1) Backwards Euler

$$y' = 4t - 3t^2 \qquad y(t_0) = 6 \qquad dt = 0.33$$
$$t_0 = 1$$

$t_1 = 1.33$ $\qquad Y_1 = Y_0 + dt\, Y_1'$

$$= 6 + 0.33\left(4(1.33) - 3(1.33)^2\right) = 6.004$$

$t_2 = \cancel{t}1.67$ $\qquad Y_2 = Y_1 + d\cancel{t}\, Y_2'$

$$= 6.004 + 0.33\left(4(1.67) - 3(1.67)^2\right) = 5.447$$

$t_3 = 2.0$ $\qquad Y_3 = Y_2 + dt\, Y_3'$

$$= 5.447 + 0.33\left(4(2) - 3(2)^2\right) = \underline{4.127}$$

2) Zircaloy cladding w/ $r_i = 0.5$ cm $\quad r_o = 0.55$ cm

$p = 15$ MPa

- thin walled assumption: $\qquad \sigma_\theta = \frac{pR}{\delta} \qquad \sigma_r = \frac{-p}{2} \qquad \sigma_z = \frac{pR}{2\delta}$

$r_{avg} = 0.525$ cm

$$\sigma_\theta = \frac{(15)(0.525)}{0.05} = \underline{157.5 \text{ MPa}} \qquad \sigma_r = \frac{-15}{2} = \underline{-7.5 \text{ MPa}}$$

$$\sigma_z = \frac{\sigma_\theta}{2} = \underline{78.8 \text{ MPa}}$$

Thick-walled @ inner wall

$$\sigma_\theta = \frac{\left(\frac{r_o}{r}\right)^2+1}{\left(r_o/r_i\right)^2-1}\, p \qquad \sigma_r = -p\,\frac{\left(r_o/r\right)^2-1}{\left(r_o/r_i\right)^2-1}$$

$$\sigma_z = \frac{p}{\left(r_o/r\right)^2-1}$$

$$\sigma_\theta = 15\,\frac{\left(\frac{0.55}{0.5}\right)^2+1}{\left(\frac{0.55}{0.5}\right)^2-1} = \underline{157.9\ \text{MPa}} \qquad \sigma_r = -15\,\frac{\left(\frac{0.55}{0.5}\right)^2-1}{\left(\frac{0.55}{0.5}\right)^2-1}$$

$$\sigma_r = \underline{-15\ \text{MPa}}$$

$$\sigma_z = \frac{15}{\left(\frac{0.55}{0.5}\right)^2-1} = \underline{71.4\ \text{MPa}}$$

3) Max stress in fuel pellet due to thermal exp.?

$\Delta T = T_0 - T_s = 425\ K$ $\quad \alpha_F = 12\times10^{-6}\ 1/K$ $\quad R_F = 0.5\ cm$ $\quad E_F = 180\ GPa$

$\nu = 0.28$

$$\sigma^* = \frac{\alpha E\,(T_0 - T_s)}{4(1-\nu)}$$

$$\sigma_r = -\sigma^*(1-\eta^2)$$

$$\sigma_\theta = -\sigma^*(1-3\eta^2) \qquad \eta = \frac{r}{R_F}$$

$$\sigma_z = -2\sigma^*(1-2\eta^2)$$

max $= \sigma_\theta$ @ $r = R_F$

$$\sigma^* = \frac{(12\times10^{-6})(180\times10^{3})(425)}{4(1-0.28)} = 318.75\ \text{MPa}$$

$$\sigma_\theta = -\sigma^*\left(1-3(1)^2\right) = \underline{637.5\ \text{MPa}}$$

4) Cladding tube w/ thermal expansion

- no pressure

$R_i = 0.6$ cm $\quad t_c = 0.1$ cm $\quad E = 250$ GPa $\quad \nu = 0.3$

$\alpha_c = 15\times10^{-6}$ 1/K $\quad \bar{T}_c = 600$ K $\quad T_c^0 = 300$ K

Stresses @ $r = 0.62$?

$$\sigma_r = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left(\frac{r}{R_i}-1\right)\left(1-\frac{R_i}{t_c}\left(\frac{r}{R_i}-1\right)\right)$$

$$= \frac{300}{2}\frac{(15\times10^{-6})(250\times10^{3})}{1-0.3}\left(\frac{0.62}{0.6}-1\right)\left(1-\frac{0.6}{0.1}\left(\frac{0.62}{0.6}-1\right)\right)$$

$$\sigma_r = (\ 803.6\ )(0.0267) = 21.4 \text{ MPa}$$

$$\sigma_\theta = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{t_c}\left(\frac{r}{R_i}-1\right)\right) = \sigma_z$$

$$\sigma_\theta = (803.6)\left(1-2\frac{0.6}{0.1}\left(\frac{0.62}{0.6}-1\right)\right) = 482.16 \text{ MPa}$$

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5) Gap Thickness change

$\alpha_f = 12\times10^{-6}$ $\quad \alpha_c = 15\times10^{-6}$ $\quad \bar{T}_f = 925$ K $\quad \bar{T}_c = 550$ K

$R_f = 0.5$ cm $\quad \bar{R}_c = 0.58$ cm $\quad T_0^{c,f} = 300$ K

$t_g = 0.03$ cm $\quad t_c = 0.1$ cm

$$\Delta t_g = \bar{R}_c\alpha_c(\bar{T}_c - T_0^c) - R_f\alpha_f(\bar{T}_f - T_0^f)$$

$$\Delta t_c = (0.58)(15\times10^{-6})(250) = 0.002175$$

$$\Delta t_f = (0.5)(12\times10^{-6})(625) = 0.00375$$

$$t_g = 0.03 + (0.002175 - 0.00375) = \boxed{0.0284 \text{ cm}}$$