NUCE 497 Midterm
Resubmit

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* Corrections noted w/ yellow highlighter *

* Note: the answer key was consulted in making these corrections.

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1.a) U-235

Si: 28.1 u

U_n: 238u

e_n: 0.72%

$$\cancel{U_3Si_5 = 5(28.1u) + 3(238u) = 854.5u}$$

$$\cancel{\text{wt \% 235: } \frac{3 \cdot 238u \cdot .72\%}{854.5u} = \boxed{0.59\%\,wt}}$$

1.b) $Q = E_f \cdot N_f \cdot \sigma_f \cdot \phi_{th}$

$$Q_1 = Q_2$$

$$\cancel{E_f} \cdot N_{U_3Si_5} (e_{U_3Si_5}) \cdot \cancel{\sigma_f} \cdot \cancel{\phi_{th}} = \cancel{E_f} \cdot N_{U_3Si_2} (e_{U_3Si_2}) \cancel{\sigma_f} \cdot \cancel{\phi_{th}}$$

$$e_{U_3Si_5} = e_{U_3Si_2} \cdot \frac{N_{U_3Si_2}}{N_{U_3Si_5}}$$

$$= 3\% \cdot \frac{769.9}{854.5} = 2.7\%$$

1.b)

$Q = E_f N_f \sigma_f \phi_{th}$

$Q_1 = Q_2$

$N_{f_1} = N_{f_2}$

$N_f = 3 \cdot g \cdot N_a \cdot \delta \cdot \cancel{MM}$

$MM_{U_3Si_2} = (238)3 + (28)2 = \underline{770 \text{ g/mol}}$

$MM_{U_3Si_5} = (238)3 + (28)5 = \underline{854 \text{ g/mol}}$

δ_u ~~12.2 g/cm³~~ 11.31

$\cancel{3} \cdot (.03) \cdot \cancel{N_a} \cdot (\cancel{12.2 \text{ g/cm}^3}) / 770 \text{ g/mol}$

$= \cancel{3}(g) \cancel{N_a} (7.5 \text{ g/cm}^3) / 854 \text{ g/mol}$

$$\frac{.03 \cdot \cancel{12.2}\ 11.31 / 770}{7.5 / 854} = \cancel{5.4\%}\ 5.0\%$$

* But since δ_u given instead of δ, use MM of U fraction only!?

$$\frac{.03 \cdot \cancel{12.2}\ 11.31 / \cancel{770}\ (238 \cdot 3)}{7.5 / (238 \cdot 3)} = \boxed{\cancel{4.5\%}}\ \boxed{4.2\%}$$

1.b) * Resubmit *

This is just a question of $\frac{N_{U_3Si_2}}{N_{U_3Si_5}}$

as I realized early. I just got flustered or something?

$$\rho_{U_3Si_5} = \rho_{U_3Si_2} \cdot \frac{\cancel{N_a} \cdot \delta U_{U_3Si_2} \cdot \cancel{M_{U_3}}}{\cancel{N_a} \cdot \delta U_{U_3Si_5} \cdot \cancel{M_{U_3}}}$$

$$= .03 \cdot \frac{11.31}{7.5}$$

$$= \boxed{4.52 \text{ g/cc}}$$

1.c)

	Thermal Cond (W/cm/K)	Density (g U/cm^3)	Comparable Enrichment
U_3Si_5	.125	7.5	4.2%
U_3Si_2	.23	11.31	3%

~~U_{235} Fuel effiency of U_3Si_5 is higher!~~

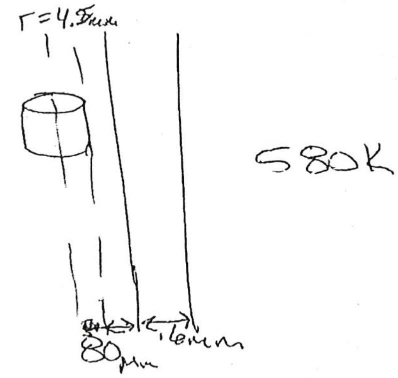
$$4.2\% \cdot 7.5\ gU/cm^3 = \underline{0.315\ g\ U_{235}/cm^3}$$

$$11.31\ g\ U/cm^3 \cdot 3\% = 0.339\ g\ U_{235}/cm^3$$

$$\Rightarrow \frac{.339 - .315}{.339} = \boxed{7\%\ \text{less}\ U_{235}\ \text{used by switching to}\ U_3Si_5}$$

U_3Si_5 thermal conductivity and power density are lower - see above table. U_3Si_5 is a poorer fuel.

