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95/100

1)

a. fissile isotope is Uranium-235 -0, 30/30

the natural enrichment form of U-236 in Uranium is 0.7 %.

b.

$M_{U_3Si_2} = 3\times(0.03\times 235 + 0.97\times 238) + 2\times 28 = 769.73$ g/mol

$N_{U235} = 3\times \gamma N_a \rho_{fuel} / M_{U_3Si_2} = \dfrac{0.03\times 6.022\times 10^{23} \times 12.2}{769.73} \times 3 = 8.59\times 10^{20}$ atoms U_{235}/cm^3

provided in (lec 3, slide 13)

$Q = E_f N_U \sigma_f \phi_{th} = (3\times 10^{-11}) \times (8.59\times 10^{20})(5.5\times 10^{-22})(3.2\times 10^{13}) = 453.56$ W/cm³

= Q from U_3Si_5 ⇒ since E_f, σ_f, ϕ_{th} are same N_{U235} should be the same

$N_{U235} = 8.59\times 10^{20}$ atoms U-235/cm³ for the fuel

$= \gamma N_a \rho_{uranium} / M_{uranium}$ ← since U_{235} has a negligible impact on the total molar mass of U.

$\gamma = \dfrac{8.59\times 10^{20} \times M_{uranium}}{7.5\times N_a} = \dfrac{8.59\times 10^{20} \times 238}{7.5\times 6.022\times 10^{23}} = 0.045$

enrichment = 4.5 %

c)

$12 \frac{W}{mk} \times \frac{1m}{100cm} = 0.12$ W/cm k for U_3Si_5

for U_3Si_2, thermal conductivity is 0.23 W/cmk

since U_3Si_5 fuel requires more fuel enrichment to provide the same amount of energy produced from U_3Si_2 and has lower thermal conductivity than U_3Si_2 fuel,

U_3Si_2 fuel is better than U_3Si_5

2 $R_f = 0.45$ cm $\quad \delta = 0.008$ cm $\quad t_{clad} = 0.06$ cm $\quad LHR = 250$ W/cm

-0, 35/35

a)

$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_f h_{cool}} = 580 + \frac{250}{2\pi \times 0.45 \times 2.5} = 615.368 [k]$$

$$T_{CI} = T_{co} + \frac{LHR \times t_{clad}}{2\pi R_f k_c} = 615.368 + \frac{250 \times 0.06}{2\pi \times 0.45 \times 0.17} = 646.575 [k]$$

(slide 11, lec 7)

$$k_{He} = 16\times10^{-6} \times 646.575^{0.79} = 0.00266 \quad [W/cmk]$$

$$k_{Xe} = 0.7\times10^{-6} \times 646.575^{0.79} = 1.163\times10^{-4} \quad [W/cmk]$$

$$k_{gap} = k_{He}^{1-0.05} \, k_{Xe}^{0.05} = 0.00227 \quad [W/cmk]$$

$$h_{gap} = \frac{k_{gap}}{\delta} = \frac{0.00227}{0.008} = 0.284 \quad [W/cm^2k]$$

$$T_s = T_{CI} + \frac{LHR}{2\pi R_f h_{gap}} = 646.575 + \frac{250}{2\pi \times 0.45 \times 0.284} = 957.8 \; [k]$$

b)

$$T_m = T_s + \frac{LHR}{4\pi k} = 957.8 + \frac{250}{4\pi \times 0.2} = 1057.272 \; [k]$$

(lec 2, slide 18)

$$\eta = \frac{r}{R_f} = 1$$

$$\sigma^* = \frac{\alpha E (T_m - T_s)}{4(1-\nu)} = \frac{(7.5\times10^{-6})(246.7)(1057.272 - 957.8)}{4(1-0.25)} = 0.06135 \text{ GPa}$$

hoop stress = max

$$\Rightarrow \sigma_{\theta\theta} = -\sigma^*(1-3\eta^2) = 2\sigma^* = 0.1227 \text{ GPa}$$

c) Since the termal conductivity of UO_2 is much lower than that of UN, the term, $(T_m - T_s)$, in σ^* of UO_2 will be greater.
Therefore, the stress will be higher than the UN fuel pellet

d) - for part a, those are the assumptions.

