})(246.7)(1057.4-957.9)/4(1-.25) = .06$

$\sigma_{\theta\theta} = -.06(1-3(1)^2)$ ← $\eta = 1$ when $r = R_f$

$\boxed{= .12\text{ GPa}}$

max stress = hoop stress

c) Would you expect this stress to be higher or lower if the pellet was $UO_2$? Why? (5 points)

$k_{UO_2} \ll k_{UN}$

$\alpha_{UO_2} > \alpha_N$

$\boxed{\sigma_{UO_2} > \sigma_{UN}}$

d) What assumptions were made in your calculations for a) and b)? (5 points)

constant k

constant gap size

constant fuel radius

steady state

constant LHR

-13, 22/35

**Question 3:**
Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

$F_{pressure} = F_{stress}$

All stresses act parallel to vessel

wall thickness → 0

-3, Stress is constant though the thickness

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$P = 6\,MPa$
$R = 5.6\,mm$
$\delta = .6\,mm$

$$\sigma_\theta = \frac{PR}{\delta} = \frac{(6\,MPa)(5.6\,mm)}{.6\,mm} = 56\,MPa$$

$$\sigma_z = \frac{PR}{2\delta} = \frac{(6\,MPa)(5.6\,mm)}{2(.6\,mm)} = 28\,MPa$$

$$\sigma_r = \frac{-P}{2} = \frac{-6\,MPa}{2} = -3\,MPa$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

hoop stress is max stress so I compare only hoop stresses.

$$\sigma_{\theta\theta} = \frac{P\left(\left(\frac{R_o}{r}\right)^2 + 1\right)}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

Max when $r = R_i$

-1, Mistake in calculate
-4, Calculate stress at two radii to see if it is constant

$$\sigma_{\theta\theta} = \frac{P\left(\left(\frac{R_o}{R_i}\right)^2 + 1\right)}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$$\sigma_{\theta\theta} = \frac{6\,MPa\left(\left(\frac{5.9}{5.3}\right)^2 + 1\right)}{\left(\frac{5.9}{5.3}\right)^2 - 1}$$

$\sigma_{\theta\theta} = 35.26\,MPa$

$$\%\,error = \frac{56 - 35.26}{35.26} \times 100 = \boxed{58\%}$$

thin walled is conservative in that it predicts a larger stress.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\begin{bmatrix} \sigma_r \\ \sigma_\theta \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 56 \end{bmatrix} = \frac{70}{(1+.41)(1-.82)} \begin{bmatrix} 1-.41 & .41 \\ .41 & 1-.41 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

-2, Stress and strain are missing zz component
-3, Calculate strain from stress from part b

$$-3 = 275.81\,(.59)(u_{r,r}) + 275.81\,(.41)(u_r/r)$$

$$56 = 275.81\,(.41)(u_{r,r}) + 275.81\,(.59)(u_r/r)$$

$$\sigma = \begin{bmatrix} -3 & 0 \\ 0 & 56 \end{bmatrix} \qquad \varepsilon = \begin{bmatrix} -.49 & 0 \\ 0 & .69 \end{bmatrix}$$