NE 591 Problem Session

1) Max stress in a fuel pellet due to thermal exp.?

$\Delta T = T_0 - T_s = 428 \text{ K}$ $\qquad \alpha_F = 12 \times 10^{-6} \ 1/K$

$R_F = 0.5 \text{ cm} \qquad E_f = 180 \text{ GPa} \qquad \nu = 0.28$

$$\sigma_r = -\sigma^* (1 - \eta^2)$$

$$\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$$

$$\sigma_\theta = -\sigma^* (1 - 3\eta^2)$$

$$\eta = \frac{r}{R_F}$$

$$\sigma_z = -2\sigma^* (1 - 2\eta^2)$$

max

σ_θ $\quad \sigma^* = \dfrac{(12\times10^{-6})(180\times10^{3})(425)}{4(1-0.28)} = 159.4\ \text{MPa}$

$\sigma_\theta\,(\text{max}) \rightarrow r = R_p \rightarrow \eta = 1$

$\sigma_\theta = -159.4\,(1-3(1)^2) = \underline{319\ \text{MPa}}$

2) Zircaloy cladding w/ $r_i = 0.5$ cm $r_o = 0.6$ cm

$p = 25$ MPa

$r_{avg} = 0.55$ cm

∃ stresses in thin-walled assumption?

$$\sigma_\theta = \frac{pR}{\delta} \qquad \sigma_r = \frac{-p}{2} \qquad \sigma_z = \frac{pR}{2\delta}$$

$$\sigma_\theta = \frac{(25)(0.55)}{0.1} = 137.5 \text{ MPa}$$

$$\sigma_z = \frac{\sigma_\theta}{2} = 68.75 \text{ MPa}$$

$$\sigma_r = -\frac{(25)}{2} = -12.5 \text{ MPa}$$

→ thick-walled assumption

→ stress states @ inner wall?

$$\sigma_r = -p \frac{\left(\frac{R_o}{r}\right)^2 - 1}{\left(\frac{R_o}{R_i}\right)^2 - 1} \qquad \sigma_\theta = p \frac{\left(\frac{R_o}{r}\right)^2 + 1}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

$$\sigma_z = \frac{p}{\left(\frac{R_o}{R_i}\right)^2 - 1} \qquad \frac{R_o}{R_i} = \frac{0.6}{0.5} = 1.2 \qquad r = R_i$$

$$\sigma_r = -(25)\frac{1.2^2 - 1}{1.2^2 - 1} = -25\ MPa$$

$$\sigma_\theta = 25 \frac{1.2^2+1}{1.2^2-1} = \underline{138.6 \text{ MPa}}$$

$$\sigma_z = \frac{25}{1.2^2-1} \rightarrow \underline{56.8 \text{ MPa}}$$

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thick vs thin

$$\sigma_\theta(R_i) = \sigma_\theta(\text{thin}) \qquad \sigma_r(R_i) = 2\,\sigma_r(\text{thin})$$

$$\sigma_z(R_i) \approx 1.2\,\sigma_z(\text{thin})$$

3) Gap thickness change

$\alpha_F = 12 \times 10^{-6}$ 1/K $\qquad \alpha_C = 5 \times 10^{-6}$ 1/K $\qquad T_F^{avg} = 925\,K$

$T_C^{avg} = 550\,K$

$T_0(\text{fuel}) = T_0(\text{clad}) = 300\,K$

$R_F = 0.5$ cm $\qquad R_C^{avg} = 0.58$ cm $\qquad t_c = 0.1$ cm

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$t_g^0 = 0.03$ cm

$$\Delta t_{gap} = R_c \alpha_c \left(T_C^{avg} - T_0^{clad}\right) - R_F T_F^{avg} \left(T_F^{avg} - T_0^{f}\right)$$

$$\Delta t_c = (0.58)(5 \times 10^{-6})(550 - 300) = 7.25 \times 10^{-4}$$

$$\Delta t_f = (0.5)(12\times10^{-6})(925-300) = -3.75\times10^{-3}$$

$$t_{gap} = 0.03\text{ cm} + 7.25\times10^{-4} - 3.75\times10^{-3}$$

$$= 0.0297\text{ cm}$$

$$T_f^{avg} \rightarrow \Delta t_g \longrightarrow \Delta T_f^{avg}$$

4) Cladding tube w/ no internal pressure but w/ thermal expansion

$\rightarrow \frac{r}{R_i} = 1.1$, σ_θ, σ_r, σ_z ?

