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92/100

Exam 1 - NucE 497

Question 1

-0, 30/30

$K = 12.5 \ W/mK$ $\quad \delta_U = 7.5 g[U]/cm^3$

a) The fissile isotope is U-235.

It would be ± 0.7 wt%.

b) $N_{U235} = x \, q \, Na \, \frac{\delta_U}{M_U}$

$Na = 6.022 \, 10^{23}$ atoms/mol

x : nº of atoms of U in the fuel

$M_U \simeq 238$ amu

$U_3Si_2 \Rightarrow N_{U235} = 3 \times q \times Na \times \frac{\delta_U}{M_U}$ $\quad \delta_U = 11.31 \ g/cm^3$

$N_{U235} = 3 \times 0.03 \times 6.022 \times 10^{23} \times \frac{11.31}{238}$

$N_{U235} = 2.5755 \times 10^{21}$

Now we calculate q that will lead to a same N_{U235}:

$U_3Si_5 \Rightarrow 2.5755 \times 10^{21} = 3 \times q \times 6.022 \times 10^{23} \times \frac{7.5}{238}$

$q = 0.0452 \Rightarrow \boxed{q = 4.52 \%}$

c) It is worst than U_3Si_2 because it has a lower density of fissile atoms and has a lower thermal conductivity.

Question 2

-4, 31/35

a) $T_{co} - T_{cool} = \dfrac{LHR}{2\pi R_f h_{cool}}$

$LHR = 250\,W/cm$

$h_{cool} = 2.5\ \dfrac{W}{cm^2 K}$

$R_f = 0.45\,cm$

$\delta_{gap} = 80 \times 10^{-4}\,cm$

$T_{ci} - T_{co} = \dfrac{LHR\, t_c}{2\pi R_f k_c}$

$T_{cool} = 580\,K$

$\delta_{clad} = 0.06\,cm$

$y = 0.05$

$T_s - T_{ci} = \dfrac{LHR}{2\pi R_f h_{gap}}$

$h_{gap} = \dfrac{K_{He}^{1-y}\ K_{xe}^{y}}{\delta_{gap}}$

summing up and uplugging the values:

Using $K_{gas} = A \times 10^{-6}\, T^{0.79}$

$A = 16$ He

$A = 0.7$ Xe

with the initial temperature T_{cool},

$K_{He} = 0.0024$ W/cmK

$K_{xe} = 1.06\ 10^{-4}$ W/cmK

$$\boxed{T_s = 985.7\ K}$$

-1, math error, Ts = 958.2 K

b) $E = 246.7$ GPa $\quad \nu = 0.25 \quad \alpha = 7.5 \times 10^{-6}\ 1/K$

calculating fuel centerline T_o:

$T_o = \dfrac{LHR}{4\pi K} + T_s = 1085.2\ K$

$K_{UN} = 0.2\ \dfrac{W}{cmK}$

Using the equations for thermal stress inside the fuel pellet and knowing that the hoop stress will be the larger:

$$\sigma_{\theta\theta}(\eta) = -\sigma^* (1 - 3\eta^2) \quad \therefore \sigma^* = \frac{\alpha E (T_0 - T_S)}{4(1-\nu)} \quad \therefore \eta = \frac{r}{R_S}$$

The largest value is when $\eta = 1$ ($r = R_S$) @ the surface

$$\sigma_{\theta\theta}(R_S) = + \frac{\alpha E \Delta T}{2(1-\nu)} = \boxed{122.7 \text{ MPa}}$$

c) $\alpha_{UO_2} = 1.2 \times 10^{-5}$ 1/K while $E = 200$ GPa

as the hoop stress is proportional to these two, it would be lower since $\alpha_{UO2} < \alpha_{UN}$ and $E_{UO2} < E_{UN}$.

-3, It is higher, but primarily because k is smaller so the DeltaT would be much higher

d)

a)
- Steady-state
- axisymmetric
- T is constant in z
- K is independent of T

b)
- Static body
- Neglect gravity
- axisymmetric
- Small strains
- Solution does not change w/ z
- Isotropic material response

Question 3

-4, 31/35

a) • Static body ← Thin wall

• neglect gravity

• axisymmetric

• small strains

• solution does not change with z

• stress is constant through the wall of the cylinder

• Isotropic material response

b) Thin wall solution:

$$\bar{\sigma}_\theta = \frac{PR}{\delta} \qquad \bar{\sigma}_z = \frac{PR}{2\delta} \qquad \bar{\sigma}_r = -\frac{P}{2}$$

$P = 6$ MPa

$R_i = 5.6$ mm

$\delta = 0.6$ mm

$\bar{\sigma}_\theta = 56.0$ MPa	$\bar{\sigma}_z = 28.0$ MPa	$\bar{\sigma}_r = -3.0$ MPa

c) for thicker walls:

	$r = R_i$	$r = R_o$
$\sigma_{rr}(r) = -P \dfrac{\left(\frac{R_o}{r}\right)^2 - 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$	-6 MPa	0 MPa
$\sigma_{\theta\theta}(r) = P \dfrac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$	56.16 MPa	50.16 MPa
$\sigma_{zz} = P \dfrac{1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$	25.08 MPa	25.08 MPa

For this case, the thin wall approximation would work.

The highest hoop stress in the thick wall approximation is @ $r = R_i$ and it is not much higher than the thin wall value (56.16 compared to 56 MPa, 0.3% higher).

The hoop stress is the highest of all three terms and it decreases with r, so thin wall solution is safe to predict failure.

Conservative but hoop stress varies by more than 10% across thickness

d)

$$\varepsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right)$$

$E = 70$ GPa

$\nu = 0.41$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right)$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{\theta\theta} + \sigma_{rr}\right)\right)$$

$$\underbrace{\begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \end{bmatrix}}_{\text{strain tensor}} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix}\underbrace{\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \end{bmatrix}}_{\text{stress tensor}}$$

plugging numerical values and using the stresses calculated in b):

$$\varepsilon_{rr} = -5.34 \times 10^{-4}$$

$$\varepsilon_{\theta\theta} = 6.53 \times 10^{-4}$$

$$\varepsilon_{zz} = 8.95 \times 10^{-5}$$

-4, Didn't write stress and strain in tensor form