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3-2-17

68/100

NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

Question 1: -8, 22/30

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm³. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

Uranium-235, 0.7 w/o ^{235}U

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm² s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

U_3Si_2

$$Q = E_f N_f \sigma_f \phi_{th}$$

$$N_f = \frac{\rho_f N_a}{M_u}$$

$$M_u = (0.03)(235) + (1-0.03)(238) = 237.91\ \text{g/mol}$$

$$N_{U235} = \frac{\rho_{U_3Si_2} N_a}{M_u} = \frac{(11.31\ \text{g/cm}^3)(6.022\times10^{23})}{237.91\ \text{g/mol}} =$$

$$N_{U235} = 2.863\times10^{22}$$

$$Q = (3\times10^{-11}\ \text{J/fission})(2.863\times10^{22})(5.5\times10^{-22}\ \text{cm}^{-2})(3.2\times10^{13}\ \text{n/cm}^2\text{s})$$

-3, Just use U density

$$Q = 15{,}116.64\ \text{W/cm}^3$$

$$N_{U235} = \frac{q N_a \delta}{M_{U_3Si_5}}$$

$$q = \frac{N_{U235}\cdot M_{U_3Si_5}}{N_a \delta}$$

$$q_{U_3Si_5} = \frac{(2.863\times10^{22})(784)}{(6.022\times10^{23})(7.5\ \text{g/cm}^3)}$$

$$\boxed{q_{U_3Si_5} = 4.97\ \text{w/o}\ ^{235}U}$$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

I would rank U_3Si_2 as a better fuel than U_3Si_5 if we are only considering the uranium density & enrichment, because U_3Si_2 has a higher uranium density than U_3Si_5 and requires a lower enrichment to get the same amount of energy.

-3, Thermal conductivity?

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-6, 29/35

Question 2: $0.0080 \times 10^{-6} = 0.0080$

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{co} = \frac{Q}{2h_{cool}} R_f + T_{cool} = \frac{250\ W/cm}{2(2.5\ W/cm^2\cdot k)} \cdot 0.45\ cm + 580k = 602.5k$$

-4, You used LHR for Q

$$T_{ci} = \frac{Q}{2k_c} R_f t_c + T_{co} = \frac{250\ W/cm}{2(0.17\ W/cm\cdot k)} \cdot 0.45\ cm \cdot 0.06\ cm + 602.5k = 622.4k$$

$$T_s = \frac{Q}{2h_g} R_f + T_{ci} = \frac{250}{2(0.275)} \cdot 0.45 + 622.4k = \boxed{826.9\ k = T_s}$$

$h_g = \frac{0.0022}{0.008}$

$h_g = 0.275$

$$h_g = k_g/t_g, \quad k_g = k_{he}^{1-y} \cdot k_{xe}^{y} = (16\times10^{-6} \cdot 622.4^{0.79})^{1-0.05} \cdot (0.7\times10^{-6} \cdot 622.4^{0.79})^{0.05} = 0.0022$$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$r = R_f$

-2, You used LHR for Q

$$\sigma_{\theta\theta} = -\sigma^* [1 - 3\eta^2]$$

$$\eta = r/R_f = 1 \text{ (for max stress)}$$

$$\sigma^* = \frac{\alpha E (T_m - T_s)}{4(1-\nu)} = \frac{(7.5e^{-6}\ 1/k)(246.7\times10^9)(63.3)}{4(1-0.25)}$$

$$\sigma^* = 39\ MPa$$

$$\sigma_{\theta\theta} = -39\times10^6[1 - 3(1)^2] = \boxed{78\ MPa}$$

$$T_m = \frac{Q}{4k_f} R_f^2 + T_s$$

$$T_m = \frac{250}{4(0.20)} 0.45^2 + 826.9k$$

$$T_m = 890.2$$

c) Would you expect this stress to be higher or lower if the pellet was UO2? Why? (5 points) I would expect the stress to be higher in UO2 because it has a lower thermal conductivity than UN. This means there is a larger temperature difference between the center of the fuel pellet & the surface which would increase the stress.

d) What assumptions were made in your calculations for a) and b)? (5 points)

assumptions:

-1, Not an assumption, this is true

1. hoop stress is largest stress
2. $\eta = 1$ because stress is greatest when $r = R_f$

-1, Not an assumption, this is true

3. isotropic material response
4. small strains
5. axisymmetric body

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Question 3: -17, 18/35

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

① static body
② gravity is negligible
③ axisymmetric body
④ small strain
⑤ isotropic material response

-3, Stress is constant across thickness

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\sigma_\theta = \frac{PR}{\delta} = \frac{(6\times10^6 Pa)(0.56 cm)}{0.06 cm} = \boxed{56 MPa}$$

$$\sigma_z = \frac{PR}{2\delta} = \frac{(6\times10^6 Pa)(0.56 cm)}{2(0.06 cm)} = \boxed{28 MPa}$$

$$\sigma_r = -\tfrac{1}{2}P = -0.5(6.0\times10^6 Pa) = \boxed{-3 MPa}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

-4, Calculate stress at two radii and compare to see if stress is constant

$$\sigma_{\theta\theta} = P\frac{\left(\frac{R_o}{r}\right)^2+1}{\left(\frac{R_o}{R_i}\right)^2-1} = (6\times10^6 Pa)\frac{\left(\frac{0.62}{0.56}\right)^2+1}{\left(\frac{0.62}{0.56}\right)^2-1} = \frac{6\times10^6 Pa\,(2.226)}{0.226}$$

$$\sigma_{\theta\theta} = 59.1 MPa$$

$$\text{error} = \left|\frac{56 MPa - 59.1 MPa}{59.1 MPa}\right| \times 100 = \boxed{5.25\% \text{ error}}$$

The thin walled approximation would not be conservative if used to estimate (when the cladding would fail.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

-4, Write stress and strain in tensor form
-4, Calculate strain from stress from part b

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{zz}\\ \sigma_{\theta\theta}\\ \sigma_{rz}\end{bmatrix} = 8.41\times10^{10}\begin{bmatrix}0.59 & 0.41 & 0.41 & 0\\ 0.41 & 0.59 & 0.41 & 0\\ 0.41 & 0.41 & 0.59 & 0\\ 0 & 0 & 0 & 0.09\end{bmatrix}\begin{bmatrix}u_{r,r}\\ u_{z,z}\\ u_r/r\\ (u_{r,z}+u_{z,r})/2\end{bmatrix}$$