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NucE 497 Fuel Performance Exam 1 covering modules 1 - 3

-8, 22/30

Question 1:

U_3Si_5 is a uranium silicide fuel being considered for use in light water reactors. It has a thermal conductivity of 12.5 W/(m K) and a density of Uranium metal of 7.5 g of U/cm^3. Answer the following questions

a) What is the fissile isotope in U_3Si_5? What would be the enrichment of this isotope in the natural (unenriched) form of the fuel? (7 points)

U-235

0.75%

b) What enrichment would be required for U_3Si_5 to have the same energy release rate of U_3Si_2 enriched to 3% with a neutron flux of 3.2e13 n/(cm^2 s)? You can assume that U_{235} has a negligible impact on the total molar mass of U in the fuel (15 points)

Assume $Q_{U_3Si_5} = Q_{U_3Si_2}$

$$Q = E_f N_f \sigma_f \varphi_{th}$$

$$\cancel{E_1} N_1 \cancel{\sigma_1} \cancel{\varphi_{th}} = \cancel{E_2} N_2 \cancel{\sigma_2} \cancel{\varphi_{th}}$$

$$N_1 = N_2 \qquad N_f = \frac{q N_a \rho}{M_f}$$

$$\rho_{U_3Si_5} = 7.5 \text{ g/cm}^3$$

$$\rho_{U_3Si_2} = 11.31 \text{ g/cm}^3$$

$$M_{U_3Si_5} = 854 \text{ g/mole}$$

$$M_{U_3Si_2} = 770 \text{ g/mole}$$

$$\frac{q_1 \cancel{N_a} \rho_{U_3Si_5}}{M_{U_3Si_5}} = \frac{q_2 \cancel{N_a} \rho_{U_3Si_2}}{M_{U_3Si_2}}$$

$$q_1 = \frac{q_2 \rho_{U_3Si_2} M_{U_3Si_5}}{M_{U_3Si_2} \rho_{U_3Si_5}}$$

-5, the density of U was provided, not the density

$$q_1 = \frac{0.03(11.31)\,854}{(7.5)\,770} = \boxed{5.02\% \text{ enriched}}$$

c) How would you rank U_3Si_5 as a potential fuel compared to U_3Si_2? Why? (8 points)

U_3Si_5 is a worse fuel than U_3Si_2 because it requires a higher enrichment than U_3Si_2 to have the same energy release rate.

-3, thermal conductivity?

-1, 34/35

Question 2:

Consider a fuel rod with a pellet radius of 4.5 mm, an 80 micron gap, and a zircaloy cladding thickness of 0.6 mm. It is experiencing a linear heat rate of 250 W/cm with a coolant temperature of 580 K. The gap is filled with He and 5% Xe and the coolant conductance is 2.5 W/(cm² K).

a) What is the surface temperature of the fuel rod? (15 points)

$$T_{co} = \frac{LHR}{2\pi R_c h_{cool}} + T_{cool}$$

$$T_{co} = \frac{250}{2\pi(0.45)(2.5)} + 580 = 615.37\,K$$

$$T_{ic} = \frac{LHR \cdot t_c}{2\pi R_c k_c} + T_{co}$$

$$T_{ic} = \frac{250(0.06)}{2\pi(0.45)(0.17)} + 615.37 = 646.57\,K$$

$$h_{gap} = \frac{k_{gap}}{}$$

$$k_{gap} = k_{He}^{1-y_{Xe}} k_{Xe}^{y}$$

$$k_{gap} = (16e{-6}(646.57)^{0.79})^{0.95}(0.7e{-6}(646.57)^{0.79})^{0.05}$$

-1, Don't round until the end. Youare off by 4 K

$$h_{gap} = \frac{0.0023}{80e-4\,cm} = 0.2875$$

$$T_s = \frac{LHR}{2\pi R_f h_{gap}} + T_{ic} = \frac{250}{2\pi(0.45)(0.2875)} + 646.57 = \boxed{954.11\,K}$$

b) Assume the pellet is made from Uranium Nitride. What is the maximum stress experienced by the pellet, given that uranium nitride has E = 246.7 GPa, ν = 0.25, and α = 7.5e-6 1/K? (10 points)

