

Thermo-Mechanics

NE 591

Last Time

- Overviewed thermal expansion
- Thermal expansion can cause stress when
 - Deformation is constrained
 - There are gradients in the expansion coefficient
 - There is a temperature gradient
- We have analytical equations for thermal stresses in the cladding and in the fuel
- Demonstrated need for iteration to obtain steady-state gap thickness

Now we can solve the temperature and the displacement vector for the full thermomechanical problem

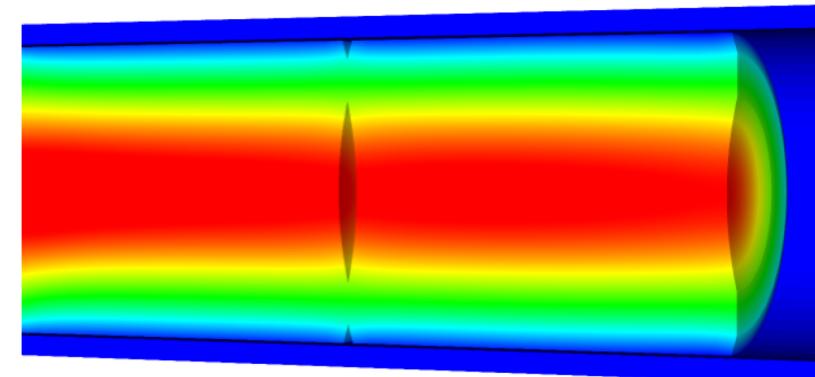
- T impacts the value of \mathbf{u} through thermal expansion
- \mathbf{u} impacts the value of T through changes in the thickness of the gap
- The value for T evolves with time
- The value for \mathbf{u} also evolves with time, even though there is not time in its PDE

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

$$\boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{\epsilon} - \alpha(T - T_{fab})\mathbf{I})$$

$$0 = \nabla \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{\epsilon} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$



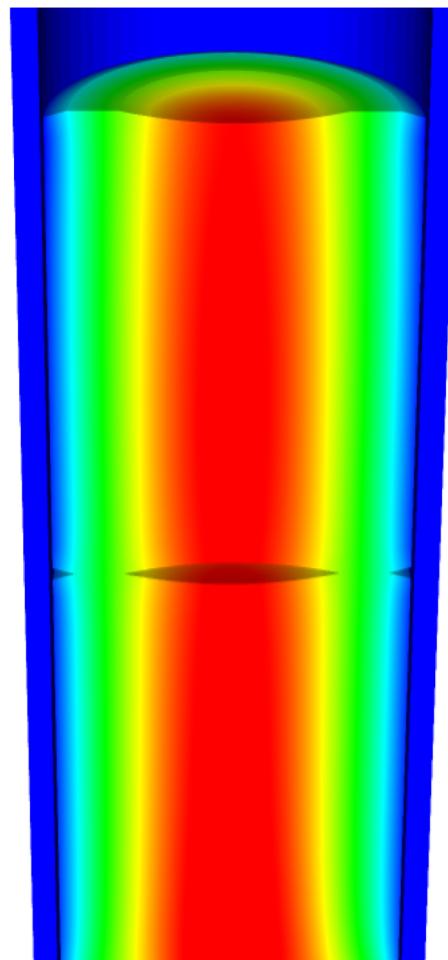
Assuming problem is axisymmetric

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r k(T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k(T) \frac{\partial T}{\partial z} \right) + Q(r, z)$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rz}}{\partial z} = 0 \quad \frac{1}{r} \frac{\partial (r \sigma_{rz})}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

$$\boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{\epsilon} - \alpha(T - T_{fab})\mathbf{I})$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} u_{r,r} & (u_{r,z} + u_{z,r})/2 & 0 \\ (u_{r,z} + u_{z,r})/2 & u_{z,z} & 0 \\ 0 & 0 & u_r/r \end{bmatrix}$$



Solve for the stress from the strain

- Assume isotropic materials
- Can perform matrix multiplication for the calculation of the stress, given the displacements

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_{z,z} \\ u_r/r \\ (u_{r,z} + u_{z,r})/2 \end{bmatrix}$$

Further simplify the problem to be 1D (1.5D)

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q \quad \boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{\epsilon} - \alpha(T - T_{fab}) \mathbf{I})$$

$$0 = \nabla \cdot \boldsymbol{\sigma} \quad \boldsymbol{\epsilon} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

- No change in Z:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r k(T) \frac{\partial T}{\partial r} \right) \quad \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \quad \boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{\epsilon} - \alpha(T - T_{fab}) \mathbf{I})$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix} \quad \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

Determine the strain and stress in the pellet for 1D case

- Assume the radial displacement in the fuel pellet is $u_r(r) = 0.05r$ cm.

- What is the strain tensor?

$$\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix} \quad \text{At the outer edge:}$$

$$\epsilon = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

- We are dealing with UO_2 , so $E = 200$ GPa and $\nu = 0.345$

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} u_{r,r} \\ u_r/r \end{bmatrix}$$

- $C_{11} = E(1-\nu)/((1+\nu)(1-2\nu)) = 200*(1-0.345)/(1.345*(1-2*.345)) = 314.2$ Gpa
- $C_{12} = E\nu/((1+\nu)(1-2\nu)) = 200*0.345/(1.345*(1-2*.345)) = 165.5$ Gpa

- Now we can calculate the stresses

- $\sigma_{rr} = 0.05*314.2 + 0.05*165.5 = 23.98$ GPa
- $\sigma_{\theta\theta} = 0.05*314.2 + 0.05*165.5 = 23.98$ GPa

$$\sigma = \begin{bmatrix} 23.98 \\ 23.98 \end{bmatrix}$$

Example problem

- Compute the stress and strain tensors in the center and at the outer edge ($r = 0.5$ cm) in 1D axisymmetric coordinates in a fuel pellet with $u_r(r) = r^2/5$. $C_{11} = 314.2$ Gpa, $C_{12} = 165.5$ Gpa. $\epsilon = \begin{bmatrix} u_{r,r} & 0 \\ 0 & u_r/r \end{bmatrix}$

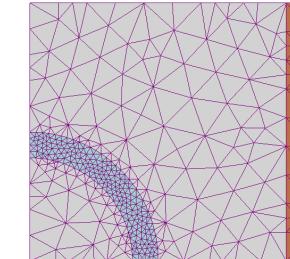
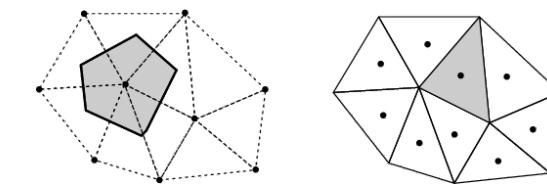
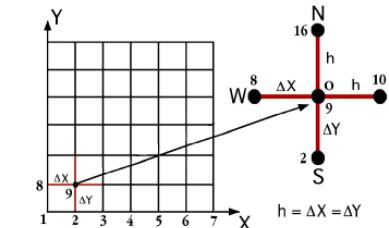
- First, calculate the strain tensor
- $\epsilon_{rr} = u_{r,r} = 2r/5$
- $\epsilon_{\theta\theta} = u_r/4 = r/5$
- At the center there is no strain; at the outer edge

$$\epsilon = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$$

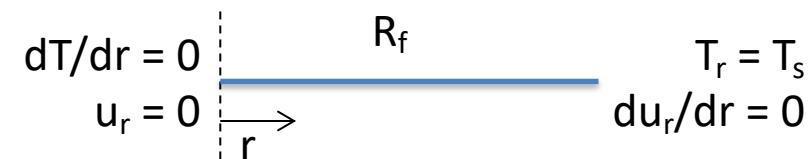
- To calculate the stress, convert to a strain vector and multiply by elastic constant matrix
- The stress in the center is zero
- On the outer edge
 - $\sigma_{rr} = 0.2*314.2 + 0.1*165.5 = 79.4$ GPa
 - $\sigma_{\theta\theta} = 0.1*314.2 + 0.2*165.5 = 64.52$ GPa

The primary tool for solving all thermomechanics problems is the finite element method

