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Exam

[93]

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#2

1. $R_f = 0.45$ cm $\quad LHR = 250$ W/cm $\quad k_f = 0.1$ W/cmK $\quad E = 290\times10^3$ MPa

$\nu = 0.25 \quad \alpha_f = 12\times10^{-6}$ 1/K

a) $$\Delta T = \frac{LHR}{4\pi k_f} = \frac{250}{4\pi(0.1)} = 198.944\,K$$

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$$\sigma^* = \frac{\alpha E \Delta T}{4(1-\nu)} = \frac{12\times10^{-6}(290\times10^3)(198.944)}{4(1-0.25)} = 230.775 \text{ MPa}$$

Max: $\sigma_\theta \rightarrow r = R_f$

$$\sigma_\theta = -\sigma^*(1-3\eta^2) \qquad \eta = \frac{r}{R_f} \rightarrow 1$$

$$\boxed{\sigma_{\theta,max} = -230.775(1-3) = 461.55 \text{ MPa}}$$

b) $\sigma_{frac} = 120$ MPa

$$-\frac{\sigma_{frac}}{\sigma^*} = 1-3\eta^2$$

- math got a bit off here

$$\rightarrow \eta = \left[\frac{1+\frac{120}{230.775}}{3}\right]^{1/2} = 0.418059$$

$$\left(\frac{\frac{120}{231}+1}{3}\right)^{1/2}$$

$$0.418059 = \frac{r}{0.45} \Rightarrow 0.1881 \text{ cm} = r$$

$R_f - 0.1881 = \boxed{0.2618 \text{ cm into pellet}}$

(or to a radius of 0.1881 cm)

3. Δt_g

$R_f = 0.52\,cm$ $\quad T_{co} = 550K$ $\quad k_{fuel} = 0.04\ W/cmK$

$t_g = 0.005cm$ $\quad R_i = 0.525$ $\quad k_{clad} = 0.15\ W/cmK$

$t_{clad} = 0.08\,cm$ $\quad R_o = 0.605$ $\quad LHR = 400\ W/cm$

$\overline{R}_c = \frac{0.525+0.605}{2}$ $\quad \alpha_c = 12\times10^{-6}\ 1/K$

$\overline{R}_c = 0.565$ ✓ $\quad \alpha_f = 8\times10^{-6}\ 1/K$

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$$\Delta t_g = \overline{R}_c\alpha_c(\overline{T}_c - T_o^c) - R_f\alpha_f(\overline{T}_f - T_o^f) \quad T_{ref} = 300K = T_o^c = T_o^f$$

$$\Delta R_c = 0.565(12\times10^{-6})[582.647 - 300] = 0.001916\ cm$$ ✓

$$\Delta T_{clad} = \frac{LHR\,t_{clad}}{2\pi R_f k_{clad}} = \frac{400(0.08)}{2\pi(0.52)(0.15)} = 65.294\,K$$ ✓

$$\overline{T} = T_{co} + \frac{\Delta T_{clad}}{2} = 582.647\,K$$ ✓

$$T_s - T_{CI} = \frac{LHR}{2\pi R_{fuel}\frac{k_{gap}}{t_{gap}}} = \frac{400(0.005)}{2\pi(0.52)(0.003)} = 204.0448\,K$$ ✓

$$T_s = 819.3388\,K$$ ✓

$$T_o - T_s = \frac{LHR}{4\pi k_f} = 795.77$$ ✓

$$\overline{T}_F = 1217.226\,K$$ ✓

$$\Delta R_F = 0.52(8\times10^{-6})[1217.226 - 300] = 0.00381566$$ ✓

$$t_g = 0.005 + 0.001916 - 0.00381566 = 0.0031$$ ✓

$$\boxed{\Delta t_g = -0.0019\ cm}$$ ✓

2. $P = 25$ MPa $\quad \bar{R}_c = 0.52$ cm $\quad t_{clad} = 0.08$ cm

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a) $$\boxed{\begin{aligned} \sigma_r &= -\frac{P}{2} = -\frac{25}{2} = -12.5 \text{ MPa} \\ \sigma_\theta &= \frac{Pr}{\delta} = \frac{25(0.52)}{0.08} = 162.5 \text{ MPa} \\ \sigma_z &= \frac{Pr}{2\delta} = \frac{\sigma_\theta}{2} = 81.25 \text{ MPa} \end{aligned}}$$

b) $r = 0.5$ Thick $\rightarrow R_i = 0.48$ cm $\quad R_o = 0.56$ cm

$$\sigma_r = -P\left[\frac{\left(\frac{r_o}{r}\right)^2 - 1}{\left(\frac{r_o}{r_i}\right)^2 - 1}\right] = -25\left[\frac{\left(\frac{0.56}{0.5}\right)^2 - 1}{\left(\frac{0.56}{0.48}\right)^2 - 1}\right] = -25\left[\frac{0.2544}{0.36111}\right]$$

