Timothy Bowles

NE 533 Test 2

1. $R_f = 0.56cm$ $LHR = 350 \frac{W}{cm}$

a) $\sigma_{max} = ?$; $k_f = 0.05 \frac{W}{cmK}$; $E = 200GPa$; $\nu = 0.35$; $\alpha = 10\times10^{-6}\ 1/K$

$\sigma_{max} = \sigma_{\theta\theta}$ @ $r = R_F$

$\sigma_{\theta\theta}(\eta) = -\frac{\alpha E (T_0 - T_S)}{4(1-\nu)}(1 - 3\eta^2)$; $T_0 - T_S = \frac{LHR}{4\pi k} = \frac{350 \frac{W}{cm}}{4\pi(0.05 \frac{W}{cmK})} = 557.04\ K$

$= -\frac{(10\times10^{-6}/K)(200\times10^3 MPa)(557.04K)}{4(1-0.35)}(1 - 3(1)^2) = 856.985 MPa$

b) $\sigma_{fracture} = 150 MPa$, how far cracks into fuel

$\frac{4\sigma_{fracture}(1-\nu)}{-\alpha E(T_0 - T_S)} = 1 - 3\eta^2 \Rightarrow \eta = \sqrt{\frac{1 + (4\sigma_f(1-\nu))/(\alpha E(T_0 - T_S))}{3}}$

$$\eta = \sqrt{\frac{1 + (4(150MPa)(1-0.35))/((10\times10^{-6}/K)(200\times10^3 MPa)(557.04K))}{3}}$$

$R_f(1 - \eta) = $ crack length

$= 0.67$; so cracks extend 0.181 cm into the pellet

2. $p = 55 MPa$ $\bar{r} = \delta = 0.55cm$ $t = 0.05cm$

a) $\bar{\sigma}_\theta = \frac{pR}{\delta} = \frac{(55MPa)(0.55cm)}{0.05cm} = 605 MPa$

$\bar{\sigma}_z = \frac{pR}{2\delta} = \frac{\bar{\sigma}_\theta}{2} = 302.5 MPa$

$\bar{\sigma}_r = -\frac{1}{2}p = -27.5 MPa$

$r = 0.525cm$

b) $\sigma_{rr} = -p\frac{(R_o/R_i)^2 - 1}{(R_o/R_i)^2 - 1} = -p = -55MPa$

$\sigma_{\theta\theta} = p\frac{(R_o/R_i)^2 + 1}{(R_o/R_i)^2 - 1} = 55MPa \frac{(0.575/0.525)^2 + 1}{(0.575/0.525)^2 - 1} = 606.25 MPa$

$\sigma_{zz} = p\frac{1}{(R_o/R_i)^2 - 1} = 275.625 MPa$

3. Gap thickness change; $R_f = 0.52cm$; $t_{gap} = 0.005cm$; $T_{co} = 550K$; $t_{clad} = 0.08cm$; $k_{fuel} = 0.04 \frac{W}{cmK}$; $h_{gap} = 0.003 \frac{W}{cmK}$; $k_{clad} = 0.15 W/cmK$; $LHR = 175 \frac{W}{cm}$; $\alpha_c = 10\times10^{-6}/K$

$\alpha_f = 14\times10^{-6}/K$; $T_{ref}^{f,c} = 300K$

$T_{CI} = \frac{LHR}{2\pi R_f}\frac{t_{clad}}{k_{clad}} + T_{co} = \frac{175 W/cm}{2\pi(0.52cm)}\frac{0.08cm}{0.15 \frac{W}{cmK}} + 550K = 578.57K$

$T_{Fo} = \frac{LHR}{2\pi R_f}\frac{t_{gap}}{h_{gap}} + T_{CI} = \frac{175 W/cm}{2\pi(0.52cm)} \cdot \frac{0.005cm}{0.003\frac{W}{cmK}} + 578.57K = 667.84K$

$T_0 = \frac{LHR}{4\pi k_{fuel}} + T_{Fo} = \frac{175 W/cm}{4\pi(0.04\frac{W}{cmK})} + 667.84K = 1015.99K$

