Cody Smith

81/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

Question 1:

-6, 24/30

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

Fissile Isotope: U^{235}

Natural (unenriched) Uranium: $U^{238} = 99.3\%$, $\boxed{U^{235} = 0.7\%}$

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

$E_f = 3\times10^{-11}\ \frac{J}{s}$, $\phi = 3.2\times10^{13}\ \frac{n}{cm^2 \cdot s}$, $\gamma = 0.03$, $\sigma = 550b$, $\rho_{U_3Si_5} = 9.1\frac{g}{cm^3}$, $\rho_{U_3Si_2} = 12.2\frac{g}{cm^3}$

$A_{U_3Si_5} = 3(238) + 5(28.1) = 854.5$, $A_{U_3Si_2} = 3(238) + 2(28.1) = 770.2$

- Set both Q values equal to each other to determine enrichment

$$Q_{U_3Si_5} = Q_{U_3Si_2}$$

$$\cancel{E_f \sigma \phi} N_{U_{235}} = \cancel{E_f \sigma \phi} N_{U_{235}}$$

-3, Just use U densities

$$\frac{\rho_{U_3Si_5} \cdot \cancel{N_A} \cdot \gamma_{U_3Si_5}}{A_{U_3Si_5}} = \frac{\rho_{U_3Si_2} \cdot \cancel{N_A} \cdot \gamma_{U_3Si_2}}{A_{U_3Si_2}} \Rightarrow \frac{(9.1)\cdot\gamma_{U_3Si_5}}{854.5} = \frac{(12.2)(0.03)}{770.2} \Rightarrow \boxed{\gamma_{U_3Si_5} = 0.04 = 4\%}$$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

Ranking these two fuels, U_3Si_2 is the better fuel when comparing its efficiency to U_3Si_5's efficiency. The efficiency of U_3Si_2 is better due the amount of enrichment of U^{235} needed in order to have the same value for Q, which was demonstrated in part b.

-3, thermal conductivity?

Given: $R_f = 0.45\text{cm}$, $t_{clad} = 0.06\text{cm}$, $t_{gap} = 0.008\text{cm}$, $LHR = 250 \frac{W}{cm}$, $T_{cool} = 580K$,
$h_{cool} = 2.5 \frac{W}{cm^2 \cdot K}$, $K_{Zr} = 0.12$, $k = 0.2$

-1, 34/35

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{oc} = \frac{LHR}{2\pi R_f h_{cool}} + T_{cool} = \frac{250}{2(\pi)(0.45)(2.5)} + 580$$

$T_{oc} = 615.4K$

T_{oc} = Temp of outer clad

$$T_{ic} = \frac{LHR \cdot t_{clad}}{2\pi R_f \cdot K_{clad}} + T_{oc} = \frac{250 \cdot 0.06}{2\pi(0.45)(0.12)} + 615.4$$

$T_{ic} = 646.6K$

T_{ic} = Temp of inner clad

$$T_S = \frac{LHR}{2\pi R_f \cdot h_{gap}} + T_{ic}$$

$h_{gap} = k_{gap}/t_{gap} \Rightarrow k_{gap} = K_{He}^{1-\gamma} K_{Xe}^{\gamma}$

$K_{gap} = \left(16\times10^{-6}(646.6)^{0.79}\right)^{1-0.05} \cdot \left(0.7\times10^{-6}(646.6)^{0.79}\right)^{0.05}$

$K_{gap} = 0.0023 \Rightarrow h_{gap} = 0.0023/0.008 = 0.284$

$\boxed{T_S = 957.8K}$, T_S = Surface temperature

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$\alpha = 7.5\times10^{-6} \frac{1}{K}$, $\nu = 0.25$, $E = 246.7\ GPa$

$T_0 = \frac{LHR}{4\pi K} + T_S \Rightarrow T_0 = 1057.4K$

$\sigma_{\theta\theta}: -\sigma^*(1-3\eta^2) \Rightarrow \sigma^* = \frac{\alpha E(T_0 - T_S)}{4(1-\nu)} = 0.06$

$\sigma_{\theta\theta} = -0.06(1-3(1)^2) = \boxed{0.12\ GPa}$ max stress occurs at $\eta = 1$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

The stress in UO_2 would be larger because K of UO_2 is less than the K of UN and UO_2 has a larger α.

d) What assumptions were made in your calculations for a) and b)? (5 points)

gap size stayed constant, Steady State, constant K, Constant LHR

-1, there are several more assumptions

-12, 23/35

Question 3:

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

-3, stress is constant through thickness

$F_{stress} = F_{pressure}$, wall thickness goes to 0, the stresses that act upon the cylinder would be parallel to it.

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$P = 6\,MPa,\ R = 5.6\,mm,\ \delta = 0.6\,mm.$

$$\sigma_\theta = PR/\delta = \frac{6(5.6)}{0.6} = \boxed{56\,MPa}$$

$$\sigma_r = -\frac{P}{2} = \frac{-6}{2} = \boxed{-3\,MPa}$$

$$\sigma_z = \frac{PR}{2\delta} = \frac{6(5.6)}{2(0.6)} = \boxed{28\,MPa}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

$$\sigma_{\theta\theta} = \frac{P\left(\left(\frac{R_o}{r}\right)^2+1\right)}{\left(\frac{R_o}{R_i}\right)^2-1} \Rightarrow \left(r = R_i \to \text{max stress}\right) \Rightarrow \frac{6\left(\left(\frac{5.9}{5.3}\right)^2+1\right)}{\left(\frac{5.9}{5.3}\right)^2-1} \Rightarrow \sigma_{\theta\theta} = 56.2\,MPa$$

-4, Calculate stress at two radii and compare to see if it is constant

$$error = \frac{|56 - 56.2|}{56.2} = 0.0036 = \boxed{0.36\%}$$

The thin walled cylinder approximation results in a smaller stress than the thick wall approx. If a stress occured equal to the thin wall approx, the cladding would not fail making it conservative.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\begin{bmatrix}\sigma_r \\ \sigma_\theta\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu \\ \nu & 1-\nu\end{bmatrix}\begin{bmatrix}U_{r,r} \\ U_r/r\end{bmatrix} \Rightarrow \begin{bmatrix}-3 \\ 56\end{bmatrix} = \frac{70}{(1+0.41)(1-2(0.41))}\begin{bmatrix}1-0.41 & 0.41 \\ 0.41 & 1-0.41\end{bmatrix}\begin{bmatrix}U_{r,r} \\ U_r/r\end{bmatrix}$$

Solve system of equations

$$\left.\begin{aligned} -3 &= 275.8(0.59)(U_{r,r}) + 275.8(0.41)(U_r/r) \\ 56 &= 275.8(0.41)(U_{r,r}) + 275.8(0.59)(U_r/r)\end{aligned}\right\} \begin{aligned} U_{r,r} &= -0.49 \\ U_r/r &= 0.69\end{aligned}$$

$$\epsilon = \begin{bmatrix}U_{r,r} & 0 \\ 0 & U_r/r\end{bmatrix} \Rightarrow \boxed{\epsilon = \begin{bmatrix}-0.49 & 0 \\ 0 & 0.69\end{bmatrix},\quad \sigma = \begin{bmatrix}-3 & 0 \\ 0 & 56\end{bmatrix}}$$

-2, Stress and strain are missing zz component

-3, Compute strain from stress from part b