PennState
College of Engineering

NucE 497: Reactor Fuel Performance

Lecture 14: Thermomechanics

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Mechanical and Nuclear Engineering

Material is taken from Dr. Motta's book, chapter 6

Today we will discuss thermal expansion and the coupling of temperature and stress

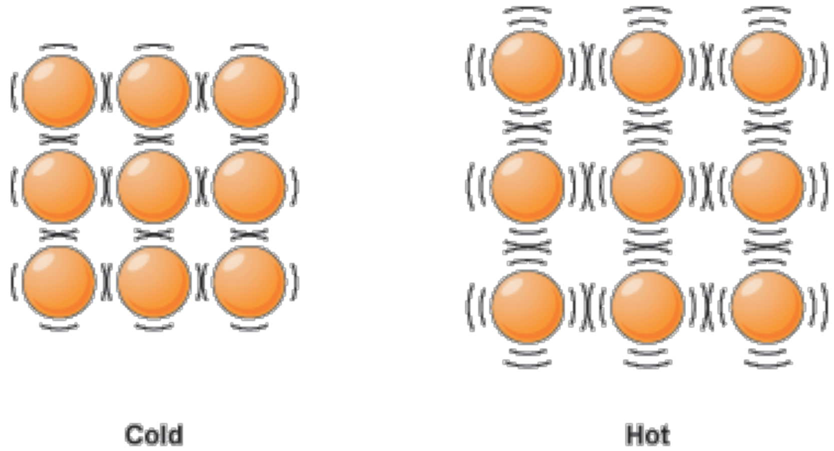
- Module 1: Fuel basics
- Module 2: Heat transport
- Module 3: Mechanical behavior
  - Introduction to solid mechanics
  - Analytical solutions of the mechanics equations
  - **Thermomechanics, thermal expansion**
  - Solving equations in 1D numerically
  - Solving in multiple dimensions with FEM
  - Summary of fuel performance codes
- Module 4: Materials issues in the fuel
- Module 5: Materials issues in the cladding
- Module 6: Accidents, used fuel, and fuel cycle

Here is some review from last time

- In a pressurized cylinder, which stress has the largest magnitude?
  a) σ_{rr}
  b) $\boxed{\sigma_{\theta\theta}}$
  c) σ_{zz}

a) What is the primary assumption for thin walled cylinders?
  a) Walls are thinner than half the radius
  b) The walls are thinner than 1/10 of the length
  c) The stress is constant through the wall of the cylinder

As the temperature increases, atoms have larger vibrations, causing the material to expand



- This expansion doesn't raise the energy of the material when unrestrained
- This expansion can be treated as a strain, but as one that doesn't cause stress

$$\boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_0)$$

In most materials, thermal expansion happens equally in all directions and is linear with temperature

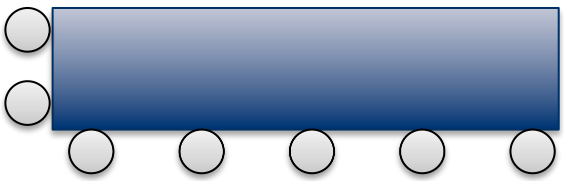
$$\boldsymbol{\epsilon}_0 = (T - T_0)\alpha\mathbf{I}$$

- In this equation
  - T is the current temperature
  - T_0 is the temperature the original size was measured
  - α is the linear thermal expansion coefficient
  - **I** is the identity tensor

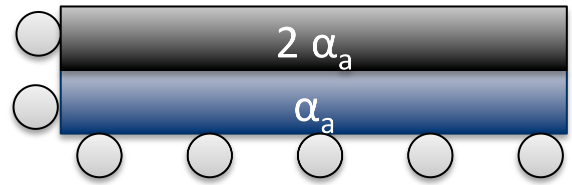
Material	α (× 10^{-6} 1/K)
Aluminum	24
Copper	17
Steel	13
UO_2	11
Zircaloy (Axial)	5.5
Zircaloy (radial)	7.1

Though thermal expansion doesn’t directly cause stress, it can still lead to thermal stress

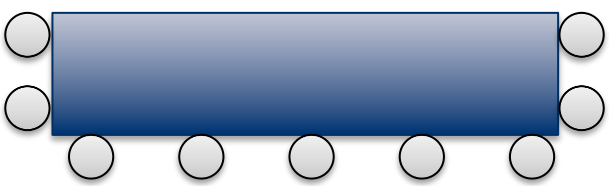
Stress? **NO**



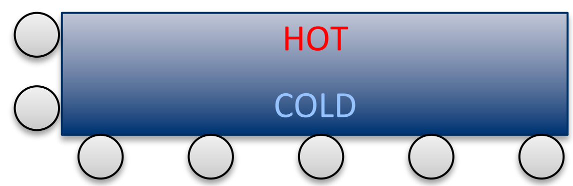
Stress? **YES**



Stress? **YES**



Stress? **Yes**



Quiz question: In which cases would thermal expansion result in stress within the material?

