Cole Manfred, NE 533 Test 2, 2/21/25

1. pellet, $R_F = 0.55\,cm$, $LHR = 350\,W/cm$

a) $K_F = 0.05\,W/cm \cdot K$, $E = 200\,GPa$, $\nu = 0.35$

$\alpha = 10 \times 10^{-6}\,1/K$

$$\sigma_{\theta\theta}(\eta) = -\sigma^*(1-3\eta^2), \quad \eta = \frac{r}{R_F} = 1 \rightarrow r = R_F$$

$$\sigma^* = \alpha E \frac{T_0 - T_s}{4(1-\nu)}$$

$$T_0 - T_s = \frac{LHR}{4\pi K_F} = 557.0\,K$$

$$\sigma^* = 428.5\,MPa$$

$$\boxed{\sigma_{\theta\theta} = 857\,MPa}$$

$$\sigma_{zz} = -2\sigma^*(1-2\eta^2) = 857\,MPa$$

$$\sigma_{rr} = -\sigma^*(1-\eta^2) = 0$$

b) $\sigma_{fr} = 150\,MPa$

$$\sigma_{\theta\theta} = \sigma_{fr} = -\sigma^*(1-3\eta^2)$$

$$\frac{\sigma_{fr}}{-\sigma^*} = 1 - 3\eta^2 \rightarrow \eta^2 = \left(1 + \frac{\sigma_{fr}}{\sigma^*}\right)\frac{1}{3} \rightarrow \eta = 0.67$$

$1 - \eta = 0.33$ $\boxed{\text{Cracks extend 33\% into fuel}}$

2. Clad rod, $p = 55\,MPa$, $\bar{R}_c = 0.55\,cm$, $t_c = 0.05\,cm$

a) $\bar{\sigma}_\theta = \dfrac{pR}{\delta} = \boxed{605\ MPa}$

$\sigma_z = \dfrac{pR}{2\delta} = \boxed{302.5\ MPa}$

$\sigma_r = -\dfrac{1}{2}p = \boxed{-27.5\ MPa}$

b) σ @ $r = R_i$, $R_i = 0.525\,cm$, $R_o = 0.575\,cm$

$$\sigma_{rr} = -p\frac{(R_o/r)^2-1}{(R_o/R_i)^2-1} = -p\frac{\cancel{(R_o/R_i)^2-1}}{\cancel{(R_o/R_i)^2-1}} = -p$$

$\boxed{\sigma_{rr} = -55\ MPa}$ $\qquad \left(\dfrac{R_o}{R_i}\right)^2 = 1.2$

$$\sigma_{\theta\theta} = p\frac{(R_o/r)^2+1}{(R_o/R_i)^2-1} = p\frac{(R_o/R_i)^2+1}{(R_o/R_i)^2-1} = \boxed{606.25\ MPa}$$

$$\sigma_{zz} = p\frac{1}{(R_o/R_i)^2-1} = \boxed{275.6\ MPa}$$

3. Gap thickness, no iteration

$R_f = 0.52\,cm$, $t_{gap} = 0.005\,cm$, $T_{oc} = 550\,K$,

$t_c = 0.08\,cm$, $k_f = 0.04\ W/cm{\cdot}K$, $k_g = 0.003\ W/cm{\cdot}K$

$k_c = 0.15\ W/cm{\cdot}K$, $LHR = 175\ W/cm$, $\alpha_c = 10\times10^{-6}\ 1/K$

$\alpha_f = 14\times10^{-6}\ 1/K$, $T_{ref} = 300\,K$

$$\Delta t_g = \bar{R}_c\alpha_c(\bar{T}_c - T_{ref}) - R_f\alpha_f(\bar{T}_f - T_{ref})$$

$$\bar{R}_c = \frac{(0.52+0.005)+(0.52+0.005+0.08)}{2} = 0.565\ cm$$

$$T_{ic} - T_{oc} = \frac{LHR}{2\pi R_f}\frac{t_c}{k_c} \rightarrow T_{ic} = \frac{LHR}{2\pi R_f}\frac{t_c}{k_c} + T_{oc}$$

