Exam 1 is in the books

- If you have any questions or comments on the grading, please let me know, office hours tomorrow at 10
- Grading scale to the right
- Test was out of 105, graded on 100 scale
- No curve
- Planned next exam for early March
- You should have received a survey this morning, please fill out
- Exam solutions posted to the drive

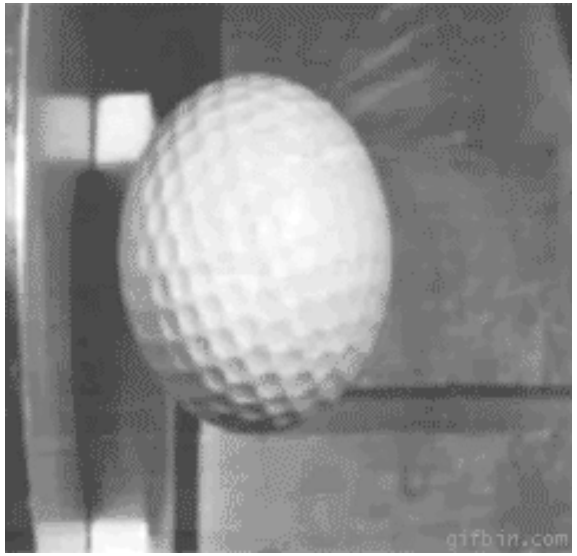
Letter Grade	Percent Grade
A+	98-100
A	93-97
A-	90-92
B+	87-89
B	83-87
B-	80-82
C+	77-79
C	73-76
C-	70-72
D+	67-69
D	63-66
D-	60-62
F	Below 60

Fuel Mechanics

NE 591

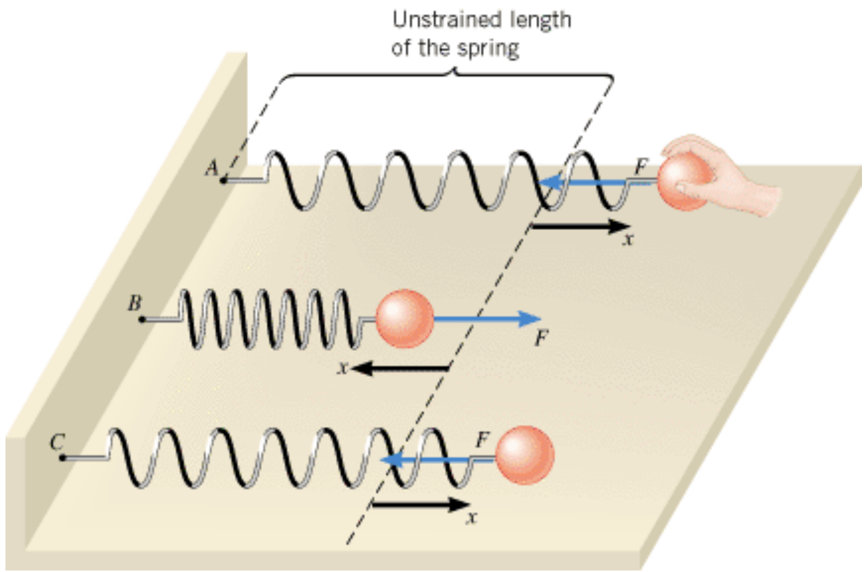
Solid mechanics

- When a load is applied to a body, it changes shape and perhaps size
- Motions throughout a body are called **displacements**
  - $\mathbf{u}(\mathbf{r}, t)$
- Rigid body displacements do not change the shape and/or size
- Changes in shape and/or size are call **deformations**
- The objective of **Solid Mechanics** is to relate loads to deformation



