Beda Kafley
NUCE 497
Exam 1:

Q1) a) The fissile material in U_3Si_5 is Uranium-235. The natural enrichment of U-235 is 0.7% and U-238 is 99.3%.

<u>My mistake</u>: I didn't make any mistake for this part.

~~Part~~ b) part b: I was unable to complete this part on my exam but the correct approach from the exam solution is shown;

<u>Solution</u>: given: U_3Si_5 as fuel material
Thermal conductivity = 12.5 W/mK
density = 7.5 grams of U/cm^3

<u>To find</u>: enrichment required for U_3Si_5 to have the same energy released rate of U_3Si_2.

<u>Approach</u>; $q_f = E_f N_f \varphi_f$

$$E_f N_{U_3Si_5} \sigma_{f235} \varphi_n = E_f N_{U_3Si_2} \sigma_{f235} \varphi_n$$

$$N_{U_3Si_5} = N_{U_2Si_2}$$

$$q_1 \delta_{U,U_3Si_5}/M_U = q_2 \delta_{U,U_3Si_2}/M_U$$

$$q_1 = q_2 \frac{\delta_{U,U_3Si_2}}{\delta_{U,U_3Si_5}}$$

$$q_1 = \frac{11.31 \times 0.03}{7.5} = \boxed{0.0452}$$

Since; I missed this part on my exam 1: I just copied the solution. ~~because~~

part c:

My answer in my exam solution:

→ Ranking of U_3Si_5 as potential fuel compared to U_3Si_2 would be;

1st choice = U_3Si_2

2nd choice = U_3Si_5.

∴ U_3Si_2 is better fuel material than U_3Si_5 because ~~we can~~ it has higher head thermal conductivity than U_3Si_5. so, U_3Si_5 cannot survive equally high temperature as of U_3Si_2. A~~lso, t~~.

answer from solution:

→ U_3Si_5 is a worse fuel because it has a lower density of Uranium and will produce less power, and because it has a lower thermal conductivity and will conduct the heat out less efficiently.

My mistakes/missings:

While, I answered the question correctly but I was missing some properties like density of Uranium. I also forget to include the power produced would be less if we use U_3Si_5 instead of U_3Si_2.

Q2) Solution: part a: Note: My approach was in terms of q.

My approach in Exam 1 solution:

→ To find the surface temperature of fuel.

$$\rightarrow T_{co} - T_{cool} = \frac{q}{2h_{cool}} R_f = \frac{250\ W/cm}{2h_{cool}} + 580\,K$$

$$T_{co} = \frac{250}{2 \times 2.5} + 280\,K = \underline{\underline{630\,K}}$$

$$\rightarrow T_{c1} = \frac{q}{2K_c} R_f \delta_c + T_{co} = \frac{250}{2(0.17)} (0.45\,cm)(0.06) + 630\,K$$

$$T_{c1} = 19.8\,K + 630\,K = \underline{\underline{649.85\,K}}.$$

$$\rightarrow T_s = \frac{q}{2h_{gap}} R_f + T_{c1} = \frac{250}{2 \times h_{gap}} \times (0.5\,cm) + 649.85\,K$$

where; $h_{gap} = \frac{K_{He}}{\delta_{gap}} = 0.7 \times 10^{-6} (668.09)^{0.79}$

$$T_s = \frac{250}{2 \times 0.03977} \times 0.5 + 649.8\,K$$

$$T_s = 314.30 + 649.8 = \boxed{964.10\,K}$$

Correct approach: Exam 1 solution was in terms of LHR

$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_f h_{cool}} = 580 + \frac{250}{2\pi \times 0.45 \times 2.5} = \underline{\underline{615.4\,K}}$$

$$T_{c1} = T_{co} + \frac{LHR}{2\pi R_f (K_{cld}/t_{cld})} = 615.4 + \frac{250}{2\pi \left(\frac{0.17}{0.06}\right)} = \underline{\underline{646.6\,K}}$$

$$K_{He} = 16 \times 10^{-6} \times T_{C1}^{.79} = 0.00266, \quad K_{Xe} = 1.16 \times 10^{-4}\ W/cmK.$$

$$T_s = T_{c1} + \frac{250}{2\pi R_f (K_{gap}/t_{gap})} = 646.6 + \frac{250}{2\pi (0.45)\left(\frac{0.00227}{80 \times 10^{-4}}\right)}$$

$\boxed{T_s = 958.2\,K}$ There was only slight difference in my answer because my approach was in terms of q.

My errors/mistakes for Q2 part a:

→ There was no mistakes made but I used the different approach; I calculated the value of T_s using approach named; interms of Q but in exam 1 solution; it was calculated interms of LHR; I think both approach are correct so we have slight difference on final answers.

my answer: $T_s = 964.10\ K$

Exam 1 Solution answer; $T_s = 958.2\ K$.

part b: My approach: to find maximum stress experienced by the fuel pallet;

→ Since $E = \cancel{246.7}\ 246.7\ GPa$, $v = 0.25$, $\alpha = 7.5\times10^{-6}\ 1/K$ was given;

I used: $\sigma_{\theta\theta}(r) = \frac{1}{2}\ \Delta T\ \frac{\alpha E}{1-v}\left(1-2\frac{R_i}{6}\cdot\left(\frac{r}{R_i}-1\right)\right)$

making some assumptions; $\sigma^* = \frac{\alpha E(T_0-T_s)}{4(1-v)}\ \cancel{\times E}$

$\sigma_\theta^* = \boxed{0.024\ GPa}$

Correct approach:

$$\Delta T = \frac{250}{4\pi K} = \frac{250}{4\pi\times0.2} = 99.5\ K.$$

$$\sigma^* = \frac{\alpha E\Delta T}{4(1-v)} = \frac{7.5\times10^{-6}\times246.7\times99.5}{4(1-0.25)} = \boxed{0.0614\ GPa}$$

$$\sigma_{\theta\theta}(r/R_f=1) = -\sigma^*(1-3\eta^2) = -0.0491(1-3) = \boxed{123\ MPa}$$

My mistake: My first mistake was not finding the temperature difference (ΔT).

