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82/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

-6, 24/30

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

$\boxed{\text{U-235}}$ is the fissile isotope

Enrichment of isotope in natural:

99.3% in U-238 ⇒ $\boxed{0.7\% \text{ in U-235}}$

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$$Q(U_3Si_2)\Big|_{\gamma=0.3} = Q(U_3Si_2)\Big|_{\gamma=x}$$

$$\cancel{E_f}\,\cancel{\sigma} N_{U235}\cancel{\phi} = \cancel{E_f}\,\cancel{\sigma} N_{U235}\cancel{\phi}$$

because same energy release rate

$$\frac{\rho_{U_3Si_2} N_A \gamma_1}{A_{U_3Si_2}} = \frac{\rho_{U_3Si_5} N_A \gamma_2}{A_{U_3Si_5}}$$

given values: $\gamma_1 = 0.03$, $\phi = 3.2\times10^{13}\left[\frac{n}{cm^2 s}\right]$, $E_f = 3\times10^{-11}$, $\sigma = 550\,[b]$

Looked up: $\rho_{U_3Si_2} = 12.2\left[\frac{g}{cm^3}\right]$, $\rho_{U_3Si_5} = 9.1\left[\frac{g}{cm^3}\right]$

$A_{U_3Si_2} = (238)3 + (28.1)2 = 770.2$

$A_{U_3Si_5} = (238)3 + (28.1)5 = 854.5$

-3, Use the U densities

$$\frac{3(12.2)(6.02\times10^{23})(0.03)}{770.2} = \frac{3(9.1)(6.02\times10^{23})\gamma_2}{854.5}$$

$$8.58\times10^{20}\ \frac{\text{atoms}}{\text{cm}^3} = (1.92\times10^{22})\gamma_2 \Rightarrow \boxed{\gamma_2 = 0.04 = 4\%}$$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

More U-235 is required when using U_3Si_5 compared to U_3Si_2 for the same heat generation rate. This indicates that U_3Si_2 is better because less U-235 is required. Another consideration is how safe it is to use based on the melting temperatures to determine fuel stability but these values are not stated.

-3, Thermal conductivity?

$R_f = 0.45[cm], t_{gap} = 0.008[cm], t_{clad} = 0.06[cm], LHR = 250\left[\frac{W}{cm}\right]$

$T_{cool} = 580[K], h_{cool} = 25\left[\frac{W}{cm^2 K}\right]$ $K_{Zr} = 0.17$

Question 2: -2, 33/35

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K). Taking surface Temp to mean outer surface of cladding

a) What is the surface temperature of the fuel rod? (15 points)

$T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool}$

$= \frac{250}{2\pi(0.45)(2.5)}$

$\boxed{T_{co} = 615.4\,K}$ ⇒ outer surface of cladding

-2, Surface temperature = Ts = surface temp of pellet

$\sigma_{\theta\theta} = -\sigma^*(1-3\eta^2)$

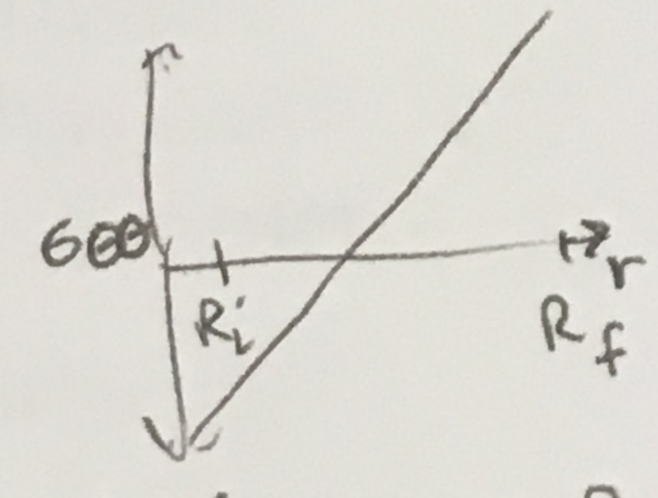
$\sigma^* = \alpha E(T_o - T_s)/4(1-\nu)$

