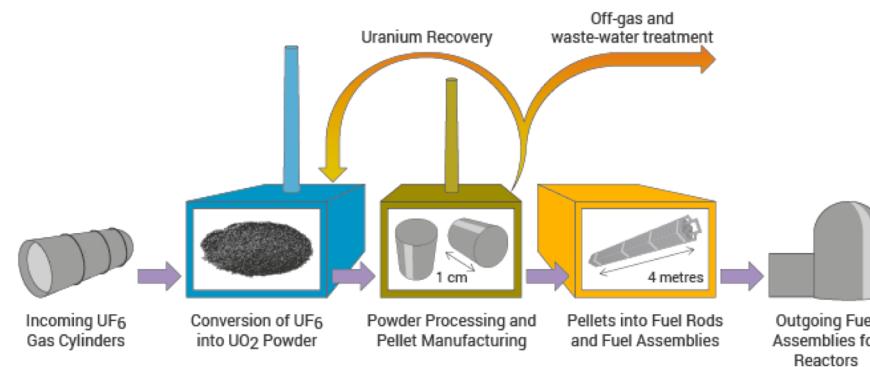


Heat Transport

NE 591

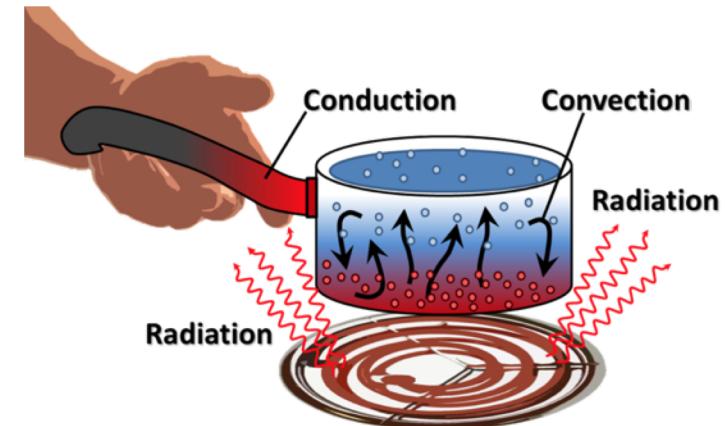
Last Time

- Mining -> Processing -> Conversion -> Enrichment -> Powder -> Compaction/Sintering -> Rod/Assembly
- U_3O_8 must be converted to UF_6 for enrichment, which is then converted to UO_2 powder for pellet manufacture
- For different fuel types, enriched UF_6 follows a different path



Last Time

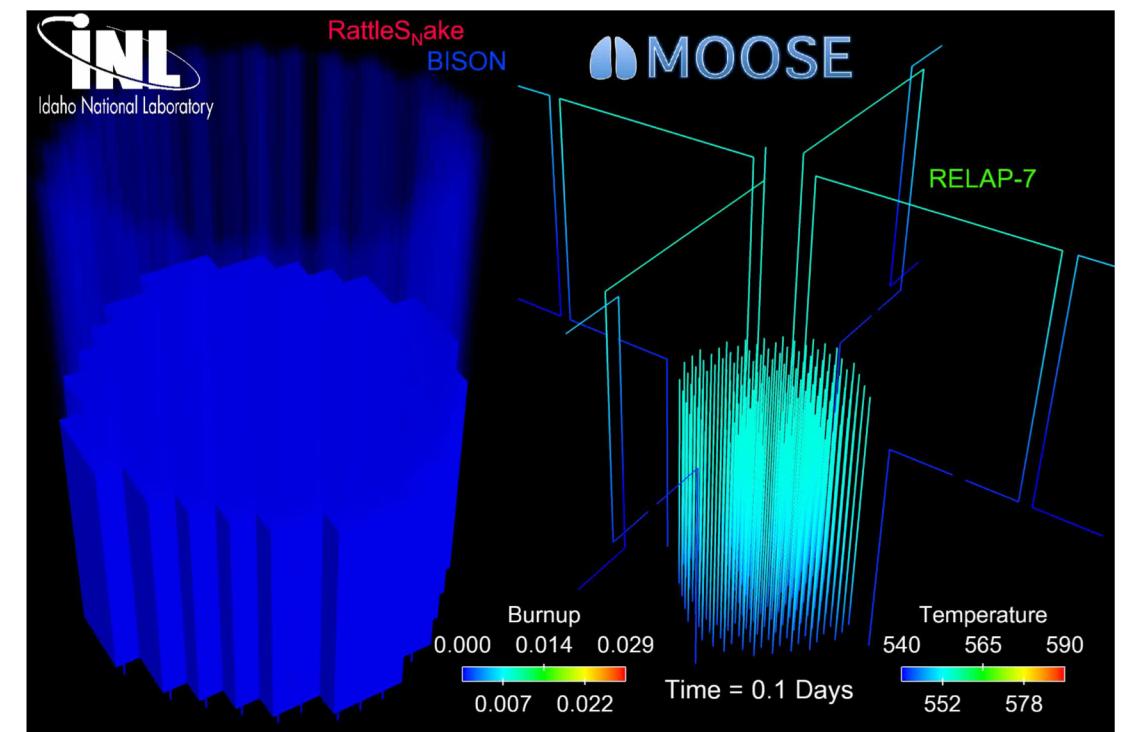
- General heat transport
- Heat is produced in the fuel, transports through the cladding and gap, and into the coolant
- The geometry of our problem
- The initial condition of T
- The boundary conditions of T
- Is each parameter is a function of T
 - Thermal conductivity, heat capacity
- Function of space/time?
 - Heat generation, dependent upon flux