$$\boxed{\sigma_r = -17.61 \text{ MPa}}$$

$$\sigma_\theta = 25\left[\frac{\left(\frac{0.56}{0.5}\right)^2 + 1}{\left(\frac{0.56}{0.48}\right)^2 - 1}\right] = \left[\frac{2.2544}{0.36111}\right]25$$

$$\boxed{\sigma_\theta = 156.07 \text{ MPa}}$$

$$\boxed{\sigma_z = \frac{25}{\left(\frac{0.56}{0.5}\right)^2 - 1} = 98.27 \text{ MPa}}$$

$$\rightarrow \frac{P}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

c) $$C_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{160(1-0.3)}{(1+0.3)(1-0.6)} = \frac{112}{0.52} = 215.385 \text{ GPa}$$

$$C_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{48}{0.52} = 92.307 \text{ GPa}$$

- strain? $\quad \epsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right)$

4. Forward Euler $\quad dt = 0.25 \quad t_0 = 0 \quad t_n = 1.0$

$y_0 = 1 \quad \frac{dy}{dt} = -5y$

$$y_{n+1} = y_n + dt\, y'_n (y_n)$$

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$$y_1 = 1 + (0.25)[-5(1)] = -0.25 \qquad y_1 = y_0 + dt\, y'_0 (y_0)$$

$$y_2 = -0.25 + 0.25[-5($$

-started on the right path, got y_1 correct

5) $\Delta T_c = 250\,K$ $\alpha_c = 8\times10^{-6}$ $E = 250\times10^3\,MPa$ $\upsilon = 0.3$

$t_c = 0.1\,cm$ $R_i = 0.55\,cm$

Max when $r = R_o$ & R_i

$R_o = R_i + t_c = 0.65\,cm$

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$$\sigma_\theta = \frac{\Delta T}{2}\cdot\frac{\alpha E}{1-\upsilon}\left(1-2\frac{R_i}{t_c}\left(\frac{r}{R_i}-1\right)\right)$$ ✓

max @ $r = R_i$

- you found the minimum

$$\sigma_\theta = \frac{250}{2}\cdot\frac{(8\times10^{-6})(250\times10^{3}\,MPa)}{1-0.3}\left[1-2\frac{0.55}{0.1}\left(\frac{0.65}{0.55}-1\right)\right]$$

$$\boxed{\sigma_{\theta,max} = 357.1428\left[1-11(0.181818)\right] = -357.14\,MPa}$$ ✓

→ Same value but positive on interior (R_i)

$$\boxed{\sigma_{\theta_0} = 0 \ @\ R_i + \frac{\delta}{2} = 0.55 + \frac{0.1}{2} = 0.6\,cm \text{ (middle of tube)}}$$ ✓

A.F.

6. 1) Finite Difference: Computationally cheap, but cannot handle mechanics/high dimensional equations
2) Finite Volume: Great for fluid mech/thermal hydraulic problems, but restricted to flux B.Cs
6/6 3) Finite Element: Can solve mechanics equations, but complex to implement/computationally expensive (relatively)

7. Strain hardening is when a material is plastically deformed under a load and then the load is removed, so that the yield strength is
8/8 increased as well as hardness. The mechanism for this is dislocation pileup, where dislocations (lines of impurity/displaced atoms) get "caught" on barriers such as precipitates, grain boundaries, etc.

8. All fuel codes must be able to predict three groups of parameters:
1) Fuel (Thermal profile, volume change, etc)
5/5 2) Clad (Thermal profile, stress, etc.)
3) Gap (Heat transport, pressure, etc.)
FRAPTRAN (NRC) and FALCON (EPRI) are currently used by various bodies for fuel perf.

9. 0-D : Vacancy
4/4 3-D : Void (Cluster of vacancies)

10. Microstructure-based fuel modeling aims to use structural relationships to connect
8/8 microstructure variables to property values as a form of quasi-mechanistic modeling. It hopes to provide more accurate modeling with more fundamental physics rather than relying on empirical curve fits, being able to calculate stress fields and more complicated phenomena.

11. High Burnup Structures (HBS) have a large amounts of voids due to fission products, which put stress on the surrounding material. During a transient or LOCA,
5/5 the additional thermal stresses (and gas pressure) may lead to this region fragmenting/pulverizing, spreading material around the fuel pin. Scientists are looking to better model this phenomenon using Phase-Field Modeling, where stress fields are constructed by tracking vacancies/fission products with sinks/sources through the material. This provides a robust methodology to predict how bubbles respond to changes in various parameters.

12. The microstructure is the orientation of species/phases at the microscopic scale, such as with how grains in a metal are. The microstructure greatly affects how the material behaves at an engineering scale. It is generally tailored by stoichiometry and kind of heat treatment process, such as cold-rolling to harden (in the case of steel). In the case of stoichiometry, an example is how UO_{2x}'s microstructure significantly changes based on O/U ratio, forming various species and lattice phases.

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OK

this is from later lectures, but technically yes.