$\bar{T}_c = \frac{T_{co} + T_{CI}}{2} = \frac{550K + 578.57K}{2} = 564.285K$

$\bar{T}_f = \frac{T_{Fo} + T_0}{2} = \frac{667.84 + 1015.99}{2} = 841.915K$

$\bar{R}_f = \frac{R_f}{2} = \frac{0.52cm}{2} = 0.26cm$

$\bar{R}_c = R_f + t_{gap} + \frac{t_{clad}}{2} = 0.565cm$

$\Rightarrow \Delta t_{gap} = 0.565\,cm(10\times10^{-6}\,1/K)(544.285-300) - 0.26cm(14\times10^{-6}\,1/K)(841.915-300)$

$= \boxed{-0.001823\ cm}$

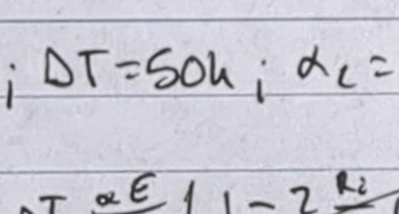
$t'_g = 0.005 - 0.001823 = 0.003176\ cm$

4. $\sigma_{\theta\theta}, \sigma_{rr} = ?\ @\ R = R_i;\ \Delta T = 50K;\ \alpha_c = 15\times10^{-6};\ E = 100GPa;\ \nu = 0.34;\ t_c = 0.06cm;\ R_i = 0.55cm$

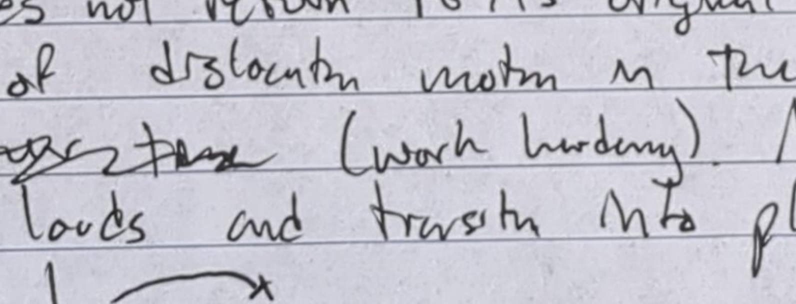
$\sigma_{\theta\theta}(R_i) = \sigma_{rr}(R_i) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1 - \cancel{2\frac{R_i}{\delta}\left(\frac{\Delta T}{R_i}-1\right)}^{\to 0}\right) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu} = \frac{50K}{2}\frac{(15\times10^{-6}\,1/K)(100\times10^{3}MPa)}{1-0.34}$

$= \boxed{56.818\ MPa}$

5. Elasticity is material ~~deformation~~ elongation that occurs when a force is applied that returns to its original length when the force is unloaded. Plasticity ~~does~~ does not return to its original length. Plastic deformation is the result of dislocation motion in the material. It hardens the material ~~over time~~ (work hardening). Most materials elastically deform for small loads and transition into plastic deformation as the load grows.



6. Strain hardening is an increase in the yield stress of a material as it is elongated (or strained). It is the result of dislocation motion in the material. As dislocations pile up on defects the material becomes harder to strain.



7. All fuel performance codes must be able to predict fuel temperature profile and volume change, cladding temperature and stress profile, and gap pressure, heat transportation, and cladding-fuel mechanical interaction. BISON and FRAPCON are currently being used.

8. A vacancy is a point defect in a crystal where one atom is missing in the pattern. A void is an example of a 3D defect in a crystaline structure where some volume is missing atoms, from the crystal/polycrystal.

9. When sintering, powder is heated and/or pressurized into a solid without melting the material. The powder forms the initial grains in the sintering process. Subsequent heat treatments may change grain size.

