Nuclear Fuel Performance

NE-533

Spring 2025

Last Time

- Solid mechanics predicts the deformation of a body from its applied load
- Elastic deformation and plastic deformation
- Concepts of stress and strain
- Elastic moduli and Elastic constants
- Work hardening/strain hardening
- Toughness, ductility, DBTT, etc.
- Thin-walled cladding stresses

MECHANICS

Small strain theory

- In continuum mechanics, the infinitesimal strain theory is a mathematical approach to the description of the deformation of a solid body in which the displacements of the material particles are assumed to be much smaller than any relevant dimension of the body so that its geometry and the constitutive properties of the material at each point of space can be assumed to be unchanged by the deformation
- With this assumption, the equations of continuum mechanics are considerably simplified
- The small strain theory is commonly adopted in civil and mechanical engineering for the stress analysis of structures built from relatively stiff elastic materials

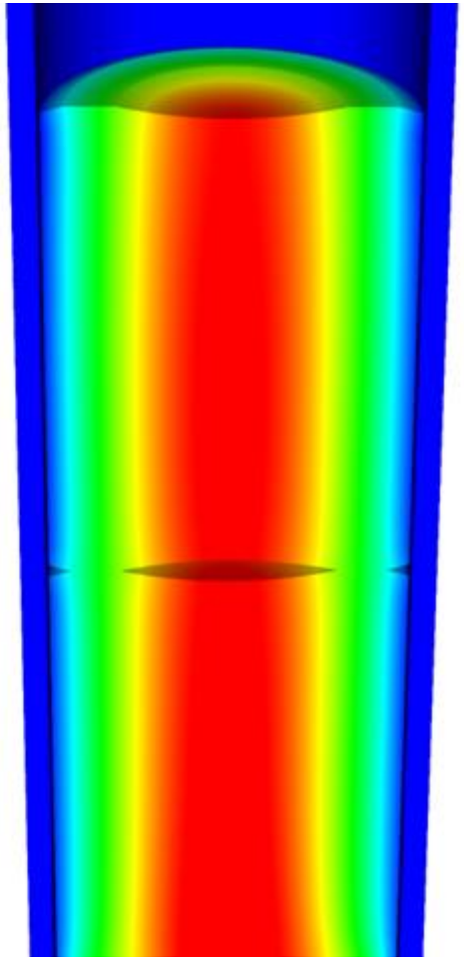
Develop constitutive relations for thick-walled

- We assume small strains, so the strain is defined with respect to displacement: u

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \ \epsilon_{\theta\theta} = \frac{u_r}{r}, \epsilon_{zz} = \frac{\partial u_z}{\partial z}, \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

- We assume isotropic material response:

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right)$$
$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right)$$
$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right)$$
$$\epsilon_{rz} = \frac{1}{2G}\sigma_{rz}$$



Stress within a pressurized cylinder that has thick walls (radius/thickness<20)

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rz}}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial (r\sigma_{rz})}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

- The cylinder as an inner radius R_i and an outer Radius R_o
- We assume there is no shear stress, so $\sigma_{rz} = 0$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad \frac{\partial \sigma_{zz}}{\partial z} = 0$$

- We will begin with the r equation

$$\sigma_{\theta\theta} = \sigma_{rr} + r\sigma_{rr,r} \qquad \sigma_{\theta\theta,r} = 2\,\sigma_{rr,r} + r\sigma_{rr,rr}$$

- Now we need our constitutive law

$$E\,\epsilon_{\theta\theta,r} = \left(\sigma_{\theta\theta,r} - \nu(\sigma_{rr,r} + \sigma_{zz,r})\right)$$

$$E\,\epsilon_{zz,r} = \left(\sigma_{zz,r} - \nu(\sigma_{\theta\theta,r} + \sigma_{rr,r})\right) = 0$$

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right)$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right)$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right)$$

- We combine these equations to get

$$E\,\epsilon_{\theta\theta,r} = \sigma_{\theta\theta,r} - \nu(\sigma_{rr,r} + \nu(\sigma_{\theta\theta,r} + \sigma_{rr,r})) \qquad E\,\epsilon_{\theta\theta,r} = \left(1 - \nu^2\right)\sigma_{\theta\theta,r} + \nu(1+\nu)\sigma_{rr,r}$$

$$E(\epsilon_{rr} - \epsilon_{\theta\theta}) = (1+\nu)(\sigma_{rr} - \sigma_{\theta\theta})$$

Develop equations for the stress within a pressurized cylinder with thick walls

- We need one more relationship from the definition of the strain

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \;\; \epsilon_{\theta\theta} = \frac{u_r}{r} \qquad \epsilon_{\theta\theta,r} = \frac{1}{r}u_{r,r} - \frac{1}{r^2}u_r = \frac{1}{r}(\epsilon_{rr} - \epsilon_{\theta\theta})$$

- From the previous slide, we have

$$E\,\epsilon_{\theta\theta,r} = \left(1-\nu^2\right)\sigma_{\theta\theta,r} + \nu(1+\nu)\sigma_{rr,r} \qquad E(\epsilon_{rr} - \epsilon_{\theta\theta}) = (1+\nu)(\sigma_{rr} - \sigma_{\theta\theta})$$

- If we combine these, we get

$$(1-\nu^2)\sigma_{\theta\theta,r} + \nu(1+\nu)\sigma_{rr,r} = \frac{1}{r}(1+\nu)(\sigma_{rr} - \sigma_{\theta\theta})$$

- Again, from the previous slide

$$\sigma_{\theta\theta,r} = 2\,\sigma_{rr,r} + r\sigma_{rr,rr} \qquad \sigma_{\theta\theta} = \sigma_{rr} + r\sigma_{rr,r}$$

- We end up with

$$r\,\sigma_{rr,rr} + 3\sigma_{rr,r} = 0$$

$$\frac{\partial}{\partial r}\left(r^3\frac{\partial\sigma_{rr}}{\partial r}\right) = 0$$

We now have an ODE so we can solve to get the stress

$$\frac{\partial}{\partial r}\left(r^3 \frac{\partial \sigma_{rr}}{\partial r}\right) = 0$$

