I Fixed-Size Data

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Every programming language comes with a language of data and a language of operations on data. The first language always provides some forms of atomic data; to represent the variety of information in the real world as data, a programmer must learn to compose basic data and to describe such compositions. Similarly, the second language provides some basic operations on atomic data; it is the programmer's task to compose these operations into programs that perform the desired computations. We use *arithmetic* for the combination of these two parts of a programming language because it generalizes what you know from grade school.

This first part of the book (I) introduces the arithmetic of BSL, the programming language used in the Prologue. From arithmetic, it is a short step to your first simple programs, which you may know as *functions* from mathematics. Before you know it, though, the process of writing programs looks confusing, and you will long for a way to organize your thoughts. We equate "organizing thoughts" with *design*, and this first part of the book introduces you to a systematic way of designing programs.

1 Arithmetic

From Prologue: How to Program, you know how to write down the kind of *expression* you know from first grade in BSL notation:

- write “(”,
- write down the name of a primitive operation `op`,
- write down the arguments, separated by some space, and
- write down “)”.

Scan this first chapter quickly, skip ahead to the second one, and return here, when you encounter “arithmetic” that you don’t recognize.

Just as a reminder, here is a primitive expression:

```
(+ 1 2)
```

It uses `+`, the operation for adding two numbers, followed by two arguments, which are plain numbers. But here is another example:

```
(+ 1 (+ 1 (+ 1 1) 2) 3 4 5)
```

This second example exploits two points in the above description that are open to interpretation. First, primitive operations may consume more than two arguments. Second, the arguments don’t have to be numbers per se; they can be expressions, too.

Evaluating expressions is also straightforward. First, BSL evaluates all the arguments of a primitive operation. Second, it “feeds” the resulting pieces of data to the operation, which produces a result. Thus,

```
(+ 1 2)
==
3
```

and

```
(+ 1 (+ 1 (+ 1 1) 2) 3 (+ 2 2) 5)
==
(+ 1 (+ 1 2 2) 3 4 5)
==
(+ 1 5 3 4 5)
==
18
```

We use == to mean “is equal to according to the laws of computation.”

These calculations should look familiar because they are the same kind of calculations that you performed in mathematics classes. You may have written down the steps in a different way; you may have never been taught how to write down a sequence of calculation steps. Yet, BSL performs calculations just like you do, and this should be a relief. It guarantees that you understand what it does with primitive operations and primitive data, so there is some hope that you can predict what your programs will compute. Generally speaking, it is critical for a programmer to know how the chosen language calculates because otherwise a program’s computation may harm the people who use them or on whose behalf the programs calculate.

The rest of this chapter introduces four forms of *atomic data* of BSL: numbers, strings, images, and Boolean values. We use the word “atomic” here in analogy to physics. You cannot peek inside atomic pieces of data, but you do have functions that combine several pieces of atomic data into another one, retrieve “properties” of them, also in terms of atomic data, and so on. The sections of this chapter introduce some of these functions, also called *primitive operations* or *pre-defined operations*. You can find others in the documentation of BSL that comes with DrRacket.

The next volume, *How to Design Components*, will explain how to design atomic data.

1.1 The Arithmetic of Numbers

Most people think “numbers” and “operations on numbers” when they hear “arithmetic.” “Operations on numbers” means adding two numbers to yield a third, subtracting one number from another, determining the greatest common divisor of two numbers, and many more such things. If we don’t take arithmetic too literally, we may even include the sine of an angle, rounding a real number to the closest integer, and so on.

The BSL language supports *Numbers* and arithmetic on them. As discussed in the Prologue, an arithmetic operation such as `+` is used like this:

```
(+ 3 4)
```

that is, in *prefix notation* form. Here are some of the operations on numbers that our language provides: `+`, `-`, `*`, `/`, `abs`, `add1`, `ceiling`, `denominator`, `exact->inexact`, `expt`, `floor`, `gcd`, `log`, `max`, `numerator`, `quotient`, `random`, `remainder`, `sqr`, and `tan`. We picked our way through the alphabet just to show the variety of operations. Explore what they compute, and then find out how many more there are.

If you need an operation on numbers that you know from your mathematics courses, chances are that BSL knows about it, too. Guess its name and experiment in the interactions area. Say you need to compute the *sin* of some angle; try

```
> (sin 0)
0
```

and use it happily ever after. Or look in the HelpDesk. You will find there that in addition to operations BSL also recognizes the names of some widely used numbers, for example, `pi` and `e`.

You might know `e` from calculus. It's a real number, close to 2.718, called "Euler's constant."

When it comes to numbers, BSL programs may use natural numbers, integers, rational numbers, real numbers, and complex numbers. We assume that you have heard of all but the last one. The last one may have been mentioned in your high school class. If not, don't worry; while complex numbers are useful for all kinds of calculations, a novice doesn't have to know about them.

A truly important distinction concerns the precision of numbers. For now, it is important to understand that BSL distinguishes *exact numbers* and *inexact numbers*. When it calculates with exact numbers, BSL preserves this precision whenever possible. For example, `(/ 4 6)` produces the precise fraction `2/3`, which DrRacket can render as a proper fraction, an improper fraction, or a mixed decimal. Play with your computer's mouse to find the menu that changes the fraction into decimal expansion.