$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T)$$

The most accurate solution is numerical, in 3D, requires modeling the entire core, and is multi-physics

- Solution is 3D and changes in time
- All the properties are functions of temperature
- The boundary conditions comes from information about the coolant flow
- The heat generation rate comes from information about the neutronics in the reactor
- No analytical solution is possible



In order to solve, make assumptions!

- #1: steady state -> $\nabla \cdot (k \nabla T) + Q = 0$

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

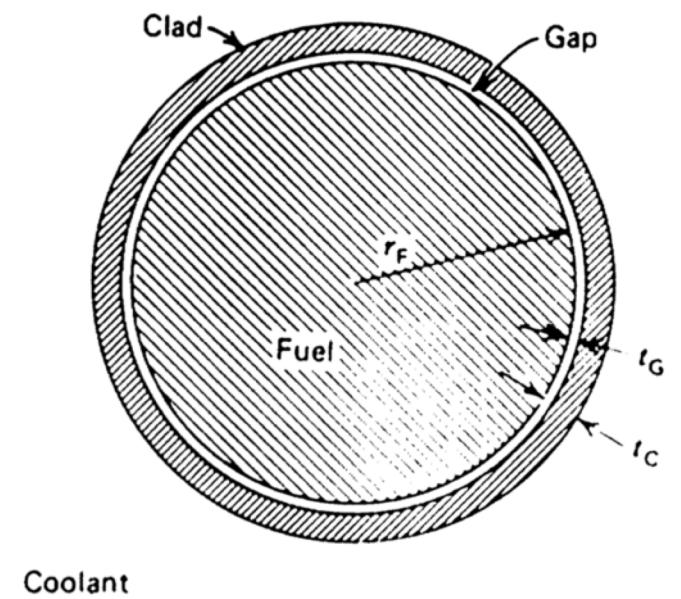
- #2: axisymmetric ->

$$\frac{1}{r} \frac{\partial}{\partial r} \left(rk(T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k(T) \frac{\partial T}{\partial z} \right) + Q(r, z) = 0$$

- #3: constant in z $\frac{1}{r} \frac{\partial}{\partial r} \left(rk(T) \frac{\partial T}{\partial r} \right) + Q(r) = 0$

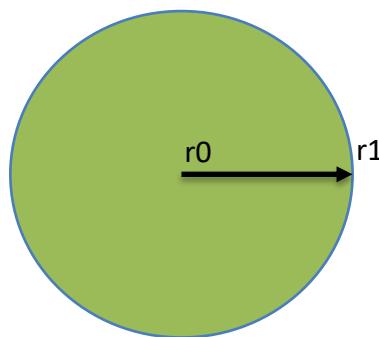
- #4: constant thermal conductivity

$$\frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial T}{\partial r} \right) + Q = 0$$



