Exam 1

NE 591 - 10

Nuclear Fuel Performance

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-1 Late

$\boxed{83}$

Md Jahid Hasan

200326449

Question 1

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Heat conduction equation :

$$\nabla \cdot (k \nabla T) + Q = \rho c_p \frac{\partial T}{\partial t}$$

If we assume the system is in steady state, the equation becomes ✓

$$\nabla \cdot (k \nabla T) + Q = 0$$

Also, if we take only one dimension (namely x-axis) in cartesian coordinates, then it becomes. ✓

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0$$

As we assumed, the system is in steady state, we can replace the partial derivatives with derivatives. - also k is constant

$$\frac{d}{dx}\left(k \frac{dT}{dx}\right) + Q = 0$$

$$\Rightarrow \frac{d}{dx}\left(k \frac{dT}{dx}\right) = -Q$$

$$\Rightarrow k \frac{dT}{dx} = -Qx + c_1$$

From $x = x_0 = 0$ and $T'(x_0) = 0$, we get

$$k(0) = -0 + c_1$$

$$\Rightarrow c_1 = 0$$

So,
$$k\frac{dT}{dx} = -Qx$$

$$\Rightarrow T = -\frac{Qx^2}{2k} + c_2$$

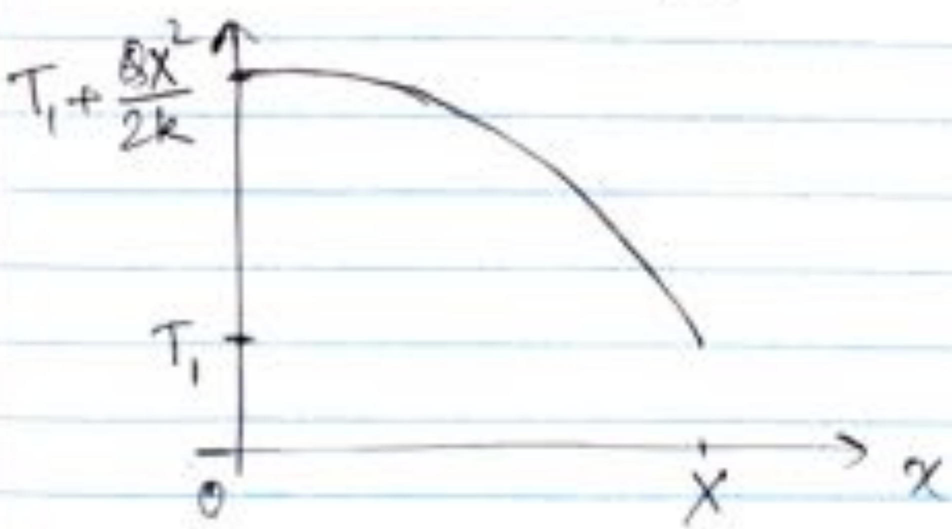
From $x = x_1 = X$ and $T(x_1) = T_1$, we get

$$T_1 = -\frac{QX^2}{2k} + c_2$$

$$\Rightarrow c_2 = T_1 + \frac{QX^2}{2k}$$

Hence,
$$T = -\frac{Qx^2}{2k} + T_1 + \frac{QX^2}{2k}$$

$$\Rightarrow T = T_1 + \frac{Q}{2k}(X^2 - x^2)$$



Question 2

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Given,

$k_{cool} = 5\ W/m\cdot K$

$k_{clad} = 15\ W/m\cdot K$

$k_{fuel} = 0.5\ W/cm\cdot K$

$k_{gap} = 25\ W/m\cdot K$

$h_{cool} = 5.5\ W/cm^2\cdot K$

$r_1 = 0.6\ cm$

$r_2 = 0.8\ cm$

$r_3 = 0.85\ cm$

$r_4 = 0.86\ cm$

$T_{cool} = 800\ K$

$Q = 400\ W/cm^3$

$k_{cool} = 0.05\ W/cm\cdot K$

$k_{clad} = 0.15\ W/cm\cdot K$

$k_{gap} = 0.25\ W/cm\cdot K$

Now,

$$LHR = Q\pi r_1^2$$

$$= 400\pi \times 0.6^2$$

$$= 452.4\ W/cm$$ ✓

$$T_{coat,1} - T_{cool} = \frac{LHR}{2\pi r_1}\ \frac{1}{h_{cool}}$$

$$\Rightarrow T_{coat,o} = \frac{452.4}{2\pi(0.6)} \cdot \frac{1}{5.5} + 800$$

$$= 821.8 \text{ K}$$

$$T_{coat,i} - T_{coat,o} = \frac{LHR}{2\pi r_1} \frac{t_{coat}}{k_{coat}}$$

$$\Rightarrow T_{coat,i} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.01}{0.05} + 821.8$$