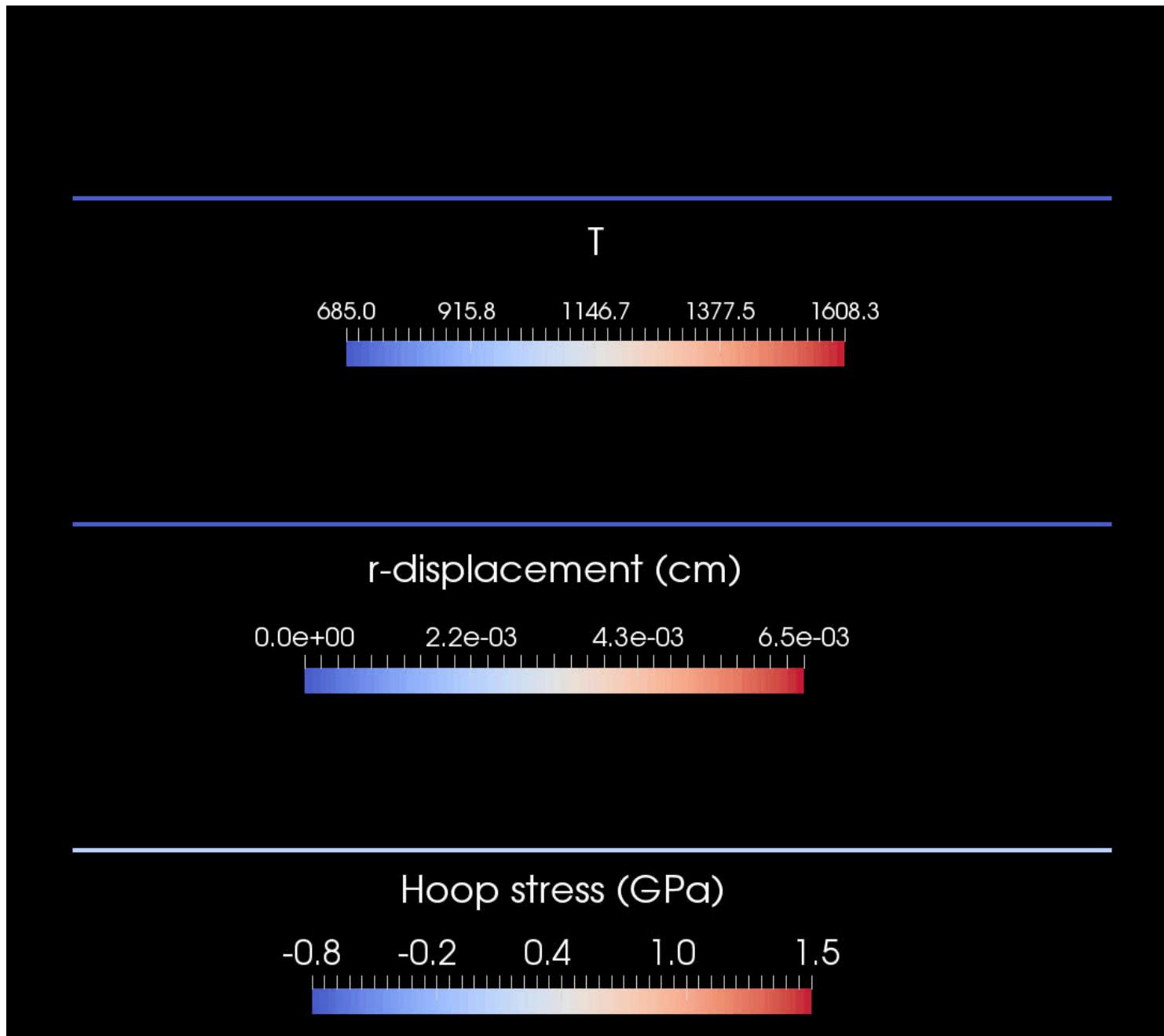
- **Finite difference**
 - Can solve the heat conduction equation
 - Can't easily solve the mechanics equations
- **Finite Volume**
 - Can solve the heat conduction equation
 - Can't easily solve the mechanics equations
- **Finite Element**
 - Can solve the heat conduction equation
 - Can solve the mechanics equations
 - Can handle any geometry
 - Can handle any boundary condition



The 1D thermomechanics problem definition

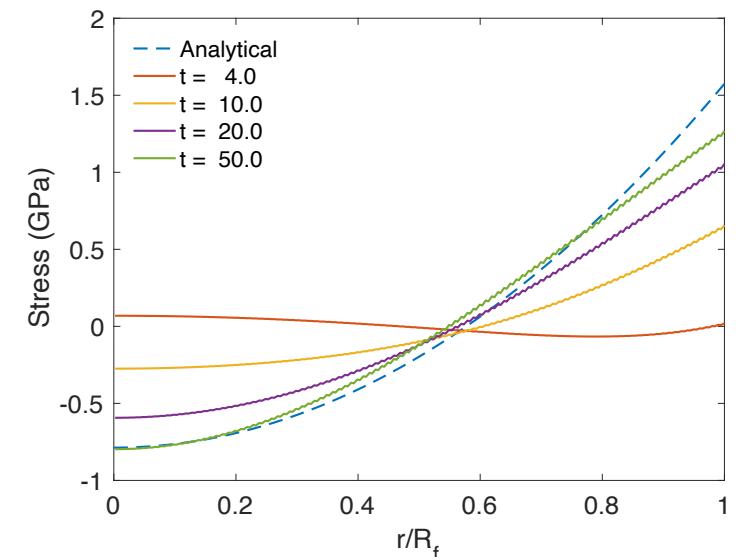
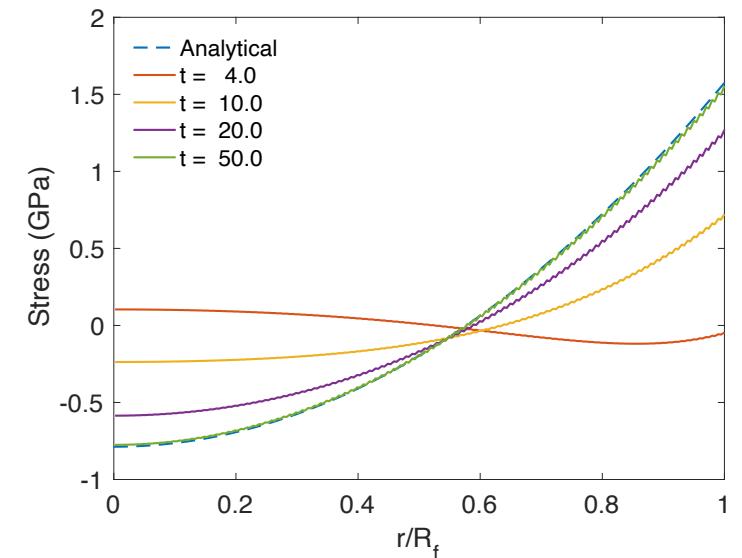


- The initial temperature is set to 273 K
- We will take 50 time steps of 0.5 s
- The full power of $Q = 450$ begins at time $t = 0$.
- UO_2 material properties are used for both the thermal and mechanics equations



Comparison to analytical theory

- If we use a constant thermal conductivity, analytical 1D model matches very well
- When k is a function of temperature, there is a difference between the FEM and analytical stress



There are various available tools for solving coupled thermomechanical problems with FEM

- Commercial tools
 - ABAQUS
 - ANSYS
 - COMSOL
- Open source
 - MOOSE
- NRC-based
 - FRAPCON/FRAPTRAN

