

Nuclear Fuel Performance

NE 533

Exam 2 Solutions

$$1) \quad R_f = 0.45 \text{ cm} \quad LHR: 250 \text{ W/cm}$$

$$\rightarrow \text{max stress?} \rightarrow \sigma_\theta$$

$$\sigma_\theta = -\sigma^* (1 - 3\eta^2)$$

$$\sigma^* = \frac{\alpha E \Delta T}{4(1-\nu)}$$

$$R_f = 0.1 \text{ W/cm} \cdot \text{K} \quad E = 290 \text{ GPa} \quad \nu = 0.25 \quad \alpha = 12 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

$$\text{max @ } r = R_f \rightarrow \eta = \frac{r}{R_f} = 1$$

$$\Delta T = \frac{LHR}{4\pi R_f} = \frac{250}{4\pi (0.1)} = 199 \text{ K}$$

$$\sigma^* = \frac{(12 \times 10^{-6})(290 \times 10^9)(199)}{4(1-0.25)}$$

$$\sigma^* = 231 \text{ MPa}$$

$$\sigma_\theta = -231 (1 - 3(1)^2)$$

$$= \underline{462 \text{ MPa}}$$

$$b) \quad \tau_{fr} = 180 \text{ MPa}$$

$$\sigma_r = -\sigma^* (1 - 3\eta^2)$$

$$180 = -231 (1 - 3\eta^2)$$

$$\eta^* = 0.506 \quad \eta = 0.71 \rightarrow 29\% \text{ into the fuel}$$

$$2) \quad p = 25 \text{ MPa} \quad \bar{r}_c = 0.50 \text{ cm} \quad t_c = 0.08 \text{ cm} \\ R_i = 0.48 \quad R_o = 0.54$$

$$a) \quad \sigma_\theta = \frac{p R}{\delta} \quad \sigma_z = \frac{p R}{2\delta} \quad \sigma_r = \frac{p}{2}$$

$$\sigma_\theta = \frac{25 (0.50)}{0.08} = \underline{162.5 \text{ MPa}} \quad \sigma_z = \frac{\sigma_\theta}{2} = \underline{81.25 \text{ MPa}}$$

$$\sigma_r = \frac{-25}{2} = \underline{-12.5 \text{ MPa}}$$

$$b) \quad r = 0.5 \text{ cm}$$

$$\sigma_\theta = p \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$$\sigma_z = \frac{p}{\left(\frac{R_o}{R_i}\right)^2 + 1}$$

$$\sigma_r = p \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$$\frac{R_o}{r} = \frac{0.54}{0.5} = 1.08 \quad \frac{R_o}{R_i} = \frac{0.54}{0.48} = 1.125$$

$$\sigma_\theta = 25 \frac{(1.08)^2 + 1}{1.125^2 - 1} = \underline{156 \text{ MPa}} \quad \sigma_z = \frac{25}{1.125^2 - 1} = \underline{69.1 \text{ MPa}}$$

$$\sigma_r = 25 \frac{1.08^2 - 1}{1.125^2 - 1} = \underline{17.6 \text{ MPa}}$$

$$c) \quad \epsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu (\sigma_r + \sigma_z)) = \frac{1}{160 \times 10^3} (156 - 0.3 (17.6 + 69.1)) \\ = \underline{8.12 \times 10^{-4}}$$

$$3) \Delta t_g = \bar{R}_c \alpha_c \Delta T_c - R_f \alpha_f \Delta T_f$$

$$R_f = 0.52 \text{ cm} \quad b_g = 0.005 \text{ cm} \quad T_c = 550 \text{ K} \quad t_c = 0.08 \text{ cm} \quad K_f = 0.04 \text{ W/cmK}$$

$$K_g = 0.003 \text{ W/cmK} \quad k_c = 0.15 \text{ W/cmK} \quad LHR = 400 \text{ W/cm} \quad \alpha_c = 12 \pi 10^{-6}$$

$$\alpha_f = 8 \pi 10^{-6}$$

$$\Delta T_c = \frac{LHR}{2\pi R_f} \frac{t_c}{k_c} = \frac{400}{2\pi(0.52)} \frac{0.08}{0.15} = 65 \text{ K}$$

$$T_{c1} = 615 \text{ K}$$

$$\Delta T_g = \frac{LHR}{2\pi R_f} \frac{b_g}{K_g} = \frac{400}{2\pi(0.52)} \frac{0.005}{0.003} = 204 \text{ K}$$

