1)

Given $r = 4.5\,mm$ $\quad K = 0.1\ W/cm\text{-}K$

$q_{HR} = 250\ W/cm$ $\quad E = 290\ GPa$

$\nu = 0.3$

$\alpha = 8.2E\text{-}6/K$

Find a) $\sigma_{max} = ?$ Max is when $y = 1$

$$\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$$

$$T_0 - T_s = \frac{q_{HR}}{4\pi K} = \frac{250\ W/cm}{(4\pi)(0.1\ W/cm\text{-}K)}$$

$$= 198.94\ K$$

$$= \frac{(8.2E\text{-}6/K)(290E3)(198.94\ K)}{4(1-0.3)}$$

$$= 168.96\ MPa$$

$$\sigma_\theta = -\sigma^*(1 - 3y^2)$$

$$= -168.96\ MPa\,(1 - 3(1)^2)$$

$$= 337.97\ MPa$$

b) Find crack depth?

using $\sigma^* = 168.96\ MPa$

Given $\sigma_r = 120\ MPa$

$y = \sqrt{\left(1 + \frac{120}{168.96}\right)\frac{1}{3}} = 0.755 = 1 - 0.755 = 25\%$

(2) $P = 50\ MPa$; $\overline{R} = 5.4\ mm$; $t_c = 1.2\ mm$

a) Find thin-walled stresses:

$$\sigma_\theta = \frac{pR}{t_c} = \frac{(50\ MPa)(0.0054\ m)}{0.0012\ m} = 225\ MPa$$

$$\sigma_r = -\tfrac{1}{2}P = -25\ MPa$$

$$\sigma_z = \frac{pR}{2t_c} = \frac{(50\ MPa)(0.0054\ m)}{2(0.0012)} = 112.5\ MPa$$

b) Find thick-wall σ @ $r = 5.6\ mm = 0.56\ cm = 0.0056\ m$

$$\sigma_\theta = P\frac{\left(\frac{R_o}{r}\right)^2 + 1}{(R_o/R_i)^2 - 1} = 50\ MPa\frac{(1.0)^2 + 1}{(1.08)^2 - 1} = 306.5\ MPa$$

$$\overline{R} = \frac{r_i + r_o}{2} = 5.4$$

assuming $r = R_o = 5.6\ mm$

$$\sigma_r = -P\frac{\left(\frac{R_o}{r}\right)^2 - 1}{(R_o/R_i)^2 - 1} = 0\ MPa$$

$r_i = 5.2$

$$\sigma_z = P\frac{1}{(R_o/R_i)^2 - 1} = 50\ MPa\frac{1}{1.08^2 - 1} = 300.5\ MPa$$

$R_o/r = 1.0$

$R_o/R_i = 1.08$

c) Find ϵ_{max}? $E = 180\ GPa = 180E3\ MPa$

$\nu = 0.28$

Given $\sigma_\theta > \sigma_z$

$\epsilon_\theta > \epsilon_z$

$$\epsilon_{max} = \epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_\theta - \nu(\sigma_r^{\,0} + \sigma_z)\right) = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_z\right) = \frac{1}{180E3}\left(306.5 - 0.28 \cdot 30\right)$$

$= .00124$

(3) Find $\Delta t_g = ?$

Given: $R_f = 0.52\,cm$; $t_g = 0.005\,cm$; $T_{co} = 550\,K$

$k_{fuel} = 0.05\ W/cm\text{-}K$; $t_{clad} = 0.08\ cm$ $LHR = 225\ W/cm$

$k_{gap} = 0.003$ " " $k_{clad} = 0.15\ W/cm\text{-}K$

$\alpha_c = 4.5E-6/K$ $\alpha_f = 15E-6/K$ $T_{ref(fuel=clad)} = 300\,K$

$$\Delta_{gap} = \bar{R}_c \alpha_c \Delta T_c - \bar{R}_f \alpha_f \Delta T_f$$

$$\bar{R}_c = R_f + t_{gap} + \frac{t_c}{2} = 0.52 + = 0.5255\,cm$$

$$\Delta R_f = \alpha_f (\bar{T}_f - T_{co})$$

$$0.52\,cm = (15E-6/K)(T_f - 550\,K)$$

$$\bar{T}_f = 550\,K =$$

$$T_{c1} = T_{co} + \frac{LHR}{2\pi R_f}\frac{t_{clad}}{k_{fuel}} = 550\,K + \frac{225\ W/cm}{2\pi(0.52cm)}\left(\frac{0.08\,cm}{0.15\ W/cm\text{-}K}\right) = 586.7K$$

