

1.a) U-235

Si: 28.1u

-10, 20/30

U: 238u

 $e_n: 0.72\%$

$$U_3Si_5 = 5(28.1u) + 3(238u) = \underline{854.5u}$$

$$\text{wt \% 235: } \frac{3 \cdot 238u \cdot 0.72\%}{854.5u} = \boxed{0.59\% \text{ wt}}$$

1.b)

$$Q = E_F \cdot N_F \cdot \sigma_F \cdot \phi_{th}$$

$$Q_1 = Q_2$$

$$\cancel{E_F} \cdot N_{U_3Si_5} \cdot \cancel{(\epsilon_{U_3Si_5})} \cdot \cancel{\sigma_F} \cdot \cancel{\phi_{th}} = \cancel{E_F} \cdot N_{U_3Si_2} \cdot \cancel{(\epsilon_{U_3Si_2})} \cdot \cancel{\sigma_F} \cdot \cancel{\phi_{th}}$$

$$\epsilon_{U_3Si_5} = \epsilon_{U_3Si_2} \cdot \frac{N_{U_3Si_2}}{N_{U_3Si_5}}$$

$$= 3\% \cdot \frac{769.9}{854.5} = 2.7\%$$

1b)

$$Q = E_F N_F \sigma_F \phi_{th}$$

$$Q_1 = Q_2$$

$$N_{F1} = N_{F2}$$

$$N_F = 3 \cdot g \cdot N_a \cdot \delta \cdot MM$$

$$MM_{U_3Si_2} = (238)3 + (28)2 = \underline{770 \text{ g/mol}}$$

$$MM_{U_3Si_5} = (238)3 + (28)5 = \underline{854 \text{ g/mol}}$$

$$\delta_U \cdot \cancel{12.2 \text{ g/cm}^3} \cdot 11.31$$

$$3 \cdot (.03) \cdot N_a \cdot (\cancel{12.2 \text{ g/cm}^3}) / 770 \text{ g/mol}$$

$$= 3(g) N_a (7.5 \text{ g/cm}^3) / 854 \text{ g/mol}$$

$$\downarrow \quad \frac{.03 \cdot \cancel{12.2} / 770}{7.5 / (854)} = \frac{5.0\%}{\cancel{5.4\%}}$$

* But since δ_U given instead of δ ,

use MM of U fraction only!?

-2, Only the dU fraction is needed

$$\frac{.03 \cdot \cancel{12.2} / (238.3)}{7.5 / (238.3)} = \boxed{\cancel{4.5\%}} \boxed{4.2\%}$$

		Thermal Cond (W/cm/K)	Density (g U/cm ³)	Comparable Enrichment
1.c)	U ₃ Si ₅	.125	7.5	4.2%
	U ₃ Si ₂	.23	11.31	3%

U₂₃₅ Fuel efficiency of U₃Si₅ is higher!

$$4.2\% \cdot 7.5 \text{ g U/cm}^3 = \frac{0.315}{\cancel{3.39}} \text{ g U}_{235}/\text{cm}^3$$

$$11.31 \text{ g U/cm}^3 \cdot \cancel{0.27} 3\% = 0.339 \text{ g U}_{235}/\text{cm}^3$$

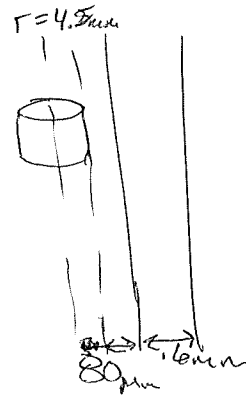
$$\Rightarrow \frac{.339 - .315}{.339} =$$

-8, U₃Si₅ is worse, because the dU
and the thermal conductivity are
worse

7% less U₂₃₅
Used by switching
to U₃Si₅

2. a)

-13, 22/35



$$T_{co} = T_{cool} + \frac{LHR}{2\pi R_F h_{cool}}$$

$$= 580 K + \frac{2500 W/cm}{2\pi(4.5 mm) 2.500 W/cm^2 \cdot K}$$

$$= \underline{615.3 K}$$

* Assumed K_c for Zr

$$T_{FE} = T_{oc} + \frac{LHR \cdot t_c}{2\pi R_F \cdot K_c}$$

$$= 615.3 K + \frac{2500 W/cm \cdot 0.6 mm}{2\pi(4.5 mm) \cdot 0.17 W/cm \cdot K}$$

$$= \underline{646.5 K}$$

$$T_s = T_{ic} + \frac{LHR}{2\pi R_F \cdot \left(\frac{K_{gap}}{t_g}\right)} ; K_{gap} = 1.6E-6 \cdot T_c^{1.79}$$