$$= 845.8 \text{ K}$$

$$T_{coat,i} = T_{clad,o} = 845.8 \text{ K}$$

$$T_{clad,i} - T_{clad,o} = \frac{LHR}{2\pi r_1} \cdot \frac{t_{clad}}{k_{clad}}$$

$$\Rightarrow T_{clad,i} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.05}{0.15} + 845.8$$

$$= 885.8 \text{ K}$$

$$T_{clad,i} = T_{gap} = 885.8 \text{ K}$$

$$T_{fuel} - T_{gap} = \frac{LHR}{2\pi r_1} \cdot \frac{t_{gap}}{k_{gap}}$$

$$\Rightarrow T_{fuel} = \frac{452.4}{2\pi(0.6)} \cdot \frac{0.2}{0.25} + 885.8$$

$$= 981.8 \text{ K}$$

Now, $T(r) = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_1^2}\right) + T_{fuel}$

$\therefore T(r=0) = \frac{452.9}{4\pi(0.5)} + 981.8$

$= 1053.8 \text{ K}$

and, $T(r=0.4) = \frac{452.4}{4\pi(0.5)}\left[1 - \left(\frac{0.4}{0.5}\right)^2\right] + 981.8$

$= 1007.7 \text{ K}$ $\left(\frac{0.4}{0.6}\right)^2$

Question 3

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(a) $N_f = (15.67 \frac{g}{cc}) \times \left(\frac{1 \text{ mol}}{A \text{ g}}\right) \times \left(\frac{6.022 \times 10^{23}}{1 \text{ mol}}\right)$

$\times \left(\frac{3 \text{ U}}{1 \text{ U}_3\text{Si}_2}\right) \times 19.5\%$

$A = (19.5\%) 235 + (80.5\%) 238$

$= 237.415 \times 3 + 2(28) = 768.2$

$\therefore N_f = 2.33 \times 10^{22}$

$\therefore\ Q = E_f N_f \sigma_f \phi_{th}$

$= (200 \times 10^6\ eV)\left(1.6 \times 10^{-19} \frac{J}{eV}\right)$

$(2.33 \times 10^{22})(570 \times 10^{-24}\ cm^2)$

$(2 \times 10^{12}\ n/(cm^2\text{-}s))$

$= 848.23\ \frac{W}{cm^3}$

- correct process
wrong N_f

(b)

$A = x\,235 + (1-x)\,238$

$\therefore\ A = 238 - 3x$

$N_f = (10.97)\left(\frac{1}{238-3x}\right)(6.02 \times 10^{23})(x)$

$\therefore\ Q = E_f N_f \sigma_f \phi_{th}$

$= (200 \times 10^6)(1.6 \times 10^{-19}) N_f (570 \times 10^{-24})$

(2×10^{12})

$\Rightarrow N_f = 2.33 \times 10^{22}$

$\Rightarrow \frac{x}{238-3x} = \frac{2.33 \times 10^{22}}{6.02 \times 10^{23} \times 10.97}$

$\Rightarrow x = 0.829 = 82.9\%$

right process
wrong N_f
?

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Question 9

(a) $L = 3\ m$

$LHR^0 = 150\ W/cm$

$\gamma = 1.1$

$$LHR = LHR^0 \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{Z_0} - 1\right)\right]$$

$$= 150 \cos\left[\frac{\pi}{2.2}\left(\frac{1.8}{1.5} - 1\right)\right]$$

$$= 143.9\ W/cm$$

(b)

