KEY

Relevant information:
Thermal neutron cross section for U-235: 570 barns
1 eV = 1.602E-19 J
Density of UO2: 10.97 g/cc
Si (Z=14, A=28)
Pay attention to units!

Total 105 pts

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Explain the difference between fertile, fissile and fissionable. 5 pts

fertile: can absorb a neutron to become a fissile nuclide

fissile: can readily undergo fission w/ neutron absorption

fissionable: can undergo fission w/ absorption of high E neutron

What is the secondary fissile element in typical commercial fuel? How is it formed? 4 pts

$^{239}_{94}Pu$ $\quad n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U \xrightarrow{\beta} {}^{239}_{93}Np \xrightarrow{\beta} {}^{239}_{94}Pu$

What does TRISO stand for? Identify the layers in a TRISO particle. Provide an example of a reactor that utilizes TRISO-based fuel. 5 pts

Tri structural Isotropic

high temperature gas reactor

Kernel
buffer
IPyC
SiC
OPyC

What is the compound that uranium is converted into for enrichment purposes? 2 pts

$UF_6$

Which noble gas has a lower thermal conductivity, He or Xe? 2 pts

Xe

Outline and describe the entire fabrication process of the fuel. 10 pts

mining → conversion → enrichment → pellets → rods

- U ore is mined & processed to make $U_3O_8$
- $U_3O_8$ is converted to $UF_6$
- $UF_6$ is utilized to increase $U^{235}$ fraction
- $UF_6$ converted back to $UO_2$, $UO_2$ powder compacted & sintered into pellets
- $UO_2$ pellets assembled into fuel rods/elements

Name two primary fission product species. Provide justification. 5 pts

Mo + Xe

Double hump fission product yield with peaks @ A = 90 + A = 135

What is the role of cladding? 2 pts

primary containment of the fuel

What are the three ways that space is discretized for numerical solutions? 4 pts

finite difference
finite volume
finite element

$U_3Si_5$ is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m-K), an enrichment of 5% and a density of 8.97 g/cm³. Answer the following questions:

10 pts

What is the heat generation rate for $U_3Si_5$ given a neutron flux of 3E13 n/cm²-s?

$$Q = \sigma N_f \phi E_f \qquad N_f = 8.97 \ g/cc \ \frac{1 \ mol}{853.55 \ g} \ \frac{6.022\times10^{23}}{1 \ mol} \ \frac{3U}{1 \ U_3Si_5} \times 0.05 = 9.5\times10^{20} \ \frac{U^{235}}{cm^3}$$

mass $U = 0.05 \times 235 + 0.95 \times 238 = 237.85$

mass $U_3Si_5 = 3(237.85) + 5(28) = 853.55$

$$Q = (570\times10^{-24} \ cm^2)\left(9.5\times10^{20} \ \frac{U^{235}}{cm^3}\right)\left(3\times10^{13} \ \frac{n}{cm^2\text{-}s}\right)(200\times10^{6} \ eV)\left(1.602\times10^{-19} \ \frac{J}{eV}\right)$$

$$Q = 520 \ \frac{J}{s\text{-}cm^3} = \boxed{520 \ W/cm^3}$$

Given a fuel radius of 0.45 cm, what is the temperature drop over the fuel pellet? 6 pts

$$T_o - T_f = \frac{LHR}{4\pi k_f} \qquad LHR = Q\pi R_f^2 = (520)(\pi)(0.45)^2 = 331 \ W/cm$$

$$T_o - T_f = \frac{331}{4\pi(0.125)} = \boxed{211 \ K}$$

$k = 12.5 \ \frac{W}{m\text{-}K}$

$k = 0.125 \ W/cm\text{-}K$

What enrichment of $UO_2$ would be required to obtain the same heat generation rate? 10 pts

$$N_f = 9.5\times10^{20} \ {}^{235}U/cm^3 \qquad \text{mass } U = 235x + (1-x)238 = 238 - 3x$$

$$\text{mass } UO_2 = 238 - 3x + 2(16) = 270 - 3x$$

$$9.5\times10^{20} \ {}^{235}U/cc = 10.97 \ g/cc \ \frac{1 \ mol}{270-3x \ g} \ \frac{6.022\times10^{23}}{1 \ mol} \ \frac{1U}{1UO_2} \times X$$

$$1.44\times10^{-4} = \frac{X}{270-3x} \rightarrow 0.0388 - 4.3\times10^{-4}x = X$$

$$X = 0.0388 \rightarrow \boxed{3.9\%}$$

Which coolant sees a larger change in outlet to inlet temperature? 1) water: $C_P$ = 4200 J/kg-K, mdot = 0.25 kg/s-rod, $Z_0$ = 1.5 m; 2) sodium: $C_P$=1404 J/kg-K, mdot = 1.2 kg/s/rod, $Z_0$=0.5 m. 10 pts

$$T_{cool} - T_{cool}^{in} = \frac{1}{1.2} \frac{Z_0 LHR^0}{\dot{m} C_p} \left\{ \sin(1.2) + \sin\left[1.2\left(\frac{z}{Z_0} - 1\right)\right] \right\}$$