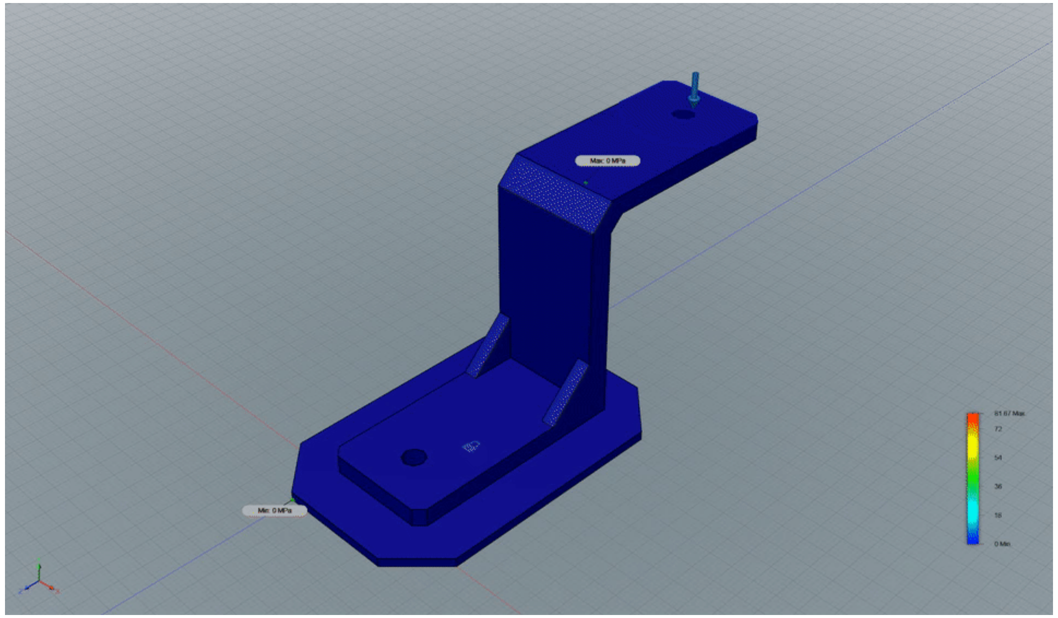
Ideal springs

- It can be instructive to view solids as a spring
- When we apply some force F, we get some displacement x
  - $F = k\,x$
- When the spring is displaced by x, there is force that responds in the opposite direction equal to kx
- Due to the displacement, there is a stored energy $E = ½\,k\,x^2$



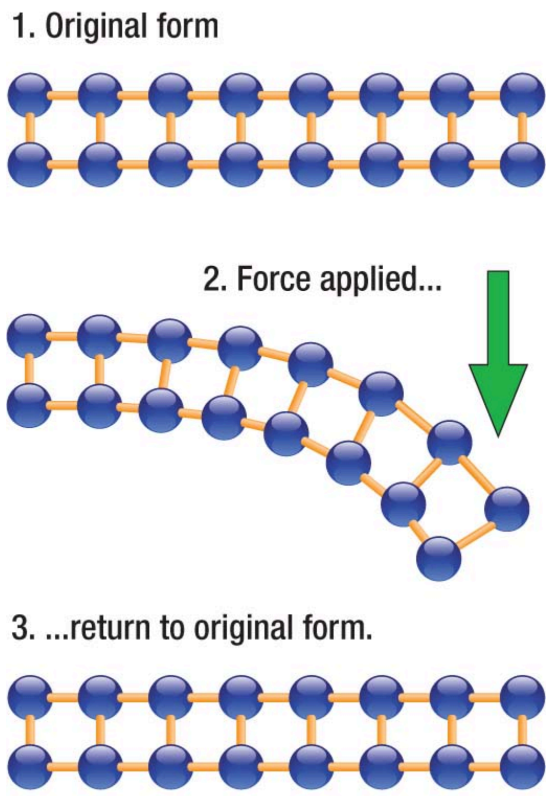
Observed deformation due to a force

- Solid mechanics is similar to the behavior of an ideal spring but throughout a body
- An applied load results results in deformation.
- The internal **strain** is like the displacements x
- The internal **stress** is like the internal force F