Directly Solving for Temperature Profile

- Boundary conditions: $r_0 = 0$, $r_1=R$,
 $T'(0) = 0$, $T(R) = T_s$



$$\frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial T}{\partial r} \right) + Q = 0$$

$$\frac{\partial}{\partial r} \left(rk \frac{\partial T}{\partial r} \right) = -Q r$$

$$rk \frac{\partial T}{\partial r} = -\frac{Q r^2}{2} + C_1 \quad 0 = -\frac{Q 0^2}{2} + C_1$$

$$\frac{\partial T}{\partial r} = -\frac{Q r}{2k}$$

$$T(r) = -\frac{Q r^2}{4k} + C_2 \quad C_2 = \frac{QR^2}{4k} + T_s$$

$$T(r) = -\frac{Q r^2}{4k} + \frac{QR^2}{4k} + T_s \quad T(r) = \frac{Q(R^2 - r^2)}{4k} + T_s$$

$$T_0 - T_s = \frac{QR^2}{4k}$$

Linear Heat Rate

- $LHR = \pi R^2 Q_{av}$
 - Where Q_{av} is the radially averaged heat generation rate, in units of per-length, i.e. W/cm
- Substitute LHR into previous equation on T0-Ts

$$T_0 - T_s = \frac{QR^2}{4k}$$

$$T_0 - T_s = \frac{R^2}{4k} \frac{LHR}{\pi R^2}$$

$$T_0 - T_s = \frac{LHR}{4\pi k}$$

Heat transport through the gap

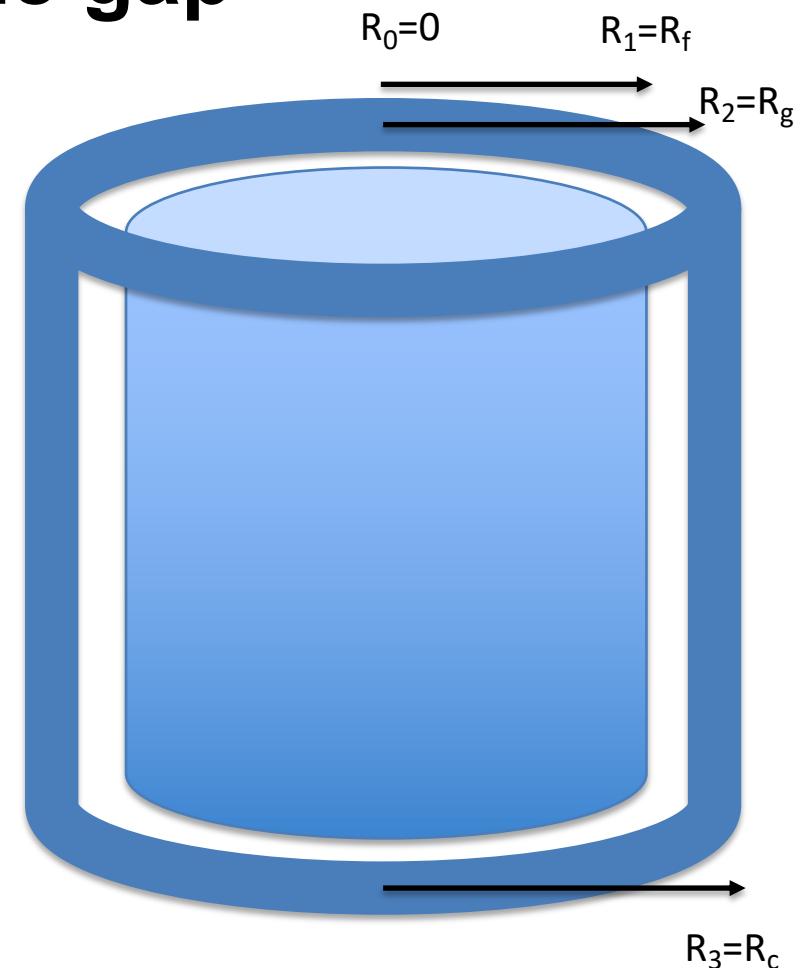
- The heat flux gives the rate, per unit area, at which heat flows in a given direction

$$q = -k \frac{T_2 - T_1}{L} \quad q_{gap} = -k_{gap} \frac{T_{gap} - T_{fuel}}{R_{gap} - R_{fuel}}$$

- The heat flux from the fuel is the LHR/pellet circumference

$$q = \frac{LHR}{2\pi R_f}$$

- Heat flux from the fuel is the same as heat flux into the gap
- Gap thickness = $R_g - R_f = t_g$
- Cladding thickness = $R_c - R_g = t_c$



Heat transport through the gap

- Set heat flux fuel/gap equal

$$\frac{LHR}{2\pi R_f} = -k_{gap} \frac{T_{gap} - T_{fuel}}{t_{gap}} \quad \frac{LHR}{2\pi R_f} = k_{gap} \frac{T_{fuel} - T_{gap}}{t_{gap}}$$

- Gap conductance is defined as:

$$h_{gap} = \frac{k_{gap}}{t_g} \quad T_{fuel} - T_{gap} = \frac{LHR}{2\pi R_f h_{gap}}$$

