NUCE497I EXAM#1 Solution

**1. (40) Problem 1:** Temperature fields in fresh and irradiated fuel

Consider two conditions for heat transfer in the pellet and the pellet–cladding gap of a BWR fuel pin:

– Initial uncracked pellet with no relocation.
– Cracked and relocated fuel.

1. For each combination, find the temperatures at the cladding inner surface, the pellet outer surface, and the pellet centerline.

2. Find for each case the temperature drop across the pellet ($T_{cl} - T_{surf}$).

*Geometry and material information:*

Cladding outside diameter = 11.20 mm
Cladding thickness = 0.71 mm
Fuel–cladding gap thickness = 180 μm
Initial solid pellet with density = 88%

*Basis for heat transfer calculations:*

Cladding conductivity is constant at 17 W/m K
Gap conductance:
Without fuel relocation - 4300 W/m$^2$ K;
With fuel relocation - 31,000 W/m$^2$ K;
Fuel conductivity (average) at 95% density:
Uncracked - 2.7 W/m K,
Cracked - 2.4 W/m K;
Volumetric heat deposition rate:
uniform in the fuel
zero in the cladding.
Do not adjust the pellet conductivity for restructuring.
Use Biancharia's porosity correction factor:

$k = \frac{(1-P)}{1+0.5P} k_{TD}$, where the porosity is defined as $P = 1 - \frac{\rho}{\rho_{TD}}$ and $\rho_{TD}$ is the theoretical density of the poreless solid.

*Operating conditions:*

Cladding outside temperature = 295°C
Linear heat generation rate = 44 kW/m

Solution:

1. **Using Biancharia's porosity correction factor, determine theoretical and 88% density fuel conductivities using 95% density fuel conductivities given.**

$$k = \frac{1-P}{1+0.5P} k_{TD} \qquad where \qquad P = 1 - \frac{\rho}{\rho_{TD}}$$

First determine cracked and uncracked theoretical conductivities (100% density).

$$k_{TD} = \frac{1+0.5P}{1-P} k_{.95} \quad where \;\; P = 1 - \frac{95\%}{100\%} = 0.05$$

$$\therefore k_{TD}^{cracked} = \frac{1+0.5*0.05}{1-0.05} \times 2.4 = 2.589 \frac{W}{m\ K}$$

$$\therefore k_{TD}^{uncracked} = \frac{1+0.5*0.05}{1-0.05} \times 2.7 = 2.913 \frac{W}{m\ K}$$

Determine uncracked and cracked conductivities (88% density).

$$k_{0.88} = \frac{1-P}{1+0.5P} k_{TD} \quad where \;\; P = 1 - \frac{88\%}{100\%} = 0.12$$

$$\therefore k_{0.88}^{uncracked} = \frac{1-0.12}{1+0.5*0.12} \times 2.913 = \mathbf{2.418} \frac{\boldsymbol{W}}{\boldsymbol{m\ K}}$$

$$.\therefore k_{0.88}^{cracked} = \frac{1-0.12}{1+0.5*0.12} \times 2.589 = \mathbf{2.149} \frac{\boldsymbol{W}}{\boldsymbol{m\ K}}$$

2. **Geometrical constants**

$$R_{co} = \frac{D_{co}}{2} = 0.0056\ m$$

$$R_{ci} = R_{co} - \Delta x_{clad} = 0.00489\ m$$

$$R_{fo} = R_{co} - \Delta x_{gap} = 0.00471\ m$$

$$R_{gap} = \frac{R_{fo} + R_{ci}}{2} = 0.0048\ m$$

3. **Thermal resistances across each surface**

Across cylindrical fuel pellet:

$$RES_{fuel}^{cracked} = \frac{1}{4\pi k_{.88}^{cracked}} = \mathbf{0.0370} \left(\frac{m\ K}{W}\right)$$

$$RES_{fuel}^{uncracked} = \frac{1}{4\pi k_{.88}^{uncracked}} = \mathbf{0.0329} \left(\frac{m\ K}{W}\right)$$

Across gas gap region:

$$RES_{gap}^{relocated} = \frac{1}{2\pi h_{gap}^{relocation}} \times \frac{1}{R_{gap}} = \mathbf{0.001069} \left(\frac{m^2\ K}{W}\right)$$

$$RES_{gap}^{no\ reloc} = \frac{1}{2\pi h_{gap}^{no\ reloc}} \times \frac{1}{R_{gap}} = \mathbf{0.007711} \left(\frac{m^2\ K}{W}\right)$$

Across cladding region:

$$RES_{clad} = \frac{1}{2\pi k_{clad}} \ln\frac{R_{co}}{R_{ci}} \quad = \mathbf{0.001269} \left(\frac{m\ K}{W}\right)$$

## 4. Temperature determination

$$\Delta T = q' \times \sum RES$$

Across cladding region:

$$T_{ci} = T_{co} + q' \times RES_{clad} = 568 + 44000 \times\ 0.001269 \ \ = \mathbf{623.8\ K}$$

