

Quantin Fawc 02/11/2021

Exam 1

NE 591 (010)

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$$1) \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + q = 0$$

$$\Rightarrow \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = -q \Rightarrow k \frac{\partial T}{\partial x} = -qx + c_1$$

$$\text{at } x=0 \Rightarrow 0 = +c_1 \quad \checkmark$$

$$\frac{\partial T}{\partial x} = -\frac{q}{k} x \Rightarrow T(x) = -\frac{q}{2} x^2 + c_2$$

$$\text{at } x_1 = X \Rightarrow T_1 = -\frac{q}{2} X^2 + c_2 \Rightarrow c_2 = T_1 + \frac{q}{2} X^2$$

$$T(x) = T_1 + \frac{q}{2} X^2 - \frac{q}{2} x^2 = T_1 + \frac{q}{2} (X^2 - x^2)$$

The assumption used are:

- Steady state ✓
- Constant in y and z ✓
- Constant thermal conductivity ✓

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dropped k $k - 2$

$$2) \quad LHR = Q \pi R_1^2 = 8400 \times \pi \times 0.6^2 = 452 \frac{W}{cm}$$

$$\cancel{T_{e0}} = \cancel{T_{cool}}$$

$$T_{n4} - T_{cool} = \frac{LHR}{2\pi R_1 h_{cool}} = \frac{452}{2 \times \pi \times 0.6 \times 5.5} = 21.8 K$$

$$\underline{T_{n4}} = T_{cool} + 21.8 = 800 + 21.8 = \underline{821.8 K}$$

$$T_{n1} - T_{n4} = \frac{LHR}{2\pi R_1 h_{coating}} = \frac{452}{2\pi \times 0.6 \times 0.05} \times \frac{n_4 - n_1}{0.85 - 0.85} = \frac{452 \times 0.85 - 0.85}{2\pi \times 0.6 \times 0.05}$$

↑
h_{coating}

$$= 234.0 K$$

$$\underline{T_{n1}} = T_{n4} + 234.0 K = 821.8 + 234 = \underline{845.8 K}$$

$$T_{n2} - T_{n1} = \frac{LHR}{2\pi R_1 h_{rad}} = \frac{LHR}{2\pi R_1} \times \frac{n_1 - n_2}{h_{rad}}$$

$$= \frac{452}{2\pi \times 0.6} \times \frac{0.85 - 0.8}{0.75}$$

$$= \underline{39.40.0 K}$$

$$\underline{T_{n2}} = T_{n1} + 40.0 = 845.8 + 40.0 = \underline{885.8 K}$$

$$T_{n3} - T_{n2} = \frac{LHR}{2\pi R_1 h_{gap}} = \frac{LHR}{2\pi R_1} \times \frac{n_2 - n_3}{h_{gap}}$$

$$= \frac{452}{2\pi \times 0.6} \times \frac{0.8 - 0.6}{0.25} = 25.9 K$$

$$T_{n_1} = T_{n_2} + 95.9 \text{ K} = 885.8 + 95.9 = \underline{981.7 \text{ K}}$$

$$T_{\text{centerline}} - T_{n_1} = \frac{L \cdot R}{4 \pi k_f} = \frac{436}{4 \times \pi \times 0.5} = 71.9 \text{ K}$$

$$T_{\text{centerline}} = T_{n_1} + 71.9 = 981.7 + 71.9 = \underline{1053.6 \text{ K}}$$

↑
centerline temperature

$$T(n=0.4) = \frac{Q}{4k} (R_i^2 - n^2) + T_{n_1}$$

$$= \frac{400}{4 \times 0.3} (0.6^2 - 0.4^2) + 981.7$$

$$T(n=0.4) = \underline{1021.7 \text{ K}}$$

$$3) \text{ i) } Q = E_f \times N_f \times \sigma_f \times \phi$$

$$E_f = 200 \text{ MeV} \quad \phi = 2 \times 10^{12} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s}}$$

