81/100

1 MATT McCLOSKEY 3/3/17 Pg: 1

Exam 1

PROFESSOR TONKS

NucE 497: Fuel Performance

-8, 22/30

1.) Given:

U_3Si_5, LWR, $k = 12.5$ [W/mK], $\rho_U = 7.5$ [g/cm³]

Find: A fissile isotope in U_3Si_5

B Enrichment Nq. for 3% w/ neutron flux = 3.2×10^{13} [n/cm²s]

C U_3Si_5 comparable to U_3Si_2?

Sol:

A) Uranium 235 = fissile isotope

Enrichment: .7%

B) Q:

$Q = E_f N_f^{35} \sigma_f^{35} \phi_{th}$

$U_3Si_2 \rightarrow 3\%$ Enrichment

$N_{U235} = \frac{3 \times (.03) \times (N_A) \times (7.5)\left[\frac{At\, g}{mol\, cm^3}\right]}{770 \left[g/mol\right]}$ $M_{U_3Si_2} \cong 770$

$N^{35}_{U_3Si_2} = 5.77 \times 10^{20}$ [At/cm³]

$Q_1 = Q_2$

$\cancel{E_f N \sigma_f \phi_{th}} = \cancel{E_f N \sigma_f \phi_{th}}$

$N^{35}_{U_3Si_2} = N^{35}_{U_3Si_5}$ for same energy output

$5.77 \times 10^{20} = \frac{3 \times (E_n) \times N_A \times 7.5}{854}$ $(M_{U_3Si_5} \cong 854\, g/mol)$

$E_{n\,U_3Si_5} = \frac{5.77 \times 10^{20} [At/cm^3] \times 854 [g/mol]}{3 \times (N_A [At/mol]) (7.5 [g/cm^3])}$

$E_n = .0364$ -5, just use U densities

$\boxed{\text{Enrichment for } U_3Si_5 \text{ is} \sim 3.64\%}$

C) U_3Si_2 is favorable due to its lower Enrichment requirement. Judging by the chart of melting points, U_3Si_2 = more easily produced but U_3Si_5 is potentially comparable due to its higher melting point, a deficiency for the 'Si' fuels. -3, thermal conductivity?

2) -2, 33/35 Pg: 2

Given:

$r_f = 4.5[mm] = .45[cm]$, $t_g = .008[cm]$, $t_c = .06[cm]$, $LHR = 250[W/cm]$

$T_{cool} = 580[K]$, $\eta_{xe} = .05$, $h_{cool} = 2.5[W/cm^2K]$, $k_c = .17[W/cmK]$ (UO2)

Find: $T_s = ?$ (A)

$\sigma_{max} = ?$ (Pellet) (B)

stress Higher or lower for UO2? (C)

sol:

A) $T_{oc} = \frac{LHR}{2\pi R_f h_{cool}} = \frac{250}{2\pi(.45)(2.5)} = 35.38 + 580 = 615.4 [K]$

$T_{IC} = LHR\, t_c / (2\pi R_f k_c) = (250 \times .06) / 2\pi(.45)(.17) = 31.22 + 615.4 = 646.6$

$h_{gap} = \frac{k_{gap}}{t_{gap}}$

$k_{gap} = k_{He}^{1-\eta} k_{xe}^{\eta}$

$k_{He} = (16 \times 10^{-6})(\cancel{615.4}\ 646.6)^{.79} = \cancel{.00256}\ .00266$

$k_{xe} = (.7 \times 10^{-6})(\cancel{615.4})^{.79} = \cancel{1.11 \times 10^{-4}}\ 1.16 \times 10^{-4}$

$k_{gap} = .00266^{.95}(1.16 \times 10^{-4\ .05}) = .00227 [W/cmK]$

$= \cancel{.00240} = \cancel{.0021}$

$h_{gap} = \frac{.00227}{.008} = .2838 [W/cm^2K]$

$T_s = LHR / (2\pi R_f h_{gap}) + T_{IC} = 250 / (2\pi(.45)(.2838)) = 311.7 + 646.6 [K]$

$\boxed{T_s = 958.3 [K]}$

B) Max stress = ?

