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77/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-5, 25/30

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

a) **What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)**

^{235}U is the fissile, in unenriched fuel the enrichment is 0.7% 235,

b) **What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)**

$$Q = E_f N_f^{25} \sigma_f^{25} \phi_{th}$$ (assume heat rate const.)

$$Q = E_f N_f^{25,U_3Si_2} \sigma_f^{25} \phi_{th} = E_f N_f^{25,U_3Si_5} \sigma_f^{25} \phi_{th} \rightarrow N_f^{25} = N_f^{25}$$

$$N_{25}^{Si_2} = \frac{\rho N_a q}{M}$$

$$M = 3[235(1-\overset{0.03}{x}) + 238(x)] + 28(2) = 761 \text{ g/mol}$$

$$N_{25}^{Si_2} = \frac{11.31(6.02e^{23})(0.03)}{761} = 2.68e^{20} \frac{1}{cm^3} = N_{25}^{Si_5}$$

$$M = 3[235(1-0.03) + 238(0.03)] + 5(28) = 845 \text{ g/mol}$$

-2, Math error

$$2.68e^{20} = \frac{7.5 N_a q}{M} \rightarrow q = \frac{2.68e^{20} M}{N_a(7.5)} \rightarrow q = 5.02\%$$

c) **How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)**

Less desirable, b/c there is need for a higher enrichment, hence, higher cost

-3, thermal conductivity?

-1, 34/35

Question 2:
Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{co} = \frac{LHR}{2\pi R_F h_{cool}} + T_{cool} = \frac{250}{2\pi(0.45)(2.5)} + 580 = 615.4 \text{ K}$$

$$T_{ci} = \frac{LHR\, t_c}{2\pi R_F h_{cool}} + T_{co} = \frac{250\,(0.06 \text{ cm})}{2\pi(0.45 \text{ cm})(0.17)} + 615.4 = 646.6 \text{ K}$$

$$\boxed{T_s} = \frac{LHR}{2\pi R_F h_{gap}} + T_{ci} = \frac{250}{2\pi(0.45 \text{ cm})(0.2837)} + 646.6 \text{ K} = \boxed{958.3 \text{ K}}$$

$k_{He} = 16e^{-6} T_{ci}^{0.79}$
$= 16e^{-6}(646.6)^{0.79}$
$= 2.66e^{-3}$
$k_{Xe} = 0.7e^{-6} T_{ci}^{0.79}$
$= 0.7e^{-6}(646.6)^{0.79}$
$= 1.16e^{-4}$
$k_{gap} = k_{He}^{1-0.05}\, k_{Xe}^{0.05}$
$= 2.27e^{-3}$
$h_{gap} = \frac{k_g}{t_g}$
$= \frac{2.27e^{-3}}{80e^{-4}}$
$= 0.2837$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

max stress → hoop @ $r = R_F$

$$\sigma_{\theta\theta} = -\sigma^*(1 - 3\eta^2) \longrightarrow r = R_F \rightarrow \eta = 1$$

$$T_0 = \frac{LHR}{4\pi k} + T_s = \frac{250}{4\pi(0.2)} + 958.3 = 1057.8 \text{ K}$$

$$\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)} = \frac{7.5e^{-6}(246.7e^9)(1057.8 - 958.3)}{4(1-0.25)}$$

$$\sigma^* = 61.4 \text{ MPa}$$

$$\boxed{\sigma_{\theta\theta}} = -61.4 \text{ MPa}(1-3) = \boxed{122.7 \text{ MPa}}$$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

Higher, b/c $k_{UO_2} >> k_{UN}$, making the value of T_0 higher ∴ greater ΔT.

d) What assumptions were made in your calculations for a) and b)? (5 points)

- Constant thermal conductivity ($k \neq f(T)$)
- Steady state
- T const. in z
- axisymmetric

-1, there are several more assumptions

Question 3:

-17, 18/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

-5, Isotropic, constan stress across thickness, small strain

$$\bar{\sigma}_\theta = \frac{pR}{\delta} \; ; \; \bar{\sigma}_z = \frac{pR}{2\delta} \; ; \; \bar{\sigma}_r = -\frac{1}{2}p$$

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\bar{\sigma}_\theta = \frac{6\,\text{MPa}\,(5.6\,\text{mm})}{0.6\,\text{mm}} = \boxed{56\ \text{MPa}}$$

$$\bar{\sigma}_r = -\frac{1}{2}p = \boxed{-3\ \text{MPa}}$$

$$\bar{\sigma}_z = \frac{6\,\text{MPa}\,(5.6\,\text{mm})}{2(0.6\,\text{mm})} = \boxed{28\ \text{MPa}}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

Thick wall: $\sigma_{\theta\theta} = p\dfrac{(R_o/r)^2-1}{(R_o/R_i)^2-1}$

$R_o = 5.9$ mm

$r(\max) = R_i = 5.3$ mm

-4, Calculate at two radii and compare to see how much they change

$$= (6\,\text{MPa})\left[\frac{(5.9/5.3)^2+1}{(5.9/5.3)^2-1}\right] = 56.16\ \text{MPa}$$

$$\%\ \text{error} = \frac{56.16-56}{56.16} = 0.285\%$$

conservative, $\sigma_{\theta\theta}(\text{thick}) > \sigma_\theta(\text{thin})$

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and v = 0.41. (10 points)

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{zz}\\ \sigma_{\theta\theta}\end{bmatrix} = \frac{1}{70\,\text{GPa}}\begin{bmatrix}1 & -0.41 & -0.41\\ -0.41 & 1 & -0.41\\ -0.41 & -0.41 & 1\end{bmatrix}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{zz}\\ \varepsilon_{\theta\theta}\end{bmatrix}$$

-4, Put stress and strain in tensor form

-4, Calculate strain from stress from part b