TOTAL TIME ~ 1 hr

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

5 min

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

1 min a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

U235 is the main fissile isotope and it has 0.7 %wt natural enrichment. (Pu239 and Pu241 are other fissile isotopes can form)

2 Mins b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$$Q_{U_3Si_5} = Q_{U_3Si_2}$$

$$\underline{\underline{E}}\,\underline{\underline{\sigma_f}}\,N_f^u\,\underline{\underline{\phi}} = \underline{\underline{E}}\,\underline{\underline{\sigma_f}}\,N_f^u\,\underline{\underline{\phi}}$$

$$\left[N_f^u\right]_{U_3Si_5} = \left[N_f^u\right]_{U_3Si_2}$$

$$\frac{\varepsilon_1 \rho_u' N_A}{M_u} = \frac{\varepsilon_2 \rho_u'' N_A}{M_u}$$

$$\varepsilon' = \varepsilon_2 \frac{\rho_u''}{\rho_u'} = 0.03\left(\frac{11.31}{7.5}\right)$$

$$\varepsilon' = 0.045 \text{ or } \boxed{4.5\ \%wt}$$

2 Mins c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

If we assume that U3Si5 and U3Si5 have similar neutronic and thermomechanical properties, U3Si5 would not be good selection as a potential fuel compared to U3Si2, since U density is lower which requires higher enrichment

21 mins

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

5 mins a) What is the surface temperature of the fuel rod? (15 points)

$$T_{ci} = T_{cool} + \frac{LHR}{2\pi R_f}\left[\frac{1}{h_{cool}} + \frac{t_c}{k_c}\right] = 580 + \frac{250}{2\pi(0.45)}\left[\frac{1}{2.5} + \frac{0.06}{0.17}\right] = 646\,K \rightarrow$$

$k_{gap} = 2.36\times10^{-3}$ W/cmK

$h_{gap} = k_{gap}/t_{gap} = 0.294$ W/cm²K

$$T_s = T_{ci} + \frac{LHR}{2\pi R_f h_{gap}} = 646 + \frac{250}{2\pi(0.45)(0.294)} = \boxed{947K}$$

10 mins b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$$\sigma_{\theta\theta} = -\sigma^*\left[1 - 3\eta^2\right]$$

$$\text{If } \eta = 1 \longrightarrow \sigma_{\theta\theta} = \sigma_{\theta\theta}^{max} = 2\sigma^* \quad \text{where} \quad \sigma^* = \frac{\alpha E\left[T_o - T_s\right]}{4(1-\nu)}$$

$$= \frac{(7.5\times10^{-6})(246.7\times10^3)(99.5)}{2(1-0.25)}$$

$$\cong \boxed{123\ MPa}$$

2 mins c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

I would expect that the stress to be higher if the pellet was UO2. Because, UO2 has lower thermal conductivity which results higher temperature gradient. Higher temperature gradient causes higher thermal stress.

4 mins d) What assumptions were made in your calculations for a) and b)? (5 points)

- constant thermal conductivity
- axisymmetric system
- steady state
- uniform heat generation
- static body
- gravity was neglected
- no shear stress

ASSUMPTIONS for (A)

ASSUMPTIONS for (B)

33mins

Question 3:

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm.

3 mins a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

- The wall is assumed to be very thin compared to the other dimensions
- The stress is constant through the wall of the cylinder
- Stress is independent of the angular coordinate
- P is constant

5 min b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\bar{\sigma}_\theta = \frac{PR}{\delta} = \frac{6\,(0.59)}{0.06} = 59 \text{ MPa}$$

$$\bar{\sigma}_z = \frac{PR}{2\delta} = \frac{6\,(0.59)}{2\,(0.06)} = 29.5 \text{ MPa}$$

$$\bar{\sigma}_r = -\frac{1}{2}P = -\frac{1}{2}6 = -3 \text{ MPa}$$

5 mins c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

If we don't use thin all assumption:

$$\bar{\sigma}_{\theta\theta} = P\frac{(R_o/R_i)^2+1}{(R_o/R_i)^2-1} = 6\,\frac{(0.59/0.53)^2+1}{(0.59/0.53)^2-1} = 56.2 \text{ MPa}$$

Thin wall approximation is pretty accurate compared to this result. In addition, it gives slightly higher value which makes it more conservative.

20 mins d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\text{Stress tensor} = \begin{bmatrix} \sigma_{zz} & \tau_{z\theta} & \tau_{zr} \\ \tau_{\theta z} & \sigma_{\theta\theta} & \tau_{\theta r} \\ \tau_{rz} & \tau_{r\theta} & \sigma_{rr} \end{bmatrix} \xrightarrow{} \text{Since } \sigma_{rr} << \text{ compared to } \sigma_{\theta\theta} \text{ and } \sigma_{zz} \Rightarrow \sigma_{rr} \approx 0$$

0 0

$$\Longrightarrow \sigma_{ij} = \begin{bmatrix} \sigma_{zz} & 0 \\ 0 & \sigma_{\theta\theta} \end{bmatrix} = \begin{bmatrix} 29.5 & 0 \\ 0 & 59 \end{bmatrix}$$

$$\sigma_{ij} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{zz} \\ u_r/r \end{bmatrix}$$

$$= \frac{70}{(1+0.41)(1-2(0.41))} \begin{bmatrix} 0.59 & 0.41 \\ 0.41 & 0.59 \end{bmatrix} \longrightarrow \varepsilon_{ij} = \begin{bmatrix} u_{zz} & 0 \\ 0 & u_r/r \end{bmatrix} = A \cdot B^{-1} = \begin{bmatrix} 0.1813 & 0 \\ 0 & 0.3626 \end{bmatrix}$$