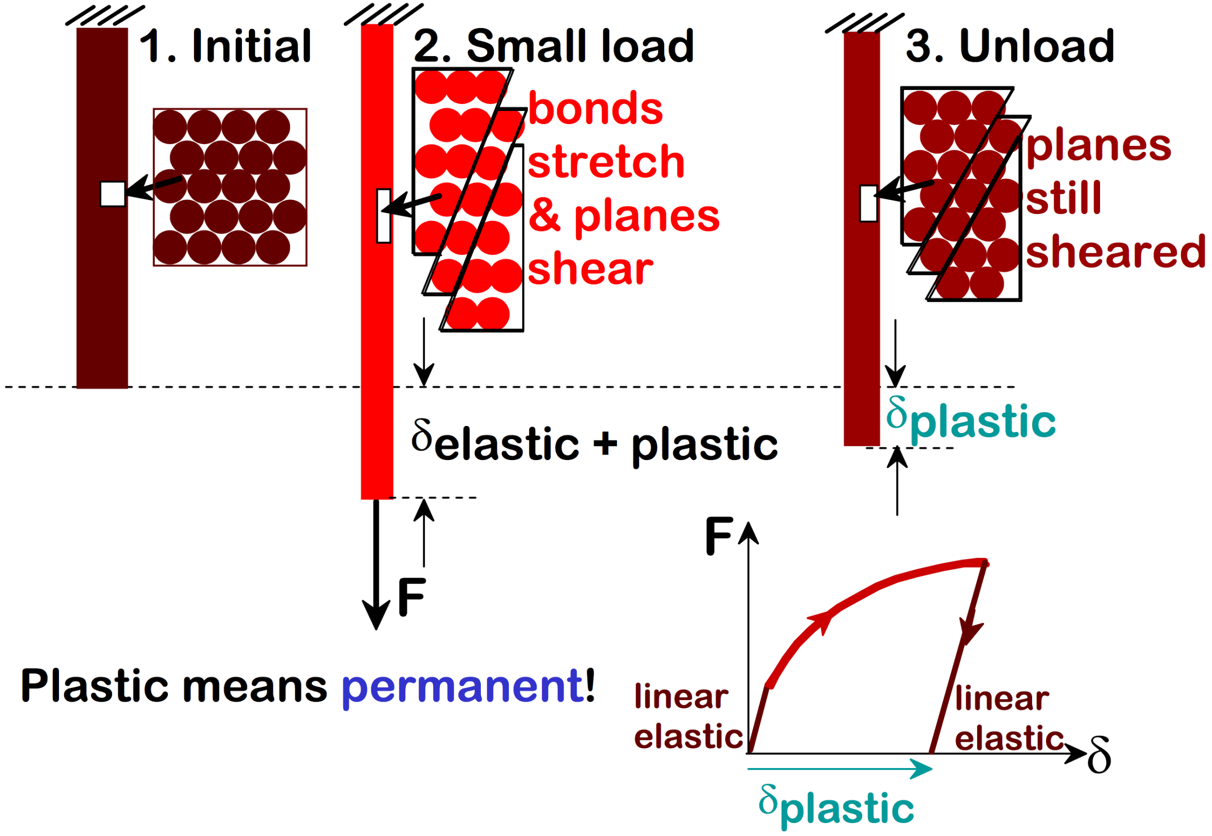
Elasticity

- Elasticity is the ability of a body to resist a distorting influence and to return to its original size and shape when that influence or force is removed
- In elastic deformation, we are stretching the atomic bonds
- The more we stretch the bonds, the more force it takes to stretch
- When we release the load, the atoms spring back into their lattice sites



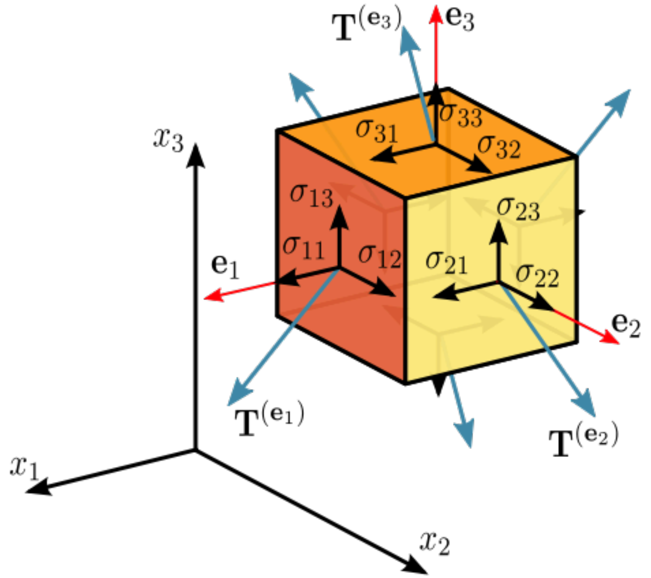
Plasticity

- Plasticity is the ability of a solid material to undergo permanent deformation, a non-reversible change of shape in response to applied forces
- Plastic deformation is observed in most materials
- The transition from elastic behavior to plastic behavior is known as yielding



Stress

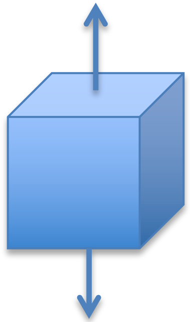
- Stress is a physical quantity that expresses the internal forces that neighboring particles of a continuous material exert on each other
- Stress is a force per unit area with SI units of Pa = N/m^2
- The stress is a 2nd order tensor (a 3 by 3 matrix): Cauchy stress tensor
- $\boldsymbol{\sigma}_{ij} = \boldsymbol{F}_{ij}/\boldsymbol{A}_i$
  - i is the face the force is applied and j is the direction it is applied
- Sigmas are normal stress components, taus are shear stress components

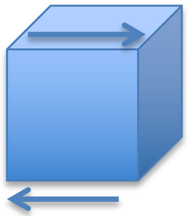


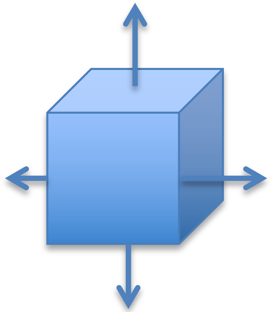
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$$

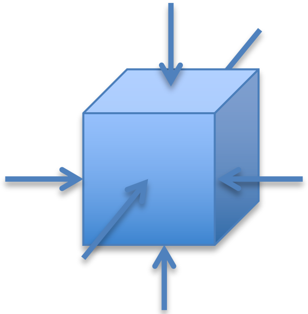
Stress as a response

- The stress in a material is the RESPONSE to an applied load (force) or an applied displacement
- Uniaxial tension or compression
  - Only one non-zero stress: σ_{ii} (σ_{11}, σ_{22}, σ_{33})
  - Tension means positive stress, compression negative
  - Examples: Cables, tension tests
- Pure shear
  - Only one non-zero stress: (σ_{12}, σ_{13}, σ_{23})
  - Examples: drive shaft
- Biaxial tension
  - Two non-zero stress (e.g. $\sigma_{11} = 1$, $\sigma_{22} = 2$)
  - Examples: Pressure cylinder or vessel
- Hydrostatic compression (pressure)
  - $\sigma_{11} = \sigma_{22} = \sigma_{33} =$ -p
  - Anything underwater