- Gap conductance depends on the gas filling the gap

- For pure He, $k_{gap} = 16 \times 10^{-6} * T^{0.79}$ (W/cm-K)
- For pure Xe, $k_{gap} = 0.7 \times 10^{-6} * T^{0.79}$ (W/cm-K)
- Simple mixing rule: $k_{gap} = k_{He}^{1-y} \times k_{Xe}^y$

Heat transport through the cladding

- Heat flux through the cladding

$$q = -k \frac{T_2 - T_1}{L}$$

$$q_{clad} = -k_{clad} \frac{T_{clad} - T_{gap}}{R_{clad} - R_{gap}}$$

$$q = \frac{LHR}{2\pi R_f}$$

$$q_{clad} = k_{clad} \frac{T_{gap} - T_{clad}}{t_{clad}}$$

- Set equal your heat fluxes

$$\frac{LHR}{2\pi R_f} = k_{clad} \frac{T_{gap} - T_{clad}}{t_{clad}}$$

$$T_{gap} - T_{clad} = \frac{LHR}{2\pi R_f} \frac{t_{clad}}{k_{clad}}$$

- My nomenclature is often also seen where $T_{gap} = T_{Cl}$, $T_{clad} = T_{Co}$

Heat transfer to the coolant

- Heat is transported from the cladding to the coolant via convection

$$T_{co} - T_{cool} = \frac{LHR}{2\pi R_F h_{cool}}$$

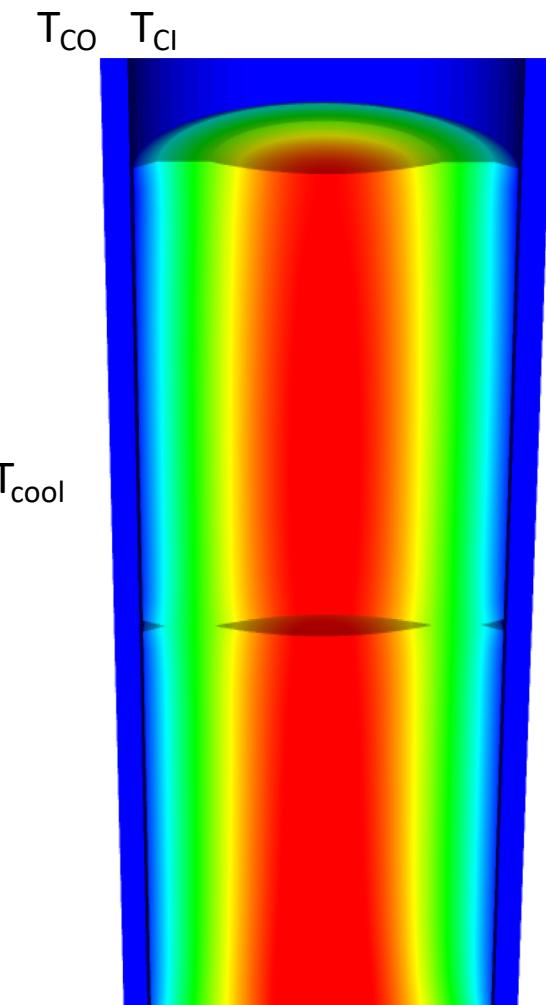
- T_{cool} is the coolant temperature, h_{cool} is the convective heat transfer coefficient between cladding wall and coolant
- Adding gap + cladding + coolant: $\frac{1}{h} = \frac{t_{gap}}{k_{gap}} + \frac{t_{clad}}{k_{clad}} + \frac{1}{h_{cool}}$

Summary of analytical solutions

- $T_0 - T_{fuel} = \frac{Q}{4k} R_{fuel}^2$ $T_0 - T_{fuel} = \frac{LHR}{4\pi k}$
- $T_{fuel} - T_{gap} = \frac{Q}{2h_{gap}} R_{fuel}$ $T_{fuel} - T_{gap} = \frac{LHR}{2\pi R_{fuel} h_{gap}}$ $h_{gap} = \frac{k_{gap}}{t_{gap}}$
- $T_{gap} - T_{clad} = \frac{Qt_{clad}}{2k_{clad}} R_{fuel}$ $T_{gap} - T_{clad} = \frac{LHRt_{clad}}{2\pi R_{fuel} k_{clad}}$ $T_{gap} = T_{CI}$ (clad inner)
 $T_{clad} = T_{CO}$ (clad outer)
- $T_{clad} - T_{cool} = \frac{Q}{2h_{cool}} R_{fuel}$ $T_{clad} - T_{cool} = \frac{LHR}{2\pi R_{fuel} h_{cool}}$

