81/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

Question 2:
Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm^2 K).

a) What is the surface temperature of the fuel rod? (15 points)

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

d) What assumptions were made in your calculations for a) and b)? (5 points)

Question 3:
Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with $E = 70$ GPa and $\nu = 0.41$. (10 points)

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1) (a) The fissile isotope in U_3Si_5 is Uranium-235. -3, 27/30
The natural enrichment of U-235 is 0.7%

(b) $\phi = 3.2\times10^{13}\ n/cm^2s$ $\quad \gamma_2 = 3\% = 0.03$ $\quad q_5 = ?$ $\quad \rho_u = 7.5\ g/cm^3$

$Q = E_f \cdot \sigma_f \cdot \phi \cdot N_f$

$Q_{U_3Si_2} = (E_f \cdot \sigma_f \cdot \phi) \cdot N_f \Rightarrow E_f \cdot \sigma_f \cdot \phi$ is same for both U_3Si_2 + U_3Si_5

$N_{f_{U_3Si_2}} = N_{f_{U_3Si_5}}$

$f_{w_2} = M_u / M_{U_3Si_2} = 238/(238\cdot3+28\cdot2) = .309$

$\rho_{U_3Si_2} = 11.31\ g/cm^3$ for U_3Si_2

$$\therefore N_{f_{U_3Si_2}} = \frac{q_2 N_a (\rho_u/f_w)}{M_{U_3Si_2}} = \frac{(0.03)(6.022\times10^{23}\ \frac{atoms}{mol})(11.31\ g/cm^3)}{(238\cdot3+28\cdot2)\ g/mol\ (.309)}$$

$$N_{f_{U_3Si_2}} = 8.59\times10^{20}\ \frac{atoms\ U\text{-}235}{cm^3} = N_{f_{U_3Si_5}}$$

$f_{w_5} = M_u / M_{U_3Si_5} = 238/(238\cdot3+28\cdot5) = 0.279$

$$q_5 = \frac{(M_{U_3Si_5})(N_{f_{U_3Si_5}})(f_{w_5})}{\rho_u \cdot N_A} = \frac{(238\cdot3+28\cdot5)\ g/mol\ (8.59\times10^{20}\ \frac{atoms}{cm^3})(.279)}{(7.5\ g/cm^3)(6.022\times10^{23}\ atoms/mol)}$$

$$q_5 = .045 = \boxed{4.5\%\ \text{enrichment}}$$

(c) U_3Si_5 would be worse than U_3Si_2, as it would likely have very similar material properties but requires substantially higher enrichment for the same energy output due to its low uranium density

-3, thermal conductivities?

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-0, 35/35

a) $R_f = .45\,cm$ $\delta_{gap} = .008\,cm$ $t_c = .06\,cm$ $LHR = 250\,W/cm$ $Y = .05$

$T_{cool} = 580\,K$ $h_{cool} = 2.5\,W/cm^2K$ $K_c = .17\,W/cm\,K$

(a) $T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250\,W/cm}{2\pi(.45\,cm)(2.5\,W/cm^2K)} + 580\,K$

$T_{co} = 615.4\,K$

$T_{ci} = \frac{LHR \cdot t_c}{2\pi R_f K_c} + T_{co} = \frac{(250\,W/cm)(.06\,cm)}{2\pi(.45\,cm)(.17\,W/cm\,K)} + 615.4\,K$

$T_{ci} = 646.6\,K$

$K_{gap} = K_{He}^{1-Y} \cdot K_{Xe}^{Y}$

$K_{He} = 16\times10^{-6}\,T_{ci}^{.79} = .00266\,W/cm\,K$

$K_{Xe} = .7\times10^{-6}\,T_{ci}^{.79} = 1.163\times10^{-4}\,W/cm\,K$

$h_{gap} = \frac{K_{gap}}{\delta_{gap}} = \frac{(.00266)^{.95}\cdot(1.163\times10^{-4})^{.05}}{.008\,cm} = .284\,\frac{W}{cm^2K}$

$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ci} = \frac{250\,W/cm}{2\pi(.45\,cm)(.284\,W/cm^2K)} + 646.6\,K$

$\boxed{T_s = 958.0\,K}$

(b) $UN \Rightarrow K = 0.2\,W/cm\,K$

max stress $\Rightarrow$ hoop stress at outer edge of pellet

$T_0 = \frac{LHR}{4\pi K} + T_s = \frac{250\,W/cm}{4\pi(.2\,W/cm\,K)} + 958.0\,K = 1057.5\,K$

$T_0 - T_s = 99.5\,K$

$\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)} = \frac{(7.5\times10^{-6}\,K^{-1})(246.7\,GPa)(99.5\,K)}{4(1-.25)} = 61.4\,MPa$

$\sigma_{\theta\theta\,max} = \sigma_{\theta\theta}\big|_{r=R_f} = -\sigma^*(1-3\eta^2) = -\sigma^*\left(1-3\left(\frac{r}{R_f}\right)^2\right)$

