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94/100

03/03/17 Exam # 1 NucE 497
Reactor Fuel Performance

-0, 30/30

Problem 1

a) The fissile isotope is uranium-235.
Natural uranium contains 0.7% of its mass that is uranium-235.
For 3 uranium atoms $\rightarrow$ $3 \times 0.7\% = 2.1\%$

b) $Q = E_f N_f \sigma_f \varphi_{th}$

$E_f = 3.0 \times 10^{-11}$ J / fission

$\sigma_{f235} = 5.50 \times 10^{-22} cm^2$

$\varphi_{th} = 3.2 \times 10^{13}$ n / (cm² . s)

For U_3Si_2:

$$N_f = 9 \times \frac{N_a \times \rho_U}{\underset{\simeq 238\ g/mol}{M(U)}} = \frac{0.03 \times 6.022 \times 10^{23} \times \overset{g/cm^3}{11.31}}{238}$$

$= 8.585 \times 10^{20}$ atoms of U235 / cm³.

$\hookrightarrow Q = 453.296\ W/cm^3$.

For U_3Si_5:

$$N_f = 9 \times \frac{N_a \times \rho_U}{M(U)} = 9 \times \frac{6.022 \times 10^{23} \times 7.5}{238}$$

$$Q = q \times \frac{6.022\times10^{23} \times 7.5}{238} \times 3\times10^{-11} \times 5.5\times10^{-22} \times 3.2\times10^{13}$$

$$= 453.296 \text{ W/cm}^3$$

→ Solving for q:

$q = 0.04524 \quad = 4.524\%$

U_3Si_5 has to be enriched to 4.5% to have the same energy release rate of U_3Si_2 enriched to 3%.

c) The thermal conductivity of U_3Si_2 is higher than the thermal conductivity of U_3Si_5, which is a great advantage of using U_3Si_2 (even comparing with UO_2) instead of U_3Si_5 (0.23 W/(cmK) > 0.125 W/(cmK)

Also, the density of Uranium metal is higher for U_3Si_2 (11.31 g/cm³ > 7.5 g/cm³)

Based on these two criteria, U_3Si_2 is definitely a better potential fuel than U_3Si_5.

-2, 33/35

Problem 2

$R_f = 4.5mm$

$t_g = 80 \mu m$

$\delta_c = 0.6 mm$

$LWR = 250 W/cm$

$T_{cool} = 580 K$

5% Xe

$h_{cool} = 2.5 W/(cm^2 . K)$

a) $T_s = ?$

$$T_{CO} - T_{cool} = \frac{LHR}{2\pi R_f h_{cool}}$$

$$T_{CO} = 580 + \frac{250}{2\pi \times 0.45 \times 2.5} = 615.37K$$

$$T_{IC} = T_{CO} + \frac{LHR \, \delta_C}{2\pi R_f K_c} = T_{CO} + \frac{250 \times 0.06}{2\pi \times 0.45 \times 0.17}$$

$$= 646.57 K$$

$$h_{gap} = \frac{K_{gap}}{t_g}$$

$$K_{gap} = \left[16 \times 10^{-6} \times T_{IC}^{0.79}\right]^{0.95} \times \left[0.7 \times 10^{-6} \times T_{IC}^{0.79}\right]^{0.05}$$

$$= 2.27 \times 10^{-3} \; W/(cm.K)$$

$$h_{gap} = 0.284 \; W/(cm^2 . K)$$

$$T_S = T_{IC} + \frac{LHR}{2\pi R_f h_{gap}} = 957.91 K$$

b) Maximum stress:

$$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2)$$

with $\eta = \frac{r}{R_f}$ and $\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$

Maximized at $r = R_f$.

$$T_0 = T_s + \frac{LHR}{4\pi k} = 957.91 + \frac{250}{4\pi \times 0.2}$$

$$= 1057.38\ K$$

$$\sigma_{\theta\theta} = -\frac{7.5\times10^{-6} \times 246.7 \times (1057.38 - 957.91)}{4(1 - 0.25)}(1 - 3)$$

$$\simeq 0.123\ GPa.$$

c) If the pellet was UO_2, then the thermal conductivity would be much lower ($k_f = 0.03\ W/(cmK)$ instead of $0.2\ W/(cm\,K)$).