Solving for the temperature profile

- You first solve for the transition temperatures.
- Start from the coolant and work inward
- Then, assume a linear profile everywhere except in the fuel
- Finally, solve for the temperature profile throughout the fuel



Fuel and Cladding Thermal Properties

Material	Density (g/cm ³)	Heat Capacity Cp (J/g-K)	Thermal Conductivity k (W/cm-K)	Thermal Expansion Coefficient a (K ⁻¹)
UO ₂	10.98	0.33	0.03	1.2 x 10 ⁻⁵
Zr	6.5	0.35	0.17	1.0 x 10 ⁻⁵
Stainless steel	8.0	0.5	0.17	9.6 x 10 ⁻⁶

Temperature profile calculation example

- $T_{cool} = 580 \text{ K}$; $LHR = 200 \text{ W/cm}$; $h_{cool} = 2.5 \text{ W/cm}^2\text{-K}$
- $R_{fuel} = 0.5 \text{ cm}$; $t_{clad} = 0.06 \text{ cm}$; $t_{gap} = 0.003 \text{ cm}$
- Work from outside->in, calculate cladding temperature

$$T_{clad} = (200)/(2\pi R_{fuel} h_{cool}) + 580$$

$$T_{clad} - T_{cool} = \frac{LHR}{2\pi R_{fuel} h_{cool}}$$

$$T_{clad} = 605.5 \text{ K}$$

- Calculate inner cladding temp

$$T_{gap} = (200 * 0.06) / (2\pi R_{fuel} k_{clad}) + 605.5$$

$$T_{gap} - T_{clad} = \frac{LHR t_{clad}}{2\pi R_{fuel} k_{clad}}$$

$$T_{gap} = 628 \text{ K}$$

Temperature profile calculation example

- Calculate fuel surface temperature
- Calculate gap conductance
 - gap with He; $k_{gap} = 16 \times 10^{-6} * T^{0.79}$ (W/cm-k); assume T_{gas} is appropriate for entire gap; $k_{gap} = 0.0026$ W/cm-K; $t_{gap} = 0.003$ cm
 - $h_{gap} = 0.87$ W/cm²-K