- A metal rod is free to deform and is heated slowly by 25 K
- A metal that is fixed on both sides is heated slowly by 25 K
- A metal rod composed of two materials with different expansion coefficients is free to deform and is heated slowly by 25 K
- A fuel pellet while the reactor is started up over several hours

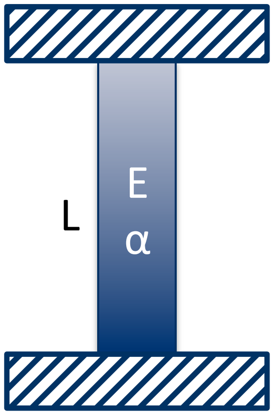
Attempts: 39 out of 39

In which cases would thermal expansion result in stress within the material?

A metal rod is free to deform and is heated slowly by 25 K	1 respondents	3 %	
A metal that is fixed on both sides is heated slowly by 25 K	38 respondents	97 %	✓
	31 respondents	79 %	✓
A fuel pellet while the reactor is started up over several hours	31 respondents	79 %	✓

Problem: What is the stress in a thin constrained rod of length L when it is heated to ΔT?

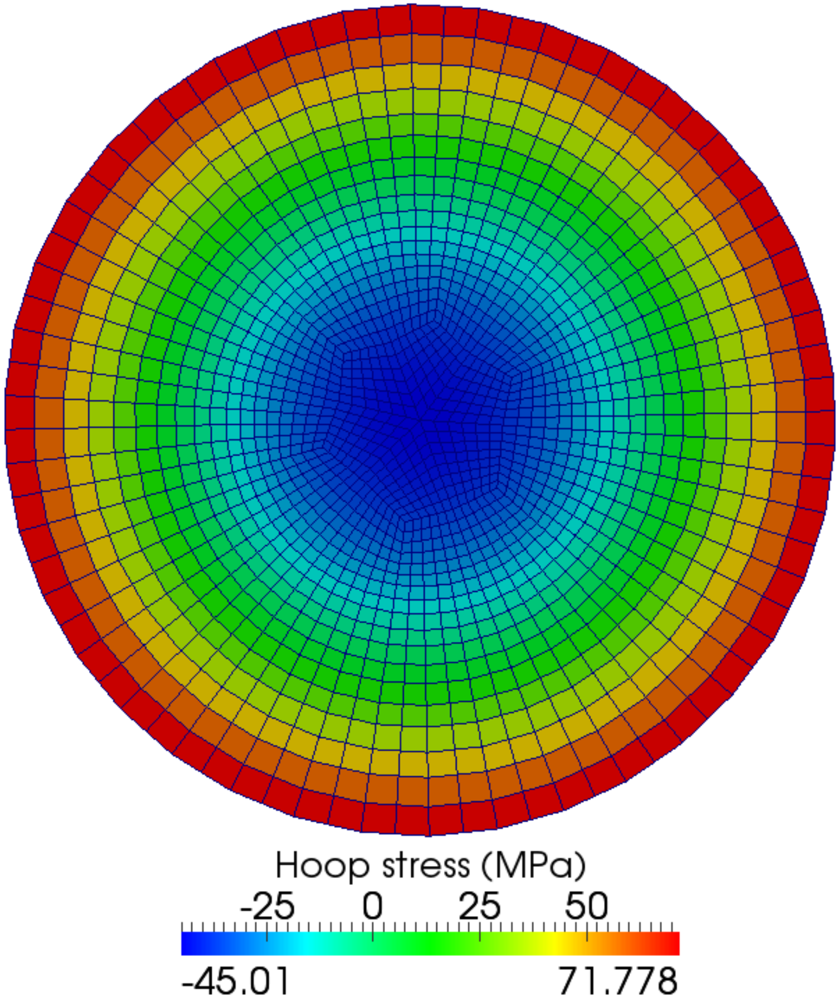
- The rod has a Young's modulus of E and an expansion coefficient of α