$$T_F = T_{c1} + \frac{LHR}{2\pi R_f}\frac{t_g}{k_g} = 586.7K + \frac{225}{2\pi(0.52cm)}\left(\frac{0.005}{0.003}\right) = 701.4K$$

$$T_o = T_f + \frac{LHR}{4\pi k_f} = 701 + \frac{225}{4\pi(0.005)} = 1054.1\,K$$

$$\Delta T_c = T_{c1} - T_{co} = 86.7\,K$$

$$\Delta T_f = T_o - T_f = 357.7\,K$$

$$\Delta t_{gap} = R_c \alpha_c \Delta T_c - R_f \alpha_f \Delta T_f$$

$$= R_c(4.5E-6/K)(86.7K) - (0.52cm)(15E-6/K)(357.7K)$$

$$= 0.5255cm\,(3.902 \times 10^{-4}) - 0.0028 = -0.0026\,cm$$

$$t_g = 0.005cm - 0.0026\,cm = 0.0024\,cm$$

(4) avg. grain size = 8 micron

$\phi = 2.0\,E\,13$ fissions/cm^3-s

$D = 2\,E\,-15$ cm^2/s $\quad \sigma_f = 550b$

$Y = 0.3017$

Find released # gas atoms @ $t = 2$ year $= 6.312\,E\,7$ s

V (assume spherical) $= \frac{\pi}{6} d^3 = \frac{\pi}{6}(8\mu m)^3 = 268.1\ mm^3$

$= 0.2681\ cm^3$

$\dot{F} = N_{U235}\,\phi\,\sigma_f\,V = (2.5\,E\,22\ atoms/cm^3)(2.0E13\ f/cm^3s)(550\,E\,-24)(0.2681c$

$= 7.37E13$ fiss/s

$gP_{tot} = Y\,\dot{F}\,t = (0.3017)(7.37E13)(6.312E7)$

$= 1.404\,E21$ gas atoms

$fr = gP_{tot} \cdot f_{gas}$

Given $\tau > \pi^{-2}$ → $f_{gas} = 1 - \frac{0.0662}{Dt/a^2}\left(1 - 0.93 \cdot e^{-\pi^2 \cdot Dt/a^2}\right)$

$\tau = Dt/a^2 = \frac{(2\,E\,-15)(6.312\,E\,7\,s)}{0.2681\ cm^3} = 0.1973$

→ $f_{gas} = 1 - \frac{0.0662}{0.1973}\left(1 - 0.93\exp(-\pi^2 \cdot 0.1973)\right)$

$= 0.709$

$gr = gP_{tot} \cdot f_{gas} = (1.404\,E21)(0.709) = 9.954 \times 10^{20}$ atoms released

(5) Strain hardening — a mat. property with a value between 0 -> 1, ...
0 = Mat. is fully plastically solid while 1 means mat. is fully elastic.
Caused by deformation beyond the mat. yield point. aka plastic deformation. ... the act hardens the mat.

(6) f(stoich) affects (1) Melting temp
(2) Thermal conductivity
(3) creep / FGR / grain growth (dependant on diff)

(7) (1) Numerically model temp in fuel
(2) " " stress in cladding
(3) Consider gap pressure, closure, and HT effects

(10) 0-D or point defects → Vacancies
3-D → large clusters of vacancies or voids

(8) FGR → Stage 1 → intragranular diff. to GB
Stage 2 → GB gas accumulation/saturation
" " 3 → once gas and connected, it is released

(9) HBS causes degradation of mat. conductivity due to change in crystal structure instability/changes

(11) Densification is caused by continuation of sintering since fuel is not @ 100% T.D. ... small pores close, pellets shrink and is driven by change in free E from decr. of surf. area of pores. (which decr. of free-E), Grain growth driven by P_d aka pressure. ... and reduction of GB-Energy. due to temp gradients Energy gradients & dislocation grads

(12) Valence state of U in UO_2 = U^{4+}.

As U oxidizes it usually goes $U^{4+} \longrightarrow U^{5+} \longrightarrow U^{6+}$ to stay electrically neutral.