Strain

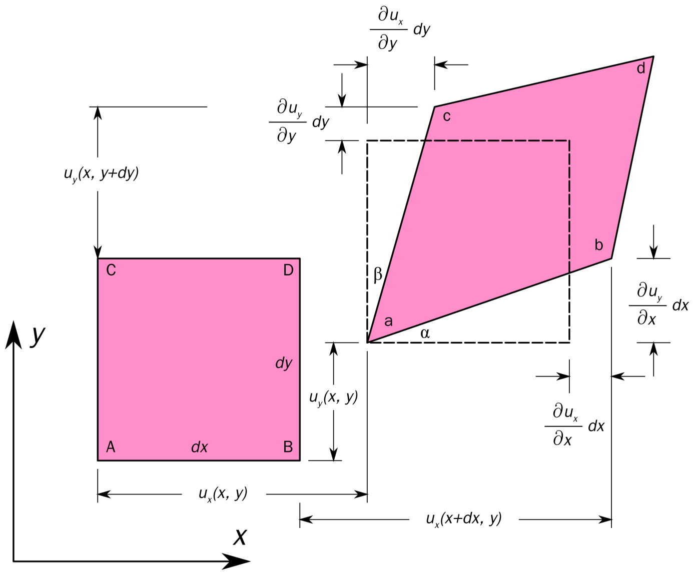
- Strain is a geometrical measure of deformation representing the relative displacement between particles in a material body
- The strain in a tensile test is the deformation divided by a representative length

$$e = \frac{\Delta L}{L} = \frac{\ell - L}{L}$$

- Strain is a second order tensor, like the stress and is computed using gradients
  - Let **u** be a vector of the displacements
  - The small strain tensor is

$$\epsilon = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$$

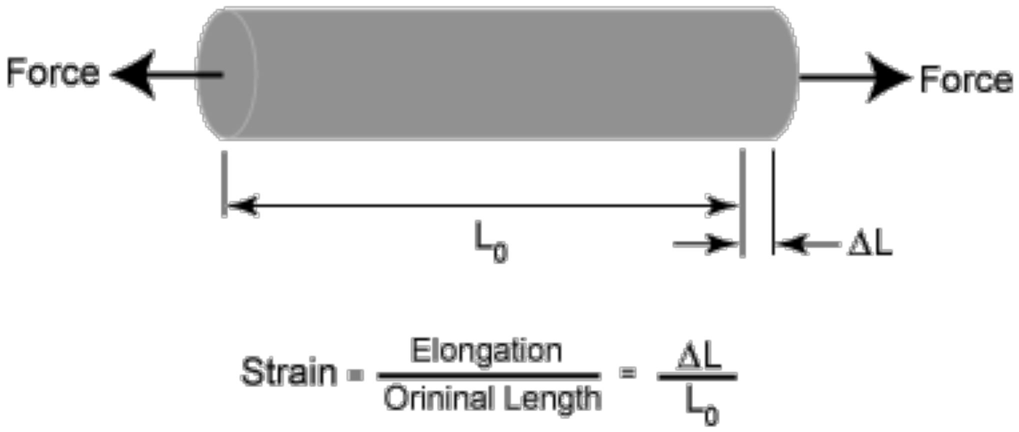
- The common strain states are the same as the stress (uniaxial tension, etc.)