$$\sigma_f = 530 \times 10^{-24} \text{ cm}^2$$

$$N_f = \text{mass } U = 0.193 \times 235 =$$

$$\text{mass } U : 0.193 \times 235 + (1 - 0.193) \times 238 = 237.4 \text{ amu}$$

$$M_{U_{235}} = 3 \times 237.4 + 2 \times 238 = 768.2 \text{ amu}$$

$$N_f = \underset{\substack{\text{Density} \\ \downarrow}}{15.67} \times \frac{1}{\underset{\substack{\text{M}_{\text{U}_3\text{Si}_2}}{261.2}}{\text{}}} \times \underset{\substack{\text{avogadro} \\ \text{number} \\ \downarrow}}{6.02 \times 10^{23}} \times 3 \times \underset{\substack{\text{3U} \\ \text{U}_3\text{Si}_2}}{0.795}$$

$$= 7.18 \times 10^{21} \frac{\text{U}^{235}}{\text{cm}^3}$$

$$Q = 200 \times 10^6 \times 1.602 \times 10^{-19} \times 7.18 \times 10^{21} \times 570 \times 10^{-24} \times 2 \times 10^{12}$$

$$Q = 262.7 \frac{\text{W}}{\text{cm}^3}$$

b) $x = \text{mole fraction}$

$$\text{mass U} : 235x + (1-x)238$$

$$\text{M}_{\text{UO}_2} = 235x + (1-x)238 + 2 \times 16 = 2270 - 3x$$

$$Q = 200 \times 10^6 \times 1.602 \times 10^{-19} \times N_f \times 570 \times 10^{-24} \times 2 \times 10^{12}$$

$$= 3.65 \times 10^{-20} \times N_f$$

$$N_f = \frac{262.7}{3.65 \times 10^{-20}} = 7.18 \times 10^{21} \frac{\text{U}^{235}}{\text{cm}^3}$$

$$7.18 \times 10^{24} = 10.94 \times \frac{1}{280.12} \times 6.02 \times 10^{23} \times 1 \times 2$$

$$1.94 \times 10^{24} - 2.15 \times 10^{22} x = 6.60 \times 10^{24} x$$

$$x = 0.294 \quad \checkmark$$

It would require an enrichment of ~~25%~~ 29.4%

$$4) a) LHR \left(\frac{y}{z_0} \right) = LHR^0 \cos \left(\frac{\pi}{28} \left(\frac{y}{z_0} - 1 \right) \right)$$

$$z_0 = 1.5 \text{ m} \quad \checkmark$$

$$y = 1.1$$

$$\text{At } y = 1.1 \text{ m}$$

$$LHR = 150 \times \cos \left(\frac{\pi}{28 \times 1.5} \left(\frac{1.1}{1.5} - 1 \right) \right)$$

$$\underline{LHR = 743.9 \frac{\text{W}}{\text{cm}}} \quad \text{at } y = 1.1 \text{ m}.$$

$$b) T_{\text{cool}}^{\text{out}} - T_{\text{cool}}^{\text{in}} = \frac{1}{\frac{\pi}{28} \times \rho \times c_{pw}} \times \frac{z_0 \times LHR_0}{\sin \left(\frac{\pi}{28} \right) - \sin \left(\frac{\pi}{28} \left(\frac{y}{z_0} - 1 \right) \right)}$$

For coolant i):

$$T_{\text{cool}}^{\text{out}} - T_{\text{cool}}^{\text{in}} = \frac{1}{\frac{\pi}{28} \times 1.3 \times 750} \times \frac{1.5 \times 750}{0.72 \times 4200} \times \left(\sin \left(\frac{\pi}{28 \times 1.5} \right) - \sin \left(\frac{\pi}{28 \times 1.5} \left(\frac{1.1}{1.5} - 1 \right) \right) \right)$$

$$\Delta T_{\text{cool}} = 0.338 \text{ K/m} \quad \checkmark$$

$$\text{For coolant ii):} \quad T_{\text{cool}}^{\text{out}} - T_{\text{cool}}^{\text{in}} = \frac{1}{\frac{\pi}{28} \times 1.3 \times 750} \times \frac{1.5 \times 750}{0.72 \times 1400} \times \left(2 \sin \left(\frac{\pi}{28 \times 1.5} \right) \right) = 1.85 \text{ K/m}$$