$$T_{fuel} = 200 / (2 * \pi * 0.5 * 0.87) + 628$$

$$T_{fuel} = 701.5 \text{ K}$$

$$T_{fuel} - T_{gap} = \frac{LHR}{2\pi R_{fuel} h_{gap}}$$

Temperature profile calculation example

- Calculate centerline temperature

$$T_0 = 200/(4\pi \cdot 0.03) + 701.5$$

$$T_0 - T_{fuel} = \frac{LHR}{4\pi k}$$

$$T_0 = 1232 \text{ K}$$

Full temperature profile

$$T(r) = \frac{Q(R^2 - r^2)}{4k} + T_s$$

$$T(r) = \frac{LHR(R_f^2 - r^2)}{4\pi k R_f} + T_s$$

$$T(r) = \frac{LHR}{4\pi k} \left(1 - \frac{r^2}{R_f^2}\right) + T_s$$

Temperature profile calculation example

- Calculate centerline temperature

$$T_0 = 200/(4\pi \cdot 0.03) + 701.5$$

$$T_0 - T_{fuel} = \frac{LHR}{4\pi k}$$

$$T_0 = 1232 \text{ K}$$

Full temperature profile

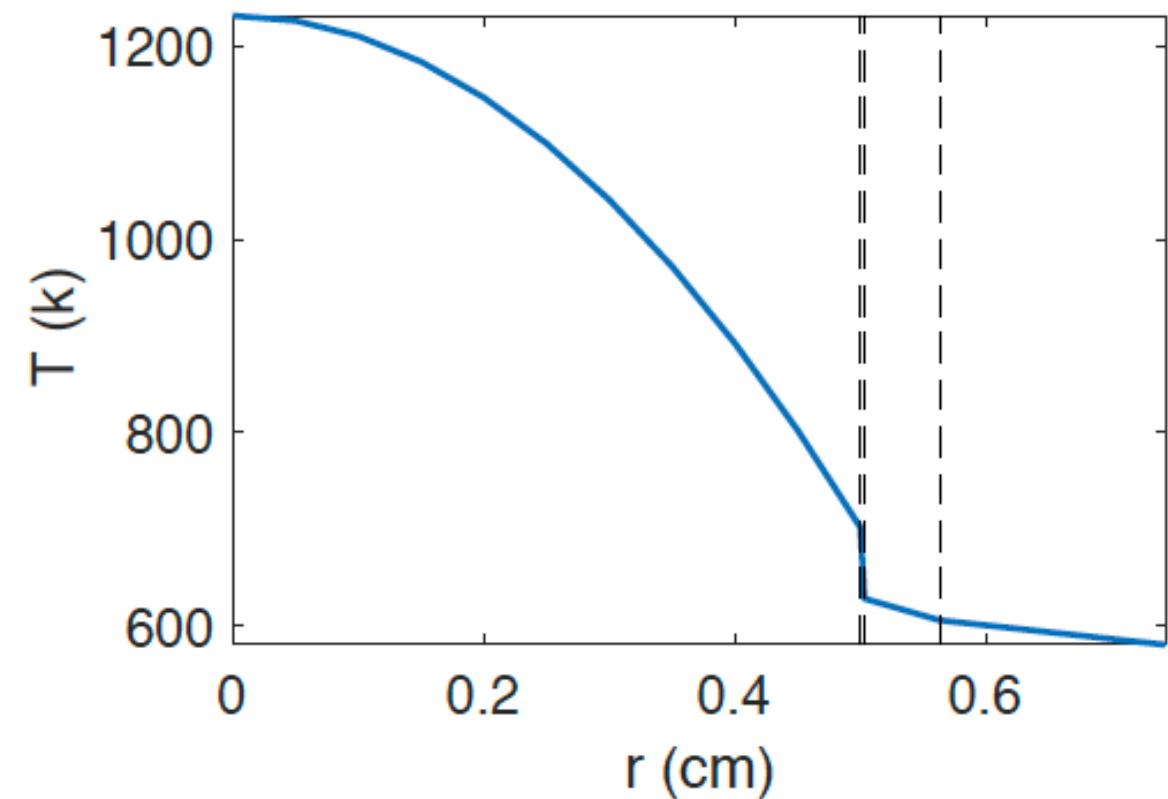
$$T(r) = \frac{Q(R^2 - r^2)}{4k} + T_s$$

$$T(r) = \frac{LHR(R_f^2 - r^2)}{4\pi k R_f} + T_s$$

$$T(r) = \frac{LHR}{4\pi k} \left(1 - \frac{r^2}{R_f^2}\right) + T_s$$

Temperature profile calculation example

- Parabolic temperature profile in fuel
- Linear in gap and clad
- Steep temperature drop over gap, very low thermal conductivity
- Smaller temperature drop over cladding

