- Find max stress in fuel pellet

LHR: $250\ W/cm$ $E = 250\ GPa$ $\nu = 0.25$

$\alpha_F = 7.5\times10^{-6}\ K^{-1}$ $K_F = 0.2\ W/cm\cdot K$

$\sigma_\theta = \sigma_{max}$ $\sigma_\theta(r = R_F) = \sigma_{max,\theta}$

$\sigma_\theta = -\sigma^*(1-3\eta^2)$ $\eta = \frac{r}{R_F}$ $\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$

$$T_0 - T_s = \Delta T_F = \frac{LHR}{4\pi K_F} = \frac{250}{4\pi(0.2)} = 99.5\,K$$

$$\sigma^* = \frac{(7.5\times10^{-6})(250\times10^3\ MPa)(99.5)}{4(1-0.25)} = 62.2\ MPa$$

$$\eta = \frac{r}{R_F} \rightarrow \eta = \frac{R_F}{R_F} = 1$$

$$\sigma_\theta = -\sigma^*(1-3\eta^2) \rightarrow -62.2(1-3(1)^2) = \boxed{124.4\ MPa}$$

$$\sigma_r = -\sigma^*(1-\eta)$$
$$\sigma_\theta = -\sigma^*(1-3\eta^2)$$
$$\sigma_z = -2\sigma^*(1-2\eta^2)$$

$\rightarrow$ min stress $\eta = \frac{r}{R_F}$

$\sigma_z(r=0)$ $r=0$, $\eta=0$

$\sigma^* = 62.2\,MPa$

$\sigma_z = -2 \times 62.2(1-2(0)^2)$

$\sigma_z = -124.4\,MPa$

Zircaloy cladding w/ $P = 6$ MPa, $R_i = 5$ mm, $R_o = 5.8$ mm
$E = 250$ GPa

→ find stress assuming thin walled approx

$$\sigma_\theta = \frac{pR}{\delta} \qquad \sigma_r = \frac{-p}{2} \qquad \sigma_z = \frac{pR}{2\delta}$$

δ = cladding thickness

$R = \bar{R} = \frac{R_o + R_i}{2}$

$$\sigma_\theta = \frac{6(5.4)}{0.8} = 40.5 \text{ MPa}$$

$$\sigma_r = \frac{-6}{2} = -3 \text{ MPa} \qquad \sigma_z = \frac{6(5.4)}{2(0.8)} = 20.3 \text{ MPa}$$

→ thick-walled, Δstress across cladding → σ_θ

$\sigma_\theta = \sigma_{max}$

$$\sigma_\theta = p \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$\sigma_\theta^i \rightarrow r = R_i$

$\sigma_\theta^o \rightarrow r = R_o$

$$\frac{R_o}{R_i} = 1.16$$

$$\sigma_\theta^i = 6 \frac{(1.16^2) + 1}{1.16^2 - 1} = 40.7 \text{ MPa}$$

$$\Delta\sigma_\theta = 6 \text{ MPa}$$

$$\sigma_\theta^o = 6 \frac{(1^2) + 1}{1.16^2 - 1} = 34.7 \text{ MPa}$$

$$\frac{\Delta\sigma_\theta}{\sigma_\theta^{max}} = 15\%$$

- assuming thin wall, strain states? $E = 250$ GPa, $\nu = 0.25$

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right) \rightarrow \frac{1}{250\times10^3}\left(-3 - 0.25(40.5 + 20.3)\right)$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right) \rightarrow \frac{1}{250\times10^3}\left(40.5 - 0.25(-3 + 20.3)\right)$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{rr} + \sigma_{\theta\theta})\right) \rightarrow \frac{1}{250\times10^3}\left(20.3 - 0.25(40.5 - 3)\right)$$

$$\epsilon_{rr} = -7.28\times10^{-5} \qquad \epsilon_{\theta\theta} = 1.45\times10^{-4} \qquad \epsilon_{zz} = 4.37\times10^{-5}$$

- given $\bar{T}_c = 450K$ $\bar{T}_F = 900K$

$\Delta \delta_{gap}$ $\alpha_c = 4\times10^{-6} K^{-1}$ $T_{ref}^c = 300K$

$\delta_{gap} = 0.01 cm$ $\alpha_F = 9\times10^{-6} K^{-1}$ $T_{ref}^F = 400K$

$R_F = 0.45 cm$ $R_i = 0.46 cm$ $R_o = 0.5 cm$

$$\Delta \delta_{gap} = \bar{R}_c \alpha_c (\bar{T}_c - T_{ref}^c) - R_F \alpha_F (\bar{T}_f - T_{ref}^F)$$

$$\Delta \delta_{gap} = 0.48(4\times10^{-6})(450-300) - 0.45(9\times10^{-6})(900-400)$$

$$\Delta \delta_{gap} = 2.88\times10^{-4} - 2.03\times10^{-3}$$

$$\Delta \delta_{gap} = -1.74\times10^{-3}$$

- find concentration of interstitials @ 1100K

$$S_f^i = 2K_B \qquad E_f^i = 3eV$$

$$C_i = \exp\left(\frac{S_f}{K_B}\right)\exp\left(\frac{-E_f}{K_B T}\right)$$

$$C_i = \exp\left(\frac{2K_B}{K_B}\right)\exp\left(\frac{-3}{(8.617\times10^{-5})(1100)}\right)$$

$$C_i = 1.33\times10^{-13}$$

→ @ what pressure will a thick-walled cladding w/

$R_o = 0.55\,cm$ $R_i = 0.5\,cm$ $\sigma_y = 400\,MPa$

exceed its yield stress?

$$\sigma_\theta = p\,\frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$\sigma_\theta = \max @ r = R_i$

$$\sigma_y = \sigma_\theta = p\,\frac{\left(\frac{0.55}{0.5}\right)^2 + 1}{\left(\frac{0.55}{0.5}\right)^2 - 1} = 400$$

$$\boxed{p = 38\,MPa}$$

→ how far do cracks extend into the fuel?

$R_F = 0.55\text{cm}$ $\nu = 0.25$ $E = 200\,\text{GPa}$

$LHR = 200\,\text{W/cm}$

$\alpha_F = 11\times10^{-6}\,K^{-1}$ $\sigma_{fr} = 120\,\text{MPa}$ $\eta = \frac{r}{R_F}$

$K_F = 0.05\,\text{W/cm-K}$

$$\sigma_\theta = \sigma^*(1-3\eta^2)$$

$$\Delta T_F = \frac{LHR}{4\pi K_F} \qquad \sigma^* = \frac{\alpha E}{4(1-\nu)}\Delta T$$

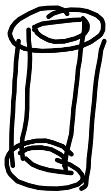
$$\sigma^* = \frac{(11\times10^{-6})(200\times10^3)(318)}{4(1-0.25)}$$

$$\Delta T_F = \frac{200}{4\pi(0.05)} = 318\,K$$

$$\sigma^* = 233\,\text{MPa}$$

$$\sigma_\theta = \sigma^*(1-3\eta^2) = \sigma_{fr} \rightarrow 120 = 233(1-3\eta^2)$$

$$\boxed{\eta = 0.402 = \frac{r}{R_F} \qquad r = 0.22\,\text{cm}}$$



cylinder, $p=0$ & TE $\quad \Delta T = (T_i - T_{ref})$

$$\sigma_r = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left(\frac{r}{R_i} - 1\right)\left(1 - \frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\sigma_\theta = \sigma_z = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left(1 - \frac{2R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right) + \alpha \Delta T$$