10. Microstructure-based fuel performance modeling combines calculated local states with emphercal constants to charecterize material performance. Using this modeling method allows for codes to accureatly predict material behavers outside/beyond validated test data. This enables less costly reactor performance modeling, and enables novel, untested fuel concepts to be explored before pursuing test data. Microstructure-based fuel performance modeling takes grain boundries, intergranular porosity, vacancies, precipitates, and more into accont.

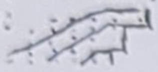
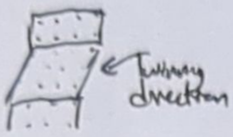
11. Microstructure is the ^patterns of^ arrangement of atoms at the x25 magnification level (~1 μm to ~100 μm). Microstructural characteristics such as grains, defects, patterns like the High Burnup Structure, heavily influence engineering-level materials performance. One example of a processing technique that effects microstructure is stress relief annealing. ~~Stre~~ In stress relief annealing a material is heated up below its melting point to allow dislocations and defects to diffuse in a material and remove residual stress that may exist from previous ~~mat~~ plastic material deformation.

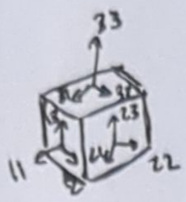
12. High Burnup Structure (HBS) is a microstructure that forms on the edge of a fuel pellet where localized power peaking drives higher ^local^ depletion over core life. This structure ~~has~~ reduces grain size from ~10 μm → 100-200 nm and creates a 20% porosity throughout the local material. This structure increases heat transmission of the [redacted] pellet because of the smaller grain size, despite the increased porosity. This structure also retains fission gasses well because the voids do not percolate well. The local increased burnup is due to fuel self shielding and lower temperatures which help capture ~~resonant~~ resonant neutrons in the U238 → Pu239 → more fissions.

NE533 Test 2 Cheat Sheet

$F=kx$; $E=\frac{1}{2}kx^2$; $u(r,t)$: displacements

Stress, strain, elastic, plastic

Plasticity: Slip:  Twinning:  ← Twinning direction



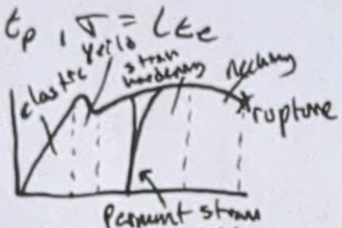
$\sigma_{ij} = F_j/A_i$ $\quad Pa = \frac{N}{m^2}$

$e = \frac{\Delta L}{L} = \frac{L-L}{L}$; $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

$$\begin{bmatrix} \epsilon_{11} & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

$\sigma = F(\epsilon)$; $\sigma = C(\epsilon)$; large ϵ: $\epsilon = \epsilon_e + \epsilon_p$, $\sigma = C\epsilon_e$

$E_{el} = \frac{1}{2}\epsilon_e \cdot \sigma$



	Engineering	True
stress σ	F/A_0	F/A
strain ϵ	$\frac{l-l_0}{l_0}$	$\int_{l_0}^{l}\frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$

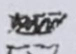
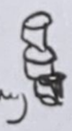
Youngs modulus $\sigma = E\epsilon$; $\nu = \frac{-d\epsilon_{trans}}{d\epsilon_{axial}} = \frac{-d\epsilon_y}{d\epsilon_x} = \frac{-d\epsilon_z}{d\epsilon_x} \approx \frac{\Delta L'}{\Delta L}$

Shear modulus: $G = \frac{E}{2(1+\nu)}$

Hooks for isotropic: $$\begin{bmatrix} \epsilon_{11} \\ 22 \\ 33 \\ 2\epsilon_{23} \\ 13 \\ 12 \end{bmatrix} = \begin{bmatrix} \epsilon_{11} \\ 22 \\ 33 \\ \gamma_{23} \\ 13 \\ 12 \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix}\begin{bmatrix} \sigma_{11} \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{bmatrix}$$

Von Mises: $\sigma_v^2 = \frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\right]$