$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \frac{Z_0 \times LHR^0}{\dot{m} C_{pw}} \left[\sin(1.2) + \sin\left[1.\left(\frac{z}{Z_0} - 1\right)\right]\right]$$ -1

here $\frac{\pi}{2\gamma} = 1.43$

(i) water

$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \cdot \frac{150 \times 150}{0.22 \times 4200} \left[2\sin(1.2)\right]$$

$$= 37.83\ K$$

(ii) sodium

$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \cdot \frac{150 \times 150}{0.12 \times 1924} \left[2\sin(1.2)\right]$$

$$= 207.45\ K$$

Question 5

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$\Delta t = 0.33$

$$\frac{dy}{dt} = 4t - 3t^2$$

$t_0 = 1 \Rightarrow y(t_0) = 6$

$$y(t_i) = y(t_{i-1}) + \frac{dy}{dt} \Delta t$$

Forward

$$y(t_1) = y(t_0) + \left.\frac{dy}{dt}\right|_{t_0} \Delta t$$

$= 6 + (0.33)$

$= 6.33$

$t_1 = t_0 + \Delta t$
$= 1.33$

$$y(t_2) = y(t_1) + \left.\frac{dy}{dt}\right|_{t_1} \Delta t$$

$= 6.33 + (0.0133)(0.33)$

$= 6.334389$ ✓

$t_2 = 1.66$

$$y(t_3) = y(t_2) + \left.\frac{dy}{dt}\right|_{t_2} \Delta t$$

$= 6.334389 + (-1.6268)(0.33)$

$= 5.7975$ ✓

$t_3 = 2$

t_4 =

Backward

right process

$$y(t_1) = y(t_0) + \frac{dy}{dt}\Big|_{t_1} \Delta t$$

$t_1 = 1.33$

$$= 6 + 0.4389$$

4×10^{-3}

$$= 6.4389$$

$$y(t_2) = y(t_1) + \frac{dy}{dt}\Big|_{t_2} \Delta t$$

$t_2 = 1.66$

$$= 6.4389 + (-1.6268)(0.33)$$

$$= 5.902$$

$$y(t_3) = y(t_2) + \frac{dy}{dt}\Big|_{t_3} \Delta t$$

$$= 5.902 + (-4)\, 0.33$$

$t_3 = 2$

$$= 4.582$$

Question 6 5/5

→ Fissionable nuclide is capable of undergoing fission after capturing a high energy neutron

→ Fissile nuclide is capable of sustaining fission chain reaction with neutrons of any energy.

→ Fertile material is not fissionable, but it can be converted into a fissile nuclide by neutron absorption.

Question 7 4/4

→ During thermal cycling pure U swells dramatically

→ α-U has both anisotropic expansion and irradiation growth.

Question 8

Linear density

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= Ratio of fuel volume to total internal volume

$$= \frac{\pi r_1^2 h}{\pi r_2^2 h}$$

$$= \frac{r_1^2}{r_2^2}$$

Swelling

→ It's necessary for thermal hydraulic and efficiency measurements.

Question 9

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why enrich?

Oxide → UF_6 → UO_2 → pellet → rod

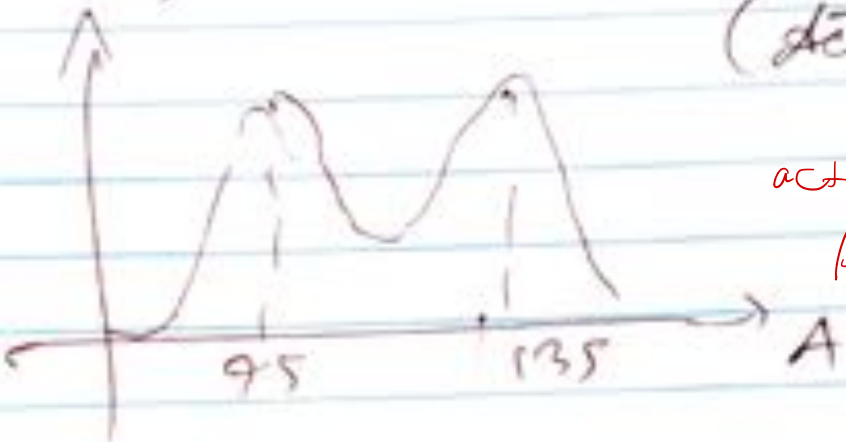
→ Because in a centrifuge U-238 gets further than U-235.

- why?

Question 10

One around $A = 95$
Another around $A = 135$
(actinides)

2/4



actinides are $A > 227$

Question 11

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1. Finite difference
2. Finite volume
3. Finite element

Finite element is used in SOA fuel performance simulations as it can model any geometry.

- other aspects?

FV can model any geometry as well