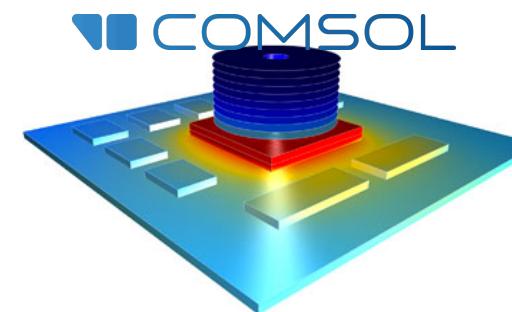
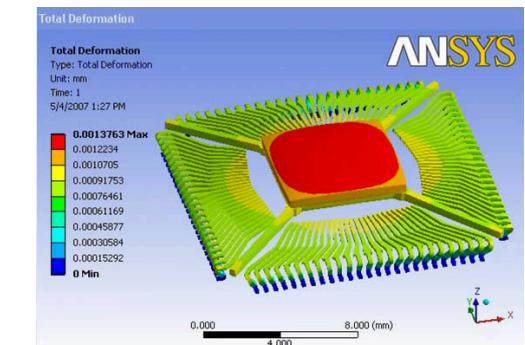
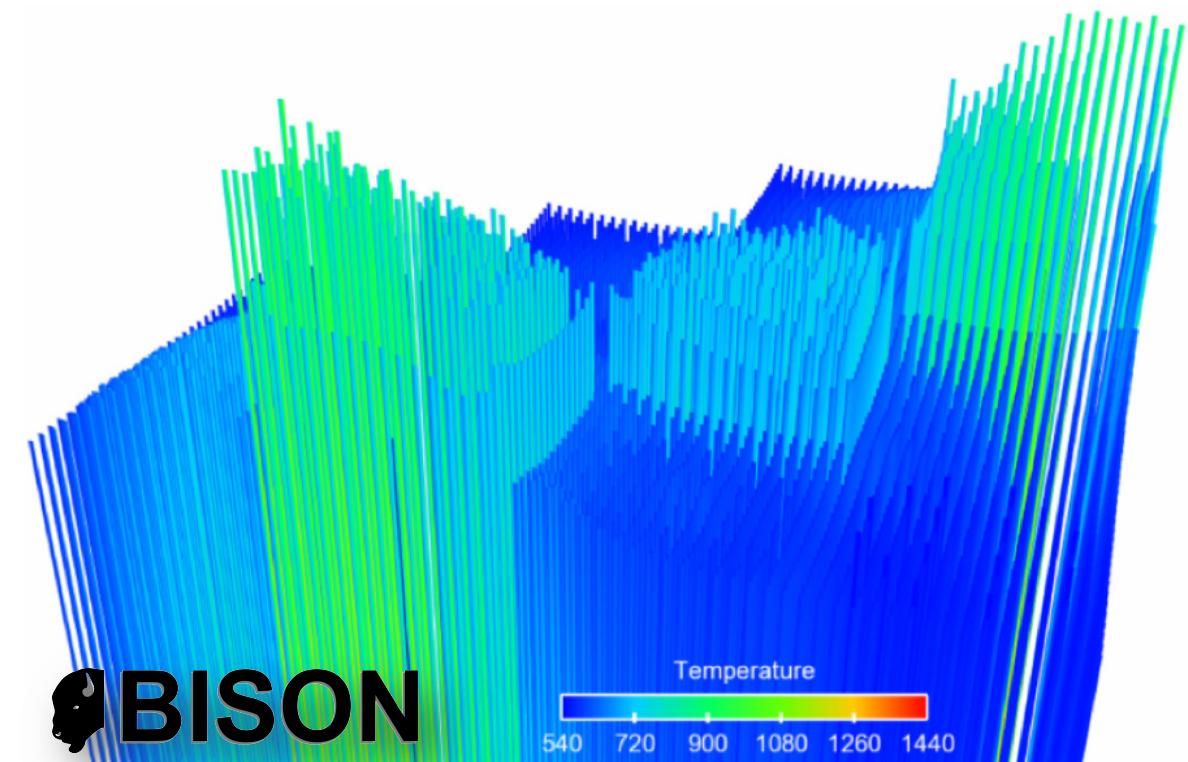


Figure 4. Temperature contour plot of exhaust manifold.



The purpose of a fuel performance code is to simulate and evaluate fuel rod behavior

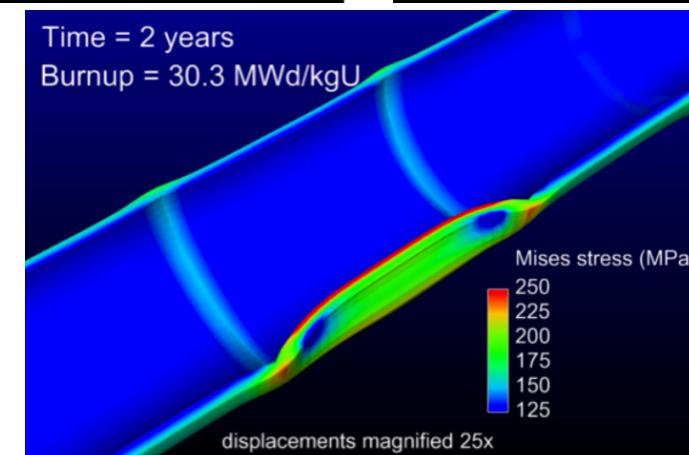
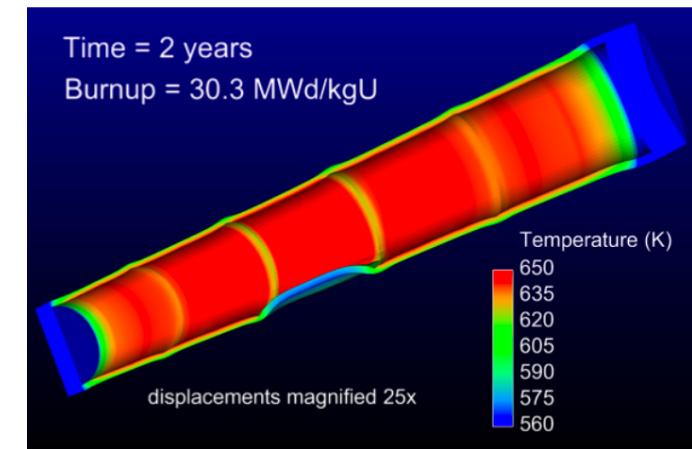
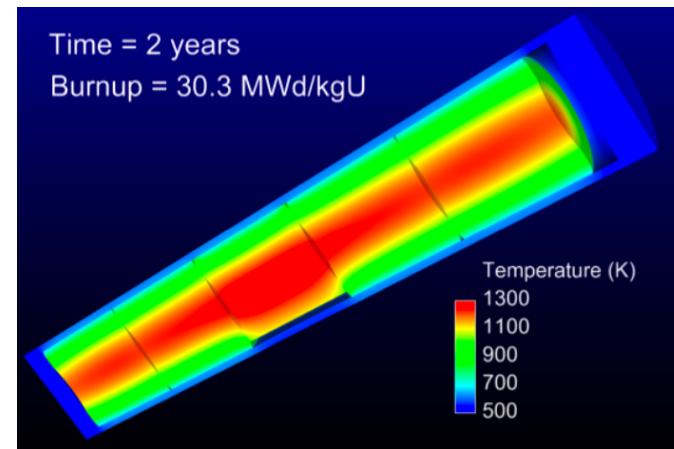
- The first fuel performance codes were developed in the mid seventies
- Advanced fuel performance codes are still under development today



KAIST-3A benchmark results, showing displacement of 3432 rods, from Gaston et al. 2014

How are fuel performance codes used?

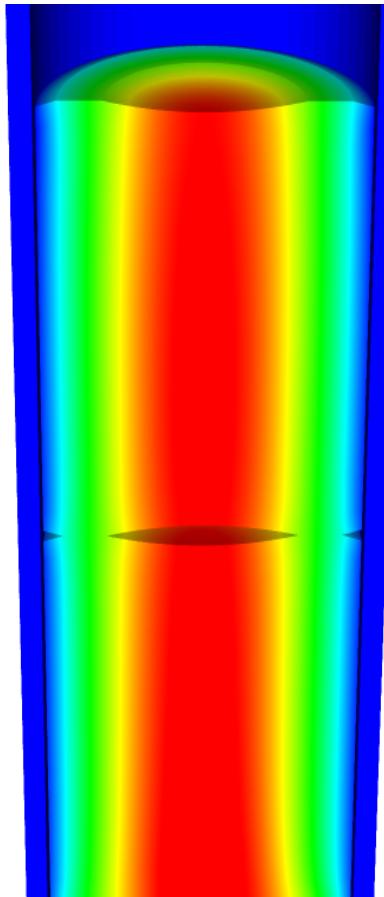
- The primary goals are to predict the fuel centerline temperature and the stress in the cladding
- Fuel performance codes aren't focused on predicting power production, but rather to predict safety margins



A fuel performance code must be able to predict:

Fuel

- Temperature profile
- Volumetric change



Cladding

- Temperature profile
- Stress

Gap

- Gap heat transport
- Mechanical interaction between fuel and cladding
- Gap pressure

The primary focus is solving the thermomechanical problem

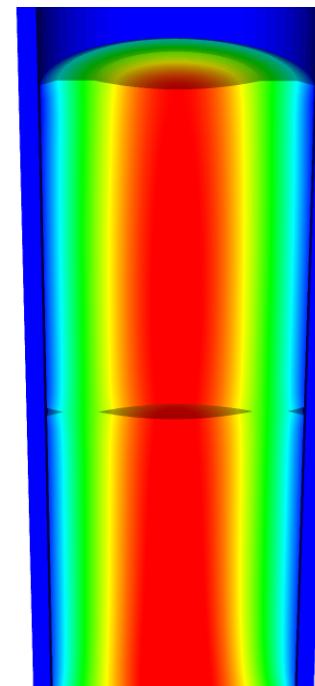
Fuel

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved Numerically

$$0 = \nabla \cdot \sigma$$

Solved Numerically or analytically



Cladding

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved Numerically or analytically

