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Exam 1

① $T'(x_0) = 0$

$x_0 = 0$

$x_1 = X$

$T(x_1) = T_1$

using:

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + Q = 0$$

Assuming: ① steady state ✓

② axissymmetric ✓

③ z = constant ✓

④ K_{th} = constant

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$$\int \frac{x^2}{2}$$

$$xk \frac{dT}{dx} = \frac{-Qx^2}{2} + C_1$$

assuming $x=0$

$$0 = -\frac{Q(0)}{2} + C_1$$

$$\frac{dT}{dx} = -\int \frac{Qx}{2k}$$

$$\boxed{T(x) = \frac{-Qx^2}{4k} + C_2}$$

- looks like you did perhaps the rz derivation, not cartesian

- need to solve for C_2 using boundary conditions

② Find $T_0 = ?$ (without coating)

Given: $k_{coat} = 0.015$ W/cm-k
$k_{clad} = 0.15$ "
$k_{fuel} = 0.05$ "
$k_{gap} = 0.004$ "

$T_{coat\_sur} = 600$ K
$Q = 250$ W/cm³
$R_f = 0.6$ cm
$t_{gap} = 0.005$ cm
$t_{clad} = 0.05$ cm
$t_{coat} = 0.01$ cm

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assuming $T_{co(w/out\ coat)} = 600$ K and coating is ... of clad

$$T_{c1} = \overset{600}{T_{co}} + \frac{\overset{250}{LHR}}{2\pi \underset{0.6}{R_f}} \cdot \frac{\overset{0.05}{t_{clad}}}{\underset{0.15}{k_{clad}}} = 622.1 \text{ K}$$

LHR ≠ Q

$LHR = \pi R_f^2 Q$

$$T_F = \underset{622.1}{T_{c1}} + \frac{LHR}{4\pi R_F} \cdot \frac{\overset{0.005}{t_{gap}}}{\underset{0.004}{k_{gap}}} = 705 \text{ K}$$

$$T_0 = T_F + \frac{LHR}{4\pi k_f} \boxed{= 1102.82 \text{ without coating}}$$

$T_{cs} = 600$ K (given)

- coating is on the surface of the cladding, not fuel

$$T_F = T_{cs} + \frac{LHR}{4\pi R_f} \cdot \cancel{\frac{\overset{0}{t_{gap}}}{k_{gap}}} = 600 + \frac{250}{2\pi(0.6)} = 666.315 \text{ K}$$

$$T_0 = T_F + \frac{LHR}{4\pi R_f} = 666.315 \text{ K} + \frac{250}{4\pi(0.6)} =$$

$\boxed{997.89 \text{ K with coating}}$

③ UN enrich = 19.5%

$\rho_{UN} = 12.3\ g/cm^3$ 9/14 UN: 50% U, 50% N

$\sigma_F = 530\ b$

$A_N = 14$

a) Find Q given $\phi = 5E12\ n/cm^2 s$

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$$Q = E_F N_f \sigma_f \phi = (200\ MeV)(N_f)(530 \times 10^{-24}\ cm^2)(5 \times 10^{12}\ n/cm^2 s)$$

$$N_f = (12.3\ g/cm^3)\left(\frac{1\ mol}{518.08\ g/mol}\right)(N_A)\left(\frac{\cancel{.92}}{1}\right)(.195)$$

assuming $M(un) = 518.08\ g/mol$ & $x_u \approx 0.92$ — where did this come from?

$= 2.565 \times 10^{21}\ u^{235}/cm^3$

$$\Rightarrow Q = (200E6)(1.662\ E{-19})(\ \ )(530 \times 10^{-24})(5 \times 10^{12})$$

$$\boxed{= 217.8\ W/cm^3}$$ off by ~2x

b) $217.8 = (200E6)(1.602\ E{-19})(N_f)(530 \times 10^{-24})(5 \times 10^{12})$

Assuming

$$N_f = \frac{\rho_{UO2} N_A}{M_{UO2}} = (10.97\ g/cm^3)\left(\frac{1\ mol}{238 + 2(16)}\right)(N_A)(.86)x$$

$\Rightarrow 217.8 = 3.204E{-11}(2.104E24\ (\text{enrich}))\ 530 \times 10^{-24}(5 \times 10^{12})$

$$\boxed{\text{Enrichment} \approx 12.25\ \%}$$

M(UO₂) includes enrichment

(4) $Z = L = 3.5\,m$

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$LHR^{o} = 350\,W/cm$

$\gamma = 1.3$

$$LHR = \left(\frac{z}{z_o}\right) = LHR^{o} \cos\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_o} - 1\right)\right) \quad \boxed{= 339.8\ W/cm}$$

b) $$\Delta T_{cool} = \frac{2\gamma}{\pi}\left(\frac{z_o \cdot LHR^{o}}{\dot{m}\ c_p}\right)\left[\sin\frac{\pi}{2\gamma} + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_o} - 1\right)\right)\right]$$

$$\boxed{\Delta T = 122.2\ K}$$

off by ~20 K

- I dont see steps so I dont know what you did wrong
- I would prefer to see implementation of values

(5) $dt = 0.5 \rightarrow t_n = 1.5$

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$$\frac{dy}{dt} = t\exp(-2t)$$

$t_0 = 0$

$y_0 = 4$

$y_{n+1} = y_n + dt\, y'_{n+1}$

$t = 0.5 \rightarrow y_{0.5} = y_0 + dt\, y'_1 = 4 + 0.5\left(t\exp(-2t)\right)$

$= 4 + 0.5\left(0.5 \cdot \exp(-1)\right)$

$\boxed{= 4.092}$

$t = 1 \rightarrow y_1 = y_{0.5} + dt\, y'_1 = 4.092 + 0.5\left(1 \cdot \exp(-2 \cdot 1)\right)$

$\boxed{= 4.1596}$

$t = 1.5 \rightarrow y_{1.5} = y_1 + dt\, y'_{1.5} = 4.1596 + 0.5\left(1.5 \cdot \exp(-2 \cdot 1.5)\right)$

$\boxed{= 4.1969}$

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(6) Fissionable = After capturing a neutron, is capable of fissioning ↑ high E neutron
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Fissile = Capable of sustaining chain reaction w/ any neutron E.

Fertile = Not fissionable nominally, but can be converted into one with neutron abs.

(7) ① To reduce swelling - yes in a sense
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② Pure U has 3 phases and its α-phase has significant CTE and irr. growth

(9) We need to enrich to increase U-235 % to sustain fission. (in LWRs) as nat U % is too low.
8/8
- We want to convert to UF_6 as it is in gaseous form at STP and able to be enriched w/ centrifuge.
- Due to variation in density of U-235 vs. U-238, the more dense/heavier molecules of U-238 move to outside and U-235 to inside due to their relative densities we can separate (or enrich)

(8) Smear density = %-fuel weight contained in vol. by unit length of cladding. Necessary due to fuel elements not having 100% theoretical dens.
- vol ratios 3/4
- fuel will swell and expand

(10) Discretized by: ① Finite difference: simple but point solution
② Finite volume: Any geometry, but more complicated
③ Finite Element: Heterogeneous but computationally complex
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(11) As the outer surface of fuel incr, H.T. mode changes 7/7

A: isolated bubbles] Nucleate Boiling
B: Boiling increases]

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C: Max flux

DNBR

D: Min heat flux

↓

E: film boiling

Critical heat flux = continuous film of steam is formed and $q = \max$

(12) Layers: (1) Fuel Kernel

(2) Buffer

(3) IPyC

(4) SiC

(5) OPyC

5/5

and used in HTGR or MSRs