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*NucE 497 Fuel Performance Exam 1 covering modules 1 - 3*

-6, 24/30

**Question 1:**

$U_3Si_5$ is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm$^3$. Answer the following questions

a) What is the fissile isotope in $U_3Si_5$? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

$\boxed{U\text{-}235}$

In natural uranium, there is a ~0.7% enrichment of U-235

b) What enrichment would be required for $U_3Si_5$ to have the same energy release rate of $U_3Si_2$ enriched to 3% with a neutron flux of 3.2e13 n/(cm$^2$ s)? You can assume that $U_{235}$ has a negligible impact on the total molar mass of U in the fuel (15 points)

$\rho_{U_3Si_2} = 12.2\ g/cm^3$

$\rho_{U_3Si_5} = 9.1\ g/cm^3$

$M_{U_3Si_2} = 3(238) + 2(28.08) = 770.16$

$M_{U_3Si_5} = 3(238) + 5(28.08) = 854.4$

$q_{known} = 0.03$

$$Q_{U_3Si_2} = Q_{U_3Si_5}$$

$$\cancel{\phi_m} \cancel{\sigma_f} N \cancel{E_f} = \cancel{\phi_m} \cancel{\sigma_f} N \cancel{E_f}$$

$$\frac{\cancel{3} \cdot \rho_{U_3Si_2} \cancel{N_A}\, q_{known}}{M_{U_3Si_2}} = \frac{\cancel{3}\, \rho_{U_3Si_5} \cancel{N_A}\, q_{unknown}}{M_{U_3Si_5}}$$

-3, Use U density that was provided

$$q_{unknown} = \frac{M_{U_3Si_5} \cdot \rho_{U_3Si_2}\, q_{known}}{M_{U_3Si_2} \cdot \rho_{U_3Si_5}} = \frac{(854.4)(12.2\ g/cm^3)(0.03)}{(770.16)(9.1\ g/cm^3)}$$

$$\boxed{q_{unknown} \approx 0.04 \sim 4\%\ \text{enrichment}}$$

c) How would you rank $U_3Si_5$ as a potential fuel compared to $U_3Si_2$? Why? (8 points)

When comparing $U_3Si_5$ to $U_3Si_2$, $U_3Si_2$ is a better option for fuel because you need less enrichment of U-235. $U_3Si_2$ is also a more dense material than $U_3Si_5$, which means a greater opportunity of interactions.

-3, thermal conductivity?

-4, 31/35

**Question 2:**

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

$R_f = 0.45cm$
$t_{gap} = 0.008cm$
$t_c = 0.06cm$
$LHR = 250\ W/cm$
$T_{coolant} = 580K$
$\gamma = 0.05$
$h_{cool} = 2.5 \frac{W}{cm^2 K}$
$K_c = 0.17$
$K_{fuel} = 0.2$

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{co} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250\ W/cm}{2\pi(0.45cm)(2.5)} + 580K$$

$$T_{co} = 615.37K$$

$$T_{ci} = \frac{LHR\, t_c}{2\pi R_f k_c} + T_{co} = \frac{250(0.06)}{2\pi(0.45)(0.17)} + 615.37K$$

$$T_{ci} : 646.58K$$

$$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ci} \qquad h_{gap} = k_{gap}/t_{gap} = 0.25$$

$$= \frac{250}{2\pi(0.45)(.25)} + 646.58K$$

$$\boxed{T_s = 1000.26K}$$

$$k_{gap} = k_{He}^{1-\gamma} k_{Xe}^{\gamma} = [(16\times10^{-6})(646.58)^{.79}]^{0.95}[(0.7\times10^{-6})(646.58)^{.79}]^{0.05} = 0.002$$