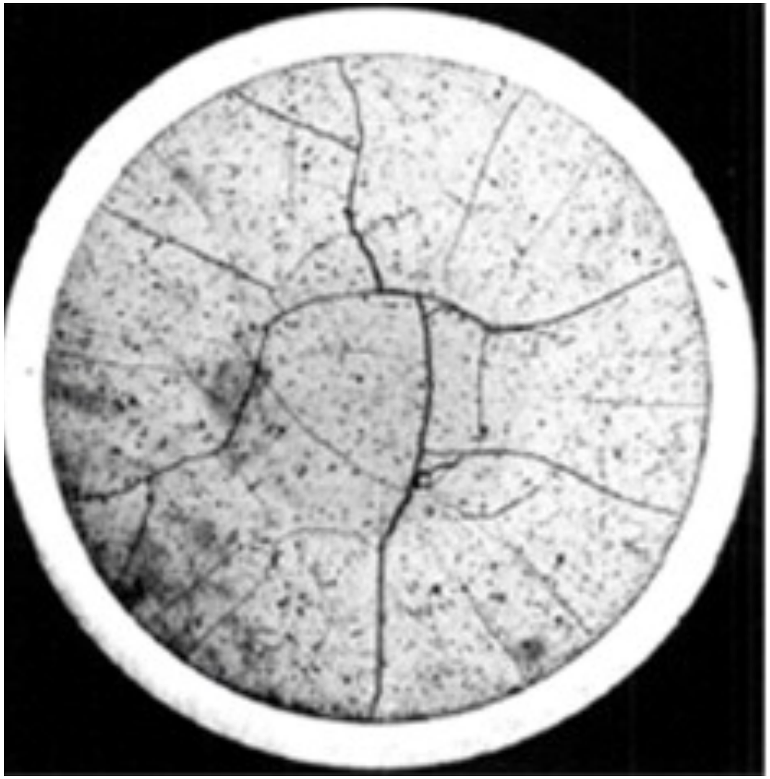


$$\boldsymbol{\epsilon}_0 = (T - T_0)\alpha\mathbf{I} \qquad \boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_0)$$

- $\varepsilon 0 = \Delta T\ \alpha$
- $\sigma = E\ (0 - \Delta T\ \alpha)$
- $\sigma = -\ E\ \Delta T\ \alpha$

The large temperature gradient within a fuel pellet results in large thermal stresses in the fuel pellet





Now we need to consider the material response of the axisymmetric body

- We assume small strains, so the strain is defined as

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r},\ \ \epsilon_{\theta\theta} = \frac{u_r}{r}, \epsilon_{zz} = \frac{\partial u_z}{\partial z}, \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

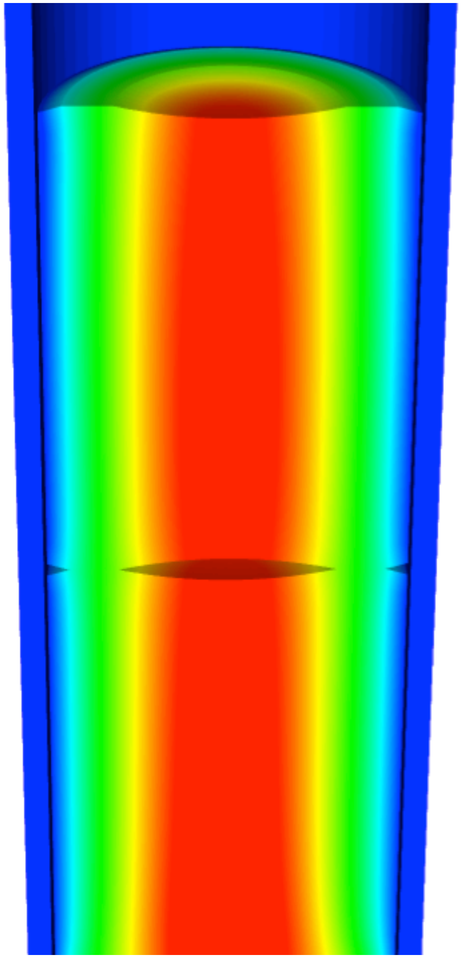
- We assume isotropic material response, so

