Nuclear Fuel Performance

NE 591-010 Spring 2021

Beeler

Date: 2/4/21

Backward Euler Time Integration (Implicit)

$$\frac{df}{dt} = 2e^{-2t} \qquad f(t_0 = 0) = 8 \qquad dt = 0.25$$

$$dt = 1 \qquad \to dt = 1$$

$$f(t_{n+1}) = f(t_n) + dt \, \frac{df}{dt_{n+1}}$$

$dt = 0.25$

$t_1 = 0.25 \qquad f(t_1) = 8 + 0.25\left(2e^{-2(0.25)}\right) = 8.30$

$t_2 = 0.5 \quad f(t_2) = 8.3 + 0.25(2e^{-2(0.5)}) = 8.49$

$t_3 = 0.75 \quad f(t_3) = 8.49 + 0.25(2e^{-2(0.75)}) = 8.6$

$t_4 = 1 \quad f(t_4) = 8.6 + 0.25(2e^{-2(1)}) = \boxed{8.67}$

$dt = 1 \quad t_1 = 1 \quad f(t_1) = 8 + 1(2e^{-2(1)}) = \boxed{8.27}$

Forward Euler Integration (Explicit)

$$\frac{df}{dt} = 2e^{-2t} \qquad dt = 0.25 \qquad \to t = 0.5 \qquad f(t_0 = 0) = 8$$

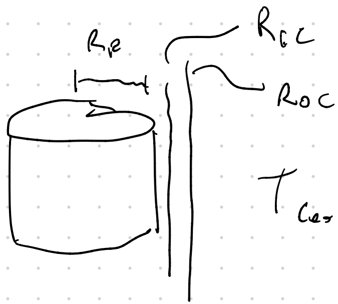
$$f(t_{n+1}) = f(t_n) + dt \frac{df}{dt}_n$$

$$t_1 = 0.25 \quad f(t_1) = 8 + 0.25\left(2e^{-2(0)}\right) = 8.5$$

$$t_2 = 0.5 \quad f(t_2) = 8.5 + 0.25\left(2e^{-2(0.25)}\right) = \boxed{8.8}$$

$$\frac{dy}{dt} = y\cos(t)$$

Solve for Centerline temperature



$T_{cool} = 625\ K$

$Q = 550\ \frac{W}{cm^3}$

$K_{fuel} = 20\ \frac{W}{m\text{-}K}$

$K_{clad} = 15\ \frac{W}{m\text{-}K}$

$R_F = 0.4\ cm$

$t_g = 0.01\ cm$

$t_c = 0.08\ cm$

$K_{gap} = 1\ \frac{W}{m\text{-}K}$

$h_{cool} = 2.5\ \frac{W}{cm\text{-}K}$

$$Q \rightarrow LHR = Q \pi R_F^2 \rightarrow (550)(\pi)(0.4^2) = 276\ W/cm$$

$$T_{co} - T_{cool} = \frac{LHR}{2\pi R_F} \frac{1}{h_{cool}} = \frac{276}{2\pi(0.4)} \frac{1}{2.5} = 44\ K$$

$$T_{co} = 625 + 44 = 669 \text{ K}$$

$$T_{c1} - T_{co} = \frac{LHR}{2\pi R_F} \frac{1}{h_{clad}} \left[\left(\frac{t_{clad}}{k_{clad}}\right)\right] \rightarrow \frac{276}{2\pi(0.4)} \left[\frac{\overbrace{0.08}^{\overbrace{t_{clad}}^{h_{clad}}}}{\underbrace{0.15}_{k_{clad}}}\right] = 58.6 \text{ K}$$

$$T_{c1} = 669 + 58.6 = 727.6 \text{ K}$$

$$T_F - T_{c1} = \frac{LHR}{2\pi R_F} \frac{1}{h_{gap}} \rightarrow \frac{276}{2\pi(0.4)} \frac{0.01}{0.01} = 109.8 \text{ K}$$

$$T_F = 727.6 + 109.8 = 837.4 \text{ K}$$

$$T_o - T_F = \frac{LHR}{4\pi K_F} = \frac{276}{4\pi(0.2)} = 109.8\ K$$

$$T_o = 837.4 + 109.8 = \boxed{947.2\ K}$$

$$T(r) = \frac{Q(R^2 - r^2)}{4K} + T_F$$

Heat Generation Rate

U_3Si_5

$\sigma = 520$ barns

5% enrichment

$\rho = 8.97$ g/cc

$\phi = 3\times10^{13}\ n/cm^2\text{-}s$

$$Q = E_f N_f \phi \sigma_f$$

mass U = $0.05 \times 235 + 0.95 \times 238 = 237.85$ amu

mass Si = 28

mass $(U_3Si_5) = 3(237.85) + 5(28) = 853.55$ amu

$$N_f \Rightarrow 8.97\ \frac{g}{cc}\ \frac{1\,mol}{853.55\,g}\ \frac{6.022\times10^{23}\,at}{1\,mol}\ \frac{3\,U}{1\,U_3Si_5} \times 0.05$$

$$N_f = 9.5\times10^{20}\ \frac{U^{235}}{cc}$$

$$Q = \left(570 \times 10^{-24}\ \text{cm}^2\right)\left(9.5 \times 10^{20}\ \frac{U^{235}}{\text{cm}^3}\right)\left(3 \times 10^{13}\ \frac{n}{\text{cm}^2\text{-s}}\right) \times$$

$$\left(200 \times 10^{6}\ \text{eV}\right)\left(1.602 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right) = \boxed{520\ \frac{\text{W}}{\text{cm}^3}}$$