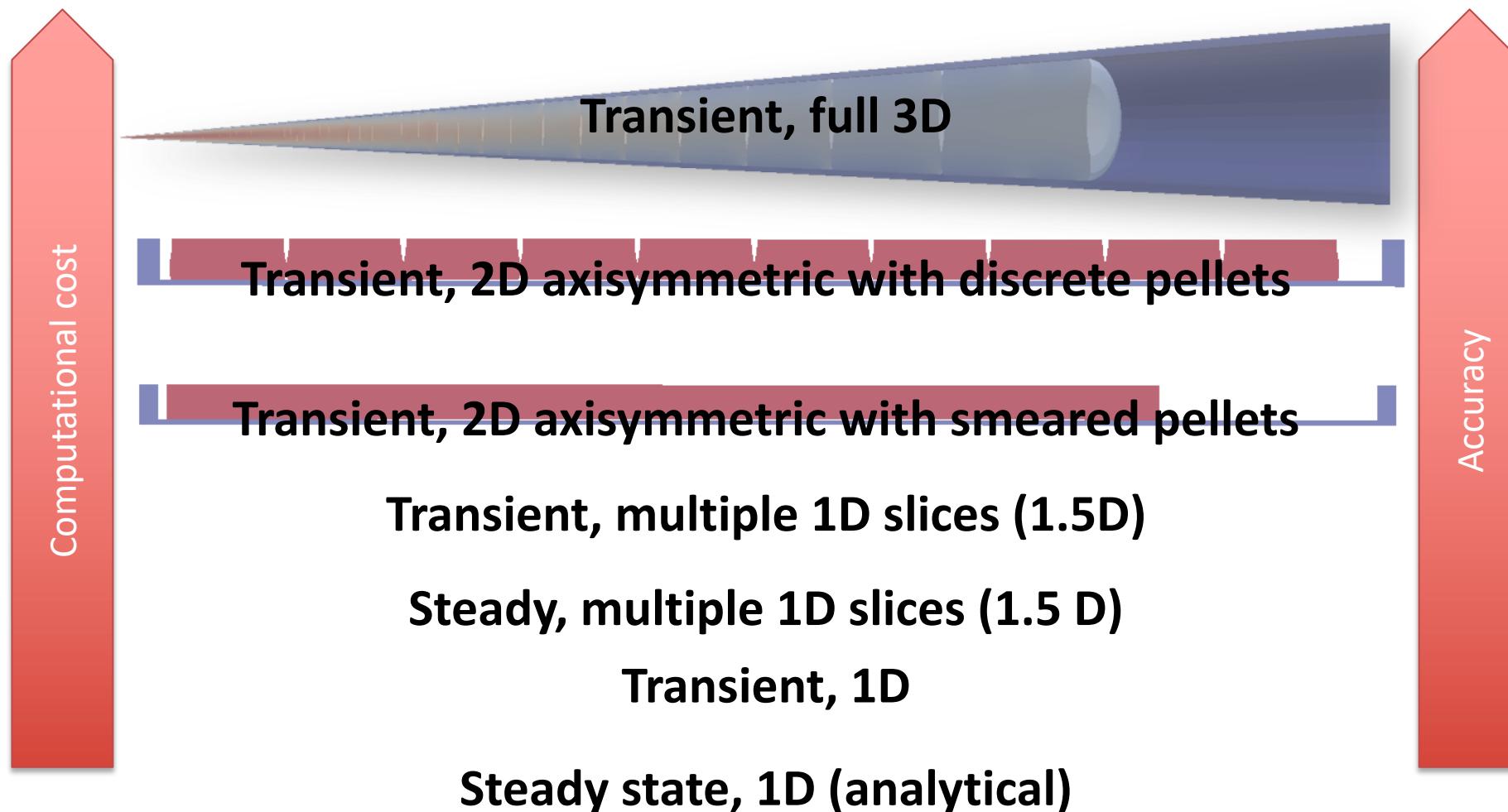
$$0 = \nabla \cdot \sigma$$

Solved Numerically

Gap

- The handling of the gap changes the most between different codes

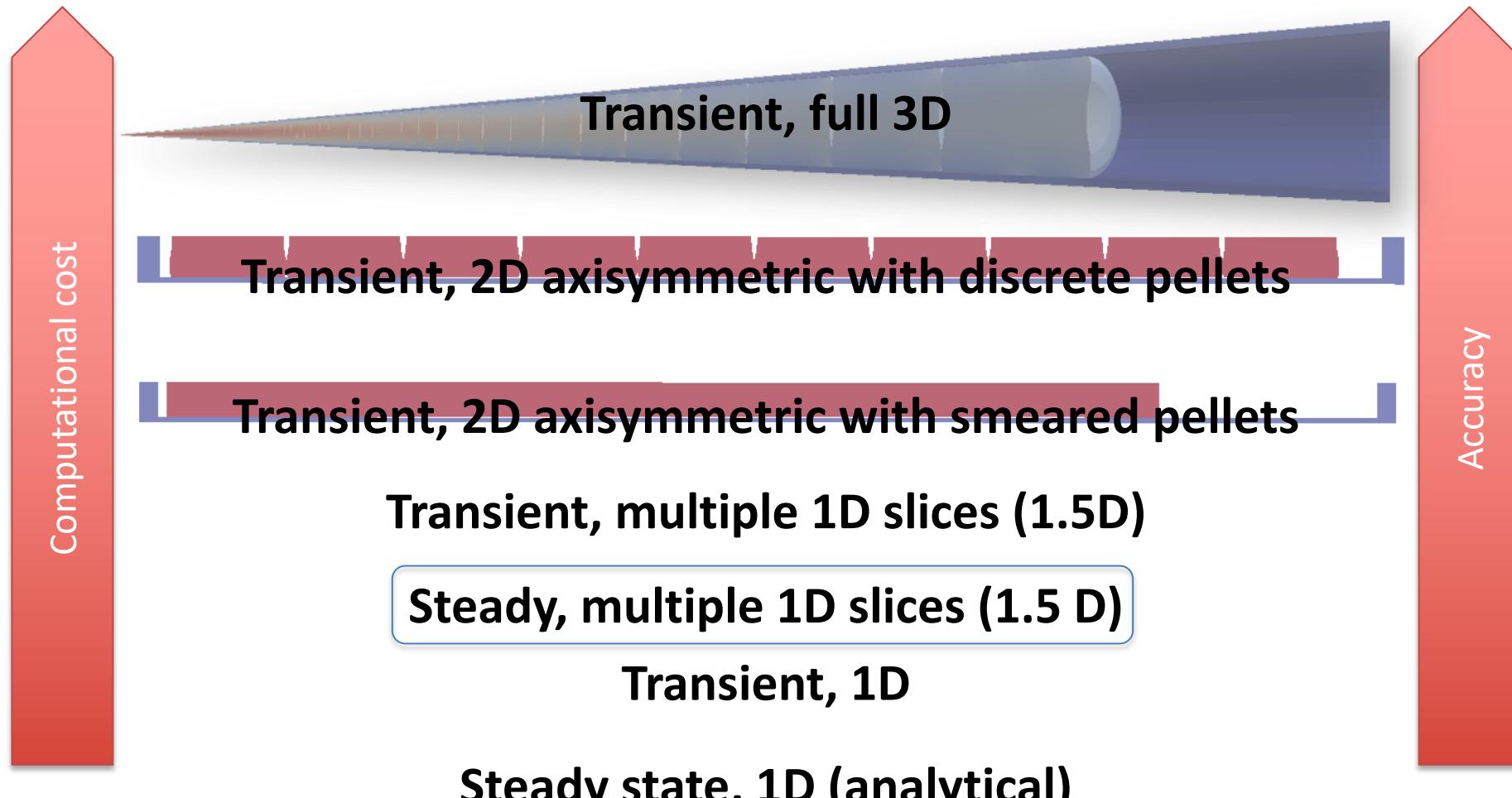
Various approaches to model the fuel rod



Early fuel performance codes were made for either steady state or transient operation

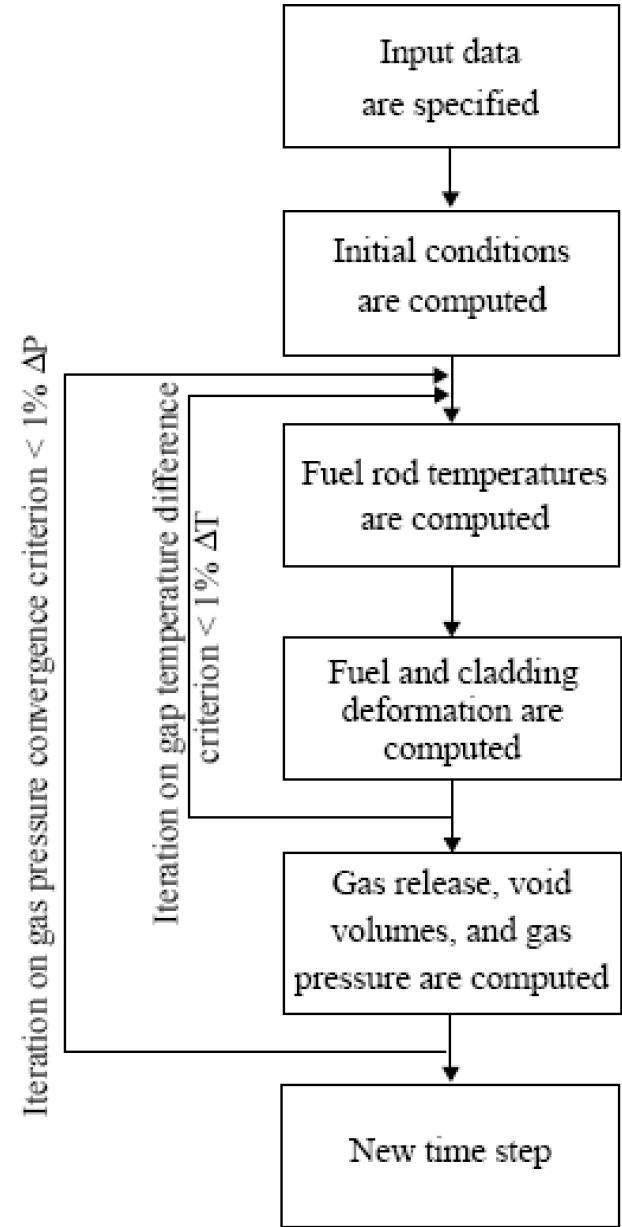
- Steady state codes
 - Leave off the time derivative part of the heat equation $\nabla \cdot (k \nabla T) + Q = 0$
 - The material properties still evolve with time as a function of burnup
 - The volumetric changes in the fuel are also a function of burnup
 - Creep of fuel and cladding change with time
- Transient codes
 - Include the time derivative $\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$
 - Have similar burnup dependent models like steady state codes, but don't include creep
 - Have additional models for rapid transients