The coolant is with sodium has the largest change in temperature. ✓

5)

Forward:

$$\Delta t = 0.33$$

$$t_0 = 1$$

$$t_1 = 1.33$$

$$y(t_1) = y(t_0) + \Delta t \left. \frac{\partial y}{\partial t} \right|_{t_0} = 6 + 0.33 \times (4 \times 1 - 3 \times 1^2)$$

$$y(t_1) = 6.33$$

$$t_2 = 1.66$$

$$y(t_2) = y(t_1) + \Delta t \left. \frac{\partial y}{\partial t} \right|_{t_1} = 6.33 + 0.33 \times (4 \times 1.33 - 3 \times 1.33^2)$$

$$y(t_2) = 6.3344$$

$$t_3 = 2$$

$$y(t_3) = y(t_2) + \Delta t \left. \frac{\partial y}{\partial t} \right|_{t_2} = 6.3344 + 0.33 \times (4 \times 1.66 - 3 \times 1.66^2)$$

$$\cancel{y(t_3) = 5.0}$$

$$y(t_3) = 6.3344 + 0.33 \times (4 \times 1.66 - 3 \times 1.66^2)$$

$$y(t_3) = 5.7976$$

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Backward:

$$t_1 = 1.33$$

$$y(t_1) = y(t_0) + dt \left. \frac{dy}{dt} \right|_{t_1}$$

$$= 6 + 0.33 \times (4 \times 1.33 - 3 \times 1.33^2)$$

$$\underline{y(t_1) = 6.0044}$$

$$t_2 = 1.66$$

$$y(t_2) = y(t_1) + dt \left. \frac{dy}{dt} \right|_{t_2}$$

$$= 6.0044 + 0.33 \times (4 \times 1.66 - 3 \times 1.66^2)$$

$$\underline{y(t_2) = 5.4676}$$

$$t_3 = 2$$

$$y(t_3) = y(t_2) + dt \left. \frac{dy}{dt} \right|_{t_3}$$

$$= 5.4676 + 0.33 \times (4 \times 2 - 3 \times 2^2)$$

$$\underline{y(t_3) = 4.7676}$$

6) fissionable: a nuclide capable to undergo ^{exo}fission with thermal or fast neutron

4/3

fissile: a nuclide capable to undergo fission with thermal neutron

fertile: a nuclide not fissionable but capable of being converted into a fissionable nuclide.

only high E neutron

u/y

- 4) - Pure uranium has swells dramatically during thermal cycle ✓
 - One of the phase (α phase) has anisotropic thermal expansion and anisotropic irradiation growth. ✓

y/y

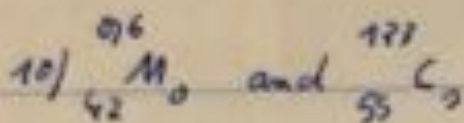
- 8) Swell density: ratio of the volume occupied by the fuel element to the total available volume inside the cladding. ✓
 It is important because all fuel element materials swell and so need more space in order to not increase the stress on the cladding.

6/8

- 9) We enriched uranium because ^{235}U natural enrichment is 0.7% ~ why is that bad?

During enrichment, the following compounds are used:
 $-\text{U}_3\text{O}_8$
 $-\text{UF}_6$
 $-\text{UO}_2$

Centrifuge based enrichment:
 gaseous UF_6 is injected at the center of the centrifuge.
 Because $^{238}\text{UF}_6$ is heavier than $^{235}\text{UF}_6$, it travels faster toward the edge of the centrifuge leaving a higher enrichment close to the center.



✓✓✓

The fission product yield curve has 2 peaks centered at $A=95$ and $A=135$ ✓

11) There are:

- finite difference ✓
- finite volume ✓
- finite element ✓

8/8

Finite element is used by state of the art software because it can be used for any geometry, BCs, and can determine the stress. ✓✓✓