NE 591 - 010

Nuclear Fuel Performance

Exam 1

1) $\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + Q = 0$ $T'(r_0) = 0$ $r_0 = 0$

$T(r_1) = T_1$ $r_1 = X$

$\int \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) = -Q \ \partial x$

$K\frac{\partial T}{\partial x} = -Qx + C_1$ $\rightarrow$ $K(0) = Q(0) + C_1$

$C_1 = 0$

$\int K\frac{\partial T}{\partial x} = -\int Qx \ dx$

$T(x) = \frac{-Q}{2K}x^2 + C_2$ $T_1 = \frac{-Q}{2K}X^2 + C_2$

$C_2 = T_1 + \frac{Q}{2K}X^2$

$T(x) - T_1 = \frac{Q}{2K}X^2 - \frac{Q}{2K}x^2 \rightarrow \frac{Q}{2K}(X^2 - x^2)$

Steady-state

1-D

K is constant

d) T_0? $T(r=0.4)$?

$$LHR = Q_{av}\,\pi R_F^2$$

$$LHR = (400\ W/cc)\,\pi\,(0.6^2)$$

$$LHR = 452\ W/cm$$

$$T_{co_1} - T_{cool} = \frac{LHR}{2\pi R_F\, h_{cool}}$$

$$\frac{LHR}{2\pi R_F} = 120$$

$$T_{co_1} - 800 = \frac{452}{2\pi(0.6)}\,\frac{1}{5.5} = 21.8\ K$$

$$T_{co_1} = 821.8\ K$$

$$T_{co} - T_{co_1} = \frac{LHR}{2\pi R_F}\,\frac{t_{co_1}}{K_{co_1}} = (120)\left(\frac{0.01}{0.05}\right) = 24\ K$$

$$T_{co} = 845.8\ K$$

$$T_{c_1} - T_{co} = \frac{LHR}{2\pi R_F}\,\frac{t_c}{K_c} = (120)\left(\frac{0.05}{0.15}\right) = 40\ K$$

$$T_{c_1} = 885.8\ K$$

$$T_F - T_{c_1} = \frac{LHR}{2\pi R_F}\,\frac{t_g}{K_g} = (120)\left(\frac{0.2}{0.25}\right) = 96\ K$$

$$T_F = 981.8\ K$$

$$T_0 - T_F = \frac{LHR}{4\pi K_F} = \frac{452}{4\pi(0.5)} = 72\ K \qquad T_F = 1053.7\ K$$

$$T(r=0.4) = \frac{Q}{4K}\left(R^2 - r^2\right) + T_s = \frac{400}{4(0.5)}\left(0.6^2 - 0.4^2\right) + 981.8\ K$$

$$\underline{T(r=0.4) = 1021.8\ K}$$

3) U_3Si_2 $\quad K = 14.5 \ \frac{W}{m\text{-}K}$ $\quad$ enrich = 19.5%

$\rho = 15.67 \ g/cc$ $\quad \phi = 2\times10^{14} \ \frac{n}{cm^2\text{-}s}$

Q? $\quad Q = E_f N_f \sigma_f \phi$

N_f? $\quad m(U) = 235\times0.195 + 238\times0.805 = 237.415$

$m(U_3Si_2) = 3(237.415) + 2(28) = 768.245 \ g/mole$

$$15.67 \ g/cc \ \frac{1 \text{ mole}}{768.245 \ g} \ \frac{6.022\times10^{23} \text{ at}}{1 \text{ mole}} \ \frac{3U}{1 U_3Si_2} \times 0.195 = 7.19\times10^{21}$$

$$Q = (200\times10^6 \ eV)\left(1.602\times10^{-19} \ \frac{J}{eV}\right)\left(7.19\times10^{21}\right)\left(570\times10^{-24}\right)\left(2\times10^{14}\right)$$

