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1. This equation is made by the assumption "steady state",  $\left(\frac{\partial T}{\partial t} = 0\right)$  ✓

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + Q = 0$$

$$\Rightarrow k \frac{\partial T}{\partial x} = -Qx + C_1 \quad \text{--- ①}$$

use boundary condition  $T'(x=x_0=0) = 0$  into ①

$$0 = 0 + C_1$$

$$\therefore C_1 = 0$$

Therefore,  $k \frac{\partial T}{\partial x} = -Qx$

$$\Rightarrow \int \frac{\partial T}{\partial x} dx = -\frac{Q}{k} \int x dx$$

$$\Rightarrow \int dT = -\frac{Q}{k} \frac{1}{2} x^2 + C_2$$

$$\Rightarrow T = -\frac{Q}{k} \frac{1}{2} x^2 + C_2 \quad \text{--- ②}$$

use another B.C.  $T(x=x_1) = T_1$  into ②

$$T(x=x_1=X) = -\frac{Q}{k} \frac{1}{2} X^2 + C_2 = T_1$$

$$\therefore C_2 = T_1 + \frac{Q}{k} \frac{1}{2} X^2 \quad \text{Insert this into ② again}$$

$$\Rightarrow T(x) = -\frac{Q}{k} \frac{1}{2} x^2 + T_1 + \frac{Q}{k} \frac{1}{2} X^2$$

$$T(x) = \frac{Q}{2k} (X^2 - x^2) + T_1 \quad \checkmark$$

- two other  
assumptions

1-0

const. k

10/  
12

2.

- this is a fuel pellet, not

a fission  
particle

- do not need  
spherical coords here

- good derivation, but  
does not apply

6/18

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 k \frac{\partial T}{\partial r}) + Q = 0$$

$$\frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r}) = -Q r^2$$

$$r^2 \frac{\partial T}{\partial r} = \int -Q r^2 dr \Rightarrow \frac{-Q}{3} r^3 + C_1$$

$$T(r_1) = T_1$$

$$T'(r_0) = 0, r_0 = 0, r_1 = R$$

$$0 = 0 + C_1$$

$$0 = 0 + C_1$$

$$r^2 \frac{\partial T}{\partial r} = -\frac{Q}{3} r^3 \rightarrow \frac{\partial T}{\partial r} = -\frac{Q}{3} r$$

$$T(r) = \int -\frac{Q}{3} r dr \rightarrow -\frac{Q}{6} r^2 + C_2$$

$$r_1 = R, T(r) = T_1$$

$$T(R) = T_1 = -\frac{QR^2}{6} + C_2 \Rightarrow C_2 = T_1 + \frac{QR^2}{6}$$

$$T(r) = -\frac{Q}{6} r^2 + T_1 + \frac{QR^2}{6}$$

$$T(r) - T_1 = \frac{Q}{6k} (R^2 - r^2) \rightarrow \text{spherical 1-D steady state}$$

3.

(\*heat generation rate by U3Si2\*)

12/14

```
In[1]:= sigma = 570 (*barns*);
d = 15.67 (*g/cm^3*);
en = 19.5 (*%*);
phi = 2 * 10^12 (*n/cm^2-s*);
Ef = 200 (*MeV*);
M = 3 * (en * 0.01 * 235 + (1 - en * 0.01) * 238) + 2 * 28;
Nf = (d * 6.02 * 10^23 / M) * 3 * en * 0.01;
Q = Ef * (10^6 * 1.6 * 10^(-19)) * Nf * phi * (sigma * 10^-24)
```

Out[1]= 262.045

✓

(\*heat generation rate by UO2\*)

```
In[4]:= sigma = 587 (*barns*); → we were assuming 570 barns here
d = 10.97 (*g/cm^3*); → will not change if T & neutron
phi = 2 * 10^12 (*n/cm^2-s*); spectrum do not change
Ef = 200 (*MeV*);
M = (x * 0.01 * 235 + (1 - x * 0.01) * 238) + 2 * 16;
Nf = (d * 6.02 * 10^23 / M) * x * 0.01;
Solve[262.045 == Ef * (10^6 * 1.6 * 10^(-19)) * Nf * phi * (sigma * 10^-24), x]
```

Out[4]= {{x → 28.4279}}

process correct

4.