- Our boundary conditions are $\sigma_{rr}(R_i) = -p$ and $\sigma_{rr}(R_o) = 0$
- After integrating twice, we get

$$\sigma_{rr}(r) = -p\frac{(R_o/r)^2 - 1}{(R_o/R_i)^2 - 1}$$

- From σ_{rr}, we can get $\sigma_{\theta\theta}$ from

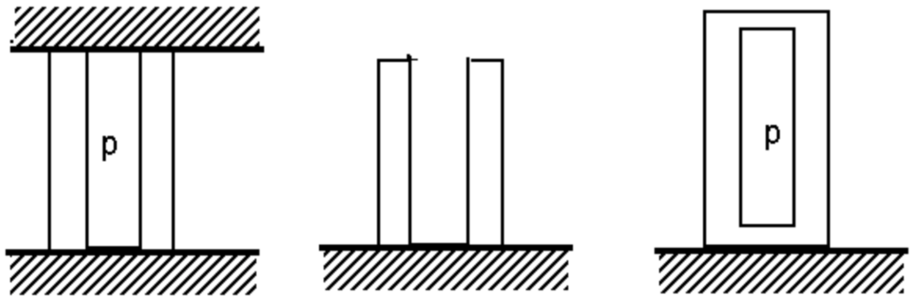
$$\sigma_{\theta\theta} = \sigma_{rr} + r\sigma_{rr,r} \qquad \sigma_{\theta\theta}(r) = p\frac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1}$$

- From the end condition, we determine σ_{zz}

(a) $$\sigma_{zz} = p\frac{2\nu}{(R_o/R)^2 - 1}$$

(b) $$\sigma_{zz} = 0$$

(c) $$\sigma_{zz} = p\frac{1}{(R_o/R)^2 - 1}$$



(a) complete axial restraint (b) no axial restraint (c) closed-end tube

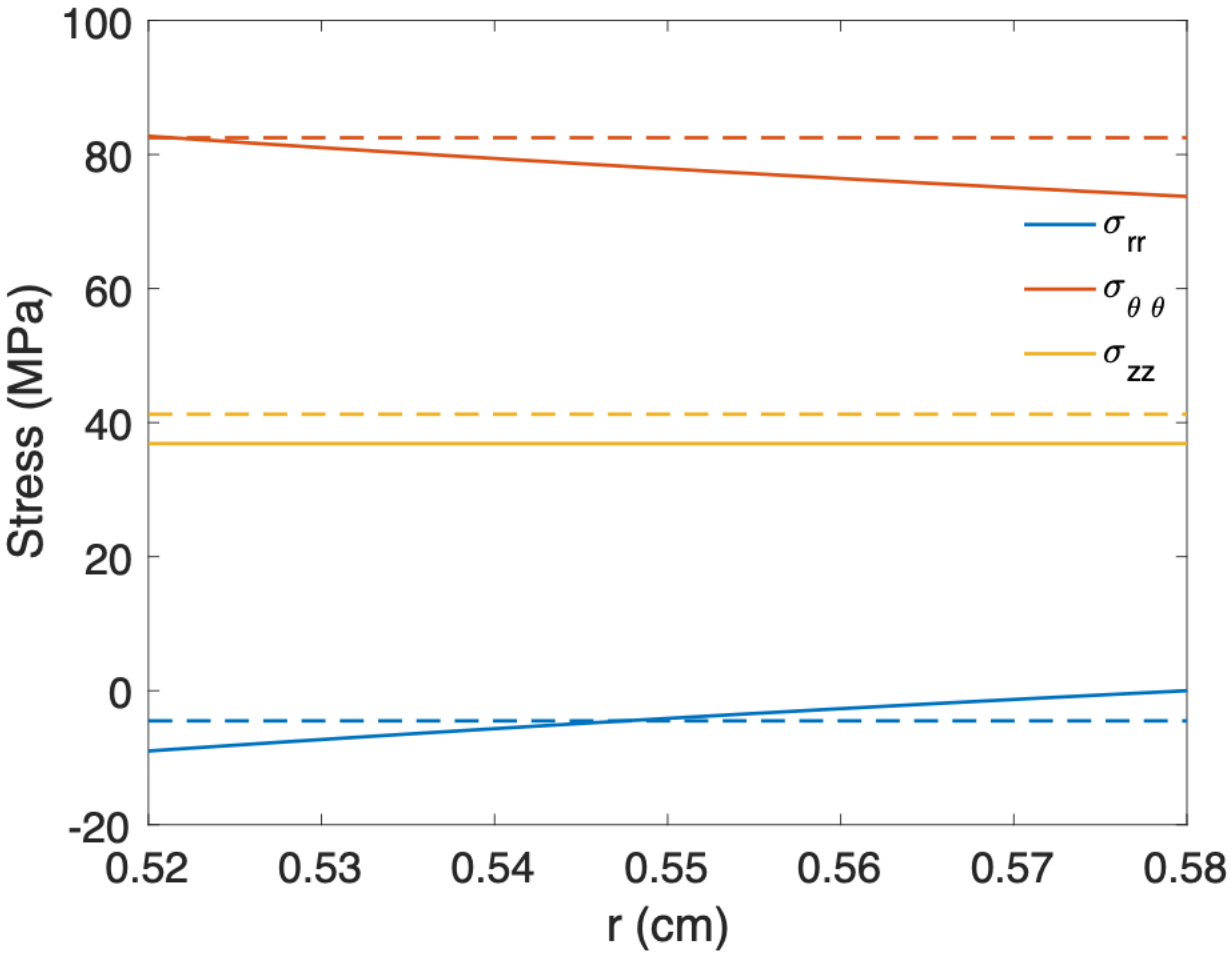
Determine if a fuel rod will exceed its yield stress

- A fuel rod has an initial pressure of 5 MPa that increases to 9 MPa during operation. Will it exceed its yield stress?
  - R_i = 0.52, R_o = 0.58 cm, σ_y = 381 MPa

$$\sigma_{rr}(r) = -p\frac{(R_o/r)^2 - 1}{(R_o/R_i)^2 - 1} \quad \sigma_{\theta\theta}(r) = p\frac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1} \quad \sigma_{zz} = p\frac{1}{(R_0/R_i)^2 - 1}$$

- Which stress is the largest?
  - $\sigma_{\theta\theta}$
- At what position r will the hoop stress be the largest?
  - $r = R_i = 0.52$ cm
- What is the stress at p = 5 MPa?
  - $\sigma_{\theta\theta}$ = 5*((0.58/0.52)^2 + 1)/((0.58/0.52)^2 – 1) = 46.0 MPa
- What is the stress at p = 9 Mpa?
  - $\sigma_{\theta\theta}$ = 9*((0.58/0.52)^2 + 1)/((0.58/0.52)^2 – 1) = 82.7 Mpa
- Again, we don't even get close to yielding