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right) + \alpha\Delta T$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right) + \alpha\Delta T$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right) + \alpha\Delta T$$

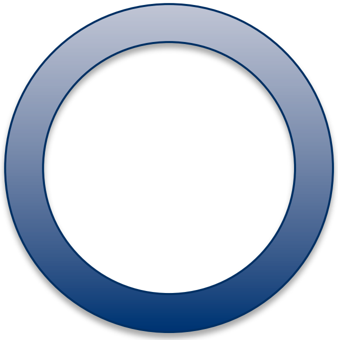
$$\epsilon_{rz} = \frac{1}{2G}\sigma_{rz}$$



We can add these equations to the analytical equations we derived previously

- Consider a cylinder with thermal expansion but no pressure
  - $\sigma_{rr}(R_i) = \sigma_{rr}(R_o) = 0$
- In a similar approach to last time, we get

$$\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\nu}\right)\frac{1}{r}\frac{dT}{dr}$$



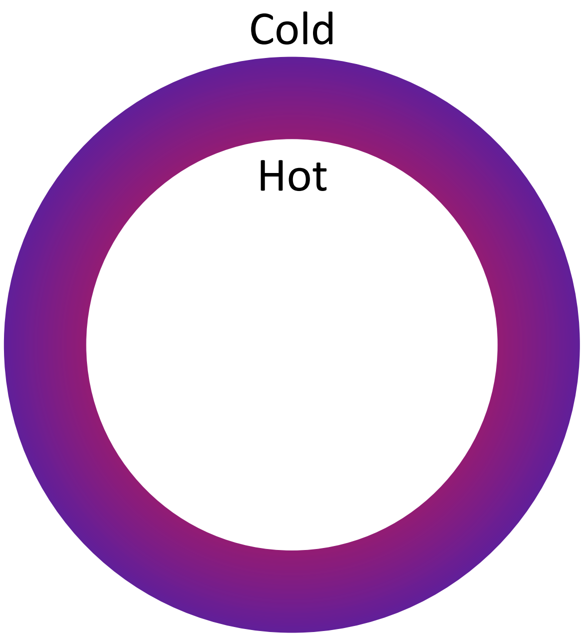
- Solving this ODE gives

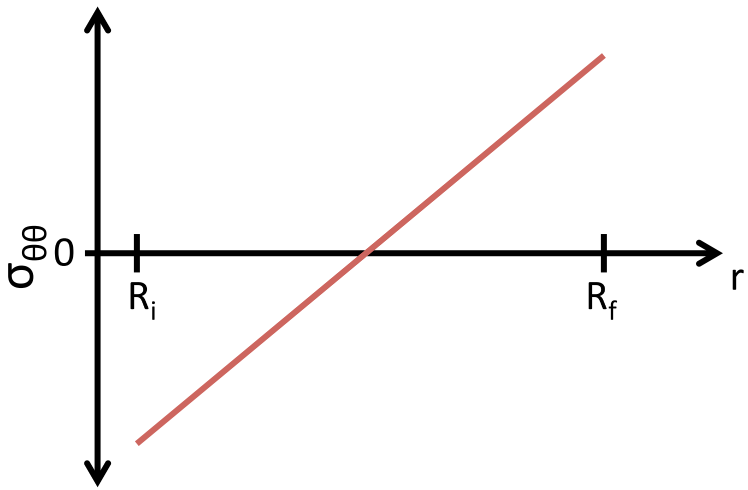
$$\sigma_{rr}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(\frac{r}{R_i}-1\right)\left(1-\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$

$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$

$$\sigma_{zz}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$

What does the hoop stress in the cladding look like?





At what radius is the hoop stress equal to zero?

- At what radius within the thickness of the cladding does the hoop stress cross zero?