$$T_0 = \frac{LHR}{4\pi k} + T_s$$

$$T_0 = \frac{250}{4\pi(0.2)} + 954.11$$

$$T_0 = 1053.58\,K$$

$$\sigma^* = \frac{\alpha E (T_0 - T_s)}{4(1-\nu)}$$

$$\sigma^* = \frac{(7.5e{-6}\ 1/K)(246.7\,GPa)(1053.58 - 954.11)K}{4(1-0.25)}$$

$$\sigma^* = 0.0613$$

Maximum Stress will be hoop stress $\eta = 1$ at max

$$\sigma_{\theta\theta} = -0.0613(1-3(1)^2) = \boxed{0.1226\ GPa}$$

c) Would you expect this stress to be higher or lower if the pellet was UO_2? Why? (5 points)

The stress would be higher for UO_2 because it has a lower thermal conductivity which would make the centerline temperature much hotter than the surface temperature and cause greater stress.

d) What assumptions were made in your calculations for a) and b)? (5 points)

- linear temperature profile across the gap
- Steady state
- axisymmetric
- T is constant in Z
- Thermal conductivity is independant of temperature

-14, 21/35

Question 3:

Consider the stress state in a zircaloy fuel rod pressurized to 6 MPa with an average radius of 5.6 mm and a cladding thickness of 0.6 mm.

a) What assumptions are made in the thin walled cylinder approximation for the stress state? (5 points)

$$\overline{\sigma_\theta} = \frac{pR}{\delta} \qquad \overline{\sigma_z} = \frac{pR}{2\delta} \qquad \overline{\sigma_r} = -\frac{1}{2}p$$

-5, isotropic, small strain, stress constant through thickness

b) Calculate all three components of the stress using the thin walled cylinder approximation. (10 points)

$$\overline{\sigma_\theta} = \frac{6\,MPa\,(5.6mm)}{0.6\,mm} = \boxed{56\,MPa}$$

$$\overline{\sigma_z} = \frac{6\,MPa\,(5.6mm)}{2(0.6mm)} = \boxed{28\,MPa}$$

$$\overline{\sigma_r} = -\frac{1}{2}(6\,MPa) = \boxed{-3\,MPa}$$

c) Quantify how accurate the thin walled cylinder approximation is for the cladding. Would the thin walled cylinder approximation be conservative if used to estimate if the cladding would fail? (10 points)

Using thick walls to calculate at max stress

$$\sigma_{\theta\theta}(r) = p\frac{(R_o/r)^2+1}{(R_o/R_i)^2-1} = 6\,MPa\,\frac{\left(\frac{5.9mm}{5.3mm}\right)^2+1}{\left(\frac{5.9mm}{5.3mm}\right)-1} = 56.16\,MPa$$

$$\%\ error = \frac{56.16-56}{56.16} = \boxed{2.85\%}$$

-4, Calculate stress at TWO radii and compare to see if stress is constant

Not conservative because thin walled is smaller than the thick wall calculations, so you may predict that the cladding will not fail even though it will.

d) Write the stress and strain tensors for the stress state in the thin walled cylinder, with E = 70 GPa and ν = 0.41. (10 points)

$$\sigma_{rr} = -p\frac{(R_o/r)^2-1}{(R_o/R_i)^2-1} = -6\,MPa$$

$$\sigma_{zz} = p\frac{1}{(R_o/R_i)^2-1} = 25.08\,MPa$$

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu(\sigma_{\theta\theta}+\sigma_{zz})\right) = -0.268$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu(\sigma_{rr}+\sigma_{zz})\right) = 0.691$$

$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{\theta\theta}+\sigma_{rr})\right) = 0.0645$$

$$\boxed{\sigma = \begin{bmatrix}\sigma_{\theta\theta}\\ \sigma_{rr}\\ \sigma_{zz}\end{bmatrix} = \begin{bmatrix}56.16\\ -6\\ 25.08\end{bmatrix} MPa \qquad \epsilon = \begin{bmatrix}\epsilon_{\theta\theta}\\ \epsilon_{rr}\\ \epsilon_{zz}\end{bmatrix} = \begin{bmatrix}0.691\\ -0.268\\ 0.0645\end{bmatrix}}$$

-4, Show stress and strain in tensor form

-1, math error in strain calculation