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3/3/17

86/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-3, 27/30

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

U^{235}, Natural Uranium is 0.7% U^{235}

↑ fissile isotope

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that U235 has a negligible impact on the total molar mass of U in the fuel (15 points)

For U_3Si_2: • Assume ρ_u for U_3Si_2 is $\boxed{= 11.31\, g/cm^3}$

$\boxed{M_u = w_{35}(M_{35}) + (1-w_{35})(M_{38})} = 0.03(235) + (1-0.03)(238) = \boxed{237.91\, g/mol}$

(*) ⇒ $\boxed{Q = E_f N_f^{35} \sigma_f^{35} \varphi_{th}} = \left(3\times10^{-11} \frac{J}{fission}\right) \frac{(0.03)(11.31 \frac{g}{cm^3})(6.022\times10^{23} \frac{atom}{mol})}{237.91\, g/mol} (5.5\times10^{-22} cm^2)(3.2\times10^{13} \frac{n}{cm^2 s})$

$\boxed{N_f^{35} = \frac{(w/o)\rho_u N_A}{M_u}}$

$\boxed{Q = 453.47\, W/cm^3}$

$\boxed{\text{Assume: } \sigma_f^{35} = 5.5\times10^{-22} cm^2}$

For U_3Si_5: Assume $M_u \approx 237.91\, g/mol$

⇒ rearrange Q Equation:

$(w/o) = Q(M_u) / [(E_f)(\rho_u^{U_3Si_5})(\sigma_f^{35})(\varphi_{th})(N_A)]$

$= \frac{(453.47)(237.91)}{(3E-11)(7.5)(5.5E-22)(3.2E+13)(6.022E23)} = 0.045$ ⇒ ANSWER: $\boxed{4.5\% \text{ enrichment}}$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

-3, thermal conductivity?

NOT as feasible because it requires higher enrichment for same energy output because it has less ρ_u.

• higher enrichment not preffered due to cost of fabrication (enrichment) + proliferation (weapons) concern.

-4, 31/35

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

SEE ATTACHED p. Q2 for WORK

957.91K

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

SEE ATTACHED p. Q2 for WORK

−0.0613 GPa

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points) $UO2$ $\alpha = 1.10E-5\ K^{-1}$

VS. UN $\alpha = 7.5E-6\ K^{-1}$

-2, Lower thermal conductivity and thus higher DT is a much larger effect

- σ^* for UO_2 would be larger
- $\Rightarrow \sigma_{\theta\theta} = -\sigma^*$ (at $r=0$)

$\Rightarrow \sigma_{\theta\theta}$ MORE NEG

$\Rightarrow$ MORE COMPRESSIVE STRESS.

$\Rightarrow$ this would be a HIGHER COMPRESSIVE STRESS (MORE NEGATIVE) FOR UO_2

UO_2 has a LARGER Thermal expansion coeff.

d) What assumptions were made in your calculations for a) and b)? (5 points)

FOR b:
- axisymmetric body (small strains)
- cylinder w/ thermal expansion but no pressure
- Static, Gravity negligible, Isotropic material response

For a: • Assume Zirc clad $k_{clad} = 0.17$ W/cmK, UN $k_{fuel} = 0.2$ W/cmK

* Steady State, Axisymmetric behavior, Temp constant in z direction (axial), Thermal conductivity independent of Temp.

Question 2:

A) $R_f = 0.45\,cm$
$t_{gap} = 0.008\,cm$
$t_{clad} = 0.06\,cm$
$LHR = 250\,W/cm$
$T_{cool} = 580\,K$ $h_{cool} = 2.5\,W/cm^2K$
Gap: 5% Xe

Assume Zirc cladding:
$K_{clad} = 0.17\,W/cmK$

OUTER CLAD TEMP: $\boxed{T_{co}} = \dfrac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \dfrac{250}{2\pi(0.45)(2.5)} + 580 = \boxed{615.37\,K}$

INNER CLAD TEMP: $T_{ci} = \dfrac{LHR \cdot t_{clad}}{2\pi R_f \cdot k_{clad}} + T_{co} = \dfrac{250(0.06)}{2\pi(0.45)(0.17)} + 615.37 = \boxed{646.58\,K}$

GAP Properties:
$k_{He} = (16E-6)(T_{ci}^{0.79}) = 0.0027\,W/cmK$
$k_{xe} = (0.7E-6)(T_{ci}^{0.79}) = 1.16E-4\,W/cmK$
$y = 0.05$ $k_{gap} = k_{He}^{1-y} k_{xe}^{y} = 0.00227\,W/cmK$
$\boxed{h_{gap}} = \dfrac{k_{gap}}{t_{gap}} = \boxed{0.284\,W/cm^2K}$