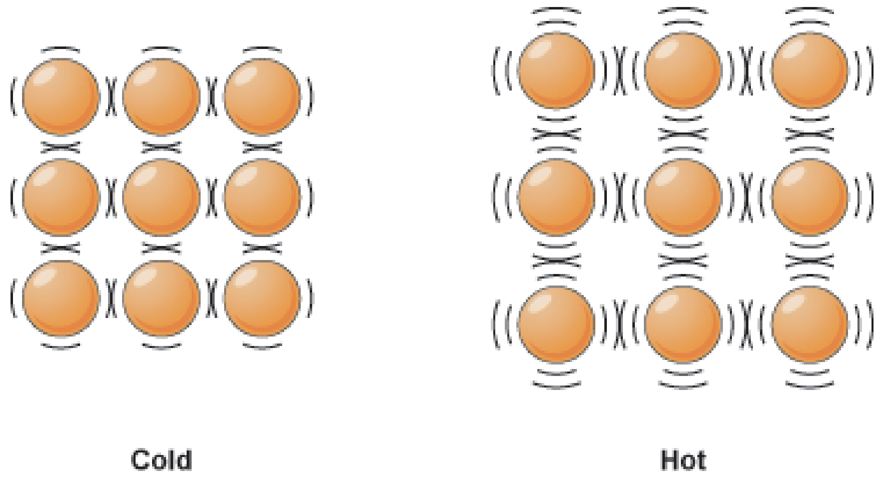
We've now solved this same problem assuming thin and thick-walled cylinders



THERMO-MECHANICS

Thermal Expansion

- As the temperature increases, atoms have larger vibrations, causing the material to expand
- This expansion doesn't raise the energy of the material when unrestrained
- This expansion can be treated as a strain, but as one that doesn't cause stress



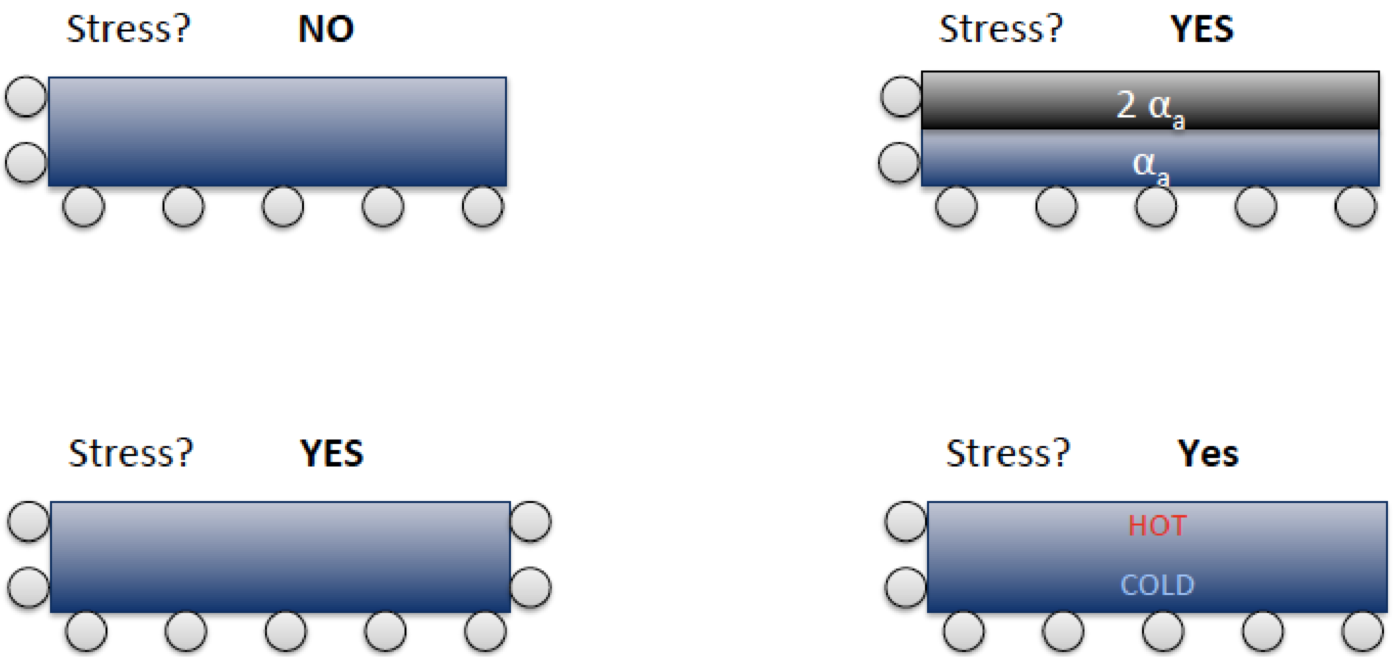
Thermal Expansion

- In isotropic materials, thermal expansion happens equally in all directions and is linear with temperature
- $\epsilon_0 = (T - T_0)\alpha I$
- In this equation
  - T is the current temperature
  - T_0 is the temperature the original size was measured
  - α is the linear thermal expansion coefficient
  - I is the identity tensor

Material	α (× 10^{-6} 1/K)
Aluminum	24
Copper	17
Steel	13
UO_2	11
Zircaloy (Axial)	5.5
Zircaloy (radial)	7.1

Thermal Expansion

- Though thermal expansion doesn't directly cause stress, it can still lead to thermal stress



What is the stress in a thin constrained rod of length L when it is heated to ΔT?

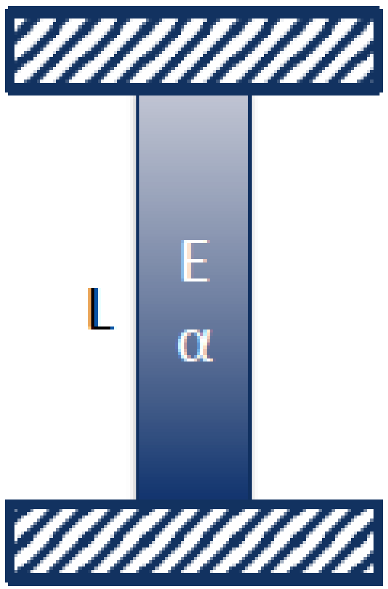
- The rod has a Young's modulus of E and an expansion coefficient of α

