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NE 591 – EXAM #1

(2) $K_{coat} = 5\ W/mK = 0.05\ W/cmK$

$K_{clad} = 15\ W/mK = 0.15\ W/cmK$

$K_f = 0.5\ W/cmK$

$K_{gap} = 25\ W/mK = 0.25\ W/cmK$

$h_{cool} = 5.5\ W/cm^2K$

$T_{cool} = 200K$

$Q = 400\ W/cm^3$

$r_1 = 0.6\ cm = r_f$

$r_2 = 0.2\ cm$

$r_3 = 0.25\ cm$

$r_4 = 0.26\ cm$

$$LHR = Q\pi R_f^2 = 400\pi(0.6)^2 = 452.4\ W/cm$$

$$T_{coat} = T_{cool} + \frac{LHR}{2\pi R_f h_{cool}} = 800 + \frac{452.4}{2\pi(0.6)(5.5)} = 821.8\ K$$

$$T_{clad} = T_{coat} + \frac{LHR\ t_{coat}}{2\pi R_f K_{coat}} \quad (t_{coat} = r_4 - r_3) = 821.8 + \frac{452.4(0.26-0.25)}{2\pi(0.6)(0.05)} = 845.8\ K$$

$$T_{gap} = T_{clad} + \frac{LHR\ t_{clad}}{2\pi R_f K_{clad}} \quad (t_{clad} = r_3 - r_2) = \frac{452.4(0.25-0.2)}{2\pi(0.6)(0.15)} + 845.8 = 885.8\ K$$

$$T_{fuel} = T_{gap} + \frac{LHR\ t_{gap}}{2\pi R_f K_{gap}} \quad (t_{gap} = r_2 - r_1) = 885.8 + \frac{452.4(0.2-0.6)}{2\pi(0.6)(0.25)} = 981.8\ K$$

$$T_o = T_{fuel} + \frac{LHR}{4\pi K_f} = 981.8 + \frac{452.4}{4\pi\ 0.5} = \boxed{1053.8\ K}$$

@ $r = 0.4\ cm$,

$$T(r) = \frac{Q(R^2 - r^2)}{4K} + T_F \Rightarrow T(0.4) = \frac{400(0.6^2 - 0.4^2)}{4(0.5)} +$$

③ $k = 14.5$ W/mK $\quad U_3Si_2$ enrichment: 19.9%

$\rho = 15.67$ g/cm^3

$\sigma_f = 570$b

a) $\phi = 2\times10^{12}$ n/cm^{2}s

mass U: $0.195(235) + 0.805(238) = 237.415$ a m u

mass Si 28 a m u

total mass: $3(237.415) + 2(28) = 768.245$ a m u

$$N_f = 15.67\ \frac{g}{cm^3}\ \frac{mol}{768.245}\ \frac{6.022\times10^{23}\ at}{mol}\ \frac{3U}{U_3Si_2}\ ,\ 0.195 = 1.19\times10^{21}\ U^{235}/g$$

$$Q = E_f N_f \phi \sigma_f$$

$$Q = (200\times10^6\ eV)\left(1.602\times10^{-19}\ \frac{J}{eV}\right)\left(1.19\times10^{21}\ \frac{U^{235}}{g}\right)\left(2\times10^{12}\ \frac{n}{cm^2 s}\right)\left(570\times10^{-24}\ cm^2\right)$$

$$\boxed{Q = 262.62\ W/cm^3}$$

b) $\rho_{UO_2} = 10.97$ g/cc

$$N_f = 1.19\times10^{21} = \left(10.97\ \frac{g}{cc}\right)\frac{1}{M}\cdot\frac{6.022\times10^{23}\ at}{mol}\ x$$

$$\frac{x}{M} = 1.09\times10^{-3} \Rightarrow M = \frac{x}{1.09\times10^{-3}}$$

$$M = \left(x\cdot 235 + (1-x)238\right) + 2\times16 = \frac{x}{1.09\times10^{-3}}$$

$$x = 0.2557x + 0.259 - 0.259x + 3.48\times10^{-2}$$

$$\boxed{x = 29.3\%\ \text{enrichment}}$$

④ $z_o = 3m$; $LHR^o = 150 W/cm$; $\delta = 1.1$

a) LHR @ $z = 1.8m$

$$LHR\left(\frac{z}{z_0}\right) = 150 \cos\left[\frac{\pi}{2(1.1)}\left(\frac{1.8}{3} - 1\right)\right] = 65.66$$

b) i) Water $c_p = 4200 J/kgK$, $\dot{m} = 0.22 kg/s$ each

$$T(z)_{cool} - T_{cool}^{in} = \frac{2.(1.1)}{\pi} \frac{3(150)}{0.22(4200)} \left[\sin\left(\frac{\pi}{2(1.1)}\right) + \sin\left(\frac{\pi}{2(1.1)}\left(\frac{1.8}{3} - 1\right)\right)\right]$$

-0.541

$$T(z)_{cool} - T_{cool}^{in} = 0.1532\ K$$

ii) sodium $c_p = 1404 J/kgK$ $\dot{m} = 0.12\ kg/s$ each

$$T(z)_{cool} - T_{cool}^{in} = 0.1532 \times \frac{0.22(4200)}{0.12(1404)} = 0.8702 K$$

⑤ Forward Euler: $f(t_{n+1}) = f(t_n) + dt\, f'(t_n)$

$t_1 = 0.33$ $f_1 = 6 + 0.33[4(0) - 3(0)^2] = 6$

$t_2 = 0.66$ $f_2 = 6 + 0.33[4(0.33) - 3(0.33)^2] = 6.33$

$t_3 = 0.99$ $f_3 = 6.33 + 0.33[4(0.66) - 3(0.66)^2] = 6.72$

$t_4 = 1.32$ $f_4 = 6.77 + 0.33[4(0.99) - 3(0.99)^2] = 7.11$

$t_5 = 1.65$ $f_5 = 7.11 + 0.33[4(1.32) - 3(1.32)^2] = 7.13$

$t_6 = 1.98$ $f_6 = 7.13 + 0.33[4(1.65) - 3(1.65)^2] = 6.61$

Backward Euler

$$f(t_{n+1}) = f(t_n) + dt\, f'(t_{n+1})$$

$t_1 = 0.33$ $\quad f_1 = 6 + 0.33\left[4(0.33) - 3(0.33)^2\right] = 6.33$

$t_2 = 0.66$ $\quad f_2 = 6.33 + 0.33\left[4(0.66) - 3(0.66)^2\right] = 6.78$

$t_3 = 0.99$ $\quad f_3 = 6.78 + 0.33\left[4(0.99) - 3(0.99)^2\right] = 7.12$

$t_4 = 1.32$ $\quad f_4 = 7.12 + 0.33\left[4(1.32) - 3(1.32)^2\right] = 7.14$

$t_5 = 1.65$ $\quad f_5 = 7.14 + 0.33\left[4(1.65) - 3(1.65)^2\right] = 6.62$

$t_6 = 1.98$ $\quad f_6 = 6.62 + 0.33\left[4(1.98) - 3(1.98)^2\right] = 5.35$

(8) → 3 phases in operation temperature cause swelling
→ low melting point (1400K)

(8) Ratio of fuel volume per total volume in fuel cell. Important due to fuel swelling during operation

(9) We enrich U to increase fissile quantity (U-235) in the fuel. Enrichment is possible due to mass difference between U-235 to U-238.