Exam 1 - NE591

Alex Chrystler
Feb 11 2021

[1] $T'(x_0) = 0 \quad x_0 = 0 \quad x_1 = X \quad T(x_1) = T_1$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0$$

Assumptions: Steady state
Axisymmetric } Equivalently, constant in y & z
Constant in z } for the Cartesian system
Constant thermal conductivity (w.r.t. Temperature)
i.e. $k \neq k(T)$

$$\int \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q \, dx = \int 0 \, dx$$

$$k \frac{\partial T}{\partial x} + Qx + C_1 = 0$$

Apply boundary condition

$$kT'(x) + Qx + C_1 = 0 \longrightarrow k\underbrace{T'(0)}_{=0} - Q(0) + C_1 = 0$$

$$\Rightarrow C_1 = 0$$

$$\int kT'(x) + Qx + C_1 \, dx = \int 0 \, dx$$

$$kT(x) + \frac{Q}{2}x^2 + C_1 x + C_2 = 0$$

$$kT(x) + \frac{Q}{2}x^2 + C_2 = 0$$

$$kT(x) + \frac{Q}{2}X^2 + C_2 = 0$$

$$kT_1 + \frac{QX^2}{2} = -C_2 \Rightarrow C_2 = -kT_1 - \frac{QX^2}{2}$$

$$\Rightarrow kT(x) + \frac{Qx^2}{2} - kT_1 - \frac{QX^2}{2} = 0$$

Final answer on pg 2

Alex Chrystler
Feb 11 2021

$$\boxed{T(x) = \frac{Q}{2k}(X^2 - x^2) + T_1}$$

2 Find Centerline temp $(r=0)$ and $T(0.4\text{ cm})$

$T_{cool} = 800\text{ K} \longrightarrow T_{cout} - 800\text{ K} = \frac{Q}{2h_{coolant}} R_{fuel} = \frac{(400 \frac{W}{cm^3})(0.46\text{ cm} - 0.45\text{ cm})}{2(5 \frac{W}{m K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm}) = 24\text{ K}$

$T_{clad} - 824\text{ K} = \frac{Q}{2h_{clad}} R_{fuel} \qquad \Rightarrow T_{cout} = 824\text{ K}$

$T_{clad} - 824\text{ K} = \frac{400 \frac{W}{cm^3}}{2(5.5 \frac{W}{K cm^2})}(0.6\text{ cm}) = 21.82\text{ K}$

$T_{clad} = 845.82\text{ K}$

$T_{gap} - 845.82\text{ K} = \frac{Q t_{clad}}{2k_{clad}} R_{fuel}$

$T_{gap} - 845.82\text{ K} = \frac{(400 \frac{W}{cm^3})(0.45\text{ cm} - 0.4\text{ cm})}{2(15 \frac{W}{m K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm}) = 40\text{ K}$

$T_{gap} = 885.82\text{ K}$

$T_{fuel} - 885.82\text{ K} = \frac{Q t_{gap}}{2k_{gap}} R_{fuel} = \frac{400 \frac{W}{cm^3}(0.65\text{ cm} - 0.6\text{ cm})}{2(25 \frac{W}{m K})(\frac{1\text{ m}}{100\text{ cm}})}(0.6\text{ cm}) = 120\text{ K}$

$T_{fuel} = 1005.82\text{ K}$

$T_0 - T_{fuel} = \frac{Q}{4k} R_{fuel}^2$

$T_0 - 1005.82\text{ K} = \frac{(400 \frac{W}{cm^3})}{4(0.5 \frac{W}{m K})}(0.6\text{ cm})^2 = 72\text{ K}$

$\boxed{T_0 = 1077.82\text{ K}}$

$T_0 - T(0.4\text{ cm}) = \frac{Q}{4k}(0.4\text{ cm})^2 = \frac{(400 \frac{W}{cm^3})}{4(0.5 \frac{W}{cm K})}(0.4\text{ cm})^2$

$1077.82\text{ K} - T(0.4\text{ cm}) = 32\text{ K} \Rightarrow \boxed{T(0.4\text{ cm}) = 1045.82\text{ K}}$

3 a) $Q = E_f N_f \sigma_f \phi$

$3\left[235(0.195) + 238(1-0.195)\right] + 2(28) = 768.25 \ \frac{g}{mol}$

$\left(15.67 \frac{g}{cm^3}\right)\left(\frac{1}{768.25 \ g/mol}\right)\left(6.022\times10^{23} \frac{molecules}{mol}\right)(0.195)\left(3 \frac{atoms}{molecule}\right) = 7.186\times10^{21} \frac{atoms}{cm^3}$ of U^{235}

$Q = (200\times10^6 \ eV)\left(1.602\times10^{-19} \frac{J}{eV}\right)\left(7.186\times10^{21} \frac{U^{235} \ atoms}{cm^3}\right)\left(2\times10^{12} \frac{n}{cm^2 \ s}\right)\left(570\times10^{-24} \ cm^2\right)$

$\boxed{Q = 262.473 \frac{J}{s \ cm^3} = 262.473 \frac{W}{cm^3}}$

b) $7.186\times10^{21} \frac{U^{235} \ atoms}{cm^3}$ is the required atom density