- steady state
- axisymmetric
- T is constant through z-direction
- thermal conductivity is independent on temperature

for part b,

- static body
- gravity is negligible
- axisymmetric
- isotropic material response

3 $\bar{R} = 0.56$ cm

$t_{clad} = 0.06$ cm

-5, 30/35

a) the stress is constant through the wall

-2, Isotropic and small strain

b)

$$\sigma_\theta = \frac{PR}{t_{clad}} = \frac{6 \times (0.56)}{(0.06)} = 56 \text{ MPa}$$

$$\sigma_r = -\frac{1}{2}P = -3 \text{ MPa}$$

$$\sigma_z = \frac{PR}{2t_{clad}} = \frac{6 \times (0.56)}{2 \times 0.06} = 28 \text{ MPa}$$

c) $\sigma_{\theta\theta} = p \dfrac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1}$

where $\bar{R} = 0.56 \Rightarrow R_o + R_i = 2 \times 0.56$.

$R_o - R_i = t_{clad} = 0.06$

$\Rightarrow R_o = R_i + 0.06$

$2R_i + 0.06 = 2 \times 0.56$

$R_i = 0.53$ cm

$R_o = 0.59$ cm

$\sigma_{rr} = -p \dfrac{(R_o/r)^2 - 1}{(R_o/R_i)^2 - 1}$

$\sigma_{zz} = p \dfrac{1}{(R_o/R_i)^2 - 1}$

for comparison, I used $r = \bar{R}$.

$$\sigma_{\theta\theta} = 6 \frac{(0.59/0.56)^2 + 1}{(0.59/0.53)^2 - 1} = \frac{2.11 \times 6}{0.239} = 52.92 \text{ MPa}$$

$$\sigma_{rr} = -6 \times \frac{(0.59/0.56)^2 - 1}{(0.59/0.53)^2 - 1} = \frac{-0.11 \times 6}{0.239} = -2.76 \text{ MPa}$$

$$\sigma_{zz} = 6 \times \frac{1}{(R_o/R_i)^2 - 1} = \frac{6}{0.239} = 25.08 \text{ MPa}$$

$$\% \text{ error} = \frac{\text{thin wall approx} - \text{any wall eq}}{\text{any wall eq}} \times 100$$

for hoop,

$$\% \text{ error} = \frac{56 - 52.92}{52.92} \times 100 = 5.82\%$$

for radial

$$\% \text{ error} = \frac{-3 + 2.76}{-2.76} \times 100 = 8.7\%$$

for axial

$$\% \text{ error} = \frac{28 - 25.08}{25.08} \times 100 = 11.64\%$$

-3, Calculate stress at TWO radii and compare to see if stress is constant as shown, thin wall approximation shows the deviation from the equations for any wall size. Therefore, the thin walled cylinder approximation is not conservative.

d) assuming there is no shear.
for stress tensor from thin wall approximation,

$$\sigma = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 56 & 0 \\ 0 & 0 & 28 \end{bmatrix} \text{ MPa}$$

$$\varepsilon_{rr} = \frac{1}{E}\left[\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right] = \frac{1}{70000}\left[-3 - 0.41(56 + 28)\right]$$

$$= -5.349 \times 10^{-4}$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left[\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right] = \frac{1}{70000}\left[56 - 0.41(-3 + 28)\right]$$

$$= 6.536 \times 10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right] = \frac{1}{70000}\left[28 - 0.41(-3 + 56)\right]$$

$$= 8.957 \times 10^{-4}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{rr} & 0 & 0 \\ 0 & \varepsilon_{\theta\theta} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} -5.349 & 0 & 0 \\ 0 & 6.536 & 0 \\ 0 & 0 & 8.957 \end{bmatrix} \times 10^{-4}$$