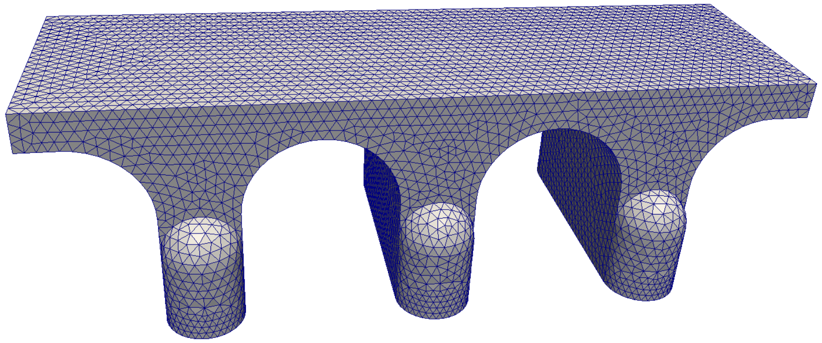
Strain produces stress

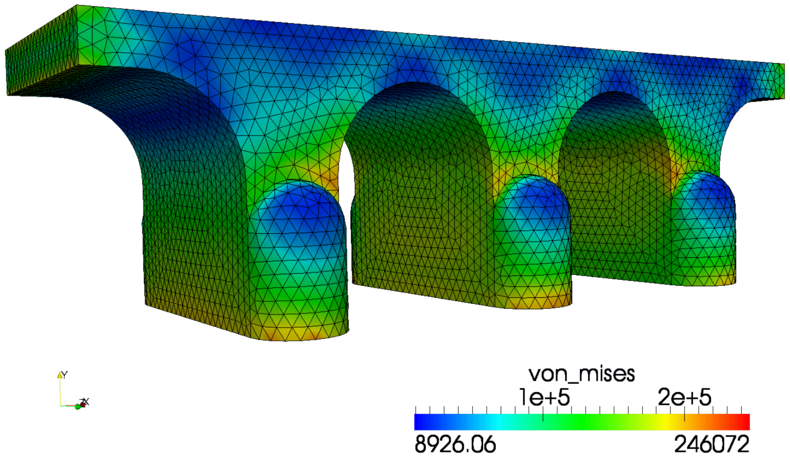
- A deformation (strain) results in stress within a material
- $\boldsymbol{\sigma} = F(\boldsymbol{\epsilon})$
- For small strains, the stress is elastic and is a linear function of the strain
  - $\boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{\epsilon})$
- For larger deformation, some of the strain is elastic and increases the stress. The rest is plastic and does NOT contribute to the stress.
  - $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_e + \boldsymbol{\epsilon}_p$

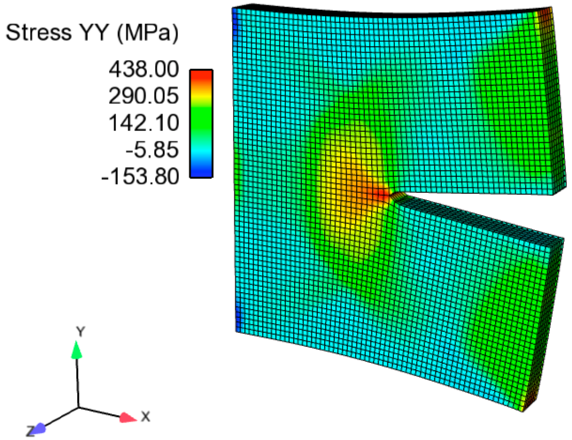
    $\boldsymbol{\sigma} = \mathcal{C}\boldsymbol{\epsilon}_e$
- The elastic energy density in a material is a scalar quantity equal to $$E_{el} = \frac{1}{2}\boldsymbol{\epsilon}_e \cdot \boldsymbol{\sigma}$$