12/12

In[\*]:= (\*Calculate LHR considering variability in Z\*)

```
In[13]:= Z = 180 (*cm*);
Z0 = 150 (*cm*);
L0 = 150 (*W/cm*);
r = 1.1;
LHR = L0 * Cos[Pi / (2 r) (Z / Z0 - 1)]
```

Out[13]= 143.924

✓ → would like to see units on this

(\*Calculate Tcool for water considering variability in Z\*)

```
Z = 300 (*cm*);
Z0 = 150 (*cm*);
L0 = 150 (*W/cm*);
r = 1.1;
m = 0.22 (*kg/s*);
Cp = 4200 (*J/kg-K*);
delta T = 2 * r / Pi (Z0 * L0 / (m Cp)) (Sin[Pi / (2 r)] + Sin[Pi / (2 r) (Z / Z0 - 1)])
```

Out[17]= 33.7575

(\*Calculate Tcool for sodium considering variability in Z\*)

```
In[19]:= Z = 300 (*cm*);
Z0 = 150 (*cm*);
L0 = 150 (*W/cm*);
r = 1.1;
m = 0.12 (*kg/s*);
Cp = 1404 (*J/kg-K*);
delta T = 2 * r / Pi (Z0 * L0 / (m Cp)) (Sin[Pi / (2 r)] + Sin[Pi / (2 r) (Z / Z0 - 1)])
```

Out[19]= 185.137

✓ — please write final answer as well!

$$\Delta T(\text{Na}) > \Delta T(\text{H}_2\text{O})$$

5.

```
(*Backward Euler*)
In[11]:= dt = 0.33;
n = 6;
Do[n = n + dt * (4 (k + dt) - 3 (k + dt)^2);
Print[n], {k, 1, 2 - dt, dt}]
6.00439
5.46755
4.17385

(*Forward Euler*)
In[10]:= dt = 0.33; n = 6; Do[n = n + dt * (4 k - 3 k^2); Print[n], {k, 1, 2 - dt, dt}]
6.33
6.33439
5.79755
```

→ I wanted to see your work here

- Yes answers are correct

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6.

5/5

- 1) A fertile material is a material that, although not itself fissionable, can be converted into a fissile nuclide by neutron absorption and subsequent nuclei conversions. ✓
- 2) A fissile nuclide is capable of sustaining a nuclear fission chain reaction with neutrons of any energy. ✓
- 3) A fissionable nuclide is capable of undergoing fission (even with a low probability) after capturing a high energy neutron. ✓

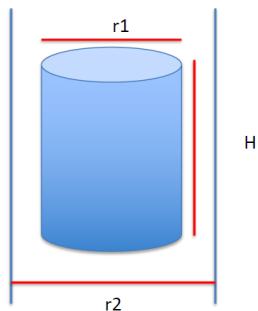
7.

4/4

We don't use pure metallic U as a fuel form because pure uranium dramatically swells during thermal cycling. ✓  
In addition, pure uranium has three phases, including α-phase is orthorhombic, β-phase is tetragonal, γ-phase is body-centered cubic. And alpha U has both anisotropic thermal expansion and anisotropic irradiation growth. ✓

8.

Smear density is the ratio of fuel volume to internal volume of the fuel element. ✓



4/4

$$\text{Smear density} = \pi r_1^2 h / \pi r_2^2 h$$

From: Professor Beeler's notes

It is important for metal fuel designs to have a low enough smear density to properly allow for fuel swelling over the course of fuel life. ✓

→ I would prefer you paraphrase me ---  
→ use your own words

9. Because water has higher neutron absorption. UF6 and UO2. Use different mass to separate U-238 and U235.

→ water doesn't absorb neutrons

3/8

10. One broad peak centered around  $A=95$ , the other around  $A=135$ .

2/4

11. Back Euler and Forward Euler. X

space discretization 2/8