$$= \underline{0.0027 W/cm \cdot K}$$

-1, 0.00227

$$= 646.5 K + \frac{2500 W/cm}{2\pi(4.5 mm) \cdot \frac{0.0027 W/cm \cdot K}{80 mm}}$$

$$= \underline{912.7 K}$$

2. b)

$$\sigma = -E \cdot \Delta T \cdot \alpha$$

$$= -246.76 Pa (-912.7 K + 300 K) 7.5E-6 \frac{1}{K}$$

Assume fuel dimensions measured at this temp

2. b) Assume the pellet is constrained:

$$\sigma = -E(\Delta T)\alpha$$

Hoop stress is largest:

$$\sigma_{\theta\theta} = \frac{1}{2} \Delta T \frac{\alpha E}{1-\nu} \left(1 - 2 \frac{R_i}{s} \left(\frac{r}{R_i} - 1 \right) \right)$$

-6 Wrong DT, should be from T_s to T_0

$$= \frac{1}{2} (912.7\text{K} - 300\text{K}) \frac{7.5 \times 10^{-6} \text{K}^{-1} \cdot 246.76 \text{Pa}}{1 - 0.25}$$

* Max
stress

-2, max stress with $\eta = 1$

$$\cdot \left(1 - 2 \frac{4.5 \text{mm}}{.6 \text{mm}} (1 - 1) \right)$$

$$= \boxed{0.7566 \text{Pa @ surface}}$$

2.c) U-D has a much \uparrow conductivity
than $\text{UO}_2 \Rightarrow$ ~~low~~ lower T_s .
 UO_2 σ_{∞} will be higher for this
reason.

-3, T_s is the same, but low UO_2 thermal conductivity results in higher T_0 and DT

2.d) Axisymmetry, isotropic material,
 k independent of T , no geometry
change (such as gap size)

-1, there are several more assumptions

3. a) Force is constant over length,
Axisymmetric,
Isotropic material response,
 $\frac{\partial v}{\partial t} = 0$, Gravity is negligible

b) $P = 6 \text{ MPa}$, $r = 5.6 \text{ mm}$, $\delta = 0.6 \text{ mm}$

$$\begin{aligned} \sigma_{\theta} &= \frac{P \cdot r}{\delta} = \frac{6 \text{ MPa} \cdot 5.6 \text{ mm}}{0.6 \text{ mm}} = 56 \text{ MPa} \\ \sigma_z &= \frac{P \cdot r}{2\delta} = \frac{6 \text{ MPa} \cdot 5.6 \text{ mm}}{2(0.6 \text{ mm})} = 28 \text{ MPa} \\ \sigma_r &= -\frac{1}{2} P = -\frac{1}{2} \cdot 6 \text{ MPa} = -3 \text{ MPa} \end{aligned}$$

c) Compare to thick wall sol'n @ $r = R_i$

-4, Calculate stress at two radii and compare

$$R_i = 5.6 \text{ mm}$$

$$R_o = 6.2 \text{ mm}$$

$$\sigma_{rr} = -P = -6 \text{ MPa}$$

$$\sigma_{\theta\theta} = P \cdot \frac{(R_o/R_i)^2 + 1}{(R_o/R_i)^2 - 1} = 59.1 \text{ MPa}$$

$$\sigma_{zz} = P \cdot \frac{1}{(R_o/R_i)^2 - 1} = 26.6 \text{ MPa}$$

Hoop stress most important, $\frac{56}{59} = 95\%$

σ_r off by factor of 2, σ_z is close.

Thin wall should be conservative (but I find it is not in this case)

correct

$$3.2) \quad \sigma_{\theta}(R) = \frac{P \cdot R}{\delta}$$

$$\epsilon = \begin{bmatrix} \frac{P \cdot R}{\delta} & 0 \\ 0 & \frac{P}{\delta} \end{bmatrix} =$$

3.2) What is the displacement??

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$$= \frac{70 \text{ GPa}}{(1.41)(0.18)} \begin{bmatrix} 0.59 & 0.41 \\ 0.41 & 0.59 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

$$= \cancel{207.8 \text{ GPa}} \begin{bmatrix} 162.7 & 113.1 \\ 113.1 & 162.7 \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

Stress tensor

Strain tensor is $\begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$

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but what is displacement?
u for this??

-2, tensors are missing zz component
-4 Calculate the strains from the stresses from part b

3d) Just looking @ Hop stress:

~~Given~~ Given elastic expansion,

$$\epsilon = \frac{\sigma}{E} \Rightarrow$$

$$\begin{bmatrix} 2.32 & 1.62 \\ 1.62 & 2.32 \end{bmatrix}$$