$\sigma_{\theta\theta\,max} = -(61.4\,MPa)(1-3) = \boxed{122.8\,MPa}$

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continued

2) (c) The stress experienced by the pellet would be much higher in UO_2 as its lower conductivity would yield a much higher temperature gradient and consequently higher thermal stress since $\sigma_{\theta\theta} \propto (T_0 - T_s)$

(d) Assumptions:

1 => Only steady state systems
2 => Axisymmetric behavior
3 => T is constant in Z direction (axial)
4 => Thermal conductivity is independent of temperature

Thermal Assumptions

5 => All strains are small
6 => Isotropic material response
7 => Static Body
8 => Negligible gravity

Mechanical assumptions

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3) $P = 60\text{ MPa}$ $\bar{R}_c = .56\text{ cm}$ $t_c = .06\text{ cm}$

-19, 16/35

(a) Thin walled pressure vessel assumptions:
Assuming very thin walls allows the stress state to be reduced and simplified into a simple force balance.

-5, Isotropic, small strain, and stress constant through thickness

(b) Using thin walled assumption:

-2, Values off by factor of 10

$$\bar{\sigma}_\theta = \frac{P \cdot R}{t_c} = \frac{(60\text{ MPa})(.56\text{ cm})}{.06\text{ cm}} = \boxed{560\text{ MPa} = \bar{\sigma}_\theta}$$

$$\bar{\sigma}_z = \frac{P \cdot R}{2 t_c} = \frac{1}{2}\bar{\sigma}_\theta = \boxed{280\text{ MPa} = \bar{\sigma}_z}$$

$$\bar{\sigma}_r = -\frac{1}{2}P = -\frac{1}{2}(60\text{ MPa}) = \boxed{-30\text{ MPa} = \bar{\sigma}_r}$$

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continued

3) (c) Quantifying accuracy by comparing to thick-walled analysis:

$R_o = \bar{R}_c + \frac{1}{2}t_c = .56\,cm + .03\,cm = .59\,cm$

$R_i = \bar{R}_c - \frac{1}{2}t_c = .56\,cm - .03\,cm = .53\,cm$

$$\sigma_{rr\,min} = \sigma_{rr}\big|_{r=R_i} = -P \cdot \frac{(R_o/R_i)^2-1}{(R_o/R_i)^2-1} = -(60\,MPa)(1) = -60\,MPa\ min$$

$$\sigma_{\theta\theta\,max} = \sigma_{\theta\theta}\big|_{r=R_i} = P \cdot \frac{(R_o/R_i)^2+1}{(R_o/R_i)^2-1} = (60\,MPa)\left(\frac{(.59/.53)^2+1}{(.59/.53)^2-1}\right)$$

-4, Calculate stress at multiple radii

$$\sigma_{\theta\theta\,max} = 561.6\,MPa$$

$$\sigma_{zz\,max} = P \cdot \frac{1}{(R_o/R_i)^2-1} = (60\,MPa)\frac{1}{(.59/.53)^2-1} = 250.8\,MPa = \sigma_{zz\,max}$$

$$\%\,err\ \theta = \left(\frac{|\sigma_{\theta\theta}-\bar{\sigma}_\theta|}{\sigma_{\theta\theta}}\right)\times 100 = \frac{561.6-560}{561.6}\times 100 = \boxed{.285\%\ \text{error in}\ \theta\theta\ \text{direction}}$$

$$\%\,err\ z = \left(\frac{|\sigma_{zz}-\bar{\sigma}_z|}{\sigma_{zz}}\right)\times 100 = \frac{|250.8-280|}{250.8}\times 100 = \boxed{11.6\%\ \text{error in}\ zz\ \text{direction}}$$

$$\%\,err\ r = \left(\frac{|\sigma_{rr}-\bar{\sigma}_r|}{\sigma_{rr}}\right)\times 100 = \left(\frac{|-60+30|}{60}\right)\times 100 = \boxed{50\%\ \text{error in}\ rr\ \text{direction}}$$

The thin walled approximation is fairly accurate in calculating maximum stresses for hoop stress, which is the most likely failure direction. While it is not conservative in calculating the MAXIMUM stress, it is conservative in calculation of stress in any other region of the cylinder other than the inside edge

(d) $E = 70\,GPa \quad \nu = .41$

Assuming no change in z-direction

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu\\ \nu & 1-\nu\end{bmatrix}\begin{bmatrix}u_{r,r}\\ u_r/r\end{bmatrix}$$

$$\boxed{\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\end{bmatrix} = (275.8\,GPa)\begin{bmatrix}.59 & .41\\ .41 & .59\end{bmatrix}\begin{bmatrix}u_{r,r}\\ u_r/r\end{bmatrix}}$$

$$\sigma = \begin{bmatrix}\sigma_{rr} & 0\\ 0 & \sigma_{\theta\theta}\end{bmatrix}$$

$$\epsilon = \begin{bmatrix}u_{r,r} & 0\\ 0 & u_r/r\end{bmatrix}$$

-8, Didn't calculate strains and didn't write full tensors