$\sigma = \sigma_y + k(\epsilon_0 + \epsilon_p)^n$ $\quad 0 \le n \le 1$; $\sigma_T = C(\epsilon_T)^n$ (F/A, $\ln(l/l_0)$) $\quad n=0.15$ some steels, $n=0.5$ some copper

Dislocation motion: Edge and screw type ; Edge:  Screw 

Dislocation ⇒ plastic deformation ⇒ dislocation pileup ⇒ hardening

$\%EL = \frac{l_f - l_0}{l_f} \times 100\%$; % Reduction in Area (RA) $= \frac{A_0 - A_f}{A_f} \times 100\%$

Simplified Cauchy: $\rho\frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + \rho g$ (static, negligible)

Axisymmetric: $\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rz}}{\partial z} = 0$; $\frac{1}{r}\frac{\partial(r\sigma_{rz})}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} = 0$

Thin walls: $F_{stress} = 2\delta\sigma_\theta$; $\sigma_\theta = \frac{pR}{\delta}$; $\sigma_z = \frac{pR}{2\delta}$; $\sigma_r = -\frac{1}{2}p$

Thick wall: $\epsilon_{rr} = \frac{1}{E}(\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz}))$; $\epsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz}))$

$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr}))$; $\epsilon_{rz} = \frac{1}{2G}\sigma_{rz}$

$E\epsilon_{\theta\theta,r} = \sigma_{\theta\theta,r} - \nu(\sigma_{rr,r} + \nu(\sigma_{\theta\theta,r} + \sigma_{rr,r})) = (1-\nu^2)\sigma_{\theta\theta,r} + \nu(1+\nu)\sigma_{rr,r}$

$E(\epsilon_{rr} - \epsilon_{\theta\theta}) = (1+\nu)(\sigma_{rr} - \sigma_{\theta\theta})$

$r\sigma_{rr,rr} + 3\sigma_{rr,r} = 0$; $\frac{\partial}{\partial r}\left(r^3\frac{\partial \sigma_{rr}}{\partial r}\right) = 0$

$\sigma_{rr}(r) = -p\frac{(R_o/r)^2 - 1}{(R_o/R_i)^2 - 1}$; $\sigma_{\theta\theta}(r) = p\frac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1}$; $\sigma_{zz} = p\frac{2\nu}{(R_o/R_i)^2 - 1}$; $\sigma_{zz} = p\frac{1}{(R_o/R_i)^2 - 1}$

$\epsilon_0 = (T - T_0)\alpha I$; $\sigma = C(\epsilon - \epsilon_T)$; $\sigma = -E\Delta T\alpha I$

$\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\nu}\right)\frac{1}{r}\frac{dT}{dr}$ ⇒

$\sigma_{rr}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(\frac{r}{R_i} - 1\right)\left(1 - \frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$; δ = thickness

$\sigma_{\theta\theta}(r) = \sigma_{zz}(r) = \frac{1}{2}\Delta T\frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$

Thermal stress in fuel pellet: $\sigma_{rr}(\eta) = -\sigma^*(1-\eta^2)$; $\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2)$

$\sigma_{zz}(\eta) = -2\sigma^*(1-2\eta^2)$; $\sigma^* = \frac{\alpha E(T_0 - T_s)}{4(1-\nu)}$; $\eta = \frac{r}{R_f}$; for fuel crack $\eta \approx 0.65$

Gap size: $\Delta\delta_{gap} = \delta_{gap} - \delta^0_{gap}$; $\Delta\delta_{gap} = \Delta R_c - \Delta R_f$; $\frac{\Delta R_f}{R_f} = \alpha_f(\bar{T}_f - T_{ref})$; $\frac{\Delta R_c}{R_c} = \alpha_c(\bar{T}_c - T_{ref})$

$\Delta\delta_{gap} = R_c\alpha_c(\bar{T}_c - T_{ref}) - R_f\alpha_f(\bar{T}_f - T_{ref})$

$\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$ $\quad \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix}\begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$

$\rho c_p\frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + Q$; $\sigma = C(\epsilon - \alpha(T - T_{ref})I)$; $0 = \nabla \cdot \sigma$; $\epsilon = \frac{1}{2}(\nabla u + \nabla u^T)$