$$\epsilon_T = (T - T_0)\alpha I \qquad \sigma = C\,(\epsilon - \epsilon_T)$$

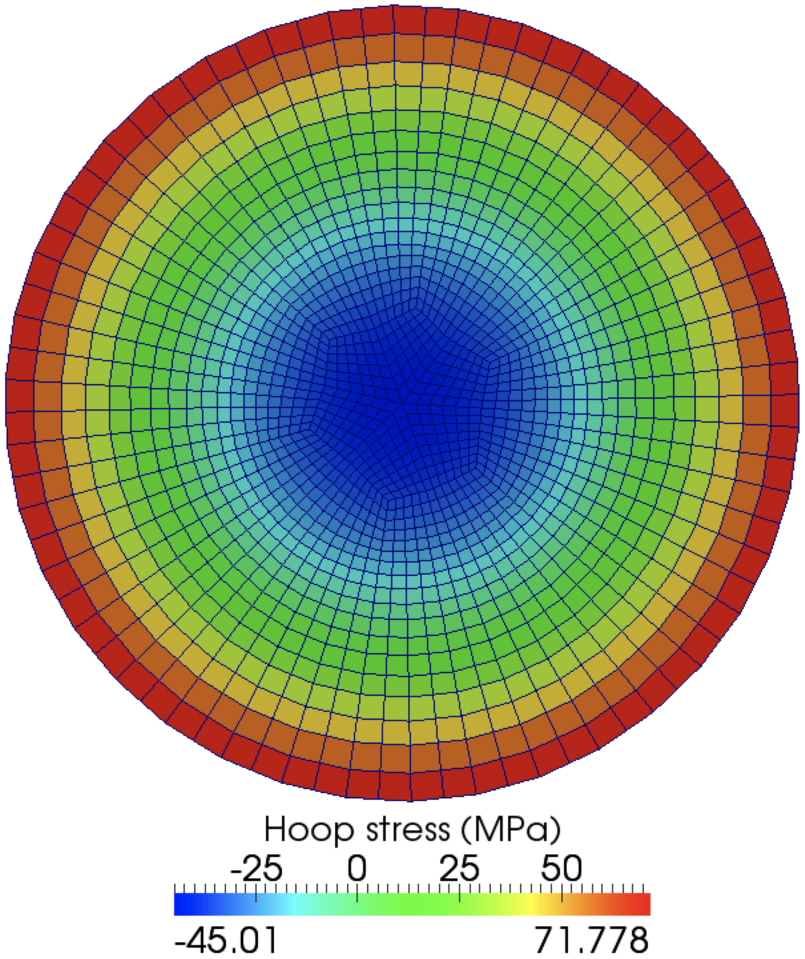
$$\epsilon_0 = (T - T_0)\alpha$$

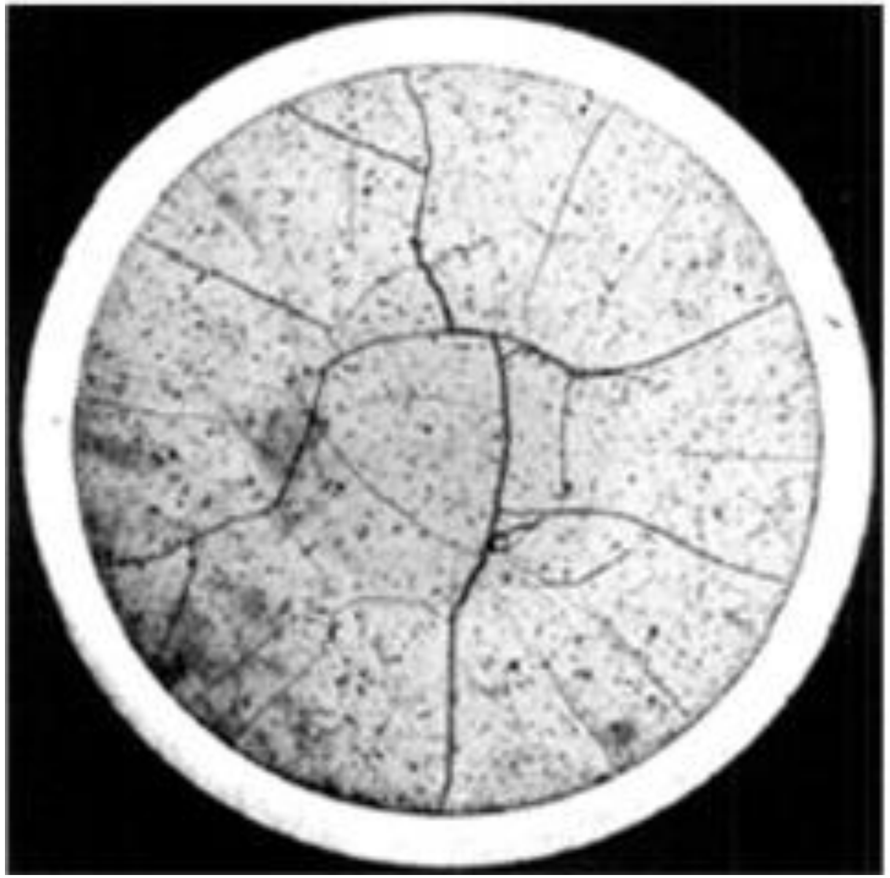
$$\sigma = E\,(0 - \Delta T\alpha)$$

$$\sigma = -E\Delta T\alpha$$



The large temperature gradient within a fuel pellet results in large thermal stresses





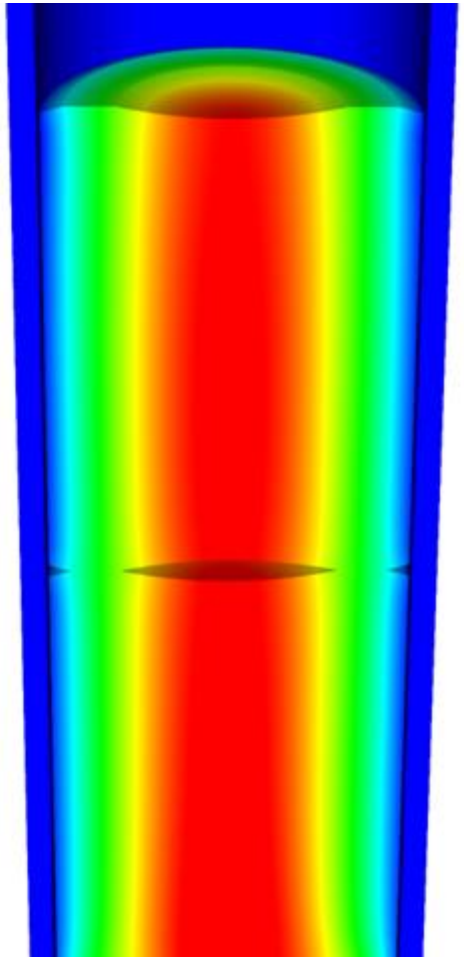
Consider the constitutive relations plus T

- We assume small strains, so the strain is defined as

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r},\ \epsilon_{\theta\theta} = \frac{u_r}{r}, \epsilon_{zz} = \frac{\partial u_z}{\partial z}, \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

- We assume isotropic material response, so

$$\begin{aligned}
\epsilon_{rr} &= \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})\right) + \alpha\Delta T \\
\epsilon_{\theta\theta} &= \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})\right) + \alpha\Delta T \\
\epsilon_{zz} &= \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})\right) + \alpha\Delta T \\
\epsilon_{rz} &= \frac{1}{2G}\sigma_{rz}
\end{aligned}$$