$T_{ic} = 578.6K$

$$\bar{T}_c = \frac{T_{ic} + T_{ce}}{2} = 564.3K$$

$$T_s = \frac{LHR}{2\pi R_f} \frac{t_g}{k_g} + T_{ic} = 667.9K$$

$$T_o = \frac{LHR}{4\pi k_f} + T_s = 1016K$$

$$\bar{T}_f = \frac{T_o + T_s}{2} = 842K$$

$$\Delta t_g = 0.565(10\times10^{-6})(564.3-300) - 0.52(14\times10^{-6})(842-300)$$

$$\boxed{\Delta t_g = -0.00245 \text{ cm}}$$

4. σ_θ, σ_r, cladding, thermal expansion

$\Delta T = 50K$, $\alpha_c = 15\times10^{-6}$ 1/K, $E = 100$ GPa, $\nu = 0.34$

$t_c = 0.06$ cm, $R_i = 0.55$ cm, $\underline{r = R_i}$

$$\sigma_\theta = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(1 - 2\frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\sigma_\theta = \frac{1}{2}(50)\frac{(15\times10^{-6})(100\times10^{3})}{1-0.34}(1-0) = \boxed{56.8 \text{ MPa}}$$

$$\sigma_r = \frac{1}{2}\Delta T \frac{\alpha E}{1-\nu}\left(\frac{r}{R_i} - 1\right)\left(1 - \frac{R_i}{\delta}\left(\frac{r}{R_i} - 1\right)\right)$$

$$\sigma_r = \frac{1}{2}(50)\frac{(15\times10^{-6})(100\times10^{3})}{1-.34}(0)(1-0) = \boxed{0 \text{ MPa}}$$

5. Any elastic deformation on loading is returned upon unloading. After the yield point, added strain is plastic or permanent.

Elastic - stretching bonds

Plastic - breaking bonds

6. Strain hardening is the increase in strength due to plastic deformation. The induced plastic strain/work adds energy in the form of defects which can raise the yield point, at the cost of making it more brittle.

7. All fuel performance codes must be able to

a) Analyze the fuel centerline temperature/temp. profile

b) Calculate the stress in the cladding

c) Simulate the gap behavior through changes in pressure, composition, thickness, etc.

Two codes currently utilized are FRAPCON (NRC) and BISON.

8. A 0-D defect would be something like a vacancy or interstitial atom. An example of a 3-D defect would be a void or precipitate.

9. UO_2 fuel pellet grains closely correspond with the individual powder particles that are sintered. Under heat and pressure, the gaps between particles are squeezed out, forming a type of grain boundary between the particles.

10. Mechanistic fuel performance modeling is modeling based on changes in microstructure and crystallinity that affect macroscopic properties like thermal conductivity and strength.
While current empirical models are useful for relating material properties to operating variables like burn-up, mechanistic models actually track the effect of radiation on the microstructure of the material. Thus these structural changes can correspond to dynamic changes in properties and thus performance.

11. Microstructure is roughly what you can see at 25x. When referencing microstructure, we mean thing like grain size, precipitates, secondary phases, voids, dislocation loops, etc.

An example of processing is hot rolling. Rolling adds work (aka energy) into the material, making it harder and stronger. However, when done above the recrystallization temperature, it can relax some of the induced strain by forming new grains. This effects a change on part geometry, microstructure, and properties. The microstructure has more equiaxed grains and can be a little bit tougher.

12. The high burnup structure of a UO_2 fuel pellet occurs on its outer rim due to a higher probability for this location to absorb fission neutrons (self shielding). The neutrons create more fissions on the outside with the U-235. Neutrons are also absorbed by U-238 in resonances, leading to production of Pu-239, another fissile species. Fissile density goes up in the outer rim as does the power. The HBS has a lot of radiation damage which creates many voids and defects. These defects/voids serve as phonon scattering centers that decrease thermal conductivity. However, radiation also sweeps defects to grain boundaries, meaning less phonon scattering and higher k.