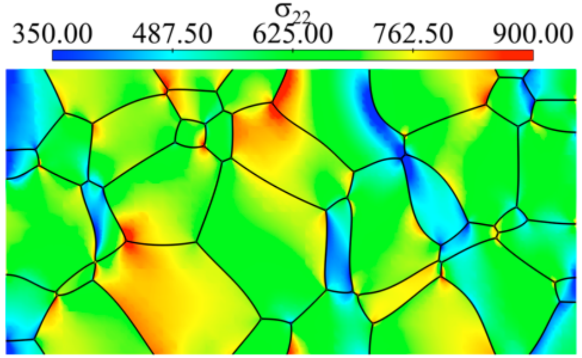


In actual materials, the stress and the strain change throughout the material

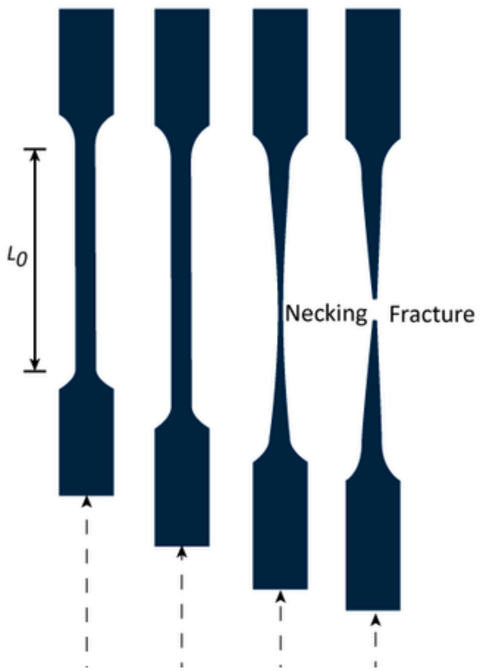




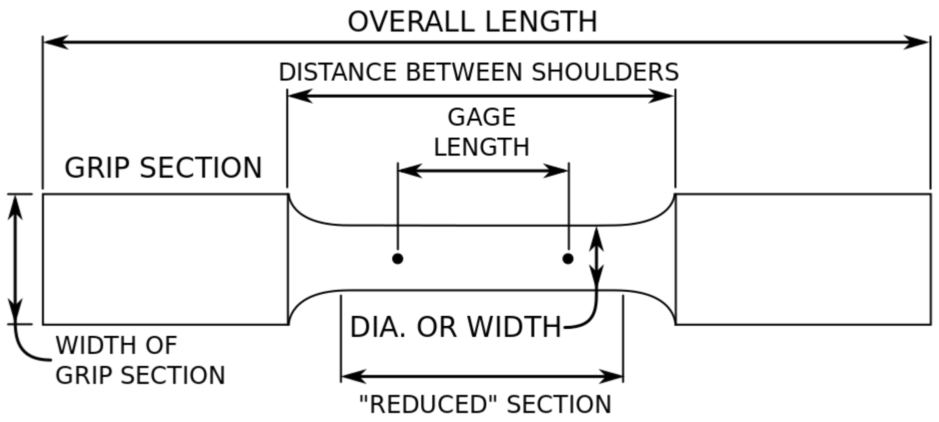




Tensile testing



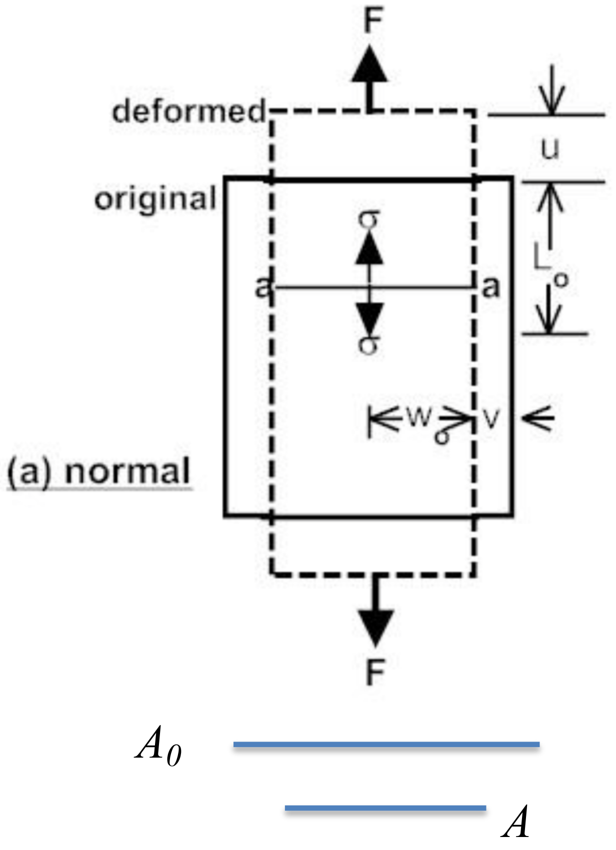
- The most common means of determining mechanical properties is a uniaxial tension test
- Apply a uniaxial load until failure
- Properties that are directly measured via a tensile test include ultimate tensile strength, maximum elongation, etc., which can be utilized to determine Young's modulus, Poisson's ratio, etc.