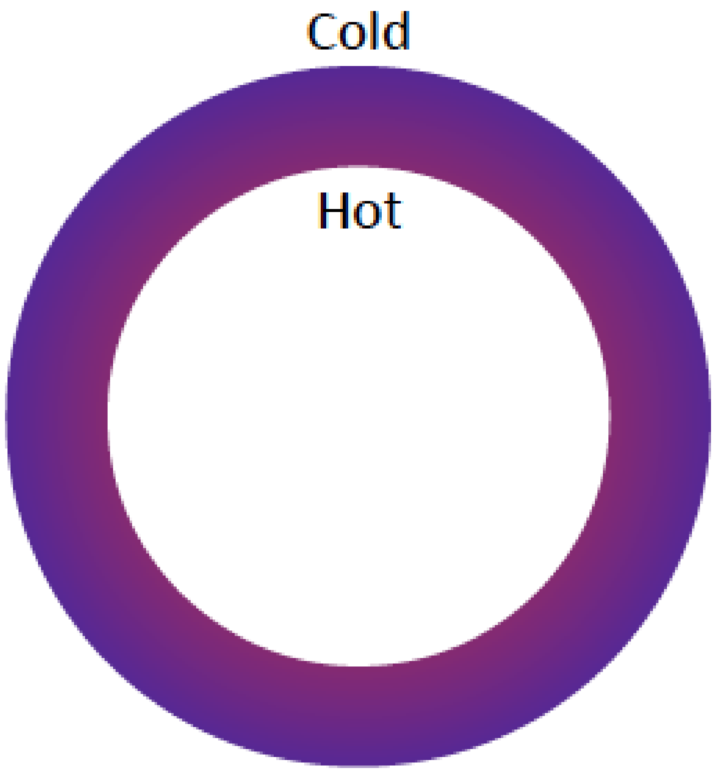


Consider a cylinder with thermal expansion but not pressure

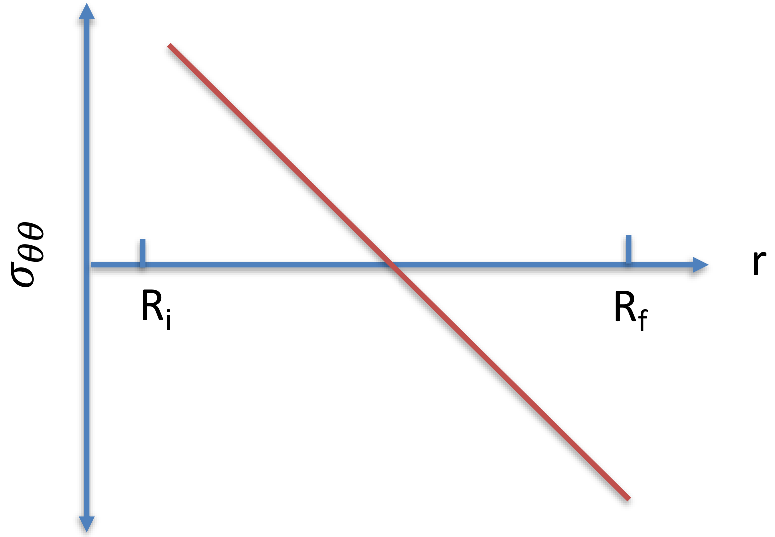
- $\sigma_{rr}(R_i) = \sigma_{rr}(R_0) = 0$
- Similar to the equations we worked through before
- $\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\nu}\right)\frac{1}{r}\frac{dT}{dr}$
- Solving this ODE:

$$\sigma_{rr}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(\frac{r}{R_i}-1\right)\left(1-\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$
$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$
$$\sigma_{zz}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1-2\frac{R_i}{\delta}\left(\frac{r}{R_i}-1\right)\right)$$

What is the hoop stress in the cladding?



$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$



Where is hoop stress equal to zero?

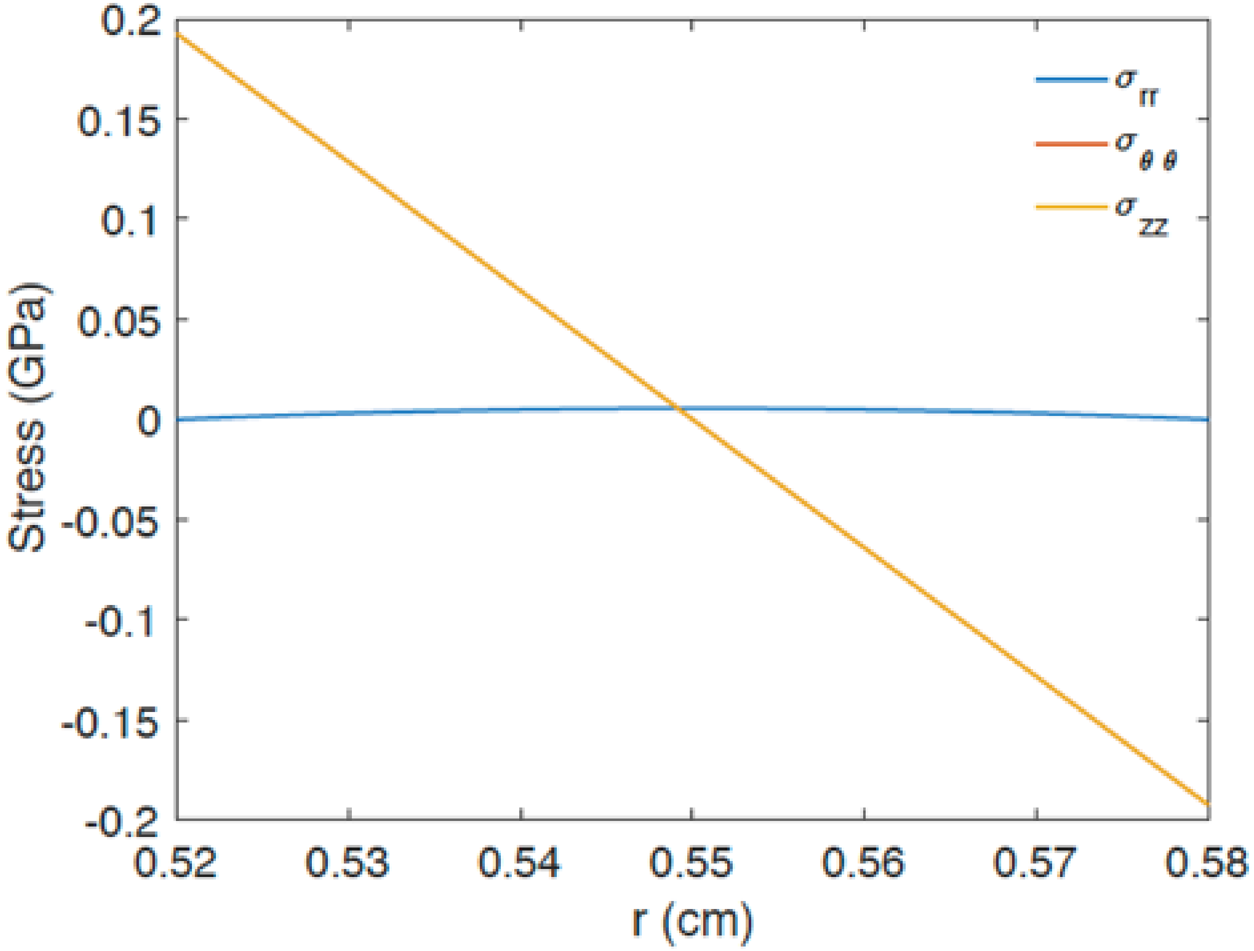
$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\sigma_{\theta\theta}(r) = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right) = 0$$