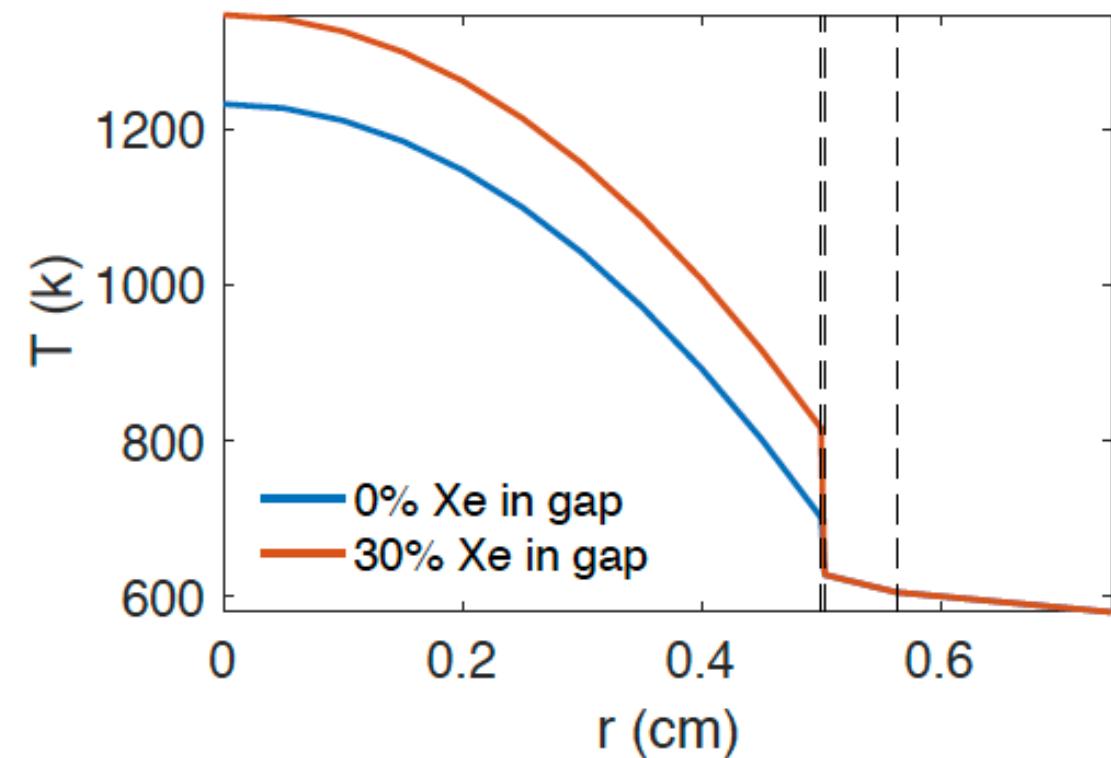


Temperature profile modification

- Instead of pure He gap, 30% of gas is Xe; how is T_0 affected?
- First, how is gap conductance affected?
- Gap conductance depends on the gas filling the gap
 - For pure He, $k_{\text{gap}} = 16 \times 10^{-6} * T^{0.79}$ (W/cm-K)
 - For pure Xe, $k_{\text{gap}} = 0.7 \times 10^{-6} * T^{0.79}$ (W/cm-K)
 - Simple mixing rule: $k_{\text{gap}} = k_{\text{He}}^{1-y} * k_{\text{Xe}}^y$
 - T_{CI} , T_{CO} , and T_{cool} is unchanged from previous example, also $T_0 - T_{\text{fuel}}$ is unchanged
 - $k_{\text{gap}} = ((16 \times 10^{-6}) * (628)^{0.79})^{(1-0.3)} * ((0.7 \times 10^{-6}) * (628)^{0.79})^{(0.3)} = 1.016 \times 10^{-3}$ W/cm-K

Temperature profile modification

- $k_{\text{gap}} = 1.016 \times 10^{-3} \text{ W/cm}\cdot\text{K}$
- $h_{\text{gap}} = 1.016 \times 10^{-3} / 0.003 = 0.34 \text{ W/cm}^2\cdot\text{K}$
- $T_{\text{fuel}} = 200/(2\pi 0.5 \times 0.34) + 628 = 816 \text{ K}$
- $T_0 - T_{\text{fuel}} = 530.5 \text{ K}$ (unchanged from before)
- $T_0 = 816 + 530.5 = 1346.5 \text{ K}$
- Increase in T_0 of 114 K

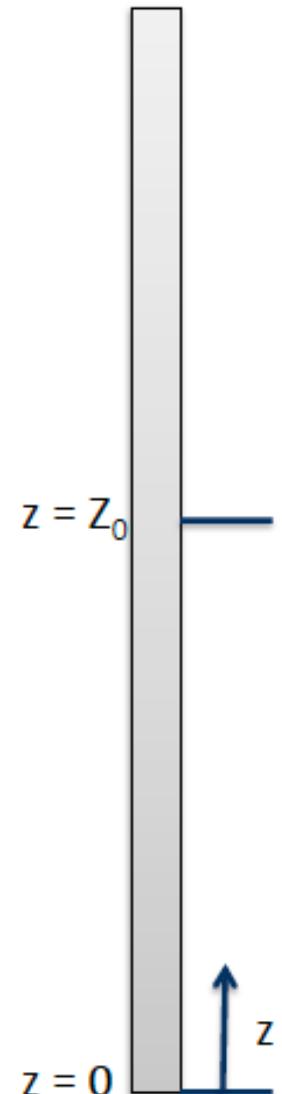


Neutron flux varies axially, so does LHR

- Taking a fuel rod with length, $L = 2*Z_0$

$$LHR\left(\frac{z}{Z_o}\right) = LHR^o \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{Z_o} - 1\right)\right] = LHR^o F\left(\frac{z}{Z_o}\right)$$

- LHR^o is the midpoint linear heat rate, i.e. @ $z=Z_0$
- $\gamma = \frac{Z_{ex}+Z_0}{Z_0}$, where Z_{ex} is the extrapolation distance
- A typical value is $\gamma = 1.3$