$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\sigma_{\theta\theta}(r) = \tfrac{1}{2}\Delta T \tfrac{\alpha E}{1-\nu}\left(1 - 2\tfrac{R}{\delta}\left(\tfrac{r}{R} - 1\right)\right) = 0$$

$$1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right) = 0$$

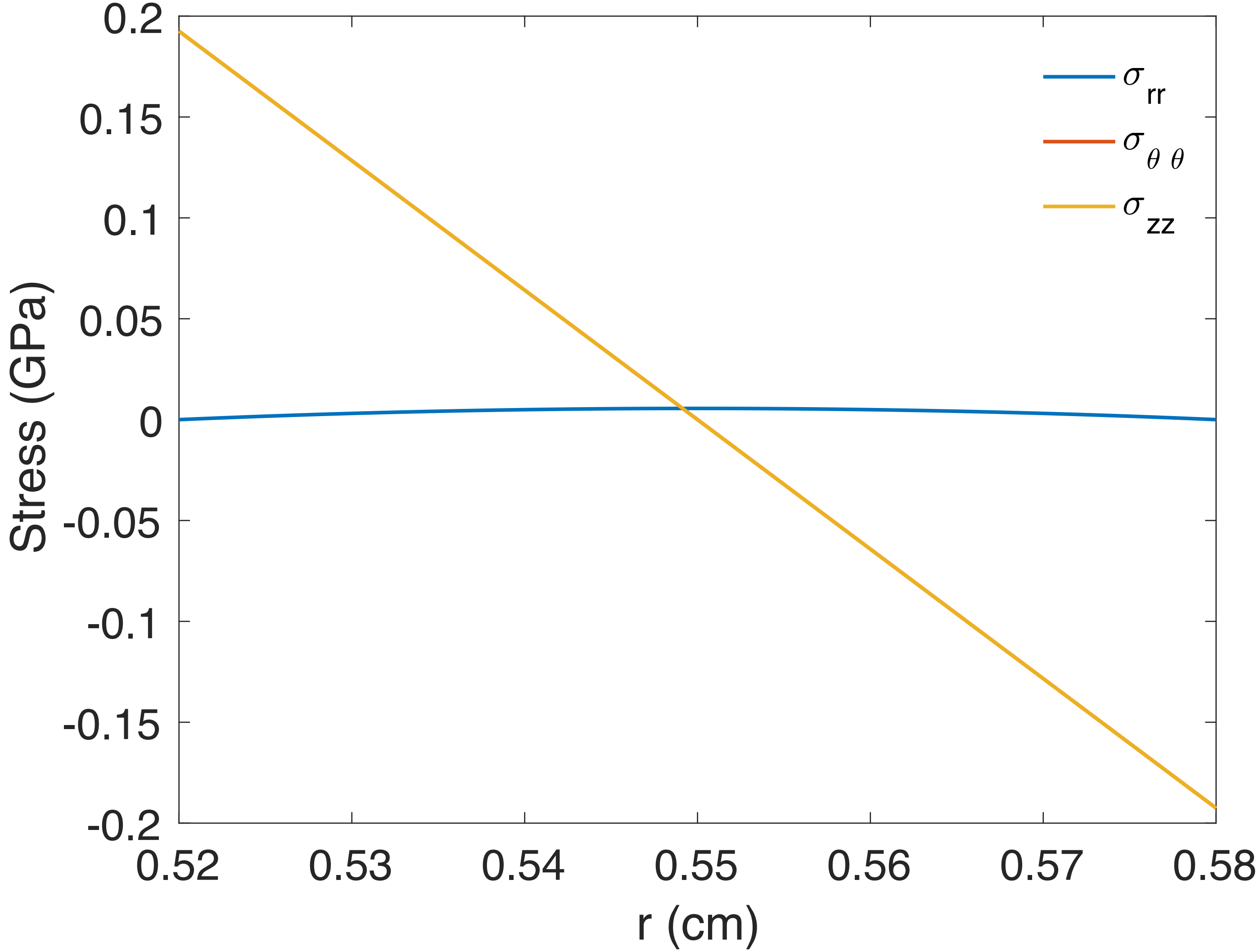
$$2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right) = 1$$

$$2\tfrac{R}{\delta}\left(\tfrac{r}{R} - 1\right) = 1$$

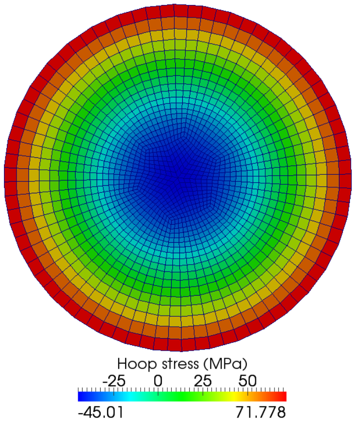
$$\frac{r}{R} - 1 = \frac{\delta}{2R_i}$$