Hoop is max (thermal)

$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2)$

$\eta = r/R_f$ @ max radius $= 1$

$*\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)} = \frac{(7.5E-6)(246.2[GPa])(694.4)}{4(1-\nu)} = .428 [GPa]$

$\nu = .25$

UN: $k = .2 [W/cmK]$

$T_0 = \frac{LHR}{4\pi k} + T_s = \frac{250}{4\pi(.2)} + 958.3$

$= 1652.7$ -2 Math error

$\sigma_{\theta\theta} = 2\sigma^* = 2(.428) = .856 = \boxed{856.5 [MPa] = \sigma_{max}}$

c)

UO_2 would be greater, bc smaller k_F value increases radial temp Gradient in fuel, thus raising ~~pressure~~ stress.

3) D) It is assumed that the change in temp through the gap, clad, & fuel is linear. Assumed that K_{gap} is independent of ~~gap thickness~~ gap temperature and that the gap isn't changing. Assumed Xe amt. is constant, the fuel, clad are uniform and in steady state and that there is only ~~radial~~ changes in the radial temperature profile.

-9, 26/35

3) Given fuel rod, $P = 6[MPa]$, $R_F = .56[cm]$, $t_c = .06[cm]$

A) thin walled approx assumptions:

- Wall is very thin relative to radius / other dimensions
- geometry & pressure loading is axisymmetric
- Isotropic material response
- Supports, end effects ignored. (Gravity too?)

-3, Stress is constant across thickness

B)

$$\boxed{\begin{aligned} \bar{\sigma}_\theta &= \frac{PR}{\delta} = \frac{6(.56)}{.06} = 56[MPa] \\ \bar{\sigma}_z &= \tfrac{1}{2}\bar{\sigma}_\theta = 28[MPa] \\ \bar{\sigma}_R &= -\tfrac{1}{2}P = -3[MPa] \end{aligned}}$$

c) /W/o Approximation/

Averages

$$\boxed{\bar{\sigma}_{rr}} = -P\frac{(R_o/\bar{r})^2 - 1}{(R_o/R_i)^2 - 1} = \frac{\left(\frac{5.9}{5.6}\right)^2 - 1}{\left(5.9/5.3\right)^2 - 1}(-6) = \boxed{-2.75[MPa]}$$

$$\boxed{\bar{\sigma}_{\theta\theta}} = P\frac{(R_o/\bar{r})^2 + 1}{(R_o/R_i)^2 - 1} = \boxed{52.9[MPa]}$$

-4, Calculate stress at two radii and compare to see if it is constant across thickness

$$\boxed{\bar{\sigma}_{zz}} = P\left(1/\left((R_o/\bar{r})^2 - 1\right)\right) = \boxed{25.1[MPa]}$$

All of the realistic values are lower than the thinwalled assumption, therefore it is not conservative if used to estimate cladding failure

C (cont)

The estimation overshoots by:

$\sigma_{\theta\theta} \rightarrow 3.1\,[MPa]$

$\sigma_{zz} \rightarrow 2.9\,[MPa]$

$\sigma_{rr} \rightarrow .25\,[MPa]$

D) Stress tensor:

$$\cancel{\sigma = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{\theta r} & \sigma_{\theta\theta} & \sigma_{\theta z} \\ \sigma_{zr} & \sigma_{z\theta} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} -2.75 & & \\ & 52.9 & \\ & & 25.1 \end{bmatrix}}$$

thin walled, Plane stress

$$\sigma = \begin{bmatrix} \sigma_{zz} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \frac{pR}{2t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 28 & 0 & 0 \\ 0 & 56 & 0 \\ 0 & 0 & 0 \end{bmatrix} [MPa] = \sigma}$$

$$\cancel{\text{Strain Tensor} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} = \frac{1}{70\,[GPa]}\begin{bmatrix} 1 & -.41 & -.41 \\ -.41 & 1 & -.41 \\ -.41 & -.41 & 1 \end{bmatrix}}$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right)$$
$$= \frac{1}{70\times10^3}\left(52.9 - .41(28-3)\right) = 6.1\times10^{-4}$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right) = 8.95\times10^{-5}$$

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{zz} + \sigma_{\theta\theta})\right) = \ldots$$

$$\boxed{\text{Strain tensor} = \begin{bmatrix} 8.95\times10^{-5} & 0 & 0 \\ 0 & 6.1\times10^{-4} & 0 \\ 0 & 0 & \epsilon_{rr} \end{bmatrix}}$$

-1 Strain in tensor form
-1 missed eps_rr