2. a)



$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_F h_{cool}}$$

$$= 580K + \frac{250W/cm}{2\pi(4.5mm)\,2.5\,W/cm^2\text{-}K}$$

$$= \underline{615.3\ K}$$

$$T_{IC} = T_{OC} + \frac{LHR \cdot t_c}{2\pi R_F \cdot K_c}$$

$$= 615.3K + \frac{250\,W/cm \cdot 0.6mm}{2\pi(4.5mm)\cdot \boxed{0.17\,W/cm/K}}$$

* Assumed K_c For Zr

$$= \underline{646.5\ K}$$

$$T_s = T_{IC} + \frac{LHR}{2\pi R_F \cdot \left(\frac{K_{gap}}{t_g}\right)} \quad ; \quad K_{gap} = 16E\text{-}6 \cdot T_K^{.79}$$

$$= .0077\ W/cm\text{-}K$$

$$= 646.5K + \frac{250\ W/cm}{2\pi(4.5mm)\ \frac{.0027\,W/cm\text{-}K}{80\mu m}}$$

* Wrong Formula

$$= \boxed{912.7\ K}$$

2.b) Assume Fuel dimensions measured at this temp

$$\sigma = -E \cdot \Delta T \cdot \alpha$$

$$= -246.7GPa(-912.7K + 300K)\,7.5E\text{-}6\,\tfrac{1}{K}$$

$$=$$

2.a) *Resubmit*

$$K_{gap} = \left(16E\text{-}6 \cdot T_{IC}^{.79}\right)^{1-X_e} \cdot \left(0.7E\text{-}6\, T_{IC}^{.79}\right)^{X_e}$$

$T_{IC} = 646.6\ K$

$X_e = 0.05$

$$\Rightarrow K_{gap} = \left(16E\text{-}6\,(646.6)^{.79}\right)^{.95} \cdot \left(.7E\text{-}6\,(646.6)^{.79}\right)^{.05}$$

$$= \underline{0.00227\ W/cm\text{-}K}$$

$$T_s = 646.6K + \frac{250\ W/cm}{2\pi(4.5mm)\,.00227\,W/cm\text{-}K/80\mu m}$$

$$= \boxed{958.2\ K}$$

2. b) Assume the pellet is constrained:

$$\sigma = -E(\Delta T)\alpha$$

<u>Hoop</u> stress is largest:

Wrong formula

$$\sigma_{\theta\theta} = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$

$$= \frac{1}{2}(912.7K-300K)\frac{7.5e^{-6}\,1/K \cdot 246.7GPa}{1-.25}$$

* Max stress

$$\cdot\left(1-2\frac{4.5mm}{.6mm}\left(1-1\right)\right)$$

$$= \boxed{0.756GPa \text{ @ surface}}$$

2.b) * Resubmit *

I used the equation for a thin constrained rod instead of for a fuel pellet. I should have used:

(1) $$T(r) = \underbrace{\frac{LHR}{4\pi k}}_{\Delta T_{fuel}} \left(1 - \frac{r^2}{R_F^2}\right) + T_s$$

$$\Rightarrow \Delta T = \frac{250\ W/cm}{4\pi \cdot (0.2\ W/cm\text{-}K)} \leftarrow \text{(for U-N)}$$

$$= \underline{99.47\ K}$$

(2) $$\sigma^* = \frac{\alpha E (T_o - T_s)}{4(1-\nu)}$$

$$= \frac{(7.5E\text{-}6\ 1/K) \cdot (246.7\ GPa) \cdot (99.47\ K)}{4(1-.28)}$$

$$= \underline{61.4\ MPa}$$

(3) $$\sigma_{\theta\theta}(\eta) = -\sigma^* (1 - 3\eta^2)$$

$$= -61.4\ MPa \cdot (1 - 3(1)^2)$$

$$= \boxed{123\ MPa}$$

2.c) U-N has a much ↑ conductivity than UO_2 => ~~big~~ lower T_S.

UO_2 $\sigma_{\theta\theta}$ will be higher for this reason.

Resubmit

* Right answer, wrong reason. I understood it in my head. T_S only determined by T_{cool}, $T_{\text{℄}}$ lower for U-N due to ↑K in the fuel.

2.a) Axisymmetry, isotropic material, K independant of T, no geometry change (such as gap size)

Resubmit

* Also, Thin wall cyl w/ therm. expansion α to ϵ but not σ. (Linear)

Also, small ϵ approximation.

- Long Rod
- No shear stress

Also, no internal pressure.

Also, σ constant through the wall.

Also, Young's Modulus: Assume $\frac{\sigma}{\epsilon}$ is linear.