Start with FRAPCON

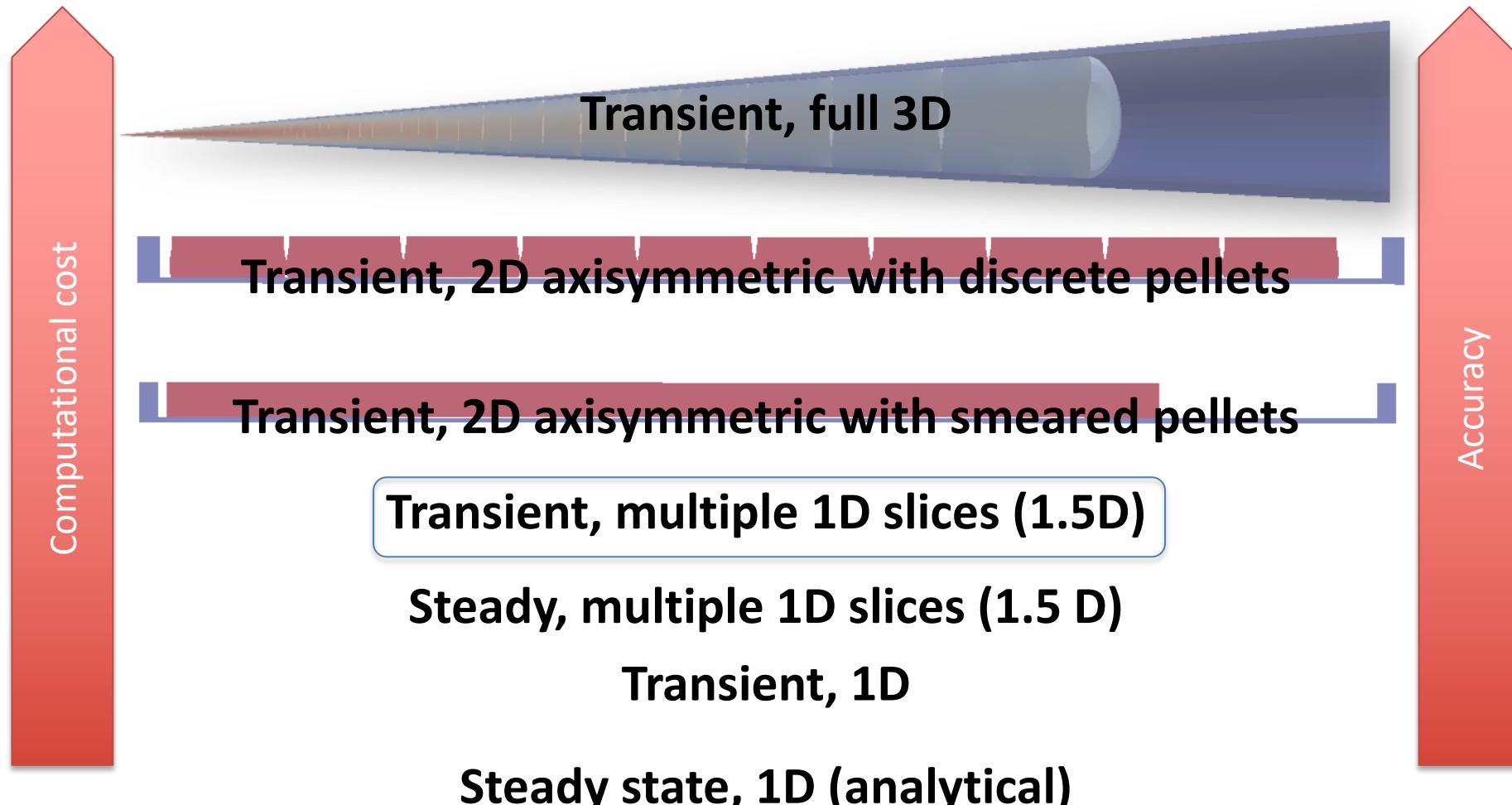


FRAPCON Flow Chart

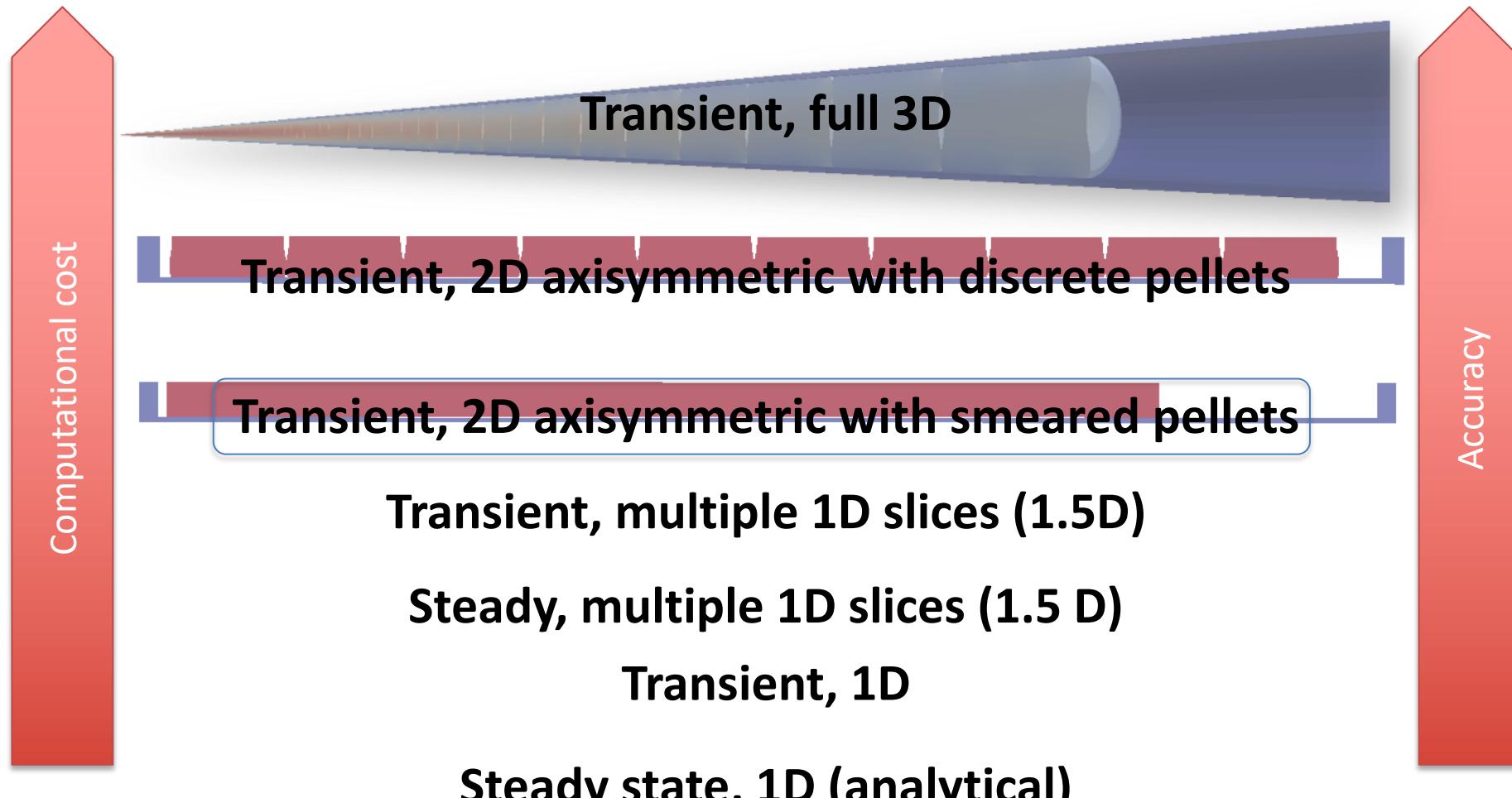
- FRAPCON iterates to determine fuel rod temperatures, fuel and cladding deformation
- This converged data is iterated to produce gas release, void volumes and plenum pressure
- Then marches forward in time