Some of BSL's numeric operations cannot produce an exact result. For example, using the `sqrt` operation on `2` produces an irrational number that cannot be described with a finite number of digits. Because computers are of finite size and BSL must somehow fit such numbers into the computer, it chooses an approximation: `1.4142135623730951`. As mentioned in the Prologue, the `#i` prefix warns novice programmers of this lack of precision. While most programming languages choose to reduce precision in this manner, few advertise it and even fewer warn programmers.

Note on Numbers The word "Number" refers to a wide variety of numbers, including counting numbers, integers, rational numbers, real numbers, and even complex numbers. For most uses, you can safely equate Number with the number line from elementary school, though on occasion this translation is too imprecise. If we wish to be precise, we use appropriate words: *Integer*, *Rational*, and so on. We may even refine these notions using such standard terms as *PositiveInteger*, *NonnegativeNumber*, *NegativeNumber*, and so on. **End**

Exercise 1. Add the following definitions for `x` and `y` to DrRacket's definitions area:

```
(define x 3)
(define y 4)
```

Now imagine that `x` and `y` are the coordinates of a Cartesian point. Write down an expression that computes the distance of this point to the origin, that is, a point with the coordinates (`0`,`0`).

The expected result for these values is `5`, but your expression should produce the correct result even after you change these definitions.

Just in case you have not taken geometry courses or in case you forgot the formula that you encountered there, the point (x,y) has the distance

$$\sqrt{x^2 + y^2}$$

from the origin. After all, we are teaching you how to design programs, not how to be a geometer.

To develop the desired expression, it is best to click *RUN* and to experiment in the interactions area. The *RUN* action tells DrRacket what the current values of `x` and `y` are so that you can experiment with expressions that involve `x` and `y`:

```
> x
3
> y
4
> (+ x 10)
13
> (* x y)
12
```

Once you have the expression that produces the correct result, copy it from the interactions area to the definitions area.

To confirm that the expression works properly, change `x` to `12` and `y` to `5`, then click *RUN*. The result should be `13`.

Your mathematics teacher would say that you computed the **distance formula**. To use the formula on alternative inputs, you need to open DrRacket, edit the definitions of x and y so they represent the desired coordinates, and click *RUN*. But this way of reusing the distance formula is cumbersome and naive. We will soon show you a way to define functions, which makes reusing formulas straightforward. For now, we use this kind of exercise to call attention to the idea of functions and to prepare you for programming with them. ▌

1.2 The Arithmetic of Strings

A widespread prejudice about computers concerns their innards. Many believe that it is all about bits and bytes—whatever those are—and possibly numbers because everyone knows that computers can calculate. While it is true that electrical engineers must understand and study the computer as just such an object, beginning programmers and everyone else need never (ever) succumb to this thinking.

Programming languages are about computing with information, and information comes in all shapes and forms. For example, a program may deal with colors, names, business letters, or conversations between people. Even though we could encode this kind of information as numbers, it would be a horrible idea. Just imagine remembering large tables of codes, such as `0` means "red" and `1` means "hello," and the like.

Instead, most programming languages provide at least one kind of data that deals with such symbolic information. For now, we use BSL's strings. Generally speaking, a *String* is a sequence of the characters that you can enter on the keyboard, plus a few others, about which we aren't concerned just yet, enclosed in double quotes. In Prologue: How to Program, we have seen a number of BSL strings: `"hello"`, `"world"`, `"blue"`, `"red"`, and others. The first two are words that may show up in a conversation or in a letter; the others are names of colors that we may wish to use.

Note We use *1String* to refer to the keyboard characters that make up a String. For example, `"red"` consists of three such 1Strings: `"r"`, `"e"`, `"d"`. As it turns out, there is a bit more to the definition of 1String, but for now thinking of them as Strings of length 1 is fine. **End**

BSL includes only one operation that exclusively consumes and produces strings: `string-append`, which, as we have seen in Prologue: How to Program, concatenates two given strings into one. Think of `string-append` as an operation that is just like `+`. While the latter consumes two (or more) numbers and produces a new number, the former consumes two or more strings and produces a new string:

```
> (string-append "what a " "lovely " "day" " 4 BSL")
"what a lovely day 4 BSL"
```

Nothing about the given numbers changes when `+` adds them up, and nothing about the given strings changes when `string-append` concatenates them into one big string. If you wish to evaluate such expressions, you just need to think that the obvious laws hold for `string-append`, similar to those for `+`:

```
(+ 1 1) == 2   (string-append "a" "b") == "ab"
(+ 1 2) == 3   (string-append "ab" "c") == "abc"
(+ 2 2) == 4   (string-append "a" " ") == "a "
...            ...
```

Exercise 2. Add the following two lines to the definitions area:

```
(define prefix "hello")
(define suffix "world")
```

Then use string primitives to create an expression that concatenates `prefix` and `suffix` and adds `"_"` between them. When you run this program, you will see `"hello_world"` in the interactions area.