I used the same formula to callute σ^* but I found different answer since (ΔT) is used was wrong

Lastly: I forget ~~to~~ to calculate $\sigma_{\theta\theta}$ (maximum stress) using $\sigma_{\theta\theta}(r/R_f=1)$ equation. $= -\sigma^*(1-3\eta^2)$

Q2; Part c:

My answer: If the pallet was UO_2 instead of Uranium nitrate the stress would be higher because the stress of fuel pellet depends on thermal conductivity of the fuel materials.

Correct answer from exam solution:

→ If the pallet was UO_2; we expect higher stress because the temperature difference would be much higher due to the much lower thermal conductivity.

→ My answer was correct but reasoning was not quite specific.

Part d: My answer:

The assumptions made in our calculation for part a and b are:

→ We only care about steady state solution.
→ The behavior is axisymmetry
→ Temperature is constant in Z-direction and;
→ The thermal conductivity K is independent of temperature

Corret answer from exam 1 solutions:

→ Axisymmetric; long rod; properties are independent; of temperature; constant q, constant T_{cool}, steady state static body; gravity was neglected; no shear stress;

∴ All of my assumptions for this part was correct but I was missing some.

Q3) Solution:

a) Assumptions made in the thin walled cylinder approximation for the stress state are:

My responses: are correct but missing the last one:

1) Small strains
2) Isotropic materials response
3) That the stress is constant through the thickness of the cylinder. (In my solution I was missing # 3 assump.).

b) My solution for this part was also correct. ~~and~~

All three components of the stress using the thin walled cylinder approximation are:

$$\sigma_{\theta\theta} = \frac{PR}{\delta} = 6\ \text{Mpa}\left(\frac{5.6}{0.6}\right) = \boxed{56\ \text{Mpa}}$$

$$\sigma_{rr} = -P/2 = \frac{-6\ \text{Mpa}}{2} = \boxed{-3\ \text{Mpa}}$$

$$\sigma_{zz} = \frac{PR}{2\delta} = 6\,(5.6/1.2) = \boxed{28\ \text{Mpa}}$$

For this part I made no mistakes on exam 1 solution.

c) My answer: / mistakes I made:

Using thin walled cylinder approximation for cladding is less accurate than that of using thick wall approximation.

→ My answer was very close and correct but I didn't quantify and give valid / correct reasoning.

→ Correct approach is shown below with correct reasoning:

↓

$R_i = 5.6 - 0.3 = 5.3$ mm, $R_o = 5.6 + 0.3 = 5.9$ mm.

For $r = R_i$, $\sigma_{\theta\theta} = P((R_o/R_i)^2 + 1)/((R_o/R_i)^2 - 1) = \dfrac{6 \times ((5.9/5.3)^2 + 1)}{((5.9/5.3)^2 - 1)}$

$\sigma_{\theta\theta} = 56.2$ MPa

For $r = R_o$; $\sigma_{\theta\theta} = P((R_o/R_o)^2 + 1)/((R_o/R_i)^2 - 1) = \dfrac{6(1+1)}{(5.9/5.3)^2 - 1}$

$\sigma_{\theta\theta} = 50.2$ MPa

In my exam solution I didn't prove like what's done above but I only state that thin walled cylinder approximation would be less accurate:

Correct: Reasoning The stress varies by more than 10% across the thickness; ~~so~~, it not very accurate. The thin walled answer is close at the inner radius but too high at the outer radius.

Part D: My approach: $E = 70$ GPa, $\nu = 0.41$ was given.

I used: $C_{11} = E(1-\nu)/(1+\nu)(1-2\nu) = 162.7265$ GPa.

$C_{12} = 70(0.41)/(1.41)(1-0.82) = 113.08$ GPa:

Correct: approach:

$$\sigma = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 56 \end{bmatrix}$$

$\varepsilon_{rr} = \dfrac{1}{E(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz}))} = \boxed{-53 \times 10^{-5}}$

$\varepsilon_{\theta\theta} = \dfrac{1}{70 \times 10^3 (56 - 0.41)(-3+28)}$

$\varepsilon_{\theta\theta} = \boxed{65 \times 10^{-5}}$

$\varepsilon_{zz} = \dfrac{1}{E(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr}))} = \dfrac{1}{70 \times 10^3 (28 - 0.41(-3+56))} = \boxed{9 \times 10^{-5}}$

$$\varepsilon = \begin{bmatrix} -53 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 65 \end{bmatrix} \times 10^{-5}.$$

My mistake: I didn't find ε_{rr}, $\varepsilon_{\theta\theta}$ and ε_{zz} using the correct formula provided on lecture: Also, I didn't put ε_{rr}, $\varepsilon_{\theta\theta}$, and ε_{zz} in matrix form.