FRAPTRAN



FALCON



FALCON

- FALCON is a 2D fuel performance code developed by EPRI
- Development of FALCON started in 1996
- The beta version was released in 2003
- It was developed by ANATECH for EPRI
- FALCON is proprietary, owned by EPRI
- It is no longer under active development in the US

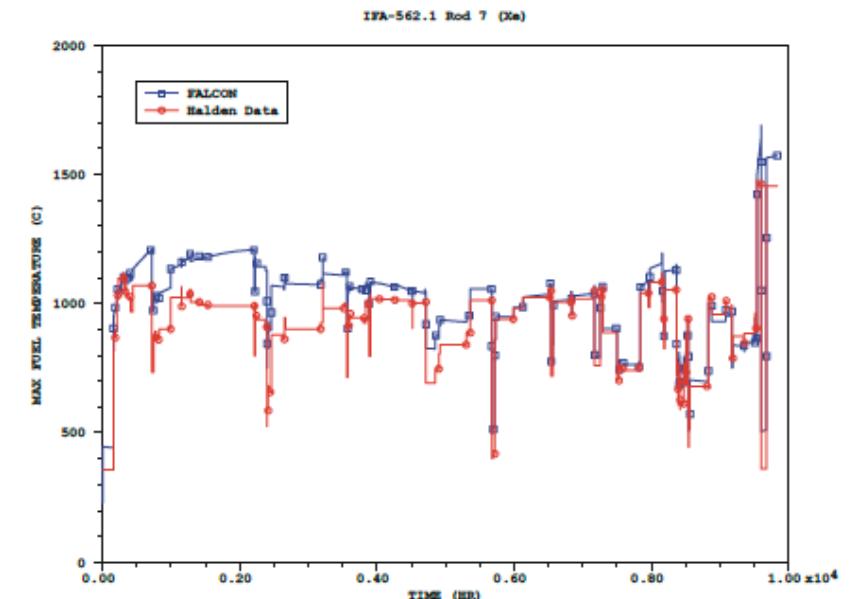


Figure 3-6
Calculated Fuel Temperatures Versus Measured Data for IFA-562.1 Rod 7 (Xe-filled)

FALCON is a 2D transient and steady state code

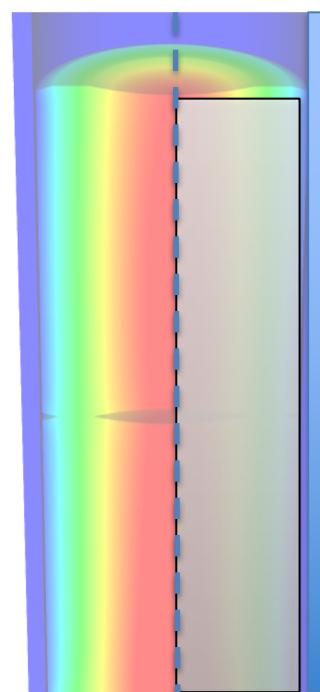
Fuel

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved with FEM

$$0 = \nabla \cdot \sigma$$

Solved with FEM



Cladding

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved with FEM

$$0 = \nabla \cdot \sigma$$

Solved with FEM

Gap

- Pressure is calculated using equation of state
- Simplified contact model is used for gap closure
- Gap heat transfer model is used

FALCON can predict the fuel performance in axisymmetric RZ space or in Rθ space

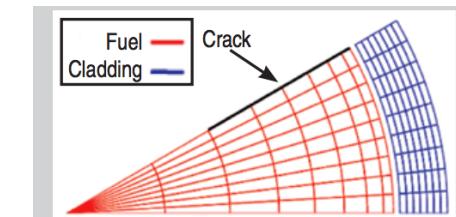
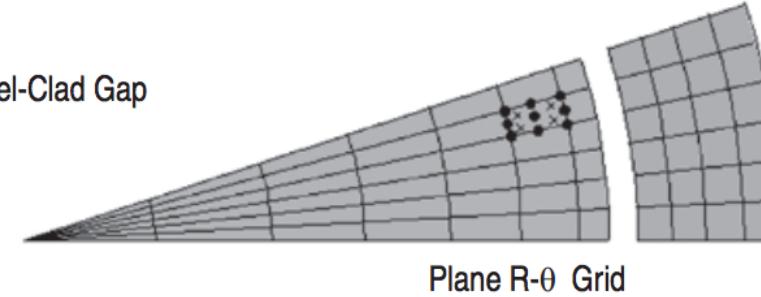
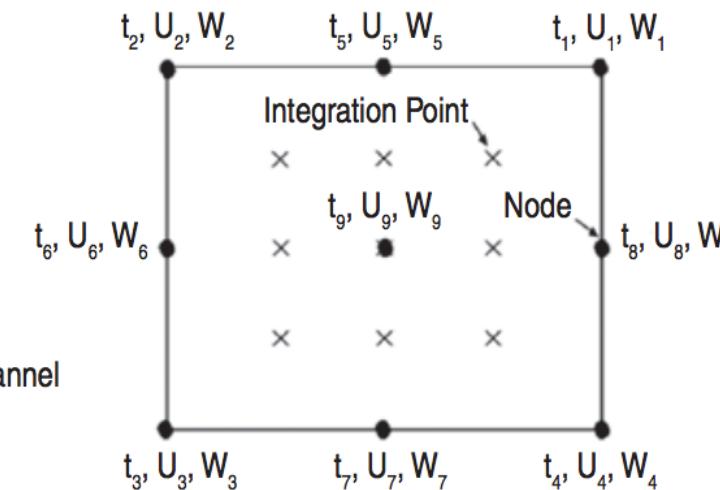
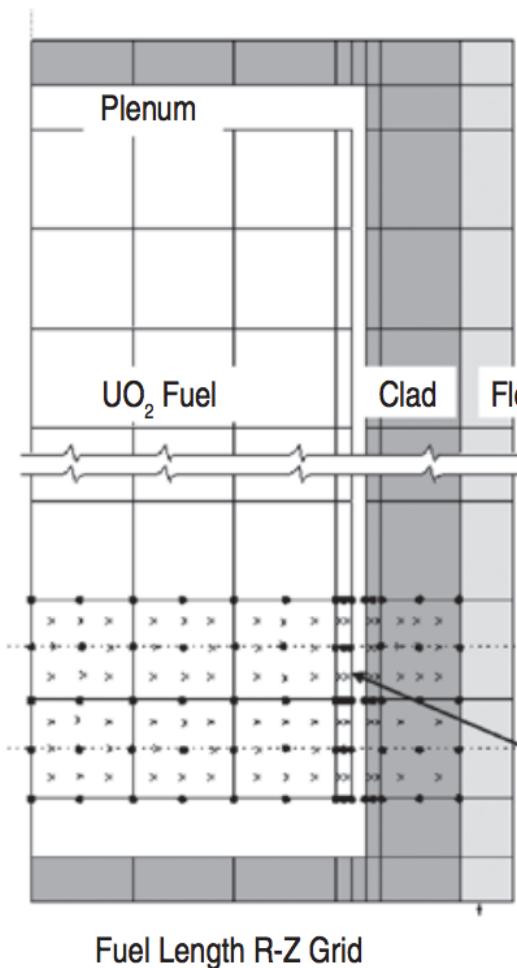


Figure 3. Standard PCI model.

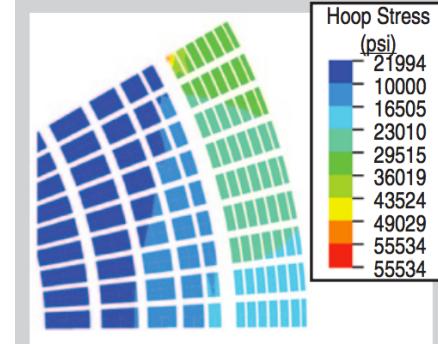


Figure 4. Calculated cladding hoop stress distribution (psi) using the standard PCI model.

