Let there be observations $x_1, x_2, \ldots, x_n$ and weights $w_1, \ldots, w_n$. Without loss of generality, $\sum_i w_i = 1$.

$$\begin{aligned}
\sum_{i \neq j} \frac{1}{2} w_i w_j (x_i - x_j)^2 &= \sum_{i \neq j} \frac{1}{2} (w_i x_i^2 w_j + w_j x_j^2 w_i - 2 w_i x_i w_j x_j) \\
&= \sum_i \frac{1}{2} w_i (1 - w_i) x_i^2 + \sum_j \frac{1}{2} w_j (1 - w_j) x_j^2 - \sum_{i \neq j} w_i x_i w_j x_j \\
&= \sum_i w_i (1 - w_i) x_i^2 - \sum_{i \neq j} w_i x_i w_j x_j.
\end{aligned}$$

As $(\sum_i w_i x_i)^2 = \sum_i w_i^2 x_i^2 + \sum_{i \neq j} w_i x_i w_j x_j$, this gives

$$\begin{aligned}
\sum_{i \neq j} \frac{1}{2} w_i w_j (x_i - x_j)^2 &= \sum_i w_i x_i^2 - (\sum_i w_i x_i)^2 \\
&= \sum_i w_i x_i^2 - 2 (\sum_i w_i x_i)^2 + (\sum_i w_i x_i)^2 \\
&= \sum_i w_i x_i^2 - 2 \sum_i w_i x_i (\sum_j w_j x_j) + \sum_i w_i (\sum_j w_j x_j)^2 \\
&= \sum_i w_i [x_i - (\sum_j w_j x_j)]^2 \\
&= \mathrm{Var}_w(x).
\end{aligned}$$

Similarly, I think,

$$\sum_{i \neq j} \frac{1}{2} w_i w_j (x_i - x_j)(y_i - y_j) = \sum_i w_i x_i y_i - (\sum_i w_i x_i)(\sum_i w_i y_i) = \mathrm{Cov}_w(x, y).$$