ANSWER:
FUEL SURF TEMP: $T_{fs} = \dfrac{LHR}{2\pi R_f h_{gap}} + T_{ci} = \dfrac{250}{2\pi(0.45)(0.284)} + 646.58 = \boxed{957.91K}$

B) UN   Max stress @ center of pellet + hoop stress
$E = 246.7\,GPa$
$\nu = 0.25$
$\alpha = 7.5E-6\,K^{-1}$
for UN: $k_{fuel} = 0.2\,W/cmK$

$\Rightarrow \eta = \frac{r}{R_f} \Rightarrow r = 0$ for max stress $\Rightarrow \underline{\eta = 0}$

$$\sigma^* = \frac{\alpha E (T_{CL} - T_{FS})}{4(1-\nu)} = \frac{(7.5E-6\,1/K)(246.7\,GPa)(1057.38 - 957.91)K}{4(1-0.25)}$$

$\boxed{T_{CL}} = \dfrac{LHR}{4\pi k_{fuel}} + T_{fs} = \dfrac{250}{4\pi(0.2)} + 957.91 = \boxed{1057.38K}$ $\boxed{\sigma^* = 0.0613\,GPa}$

$\Rightarrow \boxed{\sigma_{\theta\theta}} = -\sigma^*(1 - 3\cancel{\eta^2}^{\,0}) = \boxed{-0.0613\ GPa} \Rightarrow \boxed{-61.3\,MPa}$

-2, Max when eta = 1

Question 3:

-7, 28/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

- Static Body
- Gravity is Negligible
- Axisymmetric
- Isotropic Material Response

-3, constant stress across radius

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

SEE p.Q3 for Work

$\bar{\sigma}_\theta = 56 MPa$

$\bar{\sigma}_z = 28 MPa$

$\bar{\sigma}_r = -3 MPa$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

SEE p. Q3 for WORK

using thick walled approx, max stress (hoop @ inner radius)

$\sigma_{\theta\theta} = 53.16 MPa$

$\sigma_{rr} = -6 MPa$

$\sigma_{zz} = 23.58 MPa$

-4, Calculate stress and multiple radii to check approximation

$\Rightarrow$ Since the thin wall approx estimates larger $\sigma_{\theta\theta}$ it is more conservative.

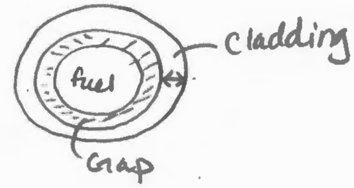
d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

SEE p. Q3

Question 3

B) $P = 6\,MPa$ avg. radius: $R = 5.6\,mm$

t_{clad}: $\delta_u = 0.6\,mm$



$$\boxed{\bar{\sigma}_\theta} = \frac{PR}{\delta} = \frac{6\,MPa\,(5.6\,mm)}{0.6\,mm} = \boxed{56\,MPa}$$

$$\boxed{\bar{\sigma}_z} = \frac{PR}{2\delta} = \frac{6\,MPa\,(5.6\,mm)}{2\,(0.6\,mm)} = \boxed{28\,MPa}$$

$$\boxed{\bar{\sigma}_r} = -\frac{1}{2}P = -\frac{1}{2}(6\,MPa) = \boxed{-3\,MPa}$$

c) USING Thick Walled Approx:

$R_o = R_{outer} = 5.6\,mm$, $R_i = R_{inner} = R_o - \delta = 5\,mm$, $\delta = 0.6\,mm$

Max stress is hoop on inner radius:

$$\boxed{\sigma_{\theta\theta}(r = R_i)} = P\,\frac{(R_o/R_i)^2 + 1}{(R_o/R_i)^2 - 1} = (6\,MPa)\,\frac{(5.6/5)^2 + 1}{(5.6/5)^2 - 1} = \boxed{53.16\,MPa}$$

$$\boxed{\sigma_{rr}(r = R_i)} = -P\,\frac{(R_o/R_i)^2 - 1}{(R_o/R_i)^2 - 1} = \boxed{-6\,MPa}$$

$$\boxed{\sigma_{zz}(r = R_i)} = P\,\frac{1}{(R_o/R_i)^2 - 1} = \boxed{23.58\,MPa}$$

d)

$$\varepsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right) = \frac{1}{70{,}000\,MPa}\left(-3 - 0.41(56 + 28)\right)$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right) =$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right) =$$

use values from B

$E = 70\,GPa = 70000\,MPa$

$\nu = 0.41$

$$\boxed{\begin{aligned} \varepsilon_{rr} &= -5.34\,E{-4} \\ \varepsilon_{\theta\theta} &= 6.54\,E{-4} \\ \varepsilon_{zz} &= 8.95\,E{-5} \end{aligned}}$$

$$\Rightarrow \varepsilon = \begin{bmatrix} \varepsilon_{rr} & 0 & 0 \\ 0 & \varepsilon_{\theta\theta} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

values from d

values from B

Assume no shear.