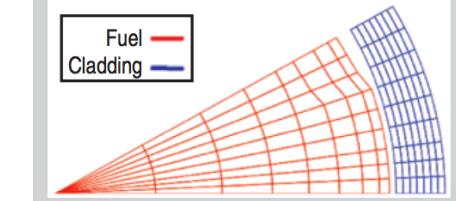
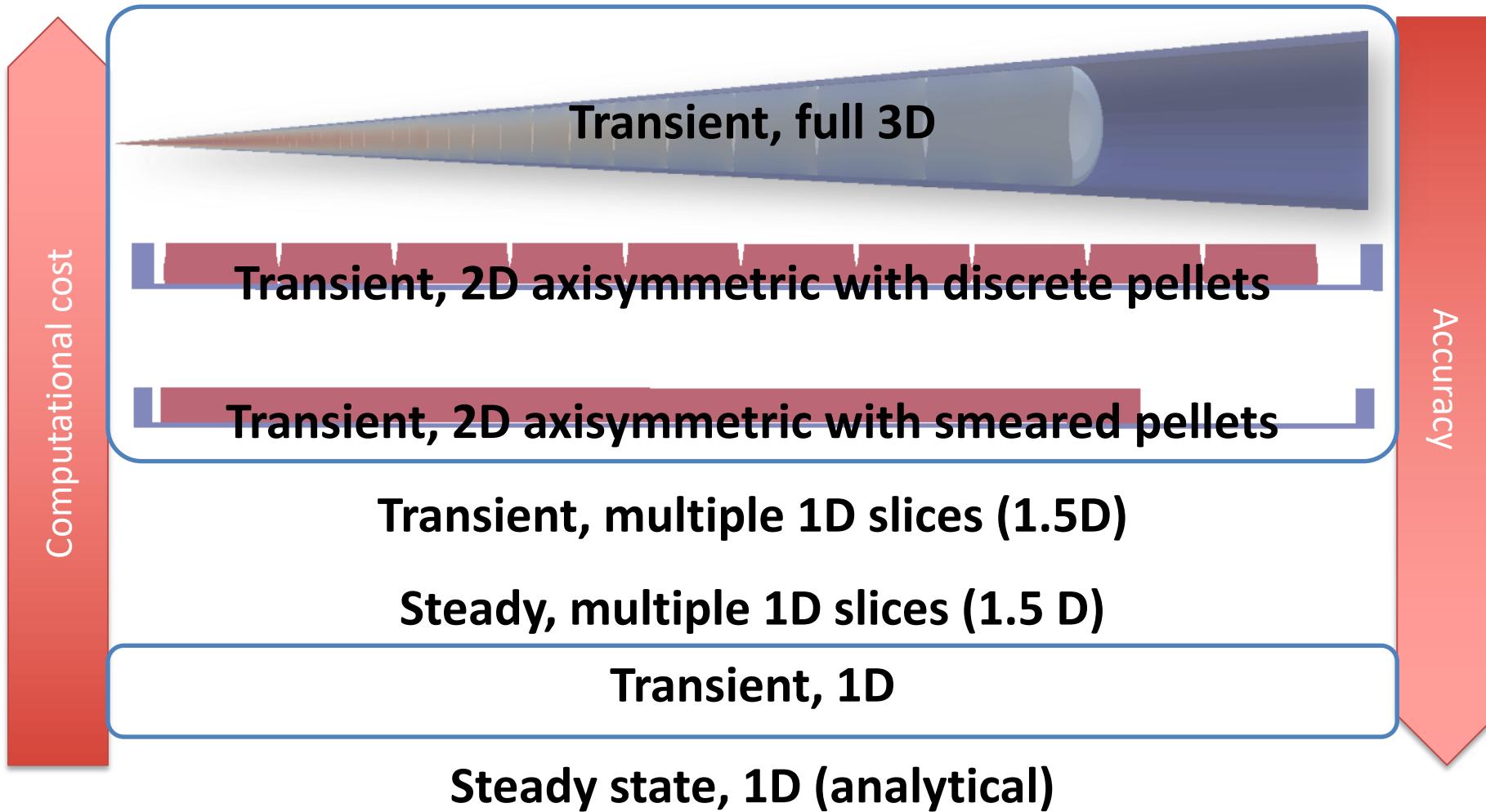


Figure 5. Missing pellet surface (MPS) PCI model.

BISON



BISON

- The next generation fuel performance code under development in the US
- It uses the MOOSE framework
- Development was begun in 2008
- The first paper using BISON was published in 2009 and the paper summarizing its full capabilities was published in 2012
- It was developed at Idaho National Laboratory, with some support by ANATECH
- BISON is available for free, but it is export controlled and requires a license agreement be signed

BISON models the fuel behavior ranging from 1D to full 3D and uses FEM

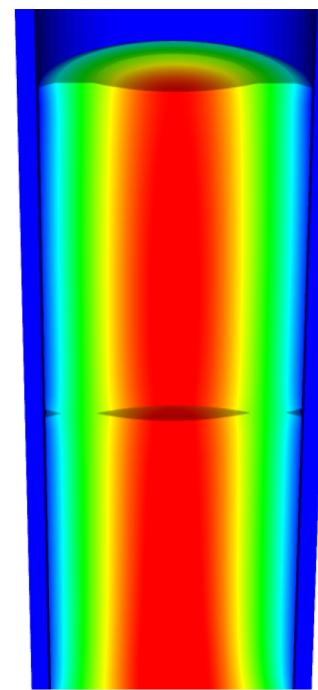
Fuel

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved with FEM

$$0 = \nabla \cdot \sigma$$

Solved with FEM

**Cladding**

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

Solved with FEM

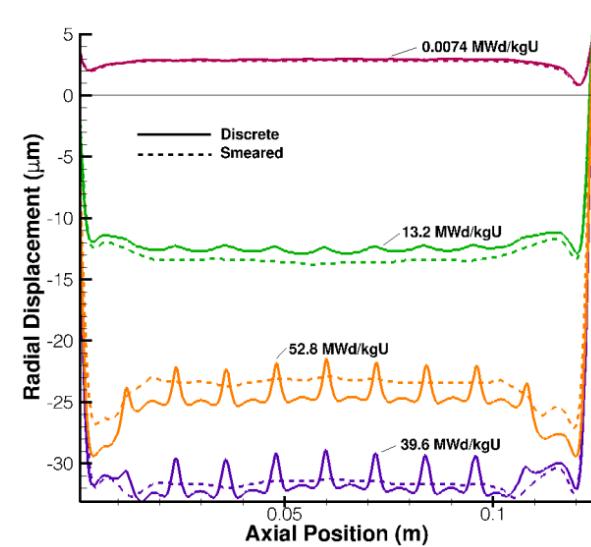
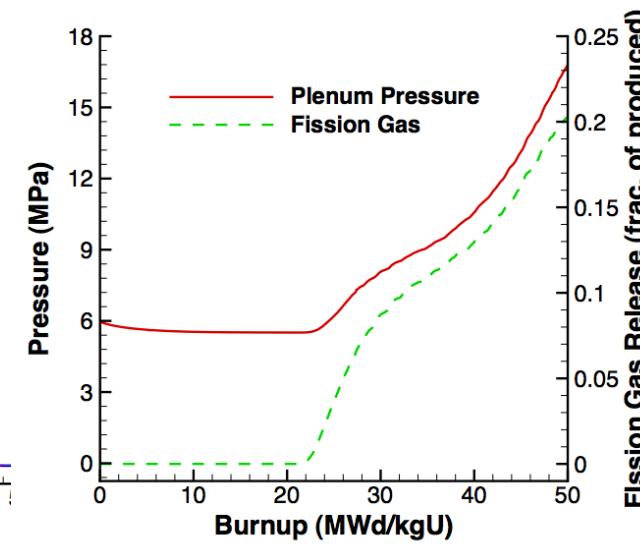
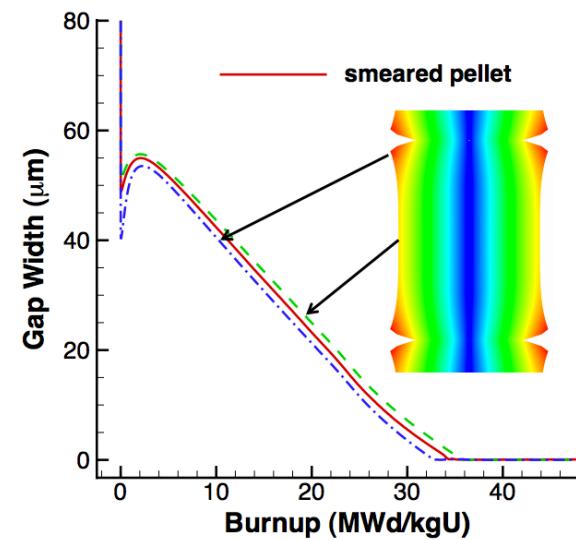
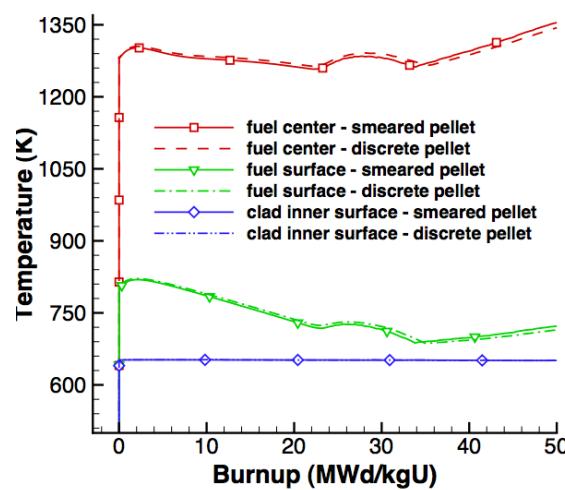
$$0 = \nabla \cdot \sigma$$