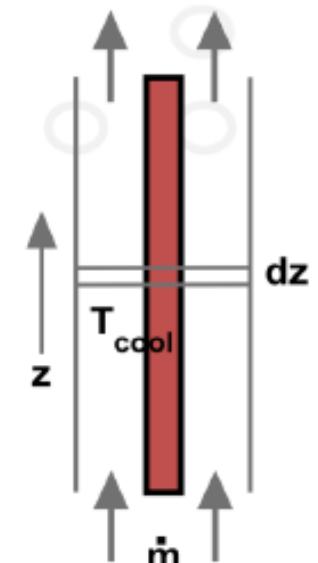
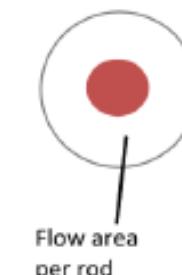
Coolant temperature varies with Z

- Each rod has a given flow area
- Mass flow rate: \dot{m}
- Coolant specific heat: C_{PW}

$$\dot{m}C_{PW} \frac{dT_{cool}}{dz} = LHR \left(\frac{z}{Z_o} \right)$$

$$\dot{m}C_{PW}(T_{cool} - T_{cool}^{in}) = Z_o \int_0^{z/Z_o} LHR \left(\frac{z}{Z_o} \right) d\left(\frac{z}{Z_o} \right)$$

$$\dot{m}C_{PW}(T_{cool} - T_{cool}^{in}) = Z_o \times LHR^o \int_0^{z/Z_o} F \left(\frac{z}{Z_o} \right) d\left(\frac{z}{Z_o} \right)$$

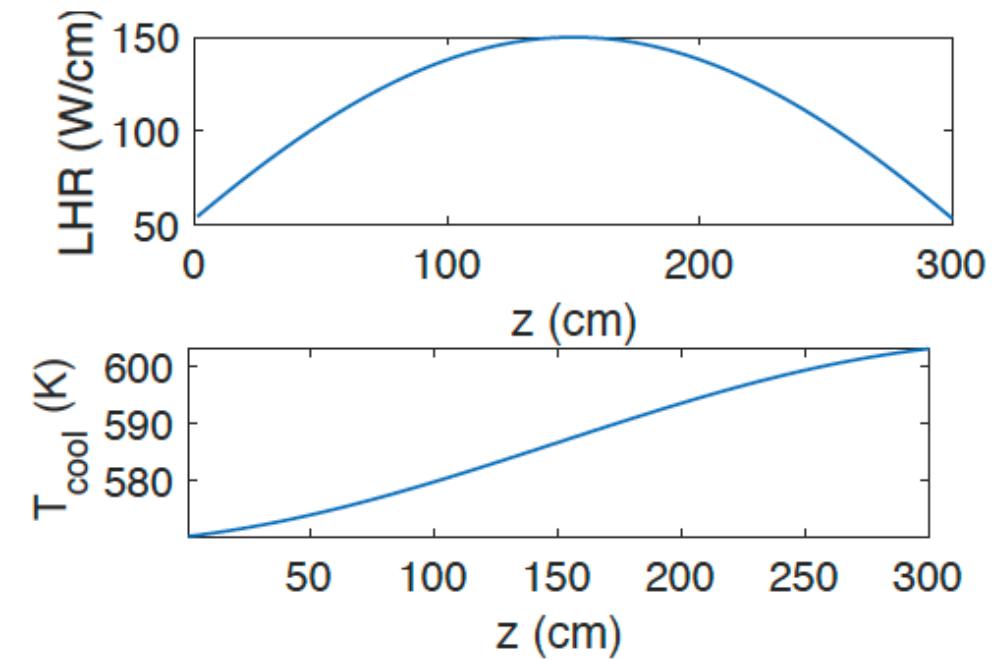


$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \frac{Z_o \times LHR^o}{\dot{m}C_{PW}} \left\{ \sin(1.2) + \sin \left[1.2 \left(\frac{z}{Z_o} - 1 \right) \right] \right\}$$

Calculate LHR and T_{cool} with axial variation

- $\dot{m} = 0.25 \text{ kg/s-rod}$; $Z_0 = 150 \text{ cm}$;
 $LHR^0 = 150 \text{ W/cm}$; $C_{PW} = 4200 \text{ J/kg-K}$;
 $T_{in} = 570 \text{ K}$

$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \frac{Z_o \times LHR^o}{\dot{m} C_{PW}} \left\{ \sin(1.2) + \sin \left[1.2 \left(\frac{z}{Z_o} - 1 \right) \right] \right\}$$



Summary

- Developed analytical expressions for the temperature profile within a fuel rod
- Required to make four assumptions:
 - Steady-state solution
 - Temperature is axisymmetric
 - T is constant in Z
 - Thermal conductivity is independent of temperature
- Temperature profile in the fuel is parabolic, assume linear profiles in gap, clad and coolant
- Can incorporate axial variation in T_{cool} with axial variation in LHR