Across gap region:

$$T_{fo}{}^{cracked} = T_{ci} + q' \times \left(RES_{gap}^{relocated} + RES_{clad}\right) = \mathbf{670.9\ K}$$

$$T_{fo}{}^{uncracked} = T_{ci} + q' \times \left(RES_{gap}^{no\ relocated} + RES_{clad}\right) = \mathbf{963.1\ K}$$

Across fuel region:

$$T_{cl}{}^{cracked} = T_{fo}{}^{cracked} + q' \times \left(RES_{gap}^{relocated} + RES_{fuel}^{cracked} + RES_{clad}\right) = \mathbf{2300.2\ K}$$

$$T_{cntr}{}^{uncracked} = T_{fo}{}^{uncracked} + q' \times \left(RES_{gap}^{no\ relocated} RES_{fuel}^{uncracked} + RES_{clad}\right) = \mathbf{2411.2\ K}$$

$$\Delta T_{fuel}^{cracked} = T_{cntr}{}^{cracked} - T_{fo}{}^{cracked} = 2300.2\ K - 670.9\ K = \mathbf{1629.3\ K}$$

$$\Delta T_{fuel}^{uncracked} = T_{cntr}{}^{uncracked} - T_{fo}{}^{uncracked} = 2411.2\ K - 963.1\ K = \mathbf{1448.1\ K}$$

**Table 4.1** Summary of Fuel Parameters

| | Cracked, relocated | Uncracked, no relocation |
|---|---|---|
| k_fuel (w/mk) | 2.149 | 2.418 |
| h_gap (w/m2k) | 31000 | 4300 |
| res_fuel (m2k/w) | 0.0370 | 0.0329 |
| res_gap (m2k/w) | 0.001069 | 0.007711 |
| res_clad (m2k/w) | 0.001269 | 0.001269 |
| T_ci (K/°C) | *623.8 / 350.85* | *623.8 / 350.85* |
| T_fo (K/°C) | *670.9 / 397.9* | *963.1 / 690.1* |
| T_cntr (K/°C) | *2300.2 / 2027.2* | *2411.2 / 2138.2* |
| deltaT_fuel (K/°C) | *1629.3 / ~~1356.3~~* | *1448.1 / ~~1175.1~~* |

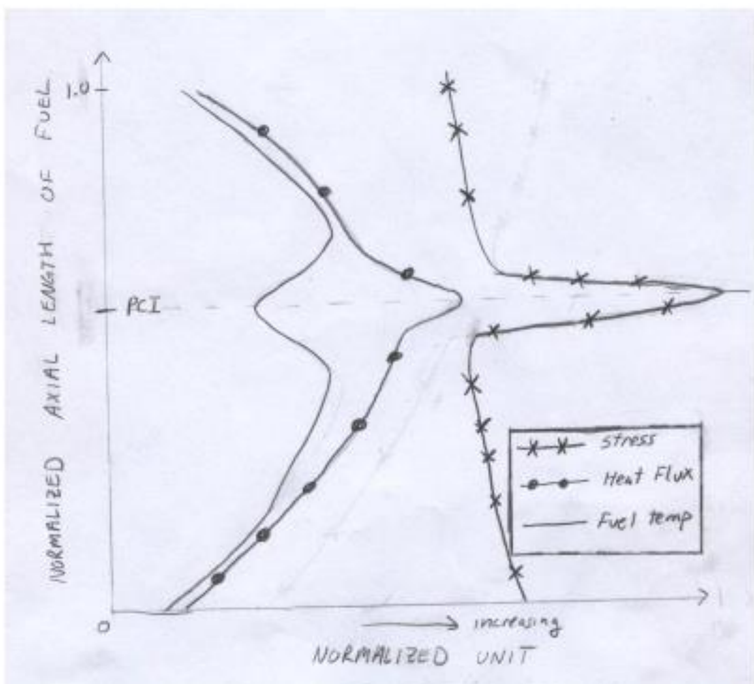
2. **(20) Problem 2:** Pellet-Cladding Contact

Assume that at the mid length of a PWR fuel pin there is a pellet-cladding contact. Sketch the corresponding axial distributions of:

- Fuel centerline temperature;
- Clad stress;
- Surface heat flux.

Solution:

In the event of pellet-cladding interaction (PCI), heat transfer between the fuel surface and inner cladding is increased as the contact serves as a point of conductive heat transfer. This ***increase in surface heat flux*** from conductive heat transfer, relative to convective gap heat transfer, allows heat to more easily propagate away from the fuel and into the cladding material. This mitigation of heat from the pellet ***reduces local fuel centerline temperature*** at the contact point while increasing surface heat flux. The interaction between pellet and cladding ***increases local stress in the interior cladding surface*** at the point of contact, however it should be noted the cladding was under radial stress prior to PCI as the pressure differential between gas gap and coolant created a distributed outward force. As the axial length of the fuel rod is progressed, the differential in internal/external pressures decreases as coolant pressure increases from heating. This decrease in pressure differential causes a slight reduction in pressure between inlet and outlet, neglecting the spike in local pressure at the PCI.



