NE 591-010

Nuclear Fuel Performance

Exam 2 Solutions

1) The true stress + strain accounts for the reduction in the cross-sectional area, as well as lengthening of the specimen. True SS will always have a "higher" SS curve than engineering SS.

True → $\sigma = \frac{F}{A}$ $\quad \epsilon = \ln\left(\frac{l}{l_o}\right)$

Eng. → $\sigma = \frac{F}{A_o}$ $\quad \epsilon = \frac{l - l_o}{l_o}$

2) Elastic deformation is temporary, reversible, and is a stretching of atomic bonds.

Plastic deformation is a breaking of bonds, that leads to permanent displacement/deformation within a material.

3) 0-D defect : vacancy, interstitial, substitutional

3-D defect : void, bubble, precipitate, etc.

4) Lattice parameter, thermal conductivity, solution energy of fission products, O diffusivity, etc.

5) Grains act to repel dislocations, Thus, a decrease in grain size can strengthen the material, increasing the yield strength. This is the Hall-Petch effect. Material switches to grain sliding at very small grain size.

6) Strain hardening is the forming of defects or dislocations within a material in order to increase the yield strength. Dislocations repel one another, thus as the dislocation density is increased, dislocation movement becomes inhibited. This inhibition manifests as an increased yield point.

7). 1) Model the temperature in the fuel

2) Model the stress in the cladding

3) Consider gap closure, pressure, and heat transfer.

8) Reduction of the free energy. Pores (and grain boundaries) have a higher energy than the ideal crystal. By reducing the porosity, the energy of the system is reduced.

9) Irradiation can accelerate grain growth. However, this effect is only pronounced at low temperature and for small grains.

An increase in the temperature also increases grain growth.

Solute atoms, pores, bubbles, second phases, etc. all serve to inhibit grain growth.

10) $p = 20$ MPa

$R_c^{avg} = 0.54$ cm $\quad t_c = 0.08$ cm

$R_i = 0.5$ cm $\quad R_o = 0.58$ cm

$$\sigma_\theta = \frac{pR}{t} \qquad \sigma_r = \frac{-p}{2} \qquad \sigma_z = \frac{pR}{2t}$$

$$\sigma_\theta = \frac{20(0.54)}{0.08} = \underline{135 \text{ MPa}} \qquad \sigma_r = \frac{-20}{2} = \underline{-10 \text{ MPa}}$$

$$\sigma_z = \frac{20(0.54)}{2(0.08)} = \underline{67.5 \text{ MPa}}$$

b) thick wall @ $r = r_{avg}$

$$\sigma_r = -p\,\frac{(R_o/r)^2 - 1}{(R_o/R_i)^2 - 1} \qquad \sigma_z = \frac{p}{(R_o/R_i)^2 - 1}$$

$$\sigma_\theta = p\,\frac{(R_o/r)^2 + 1}{(R_o/R_i)^2 - 1} \qquad R_o/R_i = \frac{0.58}{0.5} = 1.16$$

$$R_o/R = \frac{0.58}{0.54} = 1.07$$

$$\sigma_r = -20\,\frac{(1.07^2 - 1)}{(1.16^2 - 1)} = \underline{-8.4 \text{ MPa}} \qquad \sigma_z = \frac{20}{1.16^2 - 1} = \underline{58 \text{ MPa}}$$

$$\sigma_\theta = 20\,\frac{(1.07^2 + 1)}{(1.16^2 - 1)} = \underline{124 \text{ MPa}}$$

c) σ_θ = @ $r = R_i$

σ_z : no where

$\sigma_r \Rightarrow$ $$-10 = -20\,\frac{(R_o/r)^2 - 1}{1.16^2 - 1}$$

$$(R_o/r)^2 = 1.17$$

$$r = 0.536 \text{ cm}$$

11) $R_F = 0.45\ cm$ $LHR = 250\ \frac{W}{cm}$

max stress ? $\rightarrow \sigma_\theta$

$K_F = 0.1\ \frac{W}{cm \cdot K}$

$E = 290\ GPa$

$\nu = 0.3$

$\alpha_F = 8.2 \times 10^{-6}\ {}^1/_K$

$$\sigma_\theta = -\sigma^* (1 - 3\eta^2)$$

$$\eta = \frac{r}{R_F} \qquad \sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$$