Solved with FEM

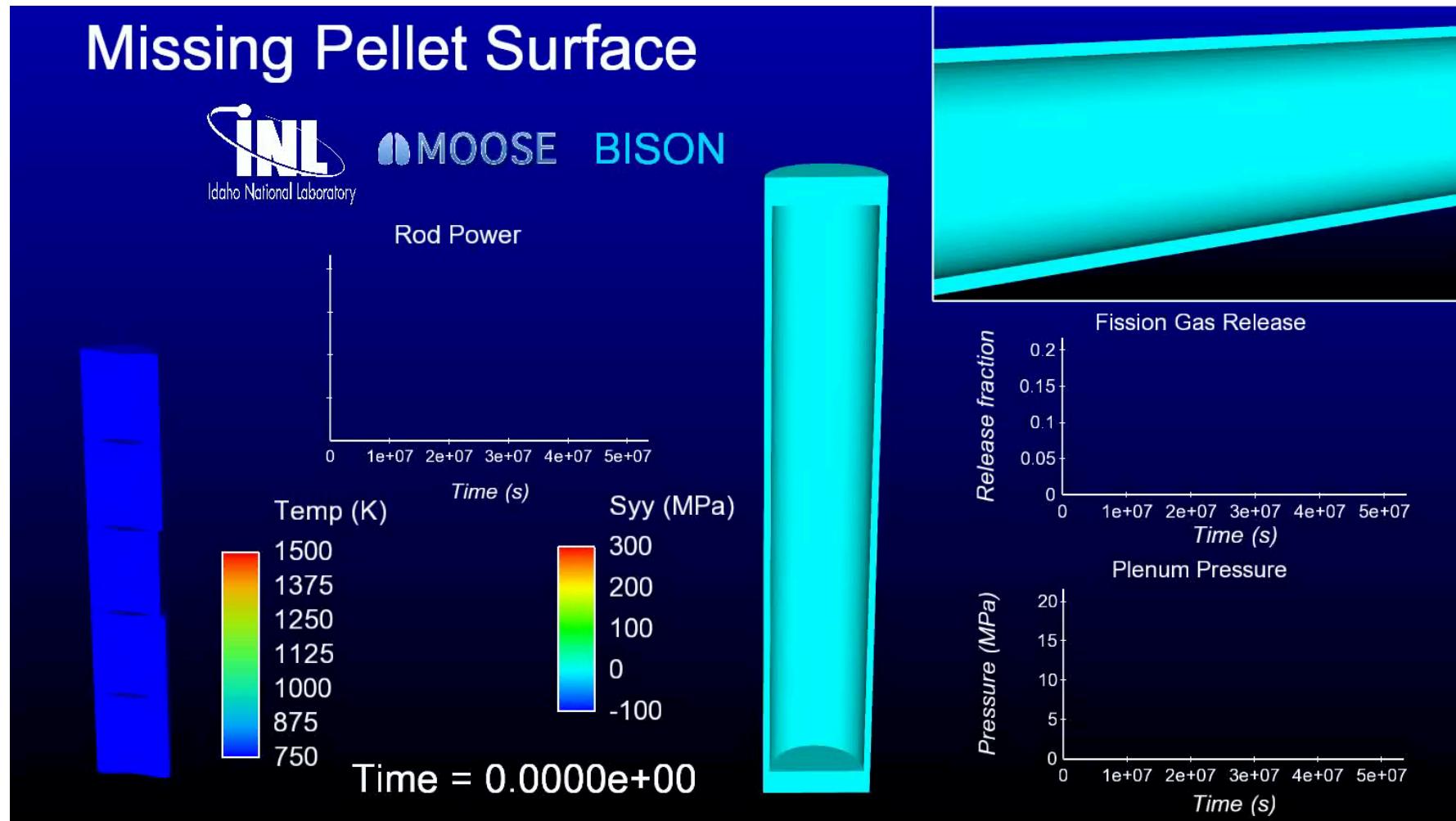
Gap

- Pressure is calculated using equation of state
- Fully implemented implicit contact algorithm
- Gap heat transfer model is used

Can handle smeared or discrete pellets, asymmetric pellet geometry and deformation

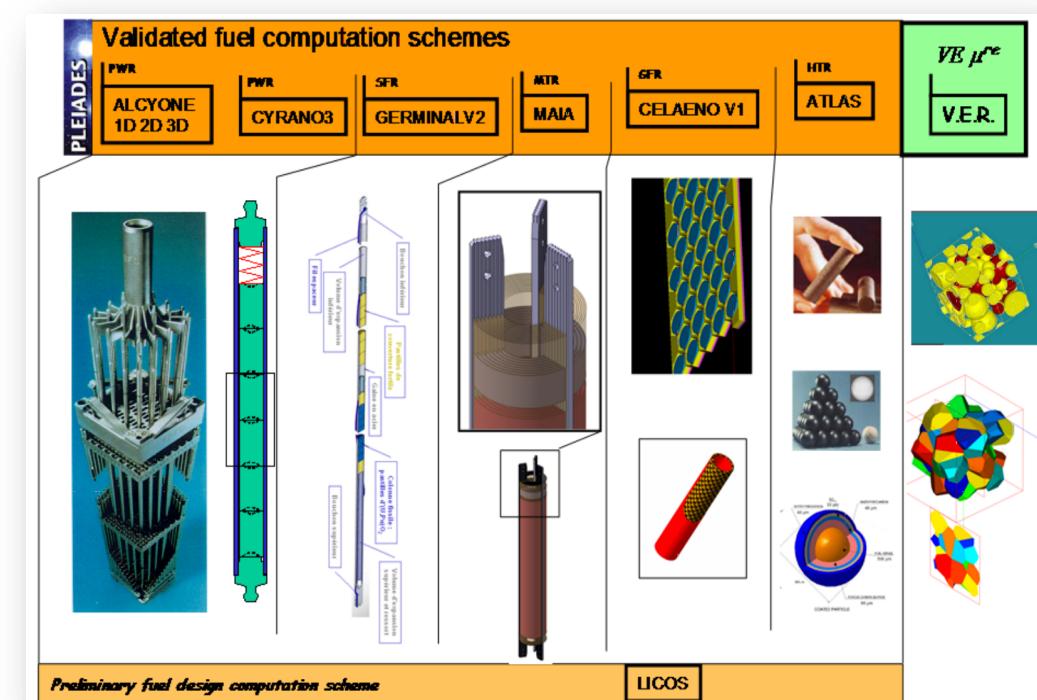


Because of its unique 3D capability, BISON can model truly 3D fuel performance problems



Other countries have other fuel performance codes

- TRANSURANUS: ITU (Germany)
- ALCYONE: CEA (France)
- ENIGMA: NNL (UK)
- FEMAXI-7: JAEA (Japan)
- SFPR: IBRAE (Russia)
- DIONISIO: (Argentina)



<http://www.materials.cea.fr/en/PDF/PLEIADES-Platform.pdf>

Summary

- Fuel performance codes are focused on predict the center temperature of the pellet and the stress in the cladding
- All fuel performance codes
 - Numerically model the temperature in the fuel
 - Numerically model the stress in the cladding
 - And consider gap pressure, closure, and heat transfer in some way
- The primary US codes are
 - FRAPCON – Steady state 1.5D, uses finite difference
 - FRAPTRAN – Transient 1.5D, uses finite difference
 - FALCON – Steady or transient 2D, uses finite element
 - BISON – Steady or transient, 1D – 3D, uses finite element

Paper assignment

- I have assigned each of you a paper to read and review
- We will have 15 minute presentations from each person on February 27 summarizing and analyzing their respective paper
- Papers will be emailed out today

MOOSE Training Extra Credit

- March 9-11
- Applied for access to FRAPCON/FRAPTRAN, but still processing, so may need to utilize MOOSE or ANSYS or ABAQUS
- INL folks are coming to NCSU for a regional MOOSE training
- Not required, but will get extra credit for attendance, +4 to total grade
- Also, this is a very good skill to have for a graduate student, MOOSE suite of tools can do a lot
 - Neutron transport, thermal hydraulics, phase-field, risk analysis, fuel performance, systems analysis, etc.