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78/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-3, 27/30

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

Fissile isotope is U-235. Natural uranium is enriched to 0.7% U-235.

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$\rho_{U_3Si_2} = 12.2\ g/cm^3$

$\rho_{U_3Si_5} = 9.1\ g/cm^3$

$A_{U_3Si_2} = 3(238) + 2(28.1)$

$A_{U_3Si_2} = 770.2$

$A_{U_3Si_5} = 3(238) + 5(28.1)$

$A_{U_3Si_5} = 854.5$

$Q_{U_3Si_2} = Q_{U_3Si_5}$

$\cancel{E_f}\, \cancel{\sigma_f}\, N_{U235} \cancel{\phi} = \cancel{E_f}\, \cancel{\sigma_f}\, N_{U235} \cancel{\phi}$

$N_{U235} = N_{U235}$

$\frac{\rho N_A \gamma}{A} = \frac{\rho N_A \gamma}{A}$

$$\frac{3(12.2)(6.02\times10^{23})(0.03)}{770.2} = \frac{3(9.1)(6.02\times10^{23})(\gamma_{U_3Si_5})}{854.5}$$

Enough info was provided (U density)

$\boxed{\gamma_{U_3Si_5} = 0.0446 = 4.46\%}$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

U_3Si_5 would be a worse ~~for a~~ fuel because it requires a higher enrichment to achieve the same energy release rate.

-3, thermal conductivity?

-4, 31/35

Question 2:
Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

$K_c = 0.17$

$h_{gap} = \frac{K_{gap}}{t_{gap}}$

$K_{gap} = K_{He}^{1-\gamma} K_{Xe}^{\gamma}$

$= [16\times10^{-6}(646.58)^{.79}]^{0.95} [0.7\times10^{-6}(646.58)^{.79}]^{0.05}$

$\boxed{K_{gap} = 0.002}$

$h_{gap} = \frac{0.002}{80\times10^{-6}} = 0.25$

a) What is the surface temperature of the fuel rod? (15 points)

$T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250\ W/cm}{2\pi(0.45)(2.5)} + 580K$

$\boxed{T_{co} = 615.37K}$

$T_{ci} = \frac{LHR\, t_c}{2\pi R_f K_c} + T_{co} = \frac{250(0.06)}{2\pi(0.45)(0.17)} + 615.37K$

$\boxed{T_{ci} = 646.58K}$

$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ci} = \frac{250}{2\pi(0.45)(0.25)} + 646.58$

$\boxed{T_s = 1000.26K}$

-2, You rounded too early, resulting in a 50 K difference in the final answer

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$T_0 = \frac{LHR}{4\pi K_{fuel}} + T_s = \frac{250}{4\pi(0.2)} + 1000.26K$

$T_0 = 1099.7K$

$\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)} = \frac{7.5\times10^{-6}(246.7)(1099.7 - 1000.26)}{4(1-0.25)}$

$\sigma^* = 61.3\ MPa$

$\sigma_{\theta\theta} = -\sigma^*(1-3\eta^2) = -61.3(1-3(1)^2)$

$\boxed{\sigma_{\theta\theta} = 122.6\ MPa}$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

Higher because UO_2 has a lower thermal conductivity, which would result in a larger temperature gradient.

d) What assumptions were made in your calculations for a) and b)? (5 points)

1) Constant properties
2) Constant LHR
3) constant Fuel radius

-2, Axisymmetry? Steady state? No Shear? etc.

20/35

Question 3:
Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

negligible wall thickness

-5, Stress is constant across radius

$F_{stress} = F_{pressure}$

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\sigma_r = -\frac{P}{2} = -\frac{6\,MPa}{2}$$
$$\boxed{\sigma_r = -3\,MPa}$$

$$\sigma_z = \frac{PR}{2t_c} = \frac{6\,MPa(5.6\,mm)}{2(0.6\,mm)}$$
$$\boxed{\sigma_z = 28\,MPa}$$

$$\sigma_\theta = \frac{PR}{t_c} = \frac{6\,MPa(5.6\,mm)}{0.6\,mm}$$
$$\boxed{\sigma_\theta = 56\,MPa}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

-4, Check stress at multiple radii

$$\sigma_{\theta\theta} = P\frac{(R_o/r)^2+1}{(R_o/R_i)^2-1} = P\frac{(R_o/R_i)^2+1}{(R_o/R_i)-1} = 6\,MPa\left(\frac{\left(\frac{5.9}{5.3}\right)^2+1}{\left(\frac{5.9}{5.3}\right)^2-1}\right)$$

$$\boxed{\sigma_{\theta\theta} = 56.16\,MPa}$$

Thin walled approximation is not conservative.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\begin{bmatrix}\sigma_r\\ \sigma_\theta\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu\\ \nu & 1-\nu\end{bmatrix}\begin{bmatrix}u_{rr}\\ u_{r/r}\end{bmatrix}$$

$$\begin{bmatrix}-3\\ 56\end{bmatrix}MPa = \frac{70\,GPa}{(1+0.41)(1-2(0.41))}\begin{bmatrix}0.59 & 0.41\\ 0.41 & 0.59\end{bmatrix}\begin{bmatrix}u_{rr}\\ u_{r/r}\end{bmatrix}$$

$$-3 = 275.81(0.59)u_{rr} + (275.81)(0.41)\,u_{r/r}$$

$$u_{r/r} = \frac{-3 - 162.73}{113.08}$$

$$56 = 275.81(0.41)u_{rr} + (275.81)(0.59)\,u_{r/r}$$

$$u_{rr} = -0.498$$
$$u_{r/r} = 0.69$$

$$\sigma = \begin{bmatrix}-3 & 0\\ 0 & 56\end{bmatrix}$$

$$\varepsilon = \begin{bmatrix}-0.498 & 0\\ 0 & 0.69\end{bmatrix}$$

-4, Need all three strains to calculate stress
-2, tensors are missing zz component