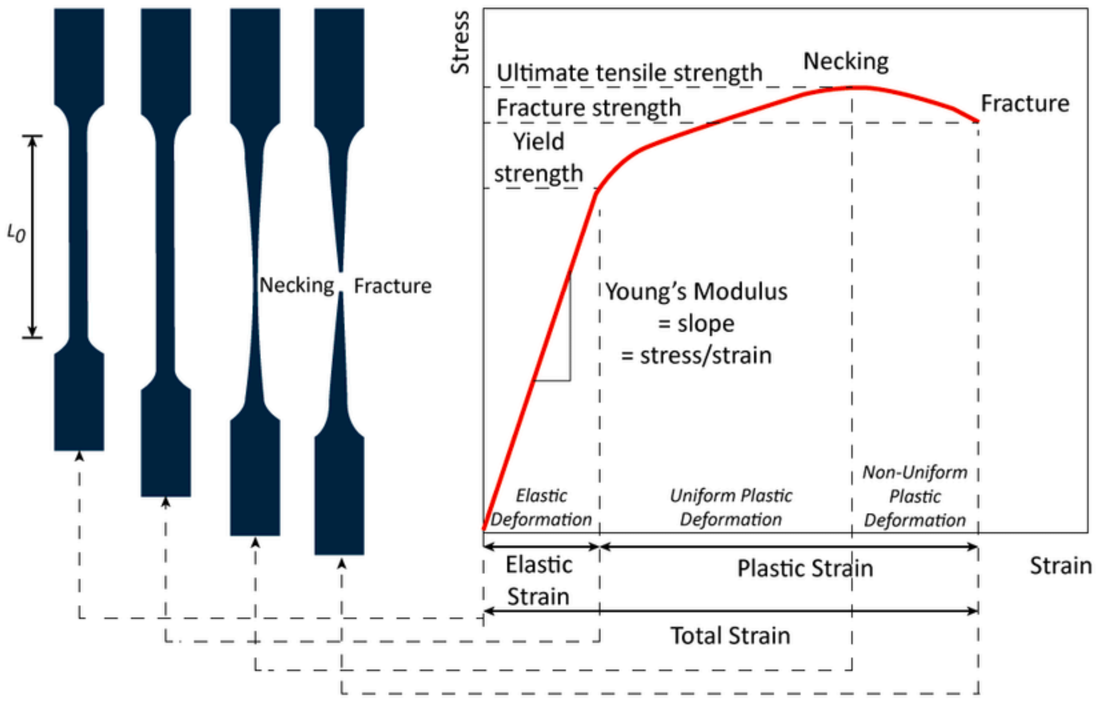
Stress/strain from a tensile test

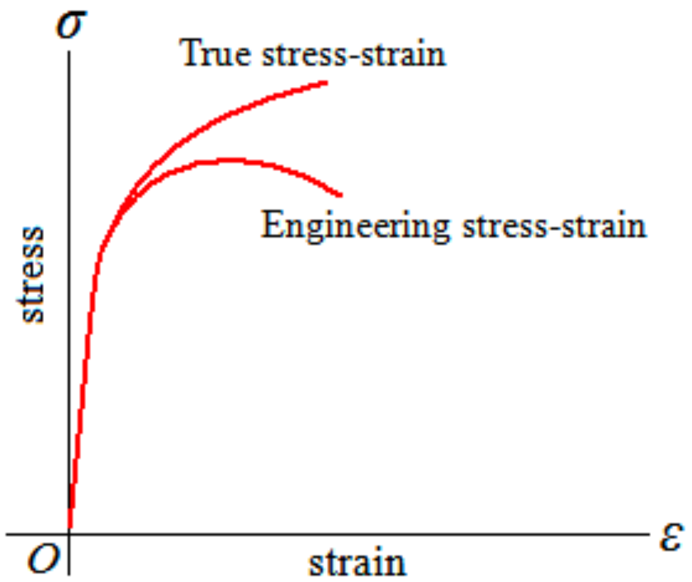
- A_0 = Initial cross section area
- A = deformed cross section area

	Engineering	True
stress σ	$\frac{P}{A_o}$	$\frac{P}{A}$
strain ε	$\frac{l-l_o}{l_o}$	$\int_{l_o}^{l}\frac{dl}{l} = ln\left(\frac{l}{l_o}\right)$



Stress vs strain curves



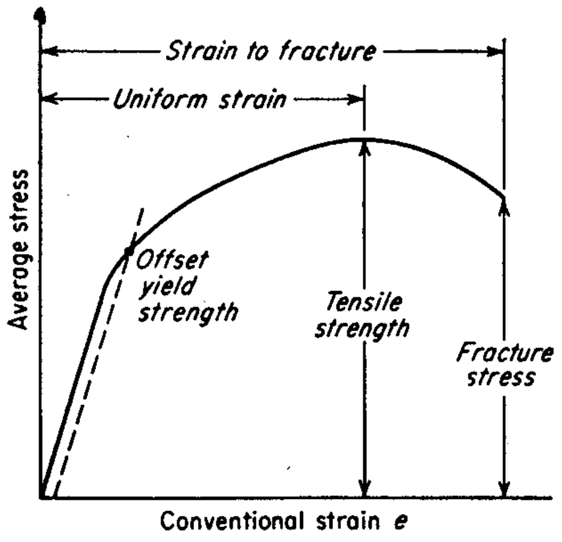


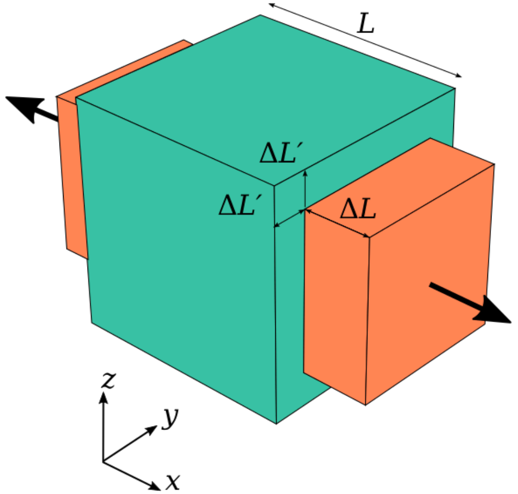
Using stress/strain curves

- In the elastic portion of the stress-strain curve, the stress varies linearly with strain
- The slope of the line is Young's Modulus, E: $\sigma = E\,\varepsilon$
- Young's modulus is an elastic constant
- Another elastic constant is Poisson's ratio, ν
- Poisson's ratio is the ratio of the shrinkage in cross section due to the extension in the pulling direction

$$\nu = -\frac{d\varepsilon_{\mathrm{trans}}}{d\varepsilon_{\mathrm{axial}}} = -\frac{d\varepsilon_{\mathrm{y}}}{d\varepsilon_{\mathrm{x}}} = -\frac{d\varepsilon_{\mathrm{z}}}{d\varepsilon_{\mathrm{x}}} \qquad \nu \approx \frac{\Delta L'}{\Delta L}.$$