$R_i = 0.6$ cm $\quad t_{cl} = 0.1$ cm $\quad E = 250$ GPa $\quad \nu = 0.3$

$\alpha_c = 15\times10^{-6}$ 1/K $\quad T_c^{avg} = 600$ K $\quad T_c^{0} = 300$ K

$$\sigma_r = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left[\left(\frac{r}{R_i} - 1\right)\left(1 - \frac{R_i}{t_c}\left(\frac{r}{R_i} - 1\right)\right)\right]$$

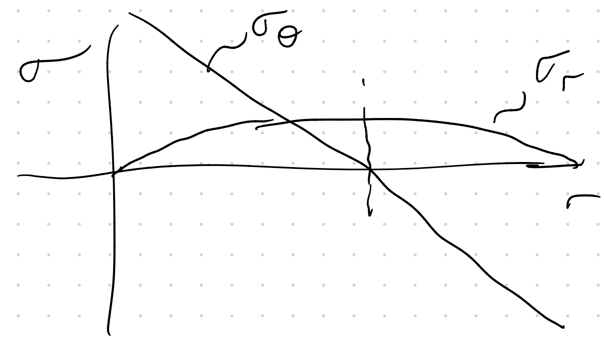
$$\sigma_\theta = \frac{\Delta T}{2}\frac{\alpha E}{1-\nu}\left(\left(1 - 2\frac{R_i}{t_c}\left(\frac{r}{R_i} - 1\right)\right)\right) = \sigma_z$$

$$\sigma_r = \frac{300}{2}\,\frac{(15\times10^{-6})(250\times10^{3})}{1-0.3}\left(1.1-1\right)\left(1-\frac{0.6}{0.1}\left(1.1-1\right)\right)$$

$$\sigma_r = 64 \text{ MPa}$$

$$\sigma_\theta = \frac{300}{2}\,\frac{(15\times10^{-6})(250\times10^{3})}{1-0.3}\left(1-2\frac{0.6}{0.1}\left(1.1-1\right)\right)$$

$$\sigma_\theta = \sigma_z = -161 \text{ MPa}$$



5) Densification strain? @ $\beta = 2 \text{WD}/\text{kg U}$ $T = 900 \text{ K}$

max densification : 2% $\beta_D = 0.005$ FIMA

$$\epsilon_D = \Delta\rho_0 \left(e^{\frac{\beta \ln 0.01}{C_D \beta_D}} - 1 \right)$$

β convert to FIMA $\rightarrow$ $\frac{2 \text{ MWD/kg U}}{950}$ $\rightarrow$ 0.002 FIMA

$C_D \Rightarrow$ $T < 750\ C$ $\rightarrow$ $7.235 - 0.0086(\overset{T(C^\circ)}{T} - 25)$

$C_D = 2.058$

$$\epsilon_0 = 0.02\left(e^{\frac{(0.002)(\ln 0.01)}{(2.058)(0.005)}} - 1\right)$$

$$\epsilon_g = -0.012$$

$$\frac{0.012}{0.02} \rightarrow \underline{60\%}$$

6) Given only strains, calculate each stress ...

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right)$$

$$\epsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right)$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right)$$

$$\epsilon_z E = \sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)$$

$$\sigma_z = \epsilon_z E + \nu\sigma_r + \nu\sigma_\theta$$

$$\epsilon_\theta E = \sigma_\theta - \nu\sigma_r - \nu\sigma_z$$

$$\epsilon_\theta E = \sigma_\theta - \nu\sigma_r - \nu\left(\epsilon_z E + \nu\sigma_r + \nu\sigma_\theta\right)$$

$$\epsilon_\theta E = \left(\sigma_\theta\right) - \nu\sigma_r - \nu\epsilon_z E - \nu^2\sigma_r \left(- \nu^2\sigma_\theta\right)$$

$$\underline{\sigma_\theta = \frac{\nu^2\sigma_r + \nu\sigma_r + \nu E\epsilon_z + \epsilon_\theta E}{1-\nu^2}}$$

$$E\epsilon_r = \sigma_r - \nu\sigma_\theta - \nu\sigma_z$$

can continue...