$\rightarrow T_0$ is higher $\rightarrow \sigma^*$ is higher $\rightarrow \sigma_{\theta\theta_{max}}$ is higher.

d) I assumed the thermal conductivity of the (and cladding) fuel is constant with temperature, steady-state solution, the behavior is axisymmetric and Temperature is constant with z.

-2, you made several more assumptions than that

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Problem 3 -4, 31/35

a) Assumptions:
- Stress is constant through the wall of the cylinder
- Static body
- Gravity is negligible
- Axisymmetric problem
- Isotropic material response

b) $\bar{\sigma}_\theta = \frac{PR}{\delta} = \frac{6 \times 5.6}{0.6} = 56 \text{ MPa}$

$\bar{\sigma}_z = \frac{PR}{2\delta} = \frac{56}{2} = 28 \text{ MPa}$

$\bar{\sigma}_r = -\frac{1}{2}P = -\frac{6}{2} = -3 \text{ MPa}$

-4 Test stress at multiple radii to see if it is constant

c) With the thin wall approximation, we get constan components of the stress, which is not accurate.
If we consider a thick wall, with $t_{gap} = 80\,\mu m$, we have

$$\underbrace{\sigma_{\theta\theta}(R_i)}_{\text{maximum hoop stress}} = 6 \times \frac{\left[(5.6 + 80\times10^{-3} + 0.6)/(5.6 + 80\times10^{-3})\right]^2 + 1}{\left[(5.6 + 80\times10^{-3} + 0.6)/(5.6 + 80\times10^{-3})\right]^2 - 1} = 59.95 \text{ MPa}$$

So there is a non-negligible change in the stress.

(I am not sure I understand the second question...)

Depending on the gap thickness, the thin wall approximat can be very inaccurate. In fuel performance codes, we care about the stress in the cladding, because safety requires that there is no rupture in the cladding. Therefore, w need to model stress in the cladding in an accurate way.

d) $E = 70$ GPa , $\nu = 0.41$

$$\varepsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right)$$

$$= \frac{1}{E}\left(-\frac{1}{2}p - \nu\left(\frac{pR}{8} + \frac{pR}{28}\right)\right)$$

$$= \frac{1}{E}\left(-\frac{1}{2}p - \frac{3}{2}\nu\frac{pR}{8}\right)$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right)$$

$$= \frac{1}{E}\left(\frac{pR}{8} - \nu\left(-\frac{1}{2}p + \frac{pR}{28}\right)\right)$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{\theta\theta} + \sigma_{rr}\right)\right)$$

$$= \frac{1}{E}\left(\frac{pR}{28} - \nu\left(\frac{pR}{8} - \frac{1}{2}p\right)\right)$$

$$\varepsilon \rightarrow \begin{pmatrix} \varepsilon_{rr} & 0 & 0 \\ 0 & \varepsilon_{zz} & 0 \\ 0 & 0 & \varepsilon_{\theta\theta} \end{pmatrix}$$

$$\sigma \rightarrow \begin{pmatrix} \overline{\sigma}_r & 0 & 0 \\ 0 & \overline{\sigma}_z & 0 \\ 0 & 0 & \overline{\sigma}_\theta \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}p & 0 & 0 \\ 0 & \frac{pR}{28} & 0 \\ 0 & 0 & \frac{pR}{8} \end{pmatrix}$$

$$\varepsilon_{rr} = \frac{1}{70 \times 10^3} \left(-3 - 0.41 \times (56 + 28) \right)$$

$$= -5.349 \times 10^{-4}$$

$$\varepsilon_{\theta\theta} = 6.536 \times 10^{-4}$$

$$\varepsilon_{33} = 8.957 \times 10^{-5}$$

$$\varepsilon \rightarrow \begin{pmatrix} -5.349 \times 10^{-4} & 0 & 0 \\ 0 & 8.957 \times 10^{-5} & 0 \\ 0 & 0 & 6.536 \times 10^{-4} \end{pmatrix}$$

$$\sigma \rightarrow \begin{pmatrix} -3 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 56 \end{pmatrix} \text{ MPa}$$