max $\sigma_\theta \rightarrow \eta = 1 \Rightarrow r = R_F$

$$T_0 - T_s = \frac{LHR}{4\pi K_f} = \frac{250}{4\pi(0.1)} = 198.9\ K$$

$$\sigma^* = \frac{(8.2 \times 10^{-6})(290 \times 10^3)(199)}{4(1-0.3)} = 169\ MPa$$

$$\sigma_\theta = -169(1 - 3(1)^2) = \underline{338\ MPa}$$

12) $R_F = 0.5 \text{ cm}$ $\quad t_g = 0.02 \text{ cm}$ $\quad R_{c1} = 0.52 \text{ cm}$ $\quad K_g = 0.04 \frac{W}{cm\text{-}K}$

$T_{c1} = 450 K$ $\quad K_f = 0.05 \frac{W}{cm\text{-}K}$ $\quad LHR = 325 \frac{W}{cm}$

$\alpha_F = 15 \times 10^{-6} \ \frac{1}{K}$ $\quad \alpha_c = 4.5 \times 10^{-4} \ \frac{1}{K}$ $\quad T_{ref} = 300 K$

$$T_0 - T_s = \frac{LHR}{4\pi K_F} = \frac{325}{4\pi(0.05)} = 517 \text{ K}$$

$$T_s - T_{c1} = \frac{LHR}{2\pi R_F} \frac{t_g}{K_g} = \frac{325}{2\pi(0.5)} \frac{0.02}{0.04} = 52 K$$

$$T_0 = 450 + 52 + 517 = \underline{1019 \text{ K}}$$

$$\Delta t_g = \Delta t_c - \Delta t_f = \alpha_c \Delta T_c R_c - \alpha_F \Delta T_F R_F$$

$$\Delta t_c = (4.5 \times 10^{-6})(450 - 300)(0.52) = 3.51 \times 10^{-4}$$

$$T_F^{avg} = \frac{T_0 + T_1}{2} = \frac{1019 + 502}{2} = 760 K$$

$$\Delta t_F = (15 \times 10^{-6})(760 - 300)(0.5) = 0.00345$$

$$\Delta t_g = 3.51 \times 10^{-4} - 3.45 \times 10^{-3} = \underline{-3.1 \times 10^{-3}}$$

$$t_g' = 0.02 - 3.1 \times 10^{-3} = 0.017 \text{ cm}$$

$$T_s - T_{c1} = \frac{325}{2\pi(0.5)} \frac{0.017}{0.04} = 44 \text{ K}$$

$$T_0 = 450 + 44 + 517 = \underline{1011 \text{ K}}$$

13) cracks extending into fuel

$$\sigma_\theta = -\sigma^* (1 - 3\eta^2) \qquad \eta = \frac{r}{R_F}$$

$R_F = 0.55$ cm $\qquad \nu = 0.25 \qquad E = 210$ GPa

$LHR = 200$ W/cm $\qquad \alpha_F = 10.5 \times 10^{-6}$ 1/K

$K_F = 0.05$ W/m-K $\qquad \sigma_{fr} = 120$ MPa

$$\sigma^* = \frac{\alpha E \Delta T}{4(1-\nu)} \qquad \Delta T = T_0 - T_s = \frac{LHR}{4\pi K_f} = \frac{200}{4\pi(0.05)}$$

$$\Delta T = 318 \text{ K}$$

$$\sigma^* = \frac{(10.5\times10^{-6})(210\times10^3)(318)}{4(1-0.25)} = \underline{233 \text{ MPa}}$$

$$\sigma_{fr} = \sigma_\theta \rightarrow 120 = -233\left(1 - 3\left(\frac{r}{R_F}\right)^2\right)$$

$$\left(\frac{r}{R_F}\right)^2 = 0.497$$

$$\boxed{r = 0.7\ R_F = 0.39 \text{ cm}}$$