$$\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right) = 0 \longrightarrow 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right) = 1 \longrightarrow \left(\frac{r}{R_i} - 1\right) = \frac{\delta}{2R_i}$$

$$r = \frac{\delta}{2} + R_i$$

The linear temperature gradient across the cladding causes axial thermal stresses



Same approach to the thermal stress in a fuel pellet

- The thermal stress is due to the temperature gradient

$$T(r) = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_f^2}\right) + T_s$$

$$T_0 - T_s = \frac{LHR}{4\pi k}$$

$$T - T_s = (T_0 - T_s)\left(1 - \frac{r^2}{R_f^2}\right)$$

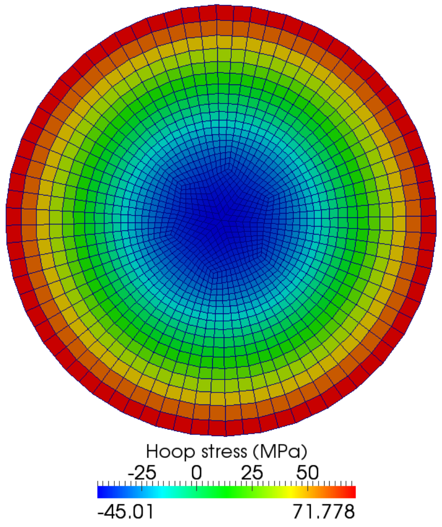
$$\frac{\partial T}{\partial r} = 2(T_0 - T_s)\left(\frac{r}{R_f^2}\right)$$

$$\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\nu}\right)\frac{1}{r}\frac{dT}{dr}$$

$$\eta = \frac{r}{R_f}$$

$$\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

$$\frac{d}{d\eta}\left(\eta^3\frac{d\sigma_{rr}}{d\eta}\right) = 8\sigma^*\eta^3$$



Solve this stress ODE

$$\frac{d}{d\eta}\left(\eta^3 \frac{d\sigma_{rr}}{d\eta}\right) = 8\sigma^*\eta^3 \qquad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$$

- The boundary conditions are:

$$\frac{d\sigma_{rr}}{d\eta} = 0 \text{ at } \eta = 0$$

$$\sigma_{rr} = 0 \text{ at } \eta = 1$$

- Once we solve it, we obtain

$$\boxed{\sigma_{rr}(\eta) = -\sigma^*(1-\eta^2)}$$

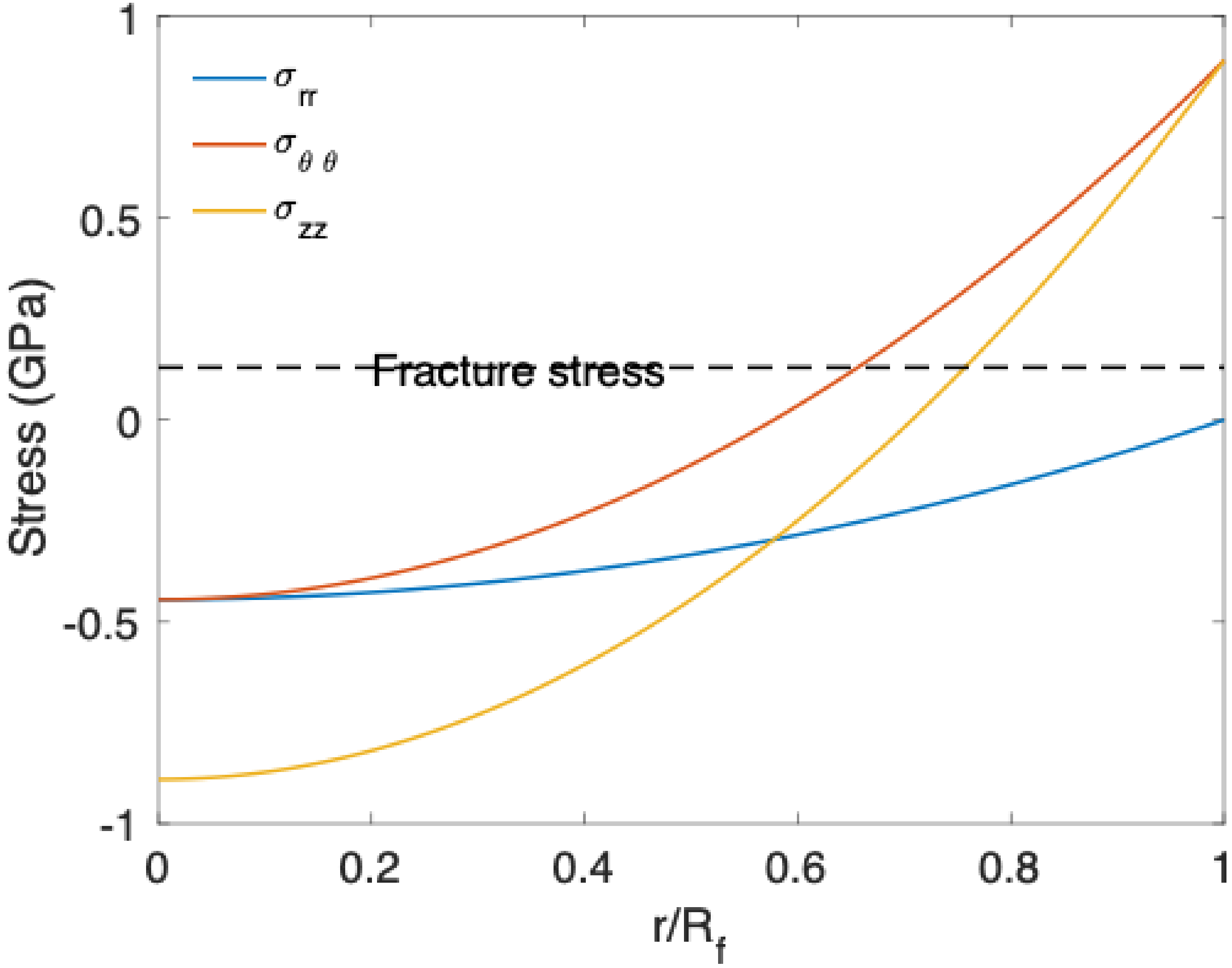
- Then we can solve the hoop stress

$$\sigma_{\theta\theta} = \sigma_{rr} + r\sigma_{rr,r} \qquad \boxed{\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2)}$$

