Joseph Guidotti

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1 (a) The fissile isotope in U_3Si_5 is $\boxed{^{235}U}$

The enrichment in natural form is $\boxed{0.7\% \ ^{235}U}$

-2, 28/30

(b) $Q = E_f N_f \sigma_f \phi_{th}$, $f_w = \frac{3M_U}{M_{U_3Si_2}}$, $q = 0.03$

U_3Si_2: $M_U = 238$, $M_{U_3Si_2} = 769.856 \rightarrow f_w = 0.9274$

from HW1

$\delta = 11.31 \ g/cm^3$

$\delta_U = f_w \delta = 0.9274 (11.31 g/cm^3) = 10.489 \ g/cm^3$

$\delta_U / M_U = \frac{10.489}{238} = 0.04407$

$N_U = N_A (\delta_U / M_U) = 6.022 \times 10^{23} (0.04407)$

$N_U = 2.654 \times 10^{22}$ atoms $/cm^3$

$N_{U235} = q N_U = 0.03 (2.654 \times 10^{22} \text{ atoms}/cm^3) \rightarrow N_{U235} = 7.962 \times 10^{20}$ atoms$/cm^3$

$\sigma_f = 5.50 \times 10^{-22} \ cm^2$, $E_f = 3.0 \times 10^{-11}$ J/fission

$Q_{U_3Si_2} = E_f N_f \sigma_f \phi_{th} = 420.41 \ W/cm^3$

U_3Si_5: Now working backwards $\rightarrow N_f = 7.962 \times 10^{20}$ atoms $/cm^3$

$N_f = q N_U \rightarrow q = N_f / N_U$, $M_{Si} = 28.0855$ (from HW1)

$M_{U_3Si_5} = 3M_U + 5M_{Si} = 3(238) + 5(28.0855)$

$M_{U_3Si_5} = 854.4275$ g/mol

1) Continued

$$f_w = \frac{3 M_U}{M_{U_3Si_5}} = \frac{3(238)}{854.4225} = 0.8356$$

$$\delta_U = 7.5 \text{ g of U/cm}^3 \rightarrow \delta_U / M_U = 7.5/238 = 0.0315$$

$$N_U = N_A (\delta_U / M_U) = 6.022 \times 10^{23} (0.0315)$$

$$N_U = 1.8977 \times 10^{22} \text{ atoms/cm}^3 \rightarrow \text{need } N_f(U_3Si_5) = N_f(U_3Si_2)$$

$$q = N_f / N_U(U_3Si_5) = (7.962 \times 10^{20} \text{ atoms/cm}^3) / (1.8977 \times 10^{22} \text{ atoms/cm}^3)$$

$$q = 0.0419 \rightarrow \boxed{q = 4.2\%}$$

-2, math error

(c) U_3Si_5 seems like a poor fuel choice to me when compared to U_3Si_2 for several reasons:

$(12.5 \text{ W/m·K} \cdot (1\text{m}/100\text{cm}) = 0.125 \text{ W/cm·K} < 0.23 \text{ W/cm·K})$ k for U_3Si_2

- it has a smaller thermal conductivity
- It requires a higher enrichment in order to produce the same power per unit volume

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2) (a) $LHR = 250\ W/cm$, $T_{cool} = 580K$, $t_{clad} = 0.6mm$, $k_c = 0.17\ W/cm\cdot k$ (from HW2)

$\gamma = 0.05$, $h_{cool} = 2.5\ W/(cm^2\cdot k)$, $t_{gap} = 80\mu m$, $R_f = 4.5mm$

$$T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250\ W/cm}{2\pi(0.45cm)(2.5\ W/(cm^2\cdot k))} + 580k = 615.37K$$

$$T_{ic} = T_{co} + \frac{LHR\, t_c}{2\pi R_f k_c} = \frac{(250)(0.06cm)}{2\pi(0.45)(0.17)} + 615.37K = 646.57K$$

$$K_{gap} = K_{He}^{1-\gamma} K_{Xe}^{\gamma},\quad K_{He} = 16\times10^{-6}\, T_{ic}^{0.79} = 0.00266$$

$$K_{Xe} = 0.7\times10^{-6}\, T_{ic}^{0.79} = 1.163\times10^{-4}$$

$$K_{gap} = K_{He}^{0.95} K_{Xe}^{0.05} = 0.002273 \rightarrow h_{gap} = \frac{K_{gap}}{t_{gap}} = 0.2841$$

$$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ic} = \frac{250}{2\pi(0.45)(0.2841)} + 646.57K$$

$$\rightarrow \boxed{T_s = 957.8K}\ (a)$$

(b) The largest stress occurs at the outer surface in the $\sigma_{\theta\theta}$ direction

($K = 0.2\ W/cm\cdot k$ from Table in Notes)

$$\eta = r/R_f = R_f/R_f = 1,\quad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

$$T_0 - T_s = \frac{LHR}{4\pi k} = \frac{250}{4\pi(0.2)} = 99.47K$$

$$\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)} = 61.349\ MPa$$

$$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2) = -0.61675(1 - 3(1))$$

$$\boxed{\sigma_{\theta\theta} = 122.70\ MPa}$$

(c)

uranium oxide has a higher thermal expansion coefficient while having about
same Young's modulus, E. For this reason I expect that the stress in urani
oxide would be larger than the stress in the uranium nitrate in this proble

-3, The stress is larger but primarily due to the much lower thermal conductivity in UO2 leading to a high DT

(d) In (a) and (b) the following assumptions were made:

1. constant K (independent of T)
2. steady state
3. axis-symmetry
4. Temp. does not change in z
5. Constant E (independent of T)

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3)

(a) Assumptions: ① a static body ② The effects of gravity are negligible ③ Axis-Symmetry ④ isotropic material response

-3, Stress is constant through the thickness

(b) $\bar{\sigma}_\theta = \frac{pR}{\delta} = \frac{(6\times10^6\ Pa)(5.6mm)}{0.6mm} \rightarrow \boxed{\bar{\sigma}_\theta = 56\ MPa}$

$\bar{\sigma}_z = \frac{pR}{2\delta} = \frac{1}{2}\left(\frac{pR}{\delta}\right) = \frac{1}{2}\bar{\sigma}_\theta = \frac{1}{2}(56MPa) \rightarrow \boxed{\bar{\sigma}_z = 28\ MPa}$

$\bar{\sigma}_r = -\frac{1}{2}p = -\frac{1}{2}(6MPa) \rightarrow \boxed{\bar{\sigma}_r = -3\ MPa}$

(c) The thin-walled approximation overestimates $\bar{\sigma}_\theta$ and $\bar{\sigma}_z$. Since $\bar{\sigma}_\theta$ is the largest stress in the cladding, the thin-walled approximation is a conservative estimate for cladding failure. While $\bar{\sigma}_z$ is underestimated for about half the range, it is the smallest stress in the cladding, so it is of the least concern.

-10, Calculate stress at two radii using thick wall equations and compare to see if they are equal

(d) $\epsilon_{rr} = \frac{1}{E}(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})) = \boxed{-5.349\times10^{-4} = \epsilon_{rr}}$

$\epsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})) = \boxed{6.534\times10^{-4} = \epsilon_{\theta\theta}}$

$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})) = \boxed{8.957\times10^{-5} = \epsilon_{zz}}$

$$\boxed{\sigma = \begin{bmatrix} 56 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix} MPa} \qquad \boxed{\epsilon = \begin{bmatrix} 65.34 & 0 & 0 \\ 0 & 8.957 & 0 \\ 0 & 0 & -53.49 \end{bmatrix} \times10^{-5}}$$