$$r = \frac{\delta}{2} + R_i$$

The linear temperature gradient across the cladding causes thermal stresses



Now we will apply this same approach to the thermal stress in a fuel pellet



- The thermal stress is due to the temperature gradient

$$T(r) = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_f^2}\right) + T_s \qquad T_0 - T_s = \frac{LHR}{4\pi k}$$

$$T - T_s = (T_0 - T_s)\left(1 - \frac{r^2}{R_f^2}\right)$$

$$\frac{\partial T}{\partial r} = 2(T_0 - T_s)\left(\frac{r}{R_f^2}\right) \qquad \frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\nu}\right)\frac{1}{r}\frac{dT}{dr}$$

$$\eta = \frac{r}{R_f} \qquad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

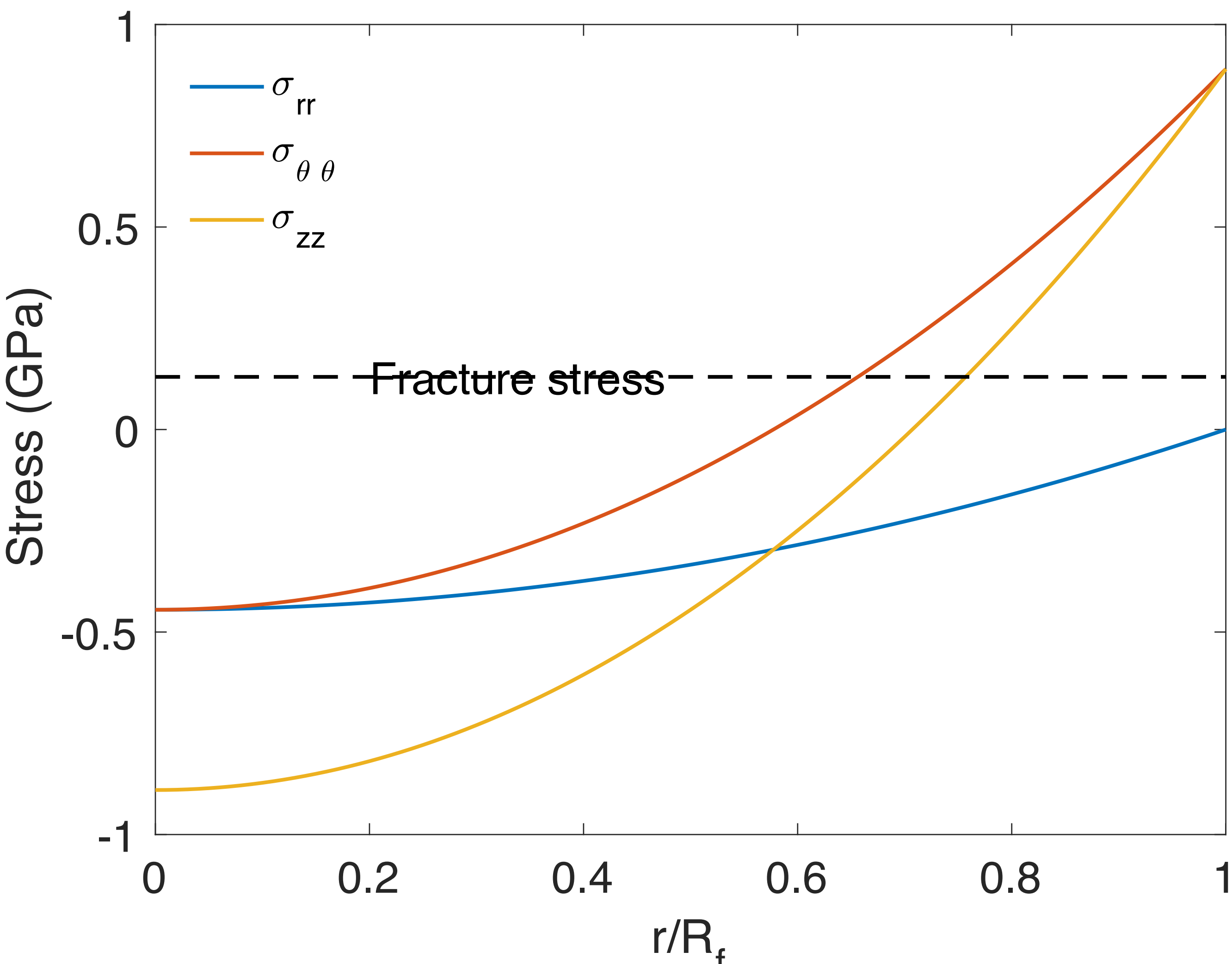
$$\frac{d}{d\eta}\left(\eta^3\frac{d\sigma_{rr}}{d\eta}\right) = 8\sigma^*\eta^3$$