- The axial stress is more complicated to obtain, but you end up with

$$\boxed{\sigma_{zz}(\eta) = -2\sigma^*(1-2\eta^2)}$$

The fuel temperature gradient causes large thermal stresses



Example

How far do fuel cracks extend?

$$\sigma_{\theta\theta}(\eta) = -\sigma^*(1 - 3\eta^2) \qquad \sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)} \qquad \eta = \frac{r}{R_f}$$

- E = 200 GPa, v = 0.345, α = 11.0e-6 1/K, σ_{fr} = 130 MPa, ΔT = 550 K
- Solve for η
    - – σ_{fr} / σ* = 1 – 3 η^2
    - 3 η^2 = 1 + σ_{fr} / σ*
    - η = ((1 + σ_{fr} / σ*)/3 $)^{½}$
- σ* = 11.0e-6*200*550/(4*(1 – 0.345)) = 461.8 MPa
- η = sqrt((1 + 130/461.8)/3) = 0.65

THERMAL EXPANSION

The gap changes as a function of time

- Both the pellet and the cladding swell

$$\Delta\delta_{gap} = \delta_{gap} - \delta_{gap}^{0}$$
$$\Delta\delta_{gap} = \Delta\bar{R}_C - \Delta R_f$$
$$\frac{\Delta R_f}{\bar{R}_C} = \alpha_f\left(\bar{T}_f - T_{fab}\right)$$
$$\frac{\Delta R_C}{\bar{R}_C} = \alpha_C\left(\bar{T}_C - T_{fab}\right)$$
$$\Delta\delta_{gap} = \bar{R}_c\alpha_C\left(\bar{T}_C - T_{fab}\right) - \bar{R}_f\alpha_f\left(\bar{T}_f - T_{fab}\right)$$

- But, as the gap decreases, the temperature changes, which again makes the gap change
- The solution using the analytical equations is iterative, due to the dependence of the gap size and temperature

Example

Calculate the steady state temperature profile in the rod, including thermal expansion

- LHR = 200 W/cm, δ^0_{gap} = 30 µm, R_f = 0.6, T_{co} = 600 K, $T_{EXP,0}$ = 373 K, k_{gap} = 0.0026 W/cm-K, t_C= 0.06 cm, α_f = 11.0e-6 1/K, α_C = 7.1e-6 1/K

$$\Delta\delta_{gap} = \bar{R}_c\alpha_C\left(\bar{T}_C - T_{fab}\right) - \bar{R}_f\alpha_f\left(\bar{T}_f - T_{fab}\right) = \Delta R_c - \Delta R_f$$

$$\Delta T_{gap} = \frac{LHR}{2\pi R_f k_{gap}/\delta_{gap}}$$

- So, T_{IC} = 600 + 18.7 = 618.7 K, T_s = 679.9 K, T_0 = 1210.5 K
- First, we will deal with expansion in the cladding
  - Av(R_c) = 0.6 + 30e-4 + 0.06/2 = 0.633 cm
  - Av(T_C) = (600+618.7)/2 = 609.4 K
  - ΔR_c = 0.633*7.1e-6*(609.4 – 373) = 0.0011 cm

Calculate the steady state temperature profile in the rod, including thermal expansion

- Second, we deal with the fuel
  - Av(Tf) = (1210.5 + 680)/2 = 945.25 K
  - ΔR_f = 0.6*11e-6*(945.3 – 373) = 0.00378 cm

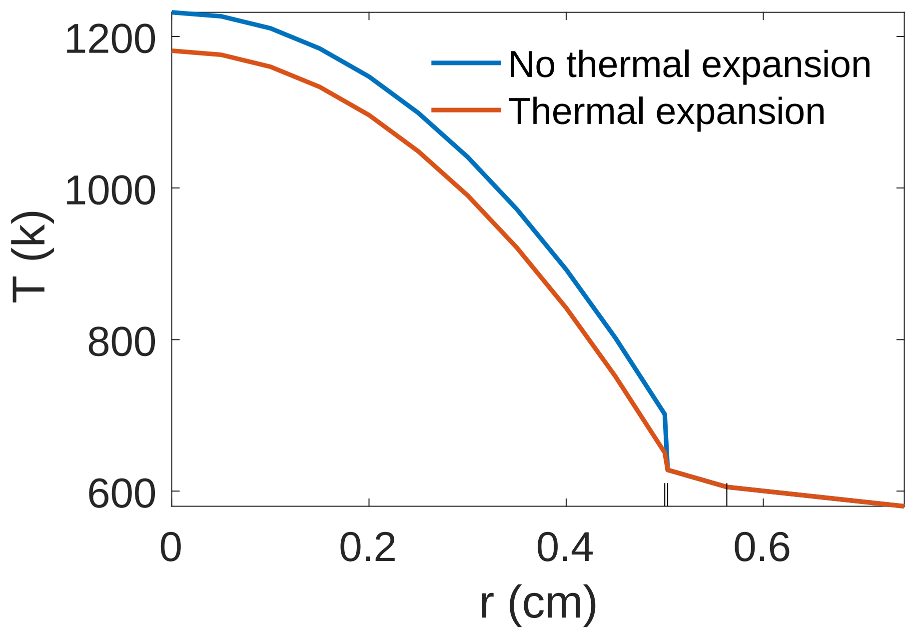
$$\Delta\delta_{gap} = \bar{R}_c \alpha_C \left(\bar{T}_C - T_{fab}\right) - \bar{R}_f \alpha_f \left(\bar{T}_f - T_{fab}\right)$$

- The total change in the gap is 0.0011 – 0.0038 = -0.0027
- However, that means the gap is smaller and so our temperatures were wrong!

This calculation is repeated until the gap width stops changing significantly

- The change in the gap does NOT affect the coolant or outer cladding temperatures, just the gap and fuel temperatures
- Does fuel centerline temp go up or down with correction?