$= (7.5\times10^{-6})(246.7)(1099.8-1000) / 4(1-0.25)$

$= 0.0613$

$\sigma_{\theta\theta} = -0.06(1-3(1)^2)$

$= \boxed{0.12\,GPa}$



$\eta = 1$ when $r = R_f$

max stress is hoop stress

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$K_{UN} = 0.2$

$T_o = \frac{LHR}{4\pi k} + T_s = 1099.8[K]$

$h_{gap} = k_{gap}/t_{gap}$

$k_{gap} = k_{He}^{1-\gamma} k_{XE}^{\gamma}$

$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{IC}$

$k_{gap} = (16\times10^{-6}(646.6)^{0.79})^{0.95} (0.7\times10^{-6}(646.6)^{.79})^{0.05}$

$k_{gap} = 0.002$

$T_{IC} = \frac{LHR \cdot t_{cl}}{2\pi R_f k_{Zr}} + T_{co}$

$h_{gap} = 0.25$

$T_{IC} = 646.6K$

Rest of part b above

$T_s = 1000.26\,[K]$

c) Would you expect this stress to be higher or lower if the pellet was UO₂? Why? (5 points)

K_{UO2} is much less than K_{UN}

$\boxed{\sigma_{UO2} > \sigma_{UN}}$

α_{UO2} is greater than α_{UN}

d) What assumptions were made in your calculations for a) and b)? (5 points)

1) constant fuel radius
2) constant K
3) constant gap size
4) constant LHR
5) steady state

-10, 25/35

Question 3:

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

-2, Stresses are constant across radius

$P = 6$ MPa
$R = 5.6$mm
$\delta = 0.6$mm

The wall thickness goes to zero
All stresses act parallel to the vessel
$F_{pressure} = F_{stress}$

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$\sigma_\theta = \frac{PR}{\delta} = \frac{(6MPa)(5.6mm)}{0.6mm}$, $\sigma_\theta = \boxed{56\ MPa}$

$\sigma_z = \frac{PR}{2\delta}$, $\sigma_z = \frac{(6MPa)(5.22mm)}{2 \cdot (0.6mm)}$, $\boxed{\sigma_z = 28 MPa}$

$\sigma_r = -\frac{P}{2} = \frac{-6\ MPa}{2}$, $\boxed{\sigma_r = -3 MPa}$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

$\sigma_{\theta\theta} = P \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$ max when $r = R_i$ $\Rightarrow$ $\sigma_{\theta\theta} = 6MPa \frac{\left(\frac{5.9}{5.3}\right)^2 + 1}{\left(\frac{5.9}{5.3}\right)^2 - 1}$

$\sigma_{\theta\theta} = 56.16$ MPa

incorrect-correct
incorrect

-4 Calculate stress at muliple radii to test approximation

% error $= \frac{56 - 56.16}{56.16} \times 100 = -0.2862$

The thin walled prediction would be slightly not conservative because it is approximated as slightly less than actual value

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\begin{bmatrix} \sigma_r \\ \sigma_\theta \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{rr} \\ u_r/r \end{bmatrix}$$

-2, Tensors are missing zz component
-2, Strains wrong

$$\begin{bmatrix} -3 \\ 56 \end{bmatrix} = \frac{70}{(1+0.41)(1-0.82)} \begin{bmatrix} 1-0.41 & 0.41 \\ 0.41 & 1-0.41 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$u_{rr} = -0.48$
$u_r/r = 0.69$
solved using Matlab

$$\boxed{\sigma = \begin{bmatrix} -3 & 0 \\ 0 & 56 \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} -0.498 & 0 \\ 0 & -0.69 \end{bmatrix}}$$