Stress components in thick-walled cylinder

$r_i = 0.5$ cm $r_o = 0.55$ cm $p = 15$ MPa @ $r = 0.52$ cm

$$\sigma_\theta = \frac{\left(\frac{r_o}{r}\right)^2 + 1}{\left(\frac{r_o}{r_i}\right)^2 - 1} p \qquad \sigma_r = -p \frac{\left(\frac{r_o}{r}\right)^2 - 1}{\left(\frac{r_o}{r_i}\right)^2 - 1}$$

$$\sigma_z = \frac{p}{\left(\frac{r_o}{r_i}\right)^2 - 1} \qquad \frac{r_o}{r_i} = \frac{0.55}{0.5} = 1.1$$

$$\sigma_\theta = \frac{\left(\frac{0.55}{0.52}\right)^2 + 1}{1.1^2 - 1}(15) = 151.7 \text{ MPa}$$

$$\sigma_b = \frac{15}{\left(\frac{0.55}{0.52}\right)^2 - 1} = 136 \text{ MPa}$$

$$\sigma_r = \frac{\left(\frac{0.55}{0.52}\right)^2 - 1}{1.1^2 - 1}(-15) = -8.5 \text{ MPa}$$

thin walled cylinder

$$r = R = \frac{0.5 + 0.55}{2} = 0.525 \text{ cm}$$

$$\sigma_\theta = \frac{pR}{t} \qquad \sigma_r = \frac{-p}{2} \qquad \sigma_z = \frac{pR}{2t}$$

$$\sigma_\theta = \frac{(15)(0.525)}{0.05} = 157.5 \text{ MPa} \qquad \sigma_r = \frac{-15}{2} = -7.5 \text{ MPa}$$

$$\sigma_z = \frac{(15)(0.525)}{2(0.05)} = 78.8 \text{ MPa}$$

max stress in fuel pellet from thermal expansion

$\Delta T = T_0 - T_s = 425\ K$ $\quad \alpha_F = 12 \times 10^{-6}\ 1/K$

$R_F = 0.5\ cm$ $\quad E = 180\ GPa$ $\quad \nu = 0.25$

$$\sigma_r = -\sigma^* (1 - \eta^2)$$

$$\sigma_\theta = -\sigma^* (1 - 3\eta^2)$$

$$\sigma_t = -2\sigma^* (1 - 2\eta^2)$$

$$\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$$

$$\eta = \frac{r}{R_F}$$

max stress? $\rightarrow$ σ_θ $\rightarrow$ $r = R_F$ $\rightarrow$ $\eta = 1$

$$\sigma^* = \frac{(12 \times 10^{-6})(180 \times 10^3\ MPa)(425)}{4(1 - 0.25)} = 318.8\ MPa$$

$$\sigma_\theta = -318.8 (1 - 3(1^2)) = \underline{637.5\ MPa}$$

Cladding tube w/ thermal expansion

$R_i = 0.6$ cm $t_c = 0.1$ cm $E = 250$ GPa $\nu = 0.3$

$\alpha_c = 15 \times 10^{-6}$ 1/K $T_{ci} = 600$ K $T_{co} = 580$ K

$\Delta T = 20$ K

stresses @ $r = 0.62$ cm?

$$\sigma_r = \frac{\Delta T}{\alpha} \frac{\alpha E}{1-\nu} \left(\frac{r}{R_i} - 1 \right) \left(1 - \frac{R_i}{t_c} \left(\frac{r}{R_i} - 1 \right) \right)$$

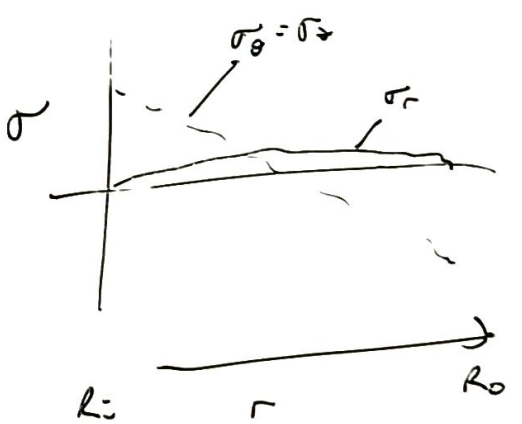
$$= \frac{20}{\alpha} \frac{(15\times10^{-6})(250\times10^{3}\ \text{MPa})}{1-0.3} \left(\frac{0.62}{0.6} - 1 \right) \left(1 - \frac{0.6}{0.1} \left(\frac{0.62}{0.6} - 1 \right) \right)$$

$\sigma_r = 1.43$ MPa

$$\sigma_\theta = \sigma_z = \frac{\Delta T}{\alpha} \frac{\alpha E}{1-\nu} \left(1 - \frac{\alpha R_i}{t_c} \left(\frac{r}{R_i} - 1 \right) \right)$$

$$= \frac{20}{\alpha} \frac{(15\times10^{-6})(250\times10^{3}\ \text{MPa})}{1-0.3} \left(1 - \frac{\alpha (0.6)}{0.1} \left(\frac{0.62}{0.6} - 1 \right) \right)$$

$\sigma_\theta = \sigma_z = 32.1$ MPa



Gap thickness change

$\alpha_F = 12\times10^{-6}$ 1/K $\alpha_C = 15\times10^{-6}$ 1/K

$\bar{T}_F = 925$ K $\bar{T}_C = 580$ K $T_0^{C,F} = 300$ K

$R_F = 0.5$ cm $\bar{R}_C = 0.58$ cm $t_g = 0.03$ cm

$$\Delta t_g = \bar{R}_C \alpha_C (\bar{T}_C - T_0^C) - R_F \alpha_F (\bar{T}_F - T_0^F)$$

$$= 0.58 (15\times10^{-6})(250) - 0.5 (12\times10^{-6})(625)$$

$$= 0.002175 - 0.00375 = -0.0016 \text{ cm}$$

$$t_g' = ? = t_g + \Delta t_g = 0.03 - 0.0016 = \underline{0.0284 \text{ cm}}$$

stress from displacements

-2D r, θ @ r = 0.45 cm

E = 200 GPa ν = 0.3 $u_r(r) = 0.5r^2 - 0.2r$

$$\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix} = \begin{bmatrix} r - 0.2 & 0 \\ 0 & 0.5r - 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.45 - 0.2 & 0 \\ 0 & 0.5(0.45) - 0.2 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.025 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \end{bmatrix}$$

$$C_{11} = \frac{E}{(1+\nu)(1-2\nu)}(1-\nu) = 269 \text{ GPa}$$

$$C_{12} = \frac{E}{(1+\nu)(1-2\nu)}\nu = 115 \text{ GPa}$$

$$\sigma_{rr} = C_{11}\epsilon_{rr} + C_{12}\epsilon_{\theta\theta} = (269)(0.25) + (115)(0.025)$$

$$\sigma_{\theta\theta} = C_{12}\epsilon_{rr} + C_{11}\epsilon_{\theta\theta} = (115)(0.25) + (269)(0.025)$$

$$\sigma_{rr} = 70 \text{ GPa}$$

$$\sigma_{\theta\theta} = 85.5 \text{ GPa}$$