Finite volume + Finite element can solve thermomechanics

Core primary goals are centerline temp and cladding stress

Code must predict: Fuel: temp + vol change ; Cladding: temp + stress profile ; Gap: pressure, heat trans, mechanical interaction

Timothy Bowes

Steady state + transient codes (include burnup effects)

Frapcon: steady, multiple 1D slices (1.5D) ; NRC baseline code ; Finite difference ; iterates to find fuel temp + fuel and cladding deformation ; iterates to gas pressure, release, void volumes

Fraptran: Same as Frapcon, but with transient capabilities

Falcon: Transient, 2D axisymmetric w/ smeared pellets ; ANATECH for EPRI (USU 2003) ; FEM fuel + cladding ; axisymmetric in RZ or Rθ space

BISON: transient, full 3D and less ; INL with ANATECH 2008–2012 ; Uses FEM

OFFBEAT ; open FOAM

Fuel Fab	Early life	Mid life	Late life	Fuel Failure
Sintered UO2	Thermal expansion	Point defect diffusion clustering	Fission product swelling	Pellet-clad interaction
h=2cm d=1cm	Fracture	Fission gas segregation to GB + voids	Bubble percolation and FG release	Clad corrosion
~10μm grain	Point defect + fission gas generation	Bubble nucleation	Clad creep	Clad fracture
density 95%	Fuel densification		Fuel creep	

Crystalline defects:

Point defects (0D): Vacancies, self interstitial atoms, interstitial impurity atoms, substitutional impurity atoms

$$C_v = \frac{N_v}{N_s} = \exp\left(\frac{S_f^v}{k}\right)\exp\left(\frac{-E_f^v}{kT}\right)$$ equilibrium point defect concentration

Radiation damage from ionizing radiation, PKA collisions cascade (primary knock-on)

Dislocations (1D): Controls plastic deformation

Grain boundaries (2D): Most are polycrystalline ; naturally form during casting ; cutting has some effect

Average grain size: Max: $P(u) = (2e)^\beta \cdot \frac{\beta u}{(2-\mu)^{2\beta}} \cdot \exp\left(\frac{-2\beta}{2-\mu}\right)$

3D defects: voids + precipitates ; point defect coalescence

Microstructure: ×25 magnification

Materials processing:

Casting: liquid material cool in mold ; sintering: powder heat/pressed into solid

Heat treatment: Annealing, quenching ; working: squeezing, bending, shearing

Radiation damage:

Frenkel defects (interstitial-vacancy pair) from secondary collision cascade

In UO2, Schottky trio formed w/ 2 anion for every cation

Loop defects are combination of screw and edge defects

slip along $\{100\}\langle110\rangle$ minimize energy

slight excess of vacancies from preferential absorption of interstitials

build up at GB + form voids for FG

Chemistry changes ~3% of fuel ; ↑defects ↓k ; ↑precipitates ↑k

↑porosity ↓k ; ↑mixed oxides ↓k

$k_0 = \frac{100}{7.5408 + 17.629t + 3.6142t^2} + \frac{6400}{t^{5/2}}\exp\left(\frac{-16.35}{t}\right)$; $t = \frac{T}{1000}$

BISON uses NFIR model

σ damage cause 0.4% swelling ; ↑Fluence ↑hardness ↓Youngs modulus

High Burnup Structure: 10% more power 200μm edge ; Pu239 generation, high local burnup

grain subdivide 10μm→100–200nm ; 20% porous ; polygonization removes defects ; k↑ from grain size despite porosity

Mechanistic modeling: Grain boundaries ; intergranular porosity ; precipitated FP

$k = \frac{k_{0B}k_p k_{pr}}{A + BT + CT^2 + C_{v}x_v + C_i x_i + C_g x_g}$; bulk conductivity ; vacancies + interstitials ; Fission gas

