Matthew J. Hanson

497 exam

93/100

(1)

a) The fissile isotope is Uranium-235.

Natural Uranium contains 0.7% Uranium-235

-0, 30/30

b) $Q = E_F N_F \sigma_F \Phi_{th}$; $E_F = 3\times10^{-11}$ J/Fission

$\sigma_F = 550b = 550\times10^{-24}\ cm^2$

$\Phi = 3.2\times10^{13}\ n\,(cm^2 s)^{-1}$

$M_U = \gamma(235) + (1-\gamma)238$

$= (0.03)(235) + (1-0.03)(238)$

$= 237.91\ g\ mol^{-1}$

$N_F = \frac{\gamma \rho N_a}{M} = \frac{(0.03)(11.31 g\ cm^{-3})(6.022\times10^{23}\ atom\ mol^{-1})}{(237.91 g\ mol^{-1})}$

$= 8.588\times10^{20}$ atom U235 $/cm^3$

$Q = (3\times10^{-11}\ J\ Fission^{-1})(8.588\times10^{20}\ atom\ U^{235}/cm^3)(550\times10^{-24}\ cm^2)(3.2\times10^{13}\ n\,(cm^2 s)^{-1})$

$= 453.467\ W\ cm^{-3}$

$N_F = \frac{Q}{E_F \sigma_F \Phi_{th}} = \frac{\gamma \rho N_a}{M} \therefore \gamma = \frac{QM}{\rho N_a E_F \sigma_F \Phi_{th}}$

$\gamma = \frac{(453.467 W cm^{-3})(237.91 g\ mol^{-1})}{(7.5 g cm^{-3})(6.022\times10^{23} atom\ mol^{-1})(3\times10^{-11}\ J\ Fission^{-1})(550\times10^{-24} cm^2)(3.2\times10^{13}\ n\,(cm^2 s)^{-1})}$

$= 0.04524 \times 100\%$

$\boxed{= 4.524\% \text{ enrichment for } U_3Si_5}$

c) U_3Si_5 would be worse than U_3Si_2 for two reasons

→ You need more enrichment to get the same energy aka less cost efficient

→ The thermal conductivity of U_3Si_5 ($0.125\ W(cm\ K)^{-1}$) is worse than that of U_3Si_2 ($0.23\ W(cm\ K)^{-1}$)

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-1, 34/35

2) $R_P = 0.45\,cm$ $LHR = 250\ W\,cm^{-1}$ $K_{clad} = 0.17\ W(cm\,K)^{-1}$

a) $gap = 80\times10^{-4}\,cm$ $T_{cool} = 580\,K$ $X_e = 5\% = 0.05$

$t_{clad} = 0.06\,cm$ $h_{cool} = 2.5\ W(cm^2K)^{-1}$ $K_{UN} = 0.2\ W(cm\,K)^{-1}$

$E = 246.7\times10^{9}\ Pa$ $\nu = 0.25$ $\alpha = 7.5\times10^{-6}\,K^{-1}$

$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_P h_{cool}} = (580K) + \frac{(250\,W cm^{-1})}{2\pi(0.45cm)(2.5W(cm^2K)^{-1})} \therefore T_{co} = 615.368\,K$$

$$T_{c1} = T_{co} + \frac{LHR\ t_{clad}}{2\pi R_P K_{clad}} = (615.368K) + \frac{(250\,Wcm^{-1})(0.06cm)}{2\pi(0.45cm)(0.17W(cmK)^{-1})} \therefore T_{c1} = 646.575K$$

$$K_{He} = 16\times10^{-6}\,T_{c1}^{\ 0.79} = \cancel{0.0085196}\ 0.0026577\ W(cm\,K)^{-1}$$

$$K_{Xe} = 0.7\times10^{-6}\,T_{c1}^{\ 0.79} = 1.1628\times10^{-4}\ W(cm\,K)^{-1}$$

$$K_{gap} = K_{He}^{(1-x_e)}\,K_{xe}^{\ x_e} = (0.0026577)^{(1-0.05)}(1.1628\times10^{-4})^{(0.05)} \therefore K_{gap} = 0.002273\ W(cm\,K)^{-1}$$

$$h_{gap} = \frac{K_{gap}}{\cancel{t_{clad}}\ gap} = \frac{0.002273\ W(cmK)^{-1}}{\cancel{0.06cm}\ 80\times10^{-4}} \therefore h_{gap} = 0.284098\ W(cm^2K)^{-1}\ \cancel{0.03788\ W(cm^2\ K)^{-1}}$$

$$T_S = T_{c1} + \frac{LHR}{2\pi R_P h_{gap}} = (646.575K) + \frac{(250\ W cm^{-1})}{2\pi(0.45cm)\cancel{(0.03788\,W(cm^2K)^{-1})}\,(0.284098W(cm^2K)^{-1})}$$

$$\boxed{T_S = 957.80K}$$

b) $$T_m = T_S + \frac{LHR}{4\pi K_{UN}} = (957.80K) + \frac{(250\ W cm^{-1})}{4\pi(0.2\,W(cm\,K)^{-1})} \therefore T_m = 1057.276K$$

$$\sigma^* = \frac{\alpha E(T_m - T_s)}{4(1-\nu)} = \frac{(7.5\times10^{-6}K^{-1})(246.7\times10^{9}Pa)(1057.276K - 957.80K)}{4(1-0.25)}$$

$\sigma^* = 61.352\ MPa$; Maximum stress for hoop is at $r = R_P \therefore \eta = \frac{r}{R_P} = 1$

$$\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2) = -\sigma^*(1-3(1)^2) = -\sigma^*(-2) = 2\sigma^* = 2(61.352\,MPa)$$

$$\boxed{\sigma_{\theta\theta} = 122.70\,MPa}$$

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2) continued

c) UO_2 stress would be higher due to the temperature gradient in the fuel, this would cause the $(T_m - T_s)$ term to increase, therefore $\sigma_{\theta\theta}$ would increase.

