88/100

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NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-8, 22/30

Question 1:
U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

- the fissile isotope is $\boxed{^{235}U}$
- ^{235}U makes up 0.7% of natural U and because there are 3 U atoms for every (3+5) = 8 fuel atoms, $\varrho_{235} = 0.7\% \frac{3}{8} = \boxed{0.2625\% \ ^{235}U}$

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points) assume $M_U = 238 \frac{g}{mol}$

- energy release rate from U_3Si_2: $Q = E_f N_f \sigma_f \Phi$; $N_f = \frac{\varrho N_a \delta}{M_F}$
- so the question is: when would the fissile atom density of U_3Si_5 be equal to that of U_3Si_2 if $\varrho_2 = 3\%$? $\varrho_5 = ?$

$N_5 = N_2$; $\frac{\cancel{3} \varrho_5 \cancel{N_A} \delta_5}{M_5} = \frac{\cancel{3} \varrho_2 \cancel{N_A} \delta_2}{M_2}$; $M_5 = 3(238 \frac{g}{mol}) + 5(28 \frac{g}{mol}) = 854 \frac{g}{mol}$

$M_2 = 3(238 \frac{g}{mol}) + 2(28 \frac{g}{mol}) = 770 \frac{g}{mol}$

- assuming that the densities are the same: $\delta_5 = \delta_2$

-5, Just use the ratio of the U density
0.03*11.31/7.5 = 4.52%

$\frac{\varrho_5}{M_5} = \frac{\varrho_2}{M_2}$; $\varrho_5 = \varrho_2 \frac{M_5}{M_2} = 3\% \left(\frac{854 \frac{g}{mol}}{770 \frac{g}{mol}} \right) = \boxed{3.33\%}$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

- U_3Si_5 would be inferior to U_3Si_2 because it has a lower fissile atom density

-3, thermal conductivity?

-> a mass of U_3Si_5 contains fewer fuel atoms than the same mass of U_3Si_2, and so would produce less power

- U_3Si_5 does have a higher melting temperature, however, which would make the fuel safer

Question 2: -0, 35/35

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

- assume that we are talking about the surface temperature of the fuel pellet

$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_f h_{cool}} = 580\,K + \frac{250\,W}{cm}\,\frac{1}{2\pi}\,\frac{1}{4.5\,mm}\,\frac{cm^2\,K}{2.5\,W}\,\frac{10\,mm}{cm} = 615.37\,K$$

$$T_{ci} = T_{co} + \frac{LHR\, t_c}{2\pi R_f K_c} = 615.37\,K + \frac{250\,W}{cm}\,\frac{0.6\,mm}{2\pi}\,\frac{1}{4.5\,mm}\,\frac{cm\,K}{0.17\,W} = 646.57\,K$$

$$\left.\begin{array}{l} K_{gas} = A \times 10^{-6}\, T_{ci}^{\,0.79} \\ A_{He} = 16\,,\ A_{Xe} = 0.7 \end{array}\right\} \begin{array}{l} K_{He} = 2.66 \times 10^{-3}\ \frac{W}{cm\,K} \\ K_{Xe} = 1.16 \times 10^{-4}\ \frac{W}{cm\,K} \end{array}$$

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b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$$T_0 = T_s + \frac{LHR}{4\pi K} = 958.18\,K + \frac{250\,W}{cm}\,\frac{1}{4\pi}\,\frac{cm\,K}{0.2\,W} = 1057.65\,K$$

$$\sigma^* = \frac{\alpha E\,(T_0 - T_s)}{4(1-\nu)} = \frac{7.5\times10^{-6}}{K}\,(1057.65 - 958.18)\,K\;246.7\,GPa\;\frac{1}{4}\;\frac{1}{1-0.25} = 0.0613\,GPa$$

- the hoop stress $\sigma_{\theta\theta}$ is the highest $\sigma_{\theta\theta} = -\sigma^*\left(1 - 3\left(\frac{r}{R_f}\right)^2\right)$ at $r = R_f$

$$\sigma_{\theta\theta} = -0.0613\,GPa\left(1 - 3\left(\frac{R_f}{R_f}\right)^2\right) = \boxed{0.1226\,GPa}$$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