1) σ @ $r = 0.52$cm ; $r_i = 0.5$cm ; $r_o = 0.55$cm ; $P = 15$MPa ;

$\sigma_\theta = \left[\frac{(0.55/0.52)^2 + 1}{(0.55/0.5)^2 - 1}\right]15 = 151.3$MPa ; $\sigma_r = \left[\frac{(0.55/0.52)^2 - 1}{(0.55/0.5)^2 - 1}\right](-15) = -8.5$MPa

$\sigma_z = 15/[(0.55/0.52)^2 - 1] = 126$MPa ; thin: $r = \frac{r_i + r_o}{2} = 0.525$cm

2) σ_{max} in pellet ; $\Delta T = T_0 - T_s = 425$K ; $\alpha_f = 12\times10^{-6}$ 1/K ; $R_f = 0.5$cm ; $E = 180$GPa ; $\nu = 0.25$

$\sigma_{max} = \sigma_\theta$ @ $r = R_f$; $\sigma^* = \frac{(12\times10^{-6})(180\times10^3 MPa)(425)}{4(1 - 0.25)} = 318.8$MPa

$\sigma_\theta = -318.8(1 - 3(1^2)) = 637.5$MPa @ $r = R_f$

3) Cladding tube stress @ $r = 0.62$; $R_i = 0.6$cm ; $t_c = 0.1$cm ; $E = 250$GPa

$\nu = 0.3$; $\alpha_c = 15\times10^{-6}$ 1/K ; $T_{ci} = 600$K ; $T_{co} = 580$K

$\sigma_r = \frac{20}{2}\frac{(15\times10^{-6})(250\times10^3 MPa)}{1 - 0.3}\left(\frac{0.62}{0.6} - 1\right)\left(1 - \frac{0.6}{0.1}\left(\frac{0.62}{0.6} - 1\right)\right) = 1.43$MPa

$\sigma_\theta = \sigma_z = \frac{20}{2}\frac{(15\times10^{-6})(250\times10^3 MPa)}{1 - 0.3}\left(1 - \frac{2(0.6)}{0.1}\left(\frac{0.62}{0.6} - 1\right)\right) = 32.1$MPa

4) Gap thickness change ; $\alpha_f = 12\times10^{-6}$ 1/K ; $\alpha_c = 15\times10^{-6}$ 1/K ; $\bar{T}_f = 925$K

$\bar{T}_c = 550$K ; $T_0^{ref} = 300$K ; $R_f = 0.5$cm ; $R_c = 0.58$cm ; $t_g = 0.03$cm

$\Delta t_g = 0.58(16\times10^{-6})(250) - 0.5(12\times10^{-6})(625) = -0.0016$cm

$t_g' = 0.03 - 0.0016 = 0.0284$cm

5) Stress @ $r = 0.45$; $E = 200$GPa ; $\nu = 0.3$; $u_r(r) = 0.5r^2 - 0.2r$

$\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix} = \begin{bmatrix} r - 0.2 & 0 \\ 0 & 0.5r - 0.2 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.025 \end{bmatrix}$

$C_{11} = \frac{E}{(1+\nu)(1-2\nu)}(1-\nu) = 269$GPa ; $C_{12} = \frac{E}{(1+\nu)(1-2\nu)}\nu = 115$GPa

$\sigma_{rr} = 269(0.25) + 115(0.025) = 70$GPa ; $\sigma_{\theta\theta} = 115(0.25) + 269(0.025) = 35$GPa

$T_0 - T_s = \frac{\dot{q}R^2}{4k} = \frac{LHR}{4\pi k}$; $T_{fs} - T_{ci} = \frac{LHR}{2\pi R_f}\frac{t_g}{k_{gap}}$; $T_{ci} - T_{co} = \frac{LHR}{2\pi R_f}\frac{t_{clad}}{k_{clad}}$

$T_{co} - T_{cool} = \frac{LHR}{2\pi R_f}\frac{1}{h_{cool}}$; $T(r) = \frac{LHR}{4\pi k_f}(1 - r^2/R_f^2) + T_s$