NE 591-010

Nuclear Fuel Performance

Exam 1

1) $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+Q=0$

$T'(r_0)=0 \quad r_0=0$

$T(r_1)=T_1 \quad r_1=X$

$\int \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=\int -Q \, \partial x$

$k\frac{\partial T}{\partial x}=-Qx+C_1 \rightarrow k(0)=Q(0)+C_1$

$C_1=0$

$\int k\frac{\partial T}{\partial x}=\int -Qx \, \partial x$

$T(x)=\frac{-Q}{2k}x^2+C_2 \qquad T_1=\frac{-Q}{2k}X^2+C_2$

$C_2=T_1+\frac{Q}{2k}X^2$

$T(x)-T_1=\frac{Q}{2k}X^2-\frac{Q}{2k}x^2 \rightarrow \frac{Q}{2k}(X^2-x^2)$

Steady-state

1-D

K is constant

a) T_o? $T(r=0.4)$?

$LHR = Q_{max} \pi R_F^2$

$LHR = \left(400 \frac{W}{cm}\right) \pi (0.6^2)$

$LHR = 452 \frac{W}{cm}$

$$T_{co_1} - T_{cool} = \frac{LHR}{2\pi R_F h_{cool}}$$

$$T_{co_1} - 800 = \frac{451}{2\pi(0.6)} \frac{1}{5.5} = 21.8\,K \qquad \frac{LHR}{2\pi R_F} = 120$$

$$T_{co_1} = 821.8\,K$$

$$T_{co} - T_{co_1} = \frac{LHR}{2\pi R_F} \frac{t_{co_1}}{K_{co_1}} = (120)\left(\frac{0.01}{0.05}\right) = 24\,K$$

$$T_{co} = 845.8\,K$$

$$T_{c_1} - T_{co} = \frac{LHR}{2\pi R_F} \frac{t_c}{K_c} = (120)\left(\frac{0.05}{0.15}\right) = 40\,K$$

$$T_{c_1} = 885.8\,K$$

$$T_F - T_{c_1} = \frac{LHR}{2\pi R_F} \frac{t_f}{K_f} = (120)\left(\frac{0.2}{0.25}\right) = 96\,K$$

$$T_F = 981.8\,K$$

$$T_o - T_F = \frac{LHR}{4\pi K_F} = \frac{452}{4\pi(0.5)} = 72\,K \qquad T_F = 1053.7\,K$$

$$T(r=0.4) = \frac{Q}{4K}\left(R^2 - r^2\right) + T_s = \frac{400}{4(0.5)}\left(0.6^2 - 0.4^2\right) + 981.8\,K$$

$$T(r=0.4) = 1021.8\,K$$

3) U_3Si_2 $K = 14.5 \frac{W}{m\text{-}K}$ enrich = 19.5%

$\rho = 15.67 \frac{g}{cc}$ $\phi = 2\times10^{14} \frac{n}{cm^2\text{-}s}$

Q? $Q = E_f N_f \sigma_f \phi$

N_f? $m(U) = 235 \times 0.195 + 238 \times 0.805 = 237.415$

$m(U_3Si_2) = 3(237.415) + 2(28) = 768.245 \frac{g}{mole}$

$$15.67 \frac{g}{cc} \frac{1\,mole}{768.245\,g} \frac{6.022\times10^{23}\,at}{1\,mole} \frac{3U}{1U_3Si_2} \times 0.195 = 7.19\times10^{21}$$

$$Q = (200\times10^6\,eV)\left(1.602\times10^{-19}\frac{J}{eV}\right)(7.19\times10^{21})(570\times10^{-24})(2\times10^{22})$$