3. **(40) Problem 3:** Zircaloy Corrosion

Zr alloy cladding undergoes corrosion and hydriding when exposed to the reactor environment. According to a simple model the growth of the oxide scale is initially cubic, changing to a linear regime at a critical oxide thickness. A Zircaloy component undergoing corrosion under this model is immersed in water. The fuel cycle is 1 year.

a) The corrosion of a cladding is often well described by the cubic law. According to the oxide growth laws discussed in class, how low does the outlet cladding temperature have to be kept at to avoid an oxide transition within the first cycle if transition happens at 1.8 micron?

b) Assuming transition happens just at the one year mark, and that in the subsequent cycles the cladding corrosion follows linear kinetics, calculate the oxide thickness at the end of the fifth cycle resulting from operation at 340 C.

c) After analysis it is decided that in order to allow the fuel to go to even higher burnup the coolant outlet temperature will be kept at 310°C, so as to reduce corrosion and consequently the hydrogen uptake. Calculate the oxide thickness at the end of the fifth cycle in that case.

This, however, would cause a loss of efficiency. A colleague proposes another option: to reduce the hydrogen pickup fraction from the current 15% using a new alloy, which, although it has the same corrosion rate, has a lower hydrogen pickup fraction.

Calculate the hydrogen pickup fraction that would cause the hydrogen uptake during the second year with the new alloy at 340°C to be the same as that with the old alloy at 310°C.

d) If the heat flux is 150 $W/m^2$, what would be the increase in cladding temperature outer wall as a result of the formation of the oxide calculated in (c)? (for extra credit)

Solution:

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Zircaloy component, 1 year cycles, ~~340C~~ what is temperature to keep transition just in second cycle?

a) $\Rightarrow \quad \delta = A \exp\left(-\frac{B}{T}\right) \cdot t^{1/3}$

$t = 365\ d$

$A = 504 \qquad B = 4600$

$\delta = 1.8\,\mu$

$1.8 = 504 \exp\left(-\frac{4600}{T}\right) 365^{1/3}$

$\exp(\ ) = \frac{1.8}{504 \times 365^{1/3}} \qquad \ln\left(\frac{1.8}{504 \times 365^{1/3}}\right) = -\frac{B}{T}$

$T = \dfrac{-4600}{\ln\left(\frac{1.8}{504 \times 365^{1/3}}\right)} = 605\,K = 332\,C$

b) $\delta_{\text{after 1 cycle}} = 1.8\,\mu m$

$\delta = K_L(t - t_0) + \delta_0 = 1.8 + 3 \times 10^9 \exp\left(\frac{-15700}{613}\right) \cdot 4 \times 365 = 34.8\,\mu$

c) $\delta = K_L(t - t_0) + \delta_0 = 1.8 + 3 \times 10^9 \exp\left(\frac{-15700}{583}\right) \times 4 \times 365 = 10.6\,\mu$

d) Hydrogen uptake during 2nd year at 340 C

$C_H = \dfrac{2 f \delta \rho_{ZrO_2} \cdot f^{o}_{ZrO_2}}{\left(t - \frac{\delta}{PBR}\right) \times \rho_{metal}} \dfrac{M_H}{M_O} \times 10^6$

$$C_H^{310} = \frac{2 \times 0.15 \times \delta_{310} \times 5.68 \times 0.26}{\left(600 - \frac{\delta_{310}}{1.56}\right) \times 6.5} \frac{1}{16} \times 10^6$$

$$\delta_{310} = 3 \times 10^9 \exp\left(\frac{-15700}{613}\right) \times 365 = 2.21\,\mu$$

$$C_H^{310} = 15.7 \text{ wt ppm}$$

$$15.7 = \frac{2 \times f_{alloy}^{ww} \times \delta_{340} \times 5.68 \times 0.26}{\left(600 - \frac{\delta_{340}}{1.56}\right) \times 6.5} \frac{1}{16} \times 10^6$$

$$f = \frac{16 \times 15.7 \times 6.5 \left(600 - 8.25/1.56\right)}{10^6 \times 8.25 \times 5.68 \times 2 \times 0.26} \simeq \underline{\underline{4\%}}$$

d) Temperature profile in cladding

$$\Delta T_{\text{across oxide}} = T_{\text{oxide-metal}} - T_{\text{outer oxide}}$$


$$\Delta T \cong \frac{q' \; \delta_{ox}}{2\pi R_c \; K_{ox}} = \frac{150\,(W/cm) \times 100 \times 10^{-4}}{2\pi \cdot 0.5 \times 0.02} =$$

[cm] [W/cm-K]

$$\simeq \underline{24\text{ K}}$$