$$T_s = 819 \text{ K}$$

$$\Delta T_f = \frac{LHR}{4\pi K_f} = \frac{400}{4\pi(0.04)} = 796 \text{ K}$$

$$T_o = 1615 \text{ K}$$

$$\bar{T}_c = \frac{T_c + T_{c1}}{2} = \frac{550 + 615}{2} = 582.5 \text{ K}$$

$$\bar{T}_f = \frac{T_o + T_s}{2} = \frac{819 + 1615}{2} = 1217 \text{ K}$$

$$\bar{R}_c = 0.52 + 0.005 + 0.08 = 0.565 \text{ cm}$$

$$\Delta t_g = 0.565 (12 \pi 10^{-6}) (582.5 - 300) - 0.52 (8 \pi 10^{-6}) (1217 - 300)$$

$$\Delta t_g = 0.0019 - 0.0028 = -0.0009$$

$$b_g = 0.005 - 0.0019 = \underline{0.003 \text{ cm}}$$

4) Forward Euler

$$\frac{dy}{dt} = -5y \quad \Delta t = 0.25 \quad t_0 = 0 \quad y_0 = 1$$

$$y_1 = y_0 + \Delta t y'_0$$

$$t_1 = 0.25 \quad y_1 = 1 + 0.25 (-5(1)) = -0.25$$

$$\begin{aligned} t_2 = 0.5 \quad y_2 &= y_1 + \Delta t y'_1 \\ &= -0.25 + 0.25 (-5(-0.25)) \\ &= 0.0625 \end{aligned}$$

$$\begin{aligned} t_3 = 0.75 \quad y_3 &= y_2 + \Delta t y'_2 \\ &= 0.0625 + 0.25 (-5(0.0625)) \\ &= -0.0156 \end{aligned}$$

$$\begin{aligned} t_4 = 1 \quad y_4 &= y_3 + \Delta t y'_3 \\ &= -0.0156 + 0.25 (-5(-0.0156)) \\ &= 0.0039 \end{aligned}$$

5) max σ_θ ?

$\Delta T_c = 250 \text{ K}$ $\alpha_c = 9 \times 10^{-6} / \text{K}$ $E = 250 \text{ GPa}$ $\nu = 0.3$

$b_c = 0.1$ $R_i = 0.55 \text{ cm}$

σ_θ max @ $r = R_i$

$$\sigma_\theta = \frac{1}{2} \Delta T \frac{\alpha E}{1-\nu} \left(1 - \frac{2R_i}{\delta} \left(\frac{r}{R_i} - 1 \right) \right)$$

$$= \frac{1}{2} \frac{(250) (9 \times 10^{-6}) (250 \times 10^9)}{1-0.3} \left(1 - \frac{2(0.55)}{0.1} \left(\frac{0.55}{0.55} - 1 \right) \right)$$

$\sigma_\theta = 357 \text{ MPa}$

$\sigma_\theta = 0$ @ $r = R_i + \frac{\delta}{2}$

$$0 = \left(1 - \frac{2R_i}{\delta} \left(\frac{r}{R_i} - 1 \right) \right)$$

$$1 = \frac{2r}{\delta} - \frac{2R_i}{\delta} \rightarrow \frac{2r}{0.1} - \frac{2(0.55)}{0.1} = 1$$

$$r = 0.6 \text{ cm} = R_i + \frac{\delta}{2}$$

6) finite difference : fast, easy to implement

finite volume : continuous, flexible, flux B.C.s

finite element : any B.C.s, continuous, computationally expensive

7) Increase in yield strength due to plastic deformation. Plasticity induces dislocation generation and diffusion, but these dislocations interact and repel one another. With an increase in plastic deformation, there are more barriers to allow further plastic deformation to take place, thus the yield strength increases.

8) Model the temperature in the fuel, describe temperature and stress in the cladding, handle gap heat transport & closure.

9) 0-0 : vacancy, interstitial, substitutional

3-0 : bubble, precipitate

10) Utilizing the underlying microstructure to guide the evolution of the state variables and material properties of the system. Removes the dependence upon empirical correlations that only account for burnup, and instead relate the actual changes in the material to the evolution of material quantities. Allows for predictive behavior of fuel performance outside of known experimental conditions. Improves descriptive performance.

11) Forster gas bubbles are present in the HDS, which is a nano-grained structure on the periphery of the fuel. Under a transient, temperature spikes, leading to increased pressure in bubbles, which can induce intergranular fracture of the HDS. Phase field and multi-scale modeling are exploring the interactions of microstructure and material properties on pulverization.

12) Anything that can be seen with $\sim 25\times$ magnification. Includes grain structure, second phases, precipitates, voids, etc. Processing, such as cold working or annealing, can refine or homogenize the microstructure, leading to changes in macro scale properties.