$Q = 262.6 \frac{W}{cm^3}$

match N_f for UO_2 x = enrich

$m(UO_2) = 235x + 238(1-x) + 32 = 270 - 3x$

$$10.97 \frac{g}{cc} \frac{1\,mole}{270-3x} \frac{6.022\times10^{23}\,at}{1\,mole} \frac{1U}{1UO_2} x = 7.19\times10^{21}$$

$$\frac{x}{270-3x} = 1.08\times10^{-3}$$

$$x = -3.26\times10^{-3}x + 0.294$$

$x = 0.293$ 29.3%

4)

$$LHR\left(\frac{z}{z_0}\right) = LHR^\circ \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right]$$

a) $z_0 = 1.5\ m \qquad LHR^\circ = 150\ W/cm \qquad \gamma = 1.1$

$$LHR(z = 1.8) = 150 \times \cos\left[\frac{\pi}{2(1.1)}\left(\frac{1.8}{1.5} - 1\right)\right]$$

$$= \underline{143.9\ W/cm}$$

b) $$\Delta T_{cool} = \frac{2\gamma}{\pi}\frac{z_0\, LHR^\circ}{\dot{m}\, C_p}\left(\sin\frac{\pi}{2\gamma} + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0} - 1\right)\right)\right)$$

$\frac{\pi}{2\gamma} = 1.43$

$$\Delta T_{cool}(water) = \frac{1}{1.43}\frac{(150)(150)}{(0.22)(4200)}\left(\sin(1.43) + \sin(1.47(2-1))\right)$$

$$\Delta T_{cool}(water) = \underline{33.7\ K}$$

$$\Delta T_{cool}(N_2) = \frac{1}{1.43}\frac{(150)(150)}{(0.12)(1404)}\left(2 \times \sin(1.43)\right)$$

$$\Delta T_{cool}(N_2) = \underline{184.9\ K} \leftarrow$$

5) $y(t_0) = 6$ $\quad t_0 = 1$ $\quad \frac{dy}{dt} = 4t - 3t^2$ $\quad dt = 0.33$ $\quad t \to 2$

Forward $\quad y_{n+1} = y_n + dt\, y'_n$

$t_1 = 1.33 \quad y_1 = y_0 + dt\, y'_0 = 6 + 0.33(4(1) - 3(1)^2) = 6.33$

$t_2 = 1.67 \quad y_2 = y_1 + dt\, y'_1 = 6.33 + 0.33\left(4(1.33) - 3(1.33)^2\right)$

$= 6.334$

$t_3 = 2 \quad y_3 = y_2 + dt\, y'_2 = 6.334 + 0.33\left(4(1.67) - 3(1.67)^2\right)$

$= 5.78$

Backward $\quad y_{n+1} = y_n + dt\, y'_{n+1}$

$t_1 = 1.33 \quad y_1 = y_0 + dt\, y'_1 = 6 + 0.33\left(4(1.33) - 3(1.33)^2\right) = 6.004$

$t_2 = 1.67 \quad y_2 = y_1 + dt\, y'_2 = 6.004 + 0.33\left(4(1.67) - 3(1.67)^2\right)$

$= 5.447$

$t_3 = 2 \quad y_3 = y_2 + dt\, y'_3 = 5.447 + 0.33\left(4(2) - 3(2)^2\right)$

$= 4.127$

(6). Fertile : can be converted into fissile via neutron capture

Fissile : capable of fission w/ any neutron energy

Fissionable : capable of undergoing fission w/ high E neutron capture

(7) Low melting point
Anisotropic thermal expansion
Anisotropic irradiation growth
Poor behavior in water
Heterogeneous phase mixture
Extreme Swelling

(8) Relative volume of fuel compared to maximum possible volume inside cladding r_1^2/r_2^2

Necessary because of inevitable fuel swelling

(9). - We enrich U to increase N_i (fissile atom number density), to have higher heat generation rate, thus higher power output of fuel.

- UF_6 is the compound that undergoes enrichment
- Spinning cylinder filled w/ UF_6
  - heavier mass of $U^{238}F_6$ preferentially moves to the outside of the canister due to centrifugal force
  - enriched $U^{235}F_6$ withdrawn from center of canister

(10) Mo + Xe

Mo has $Z = 42$ and $A = 96$

Xe has $Z = 54$ and $A = 131$

- Double hump fission product distribution has peaks at around $A = 95$ and $A = 135$
- other examples valid

(11) Finite difference

Finite volume

Finite element

FEM are used in S-O-T-A simulations because of the flexible geometry, ability to handle any boundary condition, and continuous representation of quantities of interest