NucE 497 : Exam 1

Atomic and Nuclear Physics & Interaction of Radiation with Matter

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Please print your name clearly

Read all of the following information before starting the quiz:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator (if a calculator is allowed), sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This exam has 7 problems and is worth 100 points, partial credit will be given for incomplete answers. If you are not sure how to answer a problem give as much information as you can.
- Good luck!

1. (*10 points*) Calculate the Q-value for the following two β radioactive decays:

a) ${}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{1}e + v$

b) ${}^{38}_{17}\mathrm{Cl} \rightarrow {}^{38}_{18}Ar + {}^{0}_{-1}e + \overline{v}$

$m_e = 0.51099892\ MeV/c^2$

$1u = 931.494043\ MeV/c^2$

a) $Q_{\beta^+}/c^2 = M({}^{22}_{11}Na) - M({}^{22}_{10}Ne) - 2m_e$

$M({}^{22}_{11}Na) = 33.027340\,u$, $M({}^{22}_{10}Ne) = 32.039910\,u$

$Q_{\beta^+}/c^2 = [(33.027340\,u) - (32.039910\,u)] \cdot \frac{931.494043\ MeV/c^2}{1\,u} - 2(0.51099892\ MeV/c^2) = 918.8097397\ MeV/c^2$

$\boxed{Q_{\beta^+}/c^2 = 918.810 MeV/c^2}$

b) $Q_{\beta^-}/c^2 = M({}^{38}_{17}Cl) - M({}^{38}_{18}Ar)$

$M({}^{38}_{17}Cl) = 37.96801044\,u$ $\quad M({}^{38}_{18}Ar) = 37.9627321\,u$

$Q_{\beta^-}/c^2 = [37.96801044 - 37.96273211\,u] \cdot \frac{931.494043\ MeV/c^2}{1\,u}$

$Q_{\beta^-}/c^2 = 4.916732952\ MeV/c^2$

$\boxed{Q_{\beta^-}/c^2 = 4.917\ MeV/c^2}$

2. (*15 points*) The isotope ^{132}I decays by β^- emission to ^{132}Xe with a half-life of 2.3 h.

a) How long will it take for 7/8 of the original number of ^{132}I nuclides to decay?

b) How long will it take for a sample of ^{132}I to lose 95% of its activity?

a) $T_{1/2} = \frac{\ln 2}{\lambda}$, $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{2.3h} = 0.301368339\ h^{-1}$

$\left(=\frac{1}{8} = \frac{1}{2^3} \Rightarrow \boxed{6.9h}\right.$

b) $A(t) = A_0 e^{-\lambda t} \Rightarrow \ln(0.05) = -\lambda t \Rightarrow t = \frac{-\ln(0.05)}{\lambda}$

$t = \frac{-\ln(0.05)}{0.301368339\ h^{-1}} = 9.940434618\ h$

$\boxed{t = 9.94\ h}$

3. (*10 points*) Using atomic mass data, compute the average binding energy per nucleon of the following nuclei:

a) $^{2}_{1}H$

b) $^{4}_{2}He$

c) $^{12}_{6}C$

d) $^{138}_{56}Ba$

e) $^{235}_{92}U$

a) $BE(^2_1H) = [M(^1_1H) + m_n - M(^2_1H)]c^2$

$M(^1_1H) = 1.00782503 21 u$

$m_n = 1.008664923 3 u$

$M(^2_1H) = 2.0141017780 u$

$BE(^2_1H) = (1.007825 0321 + 1.00866492 33 - 2.014101 7780) u$

$= [0.002388177 u \cdot \frac{931.494043 \, MeV/c^2}{u}] c^2$

$= 2.224572649 \, MeV$

$\frac{BE(^2_1H)}{A} = \frac{2.224572649 \, MeV}{2} = 1.112286325 \, MeV/nucleon$

$\boxed{\frac{BE(^2_1H)}{A} = 1.112 \, MeV/nucleon}$

b) $M(^4_2He) = 4.0026032497 u$

$BE(^4_2He) = [2 \cdot M(^1_1H) + 2m_n - M(^4_2He)]c^2$

$= (2 \cdot 1.0078250321 + 2 \cdot 1.0086649233 - 4.0026032497) u \cdot \frac{931.494043 \, MeV/c^2}{u}$

$= 28.29567886 \, MeV$

$\boxed{\frac{BE(^4_2He)}{A} = 7.074 \, MeV/nucleon}$

c) $M(^{12}_6C) = 12 u$, $BE(^{12}_6C) = [6 \cdot M(^1_1H) + 6m_n - M(^{12}_6C)]c^2$