Also, steady state.

3. a) Force is constant over length,

Axisymmetric,

Isotropic material response,

$\frac{\partial x}{\partial t} = 0$, Gravity is negligable

b) $P = 6MPa$, $r = 5.6mm$, $\delta = 0.6mm$

$$\sigma_\theta = \frac{P \cdot r}{\delta} = \frac{6MPa \cdot 5.6mm}{0.6mm} = \boxed{56MPa}$$

$$\sigma_z = \frac{P \cdot r}{2\delta} = \frac{6MPa \cdot 5.6mm}{2(0.6mm)} = \boxed{28MPa}$$

$$\sigma_r = -\frac{1}{2}P = -\frac{1}{2} \cdot 6MPa = \boxed{-3MPa}$$

c) Compare to thick wall sol'ns @ $r = R_i$

$R_i = 5.6mm$

$R_o = 6.2mm$

$$\sigma_{rr} = -p = \boxed{-6MPa}$$

Wrong?

$$\sigma_{\theta\theta} = p \cdot \frac{(R_o/R_I)^2 + 1}{(R_o/R_I)^2 - 1} = \boxed{\cancel{59.1}MPa}$$

Wrong

$$\sigma_{zz} = p \frac{1}{(R_o/R_I)^2 - 1} = \boxed{26.6MPa}$$

Hoop stress most important, $\frac{56}{59} = $ 95% correct

σ_r off by factor of 2, σ_z is close.

Thin wall __should__ be conservative (but I find it is not in this case)

3.c) * Resubmit *

Only need to look at $\sigma_{\theta\theta}$ b/c this will have the >> stress. Also, must compare 2 points since thin wall is a single-point approx.

$$\sigma_{\theta\theta}(r) = P \cdot \frac{(R_o/r)^2 + 1}{(R_o/R_I)^2 - 1}$$

$$\sigma_{\theta\theta}(R_o) = \frac{(1)^2 + 1}{(5.9/5.3) - 1} = \boxed{50.1\ \text{MPa}}$$

$$\sigma_{\theta\theta}(R_i) = \frac{(5.9/5.3)^2 + 1}{(5.9/5.3)^2 - 1} = \boxed{56.1\ \text{MPa}}$$

$$\sigma_{\theta\theta}(R_o) << \overline{\sigma_\theta} < \sigma_{\theta\theta}(R_i)$$

$$\frac{56 - 56.1}{56.1} = 0.2\% \text{ too low @ outer radius}$$

$$\frac{56 - 50.1}{50.1} = 12\% \text{ too high @ inner radius}$$

* As an average value, thin-wall is conservative b/c it approximately matches the max stress value.

3.2) $\sigma_\theta(R) = \frac{P \cdot R}{\delta}$

$\epsilon = \begin{bmatrix} \frac{P \cdot R}{\delta} & 0 \\ 0 & \frac{P}{\delta} \end{bmatrix} =$

3.2) What is the displacement??

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$$= \frac{70\,GPa}{(1.41)(0.18)} \begin{bmatrix} .59 & .41 \\ .41 & .59 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$$= \begin{bmatrix} 162.7 & 113.1 \\ 113.1 & 162.7 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

Stress tensor

Strain tensor is $\begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$

but what is displacement, u for this??

Next page

3(a) Just looking @ Hoop stress:

$u(r)$ isn't

(b) Given elastic expansion,

$$\varepsilon = \frac{\sigma}{E} \Rightarrow$$

$$\begin{bmatrix} 2.32 & 1.62 \\ 1.62 & 2.32 \end{bmatrix}$$

3. d) ★ Resubmit ★

This problem was just wrong. I didn't really understand this section and I'm still not sure I do. Nevertheless, the

Formulae for isotropic materials are on lecture 12, slide 19 + somehow they result in a diagonal matrix.

Pretty sure this implies there are no σ_{rz}, $\sigma_{\theta z}$, $\sigma_{r\theta}$ etc interactions...?

Anyhow:

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right) = \frac{1}{70GPa}\left(-3MPa - .41(28+56)\right)$$

$$= \boxed{-5.35E-4}$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right) = \frac{1}{70E3}\left(56 - .41(-3+28)\right)$$

$$= \boxed{6.84E-4}$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{rr} + \sigma_{\theta\theta}\right)\right) = \frac{1}{70E3}\left(28 - .41(56+28)\right)$$

$$= \boxed{8.96E-5}$$

$$\Rightarrow \boxed{\bar{\epsilon} = \begin{bmatrix} -53 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 65 \end{bmatrix} \times 10^{-5}}$$