- the thermal conductivity is lower for UO_2 so $T_0 - T_s$ is greater and so is σ^*
- therefore the $\boxed{\text{stress would be higher}}$

d) What assumptions were made in your calculations for a) and b)? (5 points)

- in deriving temperature profile:
  - steady state
  - axisymmetric
  - temperature $\neq f(z)$
  - fuel thermal conductivity $\neq f(T)$
  - linear profiles through the coolant, clad, and gap

- stress calculation:
  - body is static (not in motion)
  - gravity is negligible
  - axisymmetric
  - small strain (elastic deformation)

2 cont'd)

a cont'd) $K_{gap} = K_{He}^{1-y} \, K_{Xe}^{y} \; ; \; y = 0.05$

$$K_{gap} = 2.27 \times 10^{-3} \frac{W}{cm\,K}$$

$$T_s = T_{ci} + \frac{LHR \; t_g}{2\pi \, R_F \, K_{gap}} = \frac{250 \cancel{W}}{\cancel{cm}} \cdot \frac{80 \times 10^{-6} \cancel{m}}{2\pi} \cdot \frac{1}{4.5 \cancel{mm}} \cdot \frac{\cancel{cm}\,K}{2.27 \times 10^{-3} \cancel{W}} \cdot \frac{1000 \cancel{mm}}{1 \cancel{m}} + 646.57K$$

$$\boxed{T_s = 958.18 \; K}$$

Question 3: -4, 31/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

- to solve stress eqs in general:
  - static body
  - gravity negligible
  - axisymmetric
  - small strain
- thin wall approximation: stress state does not vary across the wall of the cylinder

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\bar{\sigma}_\theta = \frac{pR}{\delta} = \frac{6\,\text{MPa} \cdot 5.6\,\text{mm}}{0.6\,\text{mm}}$$

→ assume cylinder is bound axially

$$\bar{\sigma}_z = \frac{pR}{2\delta} = \frac{6\,\text{MPa} \cdot 5.6\,\text{mm}}{2 \cdot 0.6\,\text{mm}}$$

$$\bar{\sigma}_r = -\frac{1}{2}p = \frac{-1 \cdot 6\,\text{MPa}}{2}$$

$$\boxed{\bar{\sigma}_\theta = 56\,\text{MPa} \quad \bar{\sigma}_z = 28\,\text{MPa} \quad \bar{\sigma}_r = -3\,\text{MPa}}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

- the thick wall eqs are $\sigma_{rr} = -p \dfrac{(\frac{R_o}{r})^2 - 1}{(\frac{R_o}{R_i})^2 - 1}$, $\sigma_{\theta\theta} = p \dfrac{(\frac{R_o}{r})^2 + 1}{(\frac{R_o}{R_i})^2 - 1}$, $\sigma_{zz} = p \dfrac{1}{(\frac{R_o}{R_i})^2 - 1}$
- from the average radius and δ, $R_i = R - \frac{\delta}{2} = 5.3\,\text{mm}$, $R_o = R + \frac{\delta}{2} = 5.9\,\text{mm}$
- the hoop stress is highest at R_i, where $\sigma_{\theta\theta} = 6\,\text{MPa} \dfrac{(\frac{5.9\,\text{mm}}{5.3\,\text{mm}})^2 + 1}{(\frac{5.9\,\text{mm}}{5.3\,\text{mm}})^2 - 1} = 56.16\,\text{MPa}$

$\%\ \text{error} = \left|\dfrac{56 - 56.16}{56.16}\right| = \boxed{0.29\%\ \text{error}}$ but $\boxed{\text{NOT conservative}}$ -4, Calculate stress at TWO radii and compare

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$\varepsilon_{rr} = \frac{1}{E}(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})) = -5.35 \times 10^{-4}$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})) = 6.54 \times 10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})) = 8.96 \times 10^{-5}$$

$$\varepsilon_{rz} = \frac{1}{2G}\sigma_{rz} = 0$$

$$\sigma(r, \theta, z) = \begin{bmatrix} 28 & 0 & 0 \\ 0 & 56 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{MPa}$$

$$\varepsilon(r, \theta, z) = \begin{bmatrix} -5.35 \times 10^{-4} & 0 & 0 \\ 0 & 6.54 \times 10^{-4} & 0 \\ 0 & 0 & 8.96 \times 10^{-5} \end{bmatrix}$$