-2, kgap = 0.00227. By rounding too early your answer is off by 50 K

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$$T_o = \frac{LHR}{4\pi K_{fuel}} + T_s$$

$$T_o = \frac{250}{4\pi(0.2)} + 1000.26k$$

$$T_o = 1099.7K$$

$$\sigma_{\theta\theta} = -\sigma^*(1-3\eta^2)$$

setting $\eta = 1$ because that's where hoop stress is greatest

$$\sigma_{\theta\theta} = -(0.0613GPa)(1-3(1)^2)$$

$$\boxed{\sigma_{\theta\theta} = 0.123\ GPa = 123\ MPa}$$

$$\sigma^* = \frac{\alpha E(T_o - T_s)}{4(1-\nu)} = \frac{(7.5\times10^{-6})(246.7)(1099.7-1000.26)}{4(1-0.25)}$$

$$\sigma^* = 0.0613\ GPa$$

c) Would you expect this stress to be higher or lower if the pellet was $UO_2$? Why? (5 points)

I would expect the stress to be higher in $UO_2$ because $UO_2$'s k is lower than UN's k, making the temp difference greater. Also, $UO_2$'s α is greater than UN's α, making $\sigma^*$ greater.

d) What assumptions were made in your calculations for a) and b)? (5 points)

Assumptions

-2, There are several more assumptions

- constant thermal properties
- fuel radius and gap size are constant
- constant LHR.

-15, 20/35

**Question 3:**

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

no wall thickness

-5, stress is constant through wall thickness, isotropic, small strain

$F_{stress} = F_{pressure}$

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$P = 6\text{ MPa}$

$\bar{R} = 5.6\text{ mm}$

$t_c = 0.6\text{ mm}$

$$\sigma_r = -\frac{P}{2} = -\frac{6\text{ MPa}}{2} \qquad \boxed{\sigma_r = -3\text{ MPa}}$$

$$\sigma_z = \frac{PR}{2t_c} = \frac{(6\text{ MPa})(5.6\text{ mm})}{2(0.6\text{ mm})} \qquad \boxed{\sigma_z = 28\text{ MPa}}$$

$$\sigma_\theta = \frac{PR}{t_c} = \frac{(6\text{ MPa})(5.6\text{ mm})}{(0.6\text{ mm})} \qquad \boxed{\sigma_\theta = 56\text{ MPa}}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

-4, Compute stress at two radii and compare to see if constant

Thick walls more accurate:

Max when $r = R_i$

$R_o = 5.9\text{ mm}$

$R_i = 5.3\text{ mm}$

$$\sigma_{\theta\theta} = P\frac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1} = P\frac{(R_o/R_i)^2 + 1}{(R_o/R_i)^2 - 1} = 6\text{MPa}\frac{(5.9/5.3)^2 + 1}{(5.9/5.3)^2 - 1}$$

$\sigma_{\theta\theta} = 56.16\text{ MPa}$

As you can see, the thick walled gives a similar answer, but is much more accurate because thin-walled is an approximation. In this case, the thin-walled approximation is <u>NOT</u> conservative because if designed using that max stress, the specimen would fail.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\begin{bmatrix} \sigma_r \\ \sigma_\theta \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix}\begin{bmatrix} u_{rr} \\ u_r/r \end{bmatrix}$$

from part (a)

$$\begin{bmatrix} -3 \\ 56 \end{bmatrix} = \underbrace{\frac{70}{(1+.41)(1-2(.41))}}_{=275.81}\begin{bmatrix} 0.59 & 0.41 \\ 0.41 & 0.59 \end{bmatrix}\begin{bmatrix} u_{rr} \\ u_r/r \end{bmatrix}$$

-2, Stress and strain missing zz component

-4, Calculate strains from stresses from part b

$$-3 = (275.81)(0.59)u_{rr} + (275.81)(0.41)u_r/r \rightarrow u_r/r = \frac{-3 - 162.73u_{rr}}{113.08}$$

$$56 = (275.81)(0.41)u_{rr} + (275.81)(0.59)u_r/r \rightarrow u_{rr} = -0.498, \quad u_r/r = 0.69$$

$$\boxed{\sigma = \begin{bmatrix} -3 & 0 \\ 0 & 56 \end{bmatrix}} \qquad \boxed{\varepsilon = \begin{bmatrix} u_{rr} & 0 \\ 0 & u_r/r \end{bmatrix} = \begin{bmatrix} -0.498 & 0 \\ 0 & 0.69 \end{bmatrix}}$$