$Q = \underline{262.6 \ W/cm^3}$

match N_f for UO_2 $\quad x$ = enrich

$m(UO_2) = 235x + 238(1-x) + 32 = 270 - 3x$

$$10.97 \ g/cc \ \frac{1 \text{ mole}}{270-3x} \ \frac{6.022\times10^{23} \text{ at}}{1 \text{ mole}} \ \frac{1U}{1UO_2} \ x = 7.19\times10^{21}$$

$$\frac{x}{270-3x} = 1.08\times10^{-3}$$

$x = -3.26\times10^{-3}x + 0.294$

$\underline{x = 0.293}$ $\quad$ 29.3%

4)
$$LHR\left(\frac{z}{z_0}\right) = LHR^\circ \cos\left[\frac{\pi}{2\gamma}\left(\frac{z}{z_0}-1\right)\right]$$

a) $z_0 = 1.5\ m \qquad LHR^\circ = 150\ W/cm \qquad \gamma = 1.1$

$$LHR(z=1.8) = 150 \times \cos\left[\frac{\pi}{2(1.1)}\left(\frac{1.8}{1.5}-1\right)\right]$$

$$= \underline{143.9\ W/cm}$$

b) $$\Delta T_{cool} = \frac{2\gamma}{\pi} \frac{z_0\, LHR^\circ}{\dot{m}\, C_p}\left(\sin\frac{\pi}{2\gamma} + \sin\left(\frac{\pi}{2\gamma}\left(\frac{z}{z_0}-1\right)\right)\right)$$

$$\frac{\pi}{2\gamma} = 1.43$$

$$\Delta T_{cool}(water) = \frac{1}{1.43}\frac{(150)(150)}{(0.22)(4200)}\left(\sin(1.43) + \sin(1.43(2-1))\right)$$

$$\Delta T_{cool}(water) = \underline{33.7\ K}$$

$$\Delta T_{cool}(Na) = \frac{1}{1.43}\frac{(150)(150)}{(0.12)(1404)}\left(2 \times \sin(1.43)\right)$$

$$\Delta T_{cool}(Na) = \underline{184.9\ K} \leftarrow$$

5) $y(t_0) = 6$ $\quad \frac{dy}{dt} = 4t - 3t^2$ $\quad dt = 0.33$

$t_0 = 1$ $\quad t \to 2$

Forward $\quad y_{n+1} = y_n + dt\, y'_n$

$t_1 = 1.33 \quad y_1 = y_0 + dt\, y'_0 = 6 + 0.33(4(1) - 3(1)^2) = 6.33$

$t_2 = 1.67 \quad y_2 = y_1 + dt\, y'_1 = 6.33 + 0.33(4(1.33) - 3(1.33)^2)$

$= 6.334$

$t_3 = 2 \quad y_3 = y_2 + dt\, y'_2 = 6.334 + 0.33(4(1.67) - 3(1.67)^2)$

$= \underline{5.78}$

Backward $\quad y_{n+1} = y_n + dt\, y'_{n+1}$

$t_1 = 1.33 \quad y_1 = y_0 + dt\, y'_1 = 6 + 0.33(4(1.33) - 3(1.33)^2) = 6.004$

$t_2 = 1.67 \quad y_2 = y_1 + dt\, y'_2 = 6.004 + 0.33(4(1.67) - 3(1.67)^2)$

$= 5.447$

$t_3 = 2 \quad y_3 = y_2 + dt\, y'_3 = 5.447 + 0.33(4(2) - 3(2)^2)$

$= \underline{4.127}$

⑥. Fertile : can be converted into fissile via neutron capture

Fissile : capable of fission w/ any neutron energy

Fissionable : Capable of undergoing fission w/ high E neutron capture

⑦ Low melting point

Anisotropic thermal expansion

Anisotropic irradiation growth

Poor behavior in water

Heterogeneous phase mixture

Extreme Swelling

⑧ Relative volume of fuel compared to maximum possible volume inside cladding r_1^2/r_2^2

Necessary because of inevitable fuel swelling

⑨. - We enrich U to increase N_f (fissile atom number density), to have higher heat generation rate, thus higher power output of fuel.

- UF_6 is the compound that undergoes enrichment
- Spinning cylinder filled w/ UF_6
  - heavier mass of $U^{238}F_6$ preferentially moves to the outside of the canister due to centrifugal force,
  - enriched $U^{235}F_6$ withdrawn from center of canister

(10) Mo + Xe

Mo has $Z = 42$ and $A = 96$

Xe has $Z = 54$ and $A = 131$

- Double hump fission product distribution has peaks at around $A \approx 95$ and $A \approx 135$
- other examples valid

(11) Finite difference

Finite volume

Finite element

FEM are used in S-O-T-A simulations because of the flexible geometry, ability to handle any boundary condition, and continuous representation of quantities of interest