90/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

Question 1: 22/30

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

$k = 12.5 \left(\frac{W}{m \cdot K}\right)$ $\rho = 7.5 \left(\frac{g}{cm^3}\right)$ of U

$\frac{7.5 (g\ U)}{(cm^3)} \cdot \frac{1 (mol\ U)}{237.97862 (g\ U)} \cdot \frac{N_A (atom) U}{1 (mol)\ U} \cdot \frac{0.00711 (atom)\ U\text{-}235}{1 (atom)\ U}$

fissile isotope: U-235

unenriched: $U = \begin{cases} 0.711\%\ U\text{-}235 \\ \approx 99.289\%\ U\text{-}238 \end{cases}$

$1.349 \times 10^{20} \left(\frac{atom}{cm^3}\right)$ U-235

enr $= 0.005226$ $\approx 0.52\%$

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$Q = E_f N_f \sigma_f \varphi_{th}$; $E_f = 3 \times 10^{-11} \left(\frac{J}{fission}\right)$

$\sigma_f = 5.5 \times 10^{-22} (cm^2)$

$\varphi_{th} = 3.2 \times 10^{13} \left(\frac{n}{cm^2 \cdot s}\right)$

You didn't use density of U in U3Si5
Just use ratio of two U densities

-8

$M_{U_3Si_2} \approx 764.73 (g/mol)$

$N_{U235} \approx \frac{(3)(0.03)(N_A)(12.2)}{764} = 8.59 \times 10^{20} \left(atom \frac{U\text{-}235}{cm^3}\right)$

$Q_{U_3Si_2} = 453.552 \left(\frac{W}{cm^3}\right)$

I need $8.59 \times 10^{20} \left(\frac{atom\ U\text{-}235}{cm^3}\right)$

mass of U_3Si_5

$\frac{N_{U235}(854)}{(12.2)(N_A)(3)} = 0.332827 \approx$ 33.2% enrichment required

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

It would be not good. $K_{U_3Si_5} = 0.125 \left(\frac{W}{cm \cdot K}\right)$ and $K_{U_3Si_2} = 0.23 \left(\frac{W}{cm \cdot K}\right)$

enr = 3% enr = 33%

It conducts heat worse and you need higher enrichment to get the same results. Bad idea.

↳ Also cannot enrich ~~that~~ high in U.S.

Question 2: -2, 33/35

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$r = 0.45 \, (cm)$ 5% Xe

$t_g = 80 \times 10^{-6} m = 0.008 (cm)$

$t_c = 0.06 \, (cm)$

$LHR = 250 \left(\frac{W}{cm}\right)$

$T_{cool} = 580 \, (K)$

$h_{cool} = 2.5 \left(\frac{W}{cm^2 K}\right)$

$\boxed{T_s = 957.80 \, (K)}$

Show work!

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$E = 2.467 \times 10^{11} Pa$

$E = 2.467 \times 10^{7} \left(\frac{N}{cm^2}\right)$

$\sigma_{MAX} = \sigma_{\theta\theta}$

$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2)$; where $\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$; $\eta = \frac{r}{R_f}$

$T_0 - T_s = 99.4718 \, (K)$

$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2)$; $\eta = \frac{R_f}{R_f} = 1$

$\sigma^* = \frac{(7.5 \times 10^{-6} \frac{1}{K})(2.467 \times 10^{7} \frac{N}{cm^2})(99.4718 K)}{4(1 - 0.25)}$

$\sigma_{\theta\theta} = -\sigma^*(1-3) = -\sigma^*(-2)$

$\sigma^* = 6134.923 \left(\frac{N}{cm^2}\right)$

$\boxed{\sigma_{\theta\theta} = \sigma_{MAX} = 12,269.846 \left(\frac{N}{cm^2}\right)}$

-1, Use standard units (Pa)

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

Higher in UO_2 b/c UO_2 stored heat and conducts poorly.
↳ higher $(T_0 - T_s)$

d) What assumptions were made in your calculations for a) and b)? (5 points)

Part A assumptions:
- steady state
- axisymmetric
- T is constant in z
- K is constant

Part B assumptions:
- no other stresses except for thermal (no pressure)
- hoop stress is maximum
- no fracture or relaxation of material (which would relieve thermal stress).

-4, 31/35

Question 3:

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

Assume: – small strains
– isotropic material
– "very thin walls" means that stress is constant through wall

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$\bar{\sigma}_\theta = \frac{pR}{\delta}$; $\bar{\sigma}_z = \frac{pR}{2\delta}$ $\bar{\sigma}_r = -\frac{1}{2}p$

$r = 0.0056 (m)$
$t_c = 6 \times 10^{-4} (m)$
$p = 6 \times 10^{6} (P_a) = (\frac{N}{m^2})$

$\bar{\sigma}_\theta = 56 \times 10^{6} (P_a)$ $\boxed{\bar{\sigma}_z = 28 (MP_a)}$ $\boxed{\bar{\sigma}_r = -3.0 (MP_a)}$

$\boxed{\bar{\sigma}_\theta = 56 (MP_a)}$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

$R_i \approx 0.0053 (m)$

$R_o \approx 0.0059 (m)$

$\bar{\sigma}_{\theta\theta} = p \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$; $r = R_i$

-4, Evaluate at multiple r positions to check approximation

$\bar{\sigma}_{\theta\theta} = 56.16071429 (MP_a)$

$\bar{\sigma}_{\theta\theta} \approx 56.2 (MP_a)$ 0.16(m)

$\boxed{No}$ not conservative, but very close... off by 0.29 %

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points) $E = 70{,}000 (MP_a)$

-37,44

$\varepsilon_{rr} = \frac{1}{E}\left[\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right]$

-4, No tensors

$\boxed{\varepsilon_{rr} = -5.348571 \times 10^{-4} (m/m)}$

$\varepsilon_{\theta\theta} = \frac{1}{E}\left[\sigma_{\theta\theta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right]$

$\boxed{\varepsilon_{\theta\theta} = 6.535714 \times 10^{-4} (m/m)}$

$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{\theta\theta} + \sigma_{rr}\right)\right]$

$\boxed{\varepsilon_{zz} = 8.957143 \times 10^{-5} (m/m)}$

TENSOR EQUATION:

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{zz} \\ \varepsilon_{\theta\theta} \end{bmatrix}$$