We solve the stress ODE to obtain the stress throughout the fuel pellet

$$\frac{d}{d\eta}\left(\eta^3 \frac{d\sigma_{rr}}{d\eta}\right) = 8\sigma^*\eta^3 \qquad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

- The boundary conditions are $\frac{d\sigma_{rr}}{d\eta} = 0 \text{ at } \eta = 0$ and $\sigma_{rr} = 0 \text{ at } \eta = 1$
- Once we solve it, we obtain $\sigma_{rr}(\eta) = -\sigma^*(1-\eta^2)$
- Then we can solve the hoop stress using $\sigma_{\theta\theta} = \sigma_{rr} + r\sigma_{rr,r}$
  - $\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2)$
- The axial stress is much more complicated to obtain, but you end up with
  - $\sigma_{zz}(\eta) = -2\sigma^*(1-2\eta^2)$

The fuel temperature gradient causes large thermal stresses



How far into the fuel rod do the cracks penetrate?

$$\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2) \qquad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

- E = 200 GPa, ν = 0.345, α = 11.0e-6 1/K, σ_{fr} = 130 MPa, ΔT = 550 K

- First, we need to solve for η
  - $-\sigma_f / \sigma^* = 1 - 3\eta^2$
  - $3\eta^2 = 1 + \sigma_f / \sigma^*$
  - $\eta = ((1 + \sigma_f / \sigma^*)/3)^{½}$
- σ* = 11.0e-6*200*550/(4*(1 – 0.345)) = 461.8 MPa
- η = sqrt((1 + 130/461.8)/3) = 0.65

Summary

- Increases in temperature cause most materials to expand
- We call this thermal expansion, which is a strain that doesn't inherently cause stress
- Thermal expansion can cause stress when
  - Deformation is constrained
  - There are gradients in the expansion coefficient
  - There is a temperature gradient
- We have analytical equations for thermal stresses in the cladding and in the fuel

Thermal expansion causes a decrease in the gap

- Both the pellet and the cladding swell

$$\Delta\delta_{gap} = \delta_{gap} - \delta_{gap}^{0}$$

$$\Delta\delta_{gap} = \Delta\bar{R}_C - \Delta R_f$$

$$\frac{\Delta R_f}{\bar{R}_C} = \alpha_f \left(\bar{T}_f - T_{fab}\right)$$

$$\frac{\Delta R_C}{\bar{R}_C} = \alpha_C \left(\bar{T}_C - T_{fab}\right)$$

$$\Delta\delta_{gap} = \bar{R}_c\alpha_C \left(\bar{T}_C - T_{fab}\right) - \bar{R}_f\alpha_f \left(\bar{T}_f - T_{fab}\right)$$

- But, as the gap decreases, the temperature changes, which again makes the gap change
- The solution using the analytical equations is iterative, due to the dependence of the gap size and temperature.

We need to calculate the steady state temperature profile in the rod, including thermal expansion

- LHR = 200 W/cm, δ^0_{gap} = 30 µm, R_f = 0.5, T_{cool} = 580 K, T_0 = 373 K, k_{gap} = 0.0026 W/cm-K, δ_C= 0.06 cm, α_f = 11.0e-6 1/K, α_C = 7.1e-6 1/K

$$\Delta\delta_{gap} = \bar{R}_c\alpha_C\left(\bar{T}_C - T_{fab}\right) - \bar{R}_f\alpha_f\left(\bar{T}_f - T_{fab}\right) \quad \Delta T_{gap} = \frac{LHR}{2\pi R_f k_{gap}/\delta_{gap}}$$

- ΔT_{cool} = 25.5 K, ΔT_{clad} = 22.5 K, ΔT_{fuel} = 530.5 K
- So, T_{IC} = 580 + 25.5 + 22.5 = 628.0 K, T_s = 701.5 K, T_0 = 1232.0 K
- $\Delta\delta_{gap}$ =