$$G = \frac{E}{2(1+\nu)}$$





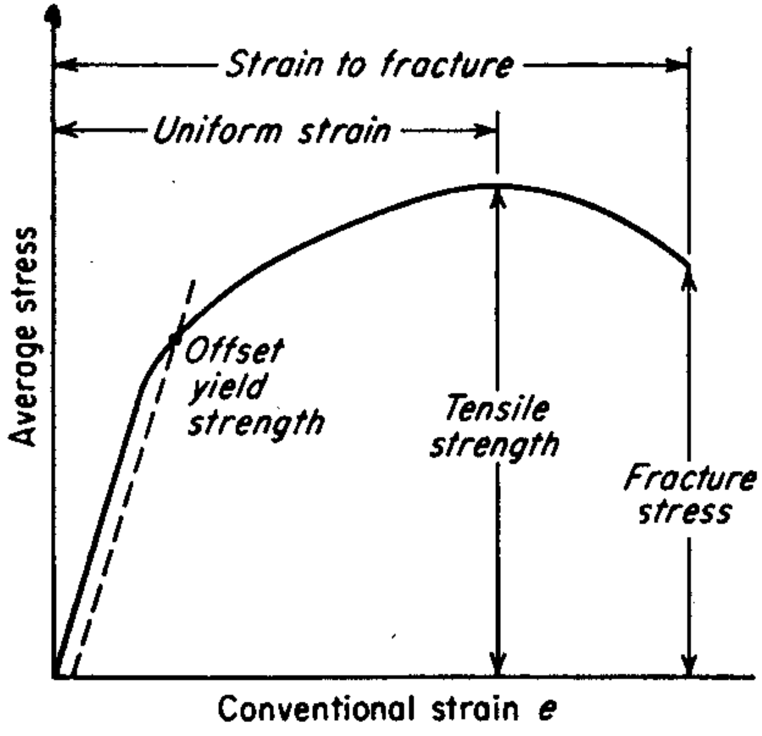
Using stress/strain curves

- The shear modulus, G, defines the stress to strain ratio in shear

$$\sigma_{12} = G\epsilon_{12}$$

- For isotropic materials, $G = E / (2 (1 + \nu))$
- In matrix form, Hooke's law for isotropic materials can be written as

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}$$



$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

Isotropic and Anisotropic

- Isotropic materials deform the same way no matter in what direction you deform them.
  - They have 2 unique elastic constants, C_{11} and C_{12}
- Anisotropic materials behave differently in different directions
  - The elasticity tensor can have 21 unique components defining anisotropy
  - Cubic structured materials have 3 unique elastic constants (UO_2)
  - Hexagonal structured materials have 5 unique elastic constants
- Polycrystalline anisotropic materials can behave as isotropic, because the various grains average out

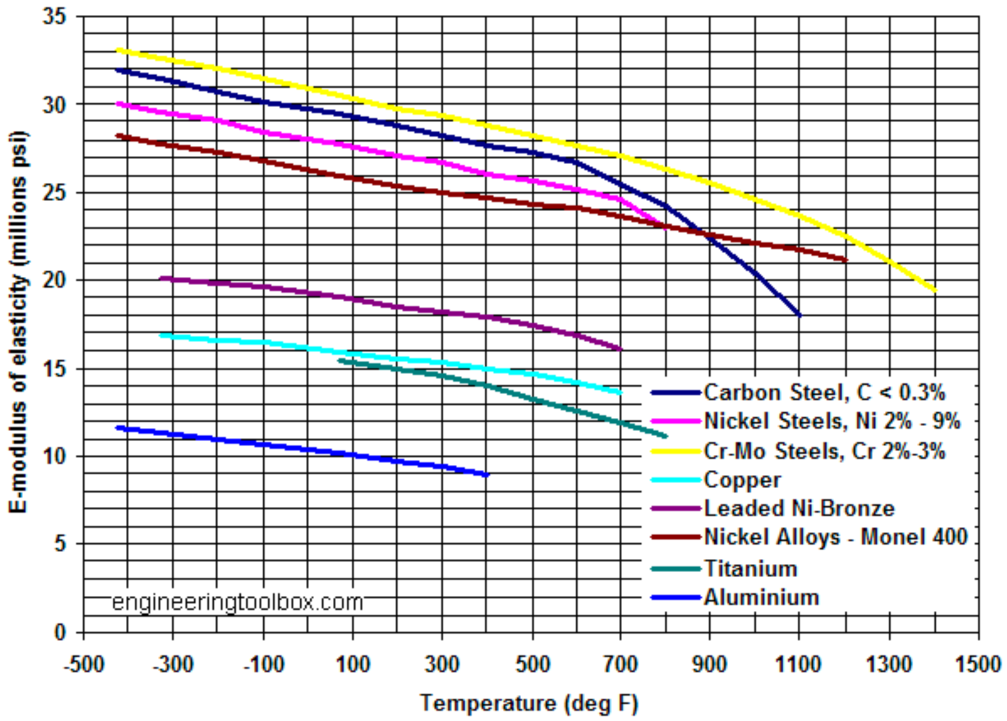
$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

Isotropic elastic constants for some materials

Material	E (Gpa)	ν
Aluminum	70.3	0.345
Gold	78.0	0.44
Iron	211.4	0.293
Nickel	199.5	0.312
Tungsten	411.0	0.28
Zircaloy	80.0	0.41
UO_2	200.0	0.345

Elastic constants are not "constant"

- Properties change with temperature
- Softening can be referred to as the decrease in elastic constants with temperature
- Young's modulus is typically a function of temperature, decreasing with increasing temperature
- Shear Modulus and Poisson's ratio can also change with T



Summary

- Solid mechanics predicts the deformation of a body from its applied load
- The strain defines the deformation
- The stress defines the material's response to the strain
- Materials can have recoverable and permanent deformation
  - Elastic deformation is recoverable and occurs due to the stretching of the atomic lattice
  - Plastic deformation is permanent and results form the breaking of bonds