Atomic mass $= 235(x) + 238(1-x) + 2(16)$

$= -3x + 238 + 32$

$= -3x + 250$

$7.186\times10^{21} \frac{U^{235} \ atoms}{cm^3} = (x)\left(6.022\times10^{23} \frac{molecules}{mol}\right)\left(10.96 \ g/cm^3\right)\left(\frac{1}{-3x + 250 \ g/mol}\right)$

$0.0011 = \frac{x}{-3x+250}$ $\quad -0.0033x + 0.272 = x$

$0.272 = 1.0033x$

$x = 0.271 \longrightarrow \boxed{27.1\% \text{ enrichment needed}}$

$\boxed{4}$ a) $LHR\left(\frac{z}{z_0}\right) = 150\ \frac{W}{cm^2} \cos\left[\frac{\pi}{2.2}\left(\frac{z}{z_0} - 1\right)\right]$

$$LHR\left(\frac{z}{z_0}\right)\Bigg|_{z=1m} = 150\ \frac{W}{cm^2}(0.99999) = \boxed{149.998\ \frac{W}{cm^2}}$$

b) $T_{cool} - T_{cool}^{in} = \frac{1}{1.2}\frac{z_0\ LHR^0}{\dot{m}C_{pw}}\left\{\sin(1.2) + \sin\left[1.2\left(\frac{z}{z_0} - 1\right)\right]\right\}$

Evaluate at $z = L = 2z_0$

$$\Delta T_{cool} = \frac{1}{1.2}\frac{z_0\ LHR^0}{\dot{m}C_{pw}}\left\{\sin(1.2) + \sin\left[1.2(1)\right]\right\}$$

As $\dot{m}C_{pw}$ increases, ΔT_{cool} decreases

$\therefore$ higher $\dot{m}C_{pw}$ indicates lower ΔT_{cool}

$\Rightarrow$ $\boxed{\text{Water sees a higher } \Delta T}$

[5] $\frac{dy}{dt} = 4t - 3t^2 \qquad dt = 0.33 \qquad y(1) = 6$

Backwards

$t_1 = 1.33 \quad f(t_1) = 6 + 0.33\left(4(1.33) - 3(1.33)^2\right) = 6.004$

$t_2 = 1.66 \quad f(t_2) = 6.004 + \underbrace{0.33\left(4(1.66) - 3(1.66)^2\right)}_{= -0.5368} = 5.467$

$t_3 = 1.99 \approx 2 \quad f(t_3) = 5.467 + \underbrace{0.33\left(4(2) - 3(2^2)\right)}_{= -1.32} = 4.147$

Forwards

$t_1 = 1.33 \quad f(t_1) = 6 + \underbrace{0.33\left(4(1) - 3(1)^2\right)}_{= 0.33} = 6.33$

$t_2 = 1.66 \quad f(t_2) = 6.33 + \underbrace{0.33\left(4(1.33) - 3(1.33)^2\right)}_{= 0.004} = 6.334$

$t_3 = 1.99 \approx 2 \quad f(t_3) = 6.334 + \underbrace{0.33\left(4(1.66) - 3(1.66)^2\right)}_{= -0.5368} = 5.797$

[6] fertile - can be converted into a fissile nuclide
fissile - capable of sustaining chain reaction w/ nt of any energy
fissionable - capable of undergoing fission w/ high energy nt's

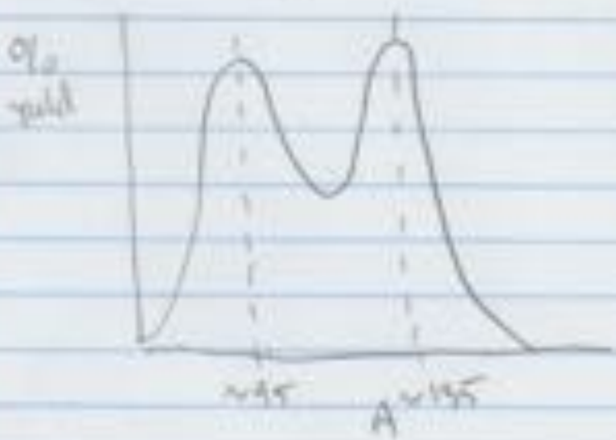
[7] Pure U experiences a very large amount of swelling, and the α phase experiences anisotropic irradiation growth. In general, too many phases on phase diagram that could be present in-core

[8] Smear density is the ratio of fuel volume to total volume of a fuel element. It allows for the accounting of material pores, gaps, etc in the fuel rods

9 We need to enrich U due to the primary component, U-238, not being fissile. We need a higher density of U-235 to be able to reliably sustain a fuel cycle that is acceptably long, as there won't be enough excess reactivity at beginning of life otherwise

Uranium is converted from "yellow-cake" into UF_6 gas in order to be enriched. The gas is centrifuged and tapped from the center
This works since U-235 is physically heavier than U-238, so it isn't thrown as far towards the outside of the centrifuge and is able to move slightly faster than U-238

10 2 primary fission product species are Molybdenum & Cesium.
Typically the 2 fission products aren't equal weights, rather 1 light & 1 heavy. The typical curve is shown below



Mo has A = 95
Cs has A = 133

11 Finite Difference
Finite Volume
Finite Element → Can model any system & any BC
→ continuous representation
→ allows for heterogeneous properties in materials

geometry

Utilized
~ S.O.A