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75/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

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Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

Fissile isotope : ^{235}U

Enrichment : 0.7%

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U235 has a negligible impact on the total molar mass of U in the fuel (15 points)

$M_U \approx 238$

$$E_{U_3Si_5} N_{U_3Si_5} \sigma_{U_3Si_5} \phi_{th} = E_{U_3Si_2} N_{U_3Si_2} \sigma_{U_3Si_2} \phi_{th}$$

$$\Rightarrow N_{U_3Si_5} = N_{U_3Si_2} \quad ; \quad N_f = \frac{\rho_U N_A}{M_U}$$

$$N_{U_3Si_2} = \frac{11.31(6.022\times10^{24})}{238} = 2.862\times10^{23} \frac{\text{atoms}}{\text{cm}^3}(0.03)$$

$$\Rightarrow N_{U_3Si_2} = 8.585\times10^{21} \frac{\text{atoms}}{\text{cm}^3}$$

$$N_{U_3Si_5} = \frac{7.5(6.022\times10^{24})}{238} = 1.898\times10^{23} \frac{\text{atoms}}{\text{cm}^3}$$

$$\Rightarrow (\text{enrichment})\, 1.898\times10^{23} = 8.585\times10^{21}$$

$$\Rightarrow \text{enrichment} = 0.452 \Rightarrow \boxed{\text{fuel enrichment of } U_3Si_5 \approx 4.5\%}$$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

I would rank it as slightly worse than U_3Si_2 due to its lower thermal conductivity and that you would need a higher enrichment to achieve the same energy, which is costly.

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$R_i = 4.5\,mm = 0.45\,cm$
$\delta_{gap} = 80\,\mu m = 80 \times 10^{-4}\,cm$
$\delta_c = 0.6\,mm = 0.06\,cm$
$LHR = 250\,\frac{W}{cm}$
$T_{cool} = 580\,K$
$h_{cool} = 2.5\,\frac{W}{cm^2 K}$
$k_c = 0.17\,\frac{W}{cm\,K}$

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ci}$; $T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250}{2\pi(0.45)(2.5)} + 580 = 615.3678\,[K]$

$\Rightarrow T_s = \frac{250}{2\pi(0.45)(0.278)} + 646.6$

$T_{ci} = \frac{LHR\,\delta_c}{2\pi R_f k_c} + T_{co} = \frac{250(0.06)}{2\pi(0.45)(0.17)} + 615.4 = 646.5746\,[K]$

$h_{gap} = \frac{k_{gap}}{\delta_g} = \frac{0.0022\,\frac{W}{cm\,K}}{80 \times 10^{-4}\,cm} = 0.278\,\frac{W}{cm^2 K}$

$\Rightarrow \boxed{T_s = 964.63\,K}$

$k_{Xe} = (0.7 \times 10^{-6})(646.5746)^{0.79} = 1.163 \times 10^{-4}$

-1, you rounded too early and your answer was off by 5 K

$k_{gap} = k_{He}^{1-x} k_{Xe}^{x} = (0.0026)^{0.95}(1.163 \times 10^{-4})^{0.05} = \boxed{0.0022}$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$\sigma_{\theta\theta}$ is largest

$\sigma_{\theta\theta} = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$ -8, DT is wrong, stress is max at eta = 1

$= \frac{1}{2}(964.63 - 580)\left(\frac{7.5 \times 10^{-6}(246.7)}{1 - 0.25}\right)\left(1 - 2\left(\frac{0.39}{0.06}\right)\left(\frac{0.45}{0.39} - 1\right)\right)$

$= -0.4744\,GPa$

$\Rightarrow \boxed{\sigma_{\theta\theta} = 474.44\,MPa}$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

lower due to the thermal expansion coefficient, α, being smaller for UO_2 ($\alpha = 11 \times 10^{-6}\,\frac{1}{K}$)

-5, higher because k is higher so DT would be much higher

d) What assumptions were made in your calculations for a) and b)? (5 points)

a

- steady-state
- axisymmetric behavior
- constant temperature in z-direction
- thermal conductivity is independent of temperature

b

-1, Assumptions in the stress calculation?

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Question 3:
Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

- Static body $(0 = \nabla \cdot \sigma \cdot \rho g)$
- negligible gravity $(0 = \nabla \cdot \sigma)$
- isotropic material
- small strains
- cladding thickness $\leq \frac{1}{10}$ radius
- asymmetric problem
- force balance
- stress is constant through wall

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\bar{\sigma}_\theta = \frac{pR}{\delta} = \frac{(6\,[MPa])(5.6[mm])}{0.6[mm]} = \boxed{56\,[MPa] = \bar{\sigma}_\theta}$$

$$\bar{\sigma}_z = \frac{pR}{2\delta} = \frac{(6\,[MPa])(5.6[mm])}{2(0.6[mm])} = \boxed{28\,[MPa] = \bar{\sigma}_z}$$

$$\bar{\sigma}_r = -\frac{1}{2}p = -\frac{1}{2}6[MPa] = \boxed{-3\,[MPa] = \bar{\sigma}_r}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

$$\delta \leq \frac{1}{10}R \Rightarrow 0.6 \leq \frac{1}{10}(5.6)$$
$$\Rightarrow \boxed{0.6\,[mm] \not\leq 0.56[mm]}$$

$\Rightarrow$ The thin-walled approximation wouldn't be that accurate for this cladding due to the cladding being 0.04[mm] too thick for the approximation. Yes, it would be conservative

-4, Check the stress at two different radii to quantify accuracy

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$C_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{70(1-0.41)}{(1.41)(1-0.82)} = \frac{41.3\text{ GPa}}{0.2538} = 162.7266\ [GPa]$$

$$C_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{70(0.41)}{0.2538} = 113.0812\ [GPa]$$

You already calculated the stresses
-2 Tensors missing zz component
-4 Calculate strains from stresses in part b

$$\varepsilon_{rr} = \frac{2r}{5} = \frac{2(0.56)}{5} \qquad \varepsilon_{\theta\theta} = \frac{r}{5} = \frac{0.56}{5}$$

$$\varepsilon_{rr} = 0.224 \qquad \varepsilon_{\theta\theta} = 0.112$$

$$\sigma_{rr} = \varepsilon_{rr}C_{11} + \varepsilon_{\theta\theta}C_{12} = 0.224(162.7) + 0.112(113.1)$$
$$\sigma_{rr} = 49.1154\text{ GPa}$$

$$\sigma_{\theta\theta} = \varepsilon_{\theta\theta}C_{11} + \varepsilon_{rr}C_{12} = 0.112(162.7) + 0.224(113$$
$$\sigma_{\theta\theta} = 43.5556\text{ GPa}$$

$$\Rightarrow \varepsilon = \begin{bmatrix} 0.224 & 0 \\ 0 & 0.112 \end{bmatrix} \qquad \Rightarrow \sigma = \begin{bmatrix} 49.12 & 0 \\ 0 & 43.56 \end{bmatrix} [GPa]$$