$= (6 \cdot 1.0078250321 + 6 \cdot 1.0086649233 - 12) u \cdot \frac{931.494043 \, MeV/c^2}{u}$

$= 92.16177135 \, MeV$

$\boxed{\frac{BE(^{12}_6C)}{A} = 7.680 \frac{MeV}{Nucleon}}$

d) $^{138}_{56}Ba$ $\quad M(^{138}_{56}Ba) = 137.905241\,u$

$$BE(^{138}_{56}Ba) = (56 \cdot 1.0078250321 + 82 \cdot 1.00866491233 - 137.905241)u \cdot \frac{931.494043\ \frac{MeV}{c^2}}{u}$$

$$= 1158.248412\ MeV$$

$$\boxed{\frac{BE(^{138}_{56}Ba)}{A} = 8.393\ \frac{MeV}{nucleon}}$$

e) $^{235}_{92}U$ $\quad M(^{235}_{92}U) = 235.0439231\,u$

$$BE(^{235}_{92}U) = (92 \cdot 1.0078250321 + 143 \cdot 1.00866491233 - 235.0439231)u \cdot \frac{931.494043\ \frac{MeV}{c^2}}{u}$$

$$= 1783.870601\ MeV$$

$$\boxed{\frac{BE(^{235}_{92}U)}{A} = 7.591\ \frac{MeV}{nucleon}}$$

4. (*20 points*) Consider the chain decay $A \rightarrow B \rightarrow C$, with no atoms of B present at $t = 0$;

a) Show that the activity of B rises to a maximum value at the time t_m given by:

$$t_m = \frac{1}{\lambda_B - \lambda_A} LN \left[\frac{\lambda_B}{\lambda_A} \right]$$

b) Show that, for $t < t_m$, the activity of B is less than that of A, whereas the reverse is the case for $t > t_m$.

$$\frac{dN_A}{dt} = -\lambda_A N_A \Rightarrow N_A(t) = N_{A,0}\, e^{-\lambda_A t}$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \Rightarrow Q_A(t) = \lambda_A N_A(t)$$

$$N_B(t) = N_{B,0}\, e^{-\lambda_B t} + \frac{\lambda_A N_{A,0}}{\lambda_B - \lambda_A} \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right)$$

5. (*15 points*) The β^- emitter ^{28}Al (half-life 2.30 *min*) can be produced by the radioactive capture of neutrons by ^{27}Al. The 0.0253 *eV* cross-section for this reaction is 0.23 *b*. Suppose that a small, 0.01 *g* aluminum target is placed in a beam of 0.0253 *eV* neutrons, $\phi = 3 \times 10^8\ neutrons/cm^2 - sec$, which strike the entire target, calculate:

a) the neutron density in the beam,

b) the rate at which ^{28}Al is produced,

c) the maximum activity (in curies) that can be produced in this experiment.

$\phi = nv$, $T = \frac{1}{2} m_e v^2$, $v = \sqrt{2T/m_e} = \left(2 \cdot 0.0253\ eV / 0.51099892\ MeV/c^2\right)^{1/2}$

$v = 0.00031467c \Rightarrow 0.00031467 \cdot 2.99792458 \cdot 10^8\ \frac{m}{s} \cdot \frac{100\ cm}{m}$

$= 94322205.746\ cm/s$

$n = \frac{\phi}{v} = \frac{3 \cdot 10^8\ n/cm^2 s}{94322205\ cm/s} = 31.80592197\ n/cm^3$ $\boxed{n \approx 32\ n/cm^3}$

b) $R = \sigma \cdot \phi = 0.23b \cdot \frac{10^{-24}\ cm^2}{b} \cdot \frac{3 \cdot 10^8\ n}{cm^2 s} = \boxed{6.9 \cdot 10^{-17}\ s^{-1}}$

c)

7. (*15 points*) A 2 MeV photon is Compton scattered through an angle of 30°.

a) What is its energy after scattering?

b) What is the recoil energy of the struck electron?

c) At what angle does the electron recoil?

a) $E' = \left[\frac{1}{E} + \frac{1}{m_e c^2}[1-\cos\theta]\right]^{-1} = \left[\frac{1}{2\,MeV} + \frac{1}{0.511\,MeV}[1-\cos(30)]\right]^{-1} = 1.31\ MeV$

$\boxed{E' = 1.31\ MeV}$

b) $(2\ MeV - 1.31\ MeV) = \boxed{0.69\ MeV}$