D) → We have a static body
→ Gravity is negligible
→ The problem is axisymmetric
→ Isotropic material response

-1, there are several more assumptions

3)

-6, 29/35

a) The thickness of the wall is so small compared to the radius that the stress does not change across the wall (constant)

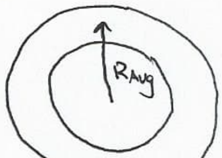
-2, Isotropic and small strain

b) $P = 6\,MPa$ $\quad R_{Avg} = 0.56\,cm$ $\quad t_c = 0.06\,cm$

$$\sigma_\theta = \frac{PR}{t_c} = \frac{(6\,MPa)(0.56\,cm)}{(0.06\,cm)} \quad \therefore \quad \boxed{\sigma_\theta = 56\,MPa}$$

$$\sigma_z = \frac{PR}{2t_c} = \frac{(6\,MPa)(0.56\,cm)}{2(0.06\,cm)} \quad \therefore \quad \boxed{\sigma_z = 28\,MPa}$$

$$\sigma_r = \frac{-1}{2}P = \frac{-1}{2}(6\,MPa) \quad \therefore \quad \boxed{\sigma_r = -3\,MPa}$$



$R_{CI} = 0.56 - 0.06/2 = 0.53\,cm$
$R_{CO} = 0.56 + 0.06/2 = 0.59\,cm$

c)
$$\sigma_{\theta\theta} = P\frac{(R_o/R_i)^2+1}{(R_o/R_i)^2-1} = (6\,MPa)\frac{\left(\frac{0.59\,cm}{0.53\,cm}\right)^2+1}{\left(\frac{0.59\,cm}{0.53\,cm}\right)^2-1} \quad \therefore \quad \sigma_{\theta\theta} = 56.16\,MPa$$

$$\sigma_{zz} = P\frac{1}{(R_o/R_i)^2-1} = \frac{(6\,MPa)}{\left(\frac{0.59\,cm}{0.53\,cm}\right)^2-1} \quad \therefore \quad \sigma_{zz} = 25.08\,MPa$$

$$\sigma_{rr} = -P\frac{(R_o/R_i)^2-1}{(R_o/R_i)^2-1} = -P \quad \therefore \quad \sigma_{rr} = -6\,MPa$$

-4 Calculate stress at two radii and see if it is constant

$\sigma_{\theta\theta}$) $\frac{56.16-56}{56.16} \times 100 = 0.285\%$ error, Yes the thin wall is close & conservative b/c it underestimated

σ_{zz}) $\frac{25.08-28}{25.08} \times 100 = -11.643\%$ error, No the thin wall overestimated, therefore not conservative but liberal

σ_{rr}) $\frac{(-6)-(-3)}{(-6)} \times 100 = 50\%$ error, Yes it was underestimated therefore conservative.

Matthew J. Hanson

NucE 497

3) Continued

D) $E = 70\times10^9 Pa$ $\quad \nu = 0.41$ $\quad \Delta T = \emptyset$

$$\varepsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})) = \frac{1}{(70\times10^9 Pa)}\left[(56.16\times10^6 Pa) - (0.41)\left[(-6\times10^6 Pa) + (25.08\times10^6 Pa)\right]\right]$$

$$= 6.91\times10^{-4} = 691\,\mu\varepsilon$$

$$\varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})) = \frac{1}{(70\times10^3 MPa)}\left[(25.08 MPa) - (0.41)\left[(56.16 MPa) + (-6 MPa)\right]\right]$$

$$= 6.45\times10^{-5} = 64.5\,\mu\varepsilon$$

$$\varepsilon_{rr} = \frac{1}{E}(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})) = \frac{1}{(70\times10^3 MPa)}\left[(-6 MPa) - (0.41)\left[(56.16 MPa) + (25.08 MPa)\right]\right]$$

$$= -5.79\times10^{-4} = -579\,\mu\varepsilon$$

$$\sigma = \begin{bmatrix}\sigma_{\theta\theta}\\ \sigma_{zz}\\ \sigma_{rr}\end{bmatrix} = \begin{bmatrix}56.16 & 0 & 0\\ 0 & 25.08 & 0\\ 0 & 0 & -6\end{bmatrix} MPa$$

$$\varepsilon = \begin{bmatrix}\varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \varepsilon_{rr}\end{bmatrix} = \begin{bmatrix}691 & 0 & 0\\ 0 & 64.5 & 0\\ 0 & 0 & -579\end{bmatrix} \mu\varepsilon$$

→ Assuming No shear so θz, θr, zr Components of stress/strain should be Zero.