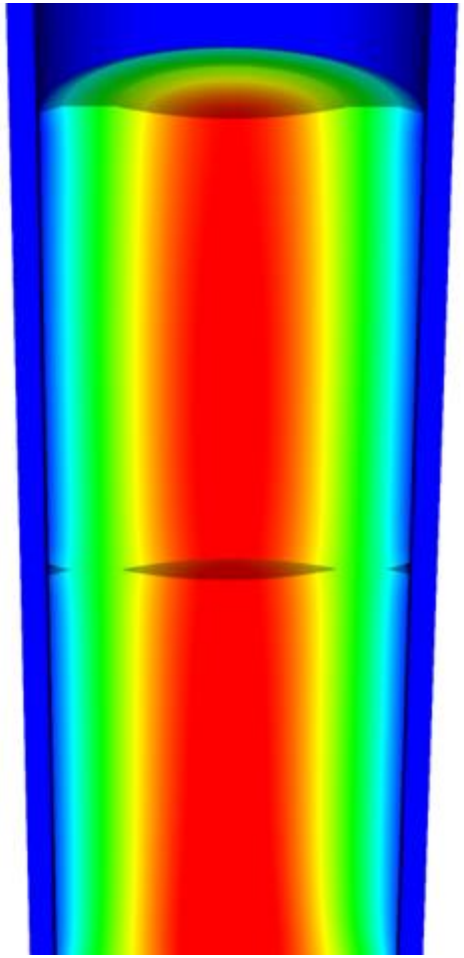
Iteration	δ_{gap} (cm)	T_s (K)	T_0 (K)
0	0.003	701	1232
1	0.00066	644	1174
2	0.00097	652	1182
3	0.00094	651	1181
4	0.00094	651	1181



LAST BIT OF MECHANICS

Relating Displacements to Stress

- We have been determining the stress due to some internal pressure (or temperature)
- Often, the information more readily obtainable are the displacements
- Utilizing the previous equations, can use displacement to stress relationships for our geometry



$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rz}}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial (r\sigma_{rz})}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

Assuming problem is axisymmetric

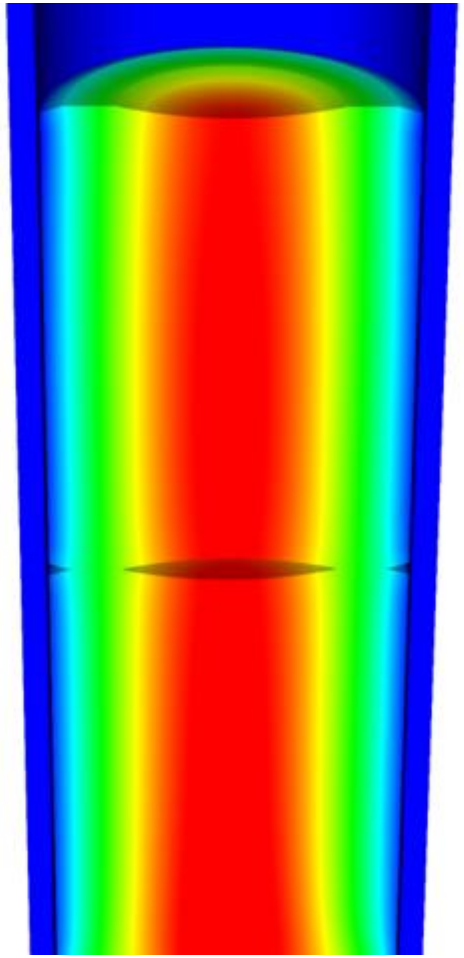
$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r k(T)\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + Q(r,z)$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rz}}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial (r\sigma_{rz})}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

$$\boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}(\boldsymbol{\epsilon} - \alpha(T - T_{fab})\mathbf{I}) \qquad \epsilon_{rr} = \frac{1}{E}(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})) + \alpha\Delta T$$

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r},\ \epsilon_{\theta\theta} = \frac{u_r}{r}, \epsilon_{zz} = \frac{\partial u_z}{\partial z}, \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} u_{r,r} & (u_{r,z} + u_{z,r})/2 & 0 \\ (u_{r,z} + u_{z,r})/2 & u_{z,z} & 0 \\ 0 & 0 & u_r/r \end{bmatrix}$$



Solve for the stress from the strain

- Assume isotropic materials
- Can perform matrix multiplication for the calculation of the stress, given the displacements

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_{z,z} \\ u_r/r \\ (u_{r,z}+u_{z,r})/2 \end{bmatrix}$$

Further simplify the problem to be 1D

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q \qquad \boldsymbol{\sigma} = \boldsymbol{\mathcal{C}}(\boldsymbol{\epsilon} - \alpha(T - T_{fab})\mathbf{I})$$

$$0 = \nabla \cdot \boldsymbol{\sigma} \qquad \boldsymbol{\epsilon} = \tfrac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$$

- No change in Z:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r k(T) \frac{\partial T}{\partial r}\right) \qquad \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix} \qquad \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

Determine the strain and stress in the pellet for 1D case

- Assume the radial displacement in the fuel pellet is $u_r(r) = 0.05r$ cm.
- What is the strain tensor? $$\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$$ At the outer edge: $$\epsilon = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$
- We are dealing with UO_2, so E = 200 GPa and v = 0.345

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

  - C_{11} = E (1 – v)/((1+v)(1–2 v)) = 200*(1–0.345)/(1.345*(1–2*.345)) = 314.2 Gpa
  - C_{12} = E v/((1+v)(1–2 v)) = 200*0.345/(1.345*(1–2*.345)) = 165.5 Gpa
- Now we can calculate the stresses
  - σ_{rr} = 0.05*314.2 + 0.05*165.5 = 23.98 GPa
  - $\sigma_{\theta\theta}$ = 0.05*165.5 + 0.05*314.2 = 23.98 GPa

$$\sigma = \begin{bmatrix} 23.98 \\ 23.98 \end{bmatrix}$$

Example problem

- Compute the stress and strain tensors in the center and at the outer edge ($r = 0.5$ cm) in 1D axisymmetric coordinates in a fuel pellet with $u_r(r) = r^2/5$. $C_{11} = 314.2$ Gpa, $C_{12} = 165.5$ Gpa. $\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$

- First, calculate the strain tensor
- $\varepsilon_{rr} = u_{r,r} = 2r/5$
- $\varepsilon_{\theta\theta} = u_r/r = r/5$
- At the center there is no strain; at the outer edge $\epsilon = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$

- To calculate the stress, convert to a strain vector and multiply by elastic constant matrix
- The stress in the center is zero
- On the outer edge
  - $\sigma_{rr} = 0.2*314.2 + 0.1*165.5 = 79.4$ GPa
  - $\sigma_{\theta\theta} = 0.1*314.2 + 0.2*165.5 = 64.52$ GPa

Summary

- We derived two analytical solutions for the stress in a pressurized cylinder
  - Thin-walled cylinder
  - Thick-walled cylinder
- Increases in temperature cause most materials to expand
- We call this thermal expansion, which is a strain that doesn't inherently cause stress
- Thermal expansion can cause stress when
  - Deformation is constrained
  - There are gradients in the expansion coefficient
  - There is a temperature gradient
- We developed analytical equations for thermal stresses
  - in the cladding
  - in the fuel