$z = 2Z_0$ → convert to cm

1) $$\Delta T_{cool} = \frac{1}{1.2} \frac{150\ LHR^0}{(0.25)(4200)} \left\{ \sin(1.2) + \sin(1.2) \right\}$$

$$\Delta T_{cool} = 0.22\ LHR^0$$

2) $$\Delta T_{cool} = \frac{1}{1.2} \frac{50\ LHR^0}{(1.2)(1404)} \left( \sin(1.2) + \sin(1.2) \right)$$

$$\Delta T_{cool} = 0.046\ LHR^0$$

#1

Given a rod of 2 m in length and an $LHR^0$ = 150 W/cm, what is the LHR at z=1.6 m? 6 pts

$Z_0 = 1$ $\quad LHR(z=1.6)$

$$LHR\left(\frac{z}{Z_0}\right) = LHR^0 \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{Z_0} - 1\right)\right]$$

$$LHR = 150\ \frac{W}{cm} \cos\left[\frac{\pi}{2 \times 1.3}\left(\frac{1.6}{1} - 1\right)\right]$$

$$\boxed{LHR = 112\ W/cm}$$

Perform a forward Euler time stepping to approximate the following function: $y(t) = e^{-2t}$ 12 pts
Compute with timesteps dt=0.5 and dt=0.25, expanding to $t_n$=1.

$y(t) = e^{-2t}$ $y'(t) = -2e^{-2t}$

$y(0) = 1$ $dt = 0.5, 0.25$

$y_{n+1} = y_n + h y_n'$

$dt = 0.5$

$t_0 = 0$

$t_1 = 0.5$

$t_2 = 1$

$y_1 = y_0 + h y_0' = 1 + (0.5)(-2e^{-2(0)}) = 0$

$y_2 = y_1 + h y_1' = 0 + (0.5)(-2e^{-2(0.5)}) = -0.37$

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$dt = 0.25$

$t_0 = 0$

$t_1 = 0.25$

$t_2 = 0.5$

$t_3 = 0.75$

$t_4 = 1$

$y_1 = y_0 + h y_0' = 1 + (0.25)(-2e^{-2(0)}) = 0.5$

$y_2 = y_1 + h y_1' = 0.5 + (0.25)(-2e^{-2(0.25)}) = 0.2$

$y_3 = y_2 + h y_2' = 0.2 + (0.25)(-2e^{-2(0.5)}) = 0.02$

$y_4 = y_3 + h y_3' = -0.02 + (0.25)(-2e^{-2(0.75)}) = -0.92$

Consider a metallic (UZr) fuel slug with a radius of 0.3 cm, a sodium gap of 0.1 cm, an HT9 cladding thickness of 0.05 cm. UZr thermal conductivity = 0.22 W/cm-K; Na thermal conductivity = 0.5 W/cm-K; $h_{cool}$ = 5.5 W/cm-K; HT9 thermal conductivity = 20 W/m-K; $T_{cool}$= 400 K; Q = 550 W/cm$^3$ 12 pts

What is the temperature at r=0.15 cm?

$$LHR = Q \pi R_F^2 = (550)(\pi)(0.3^2) = 155 \ W/cm$$ ✓

$$T_{clad} - T_{cool} = \frac{LHR}{2\pi R_F h_{cool}} = \frac{155}{2\pi (0.3)(5.5)} = 15 \ K$$ ✓ $\quad \underline{T_{clad} = 415 \ K}$

$$T_{gap} - T_{clad} = \frac{LHR}{2\pi R_F h_{clad}} = \frac{155}{2\pi (0.3)} \frac{0.05}{0.2} = 20.6 \ K \qquad \underline{T_{gap} = 435.6 \ K}$$

$$T_{fuel} - T_{gap} = \frac{LHR}{2\pi R_F h_{gap}} = \frac{155}{2\pi (0.3)} \frac{0.1}{0.5} = 16.4 \ K \qquad \underline{T_{fuel} = 452 \ K}$$

$$T(r) - T_F = \frac{LHR}{4\pi k}\left(1 - \frac{r^2}{R_F^2}\right)$$

$$T(0.15) = \frac{155}{4\pi(0.22)}\left(1 - \frac{0.15^2}{0.3^2}\right) + 452$$

$$T(0.15) = 494 \ K$$