See exercise 1 for how to create expressions using DrRacket. ▌

1.3 Mixing It Up

All other operations (in BSL) concerning strings consume or produce data other than strings. Here are some examples:

- `string-length` consumes a string and produces a number;
- `string-ith` consumes a string `s` together with a number `i` and extracts the 1String located at the `i`th position (counting from 0); and
- `number->string` consumes a number and produces a string.

Also look up `substring` and find out what it does.

If the documentation in HelpDesk appears confusing, experiment with the functions in the interactions area. Give them appropriate arguments, and find out what they compute. Also use **inappropriate** arguments for some operations just to find out how BSL reacts:

```
> (string-length 42)
string-length:expects a string, given 42
```

As you can see, BSL reports an error. The first part "string-length" informs you about the operation that is misapplied; the second half states what is wrong with the arguments. In this specific example, `string-length` is supposed to be applied to a string but is given a number, specifically `42`.

Naturally, it is possible to nest operations that consume and produce different kinds of data **as long as you keep track of what is proper and what is not**. Consider this expression from the Prologue: How to Program:

```
(+ (string-length "hello world") 20)
```

The inner expression applies `string-length` to `"hello world"`, our favorite string. The outer expression has `+` consume the result of the inner expression and `20`.

Let's determine the result of this expression in a step-by-step fashion:

```
(+ (string-length "hello world") 20)
==
(+ 11 20)
==
31
```

Not surprisingly, computing with such nested expressions that deal with a mix of data is no different from computing with numeric expressions. Here is another example:

```
(+ (string-length (number->string 42)) 2)
==
(+ (string-length "42") 2)
==
(+ 2 2)
==
4
```

Before you go on, construct some nested expressions that mix data in the **wrong** way, say,

```
(+ (string-length 42) 1)
```

Run them in DrRacket. Study the red error message but also watch what DrRacket highlights in the definitions area.

Exercise 3. Add the following two lines to the definitions area:

```
(define str "helloworld")
(define i 5)
```

Then create an expression using string primitives that adds `"_"` at position `i`. In general this means the resulting string is longer than the original one; here the expected result is `"hello_world"`.

Position means *i* characters from the left of the string, but programmers start counting at `0`. Thus, the 5*th* letter in this example is `"w"`, because the `0`th letter is `"h"`. **Hint** When you encounter such "counting problems" you may wish to add a string of digits below `str` to help with counting:

```
(define str "helloworld")
(define ind "0123456789")
(define i 5)
```

See exercise 1 for how to create expressions in DrRacket. ▮

Exercise 4. Use the same setup as in exercise 3 to create an expression that deletes the *i*th position from `str`. Clearly this expression creates a shorter string than the given one. Which values for `i` are legitimate? ▮

1.4 The Arithmetic of Images

An *Image* is a visual, rectangular piece of data, for example, a photo or a geometric figure and its frame. You can insert images in DrRacket wherever you can write down an expression because images are values, just like numbers and strings.

Remember to require the *2htdp/image* library in a new tab.

Your programs can also manipulate images with primitive operations. These primitive operations come in three flavors. The first kind concerns the creation of basic images:

- `circle` produces a circle image from a radius, a mode string, and a color string;

- `ellipse` produces an ellipse from two diameters, a mode string, and a color string;
- `line` produces a line from two points and a color string;
- `rectangle` produces a rectangle from a width, a height, a mode string, and a color string;
- `text` produces a text image from a string, a font size, and a color string; and
- `triangle` produces an upward-pointing equilateral triangle from a size, a mode string, and a color string.

The names of these operations mostly explain what kind of image they create. All you must know is that *mode strings* means `"solid"` or `"outline"`, and *color strings* are strings such as `"orange"`, `"black"`, and so on.

Play with these operations in the interactions window:

```
> (circle 10 "solid" "green")
```

```
> (rectangle 10 20 "solid" "blue")
```

```
> (star 12 "solid" "gray")
```

Stop! The above uses a previously unmentioned operation. Look up its documentation and find out how many more such operations the *2htdp/image* library comes with. Experiment with the operations you find.

The second kind of functions on images concern image properties:

- `image-width` determines the width of an image in terms of pixels;
- `image-height` determines the height of an image;

They extract the kind of values from images that you expect:

```
> (image-width (circle 10 "solid" "red"))
20
> (image-height (rectangle 10 20 "solid" "blue"))
20
```

Stop! Explain how DrRacket determines the value of this expression:

```
(+ (image-width (circle 10 "solid" "red"))
   (image-height (rectangle 10 20 "solid" "blue")))
```

A proper understanding of the third kind of image-composing primitives requires the introduction of one new idea: the *anchor point*. An image isn't just a single pixel, it consists of many pixels. Specifically, each image is like a photograph, that is, a rectangle of pixels. One of these pixels is an implicit anchor point. When you use an image primitive to compose two images, the composition happens with respect to the anchor points, unless you specify some other point explicitly:

- `overlay` places all the images to which it is applied on top of each other, using the center as anchor point;
- `overlay/xy` is like `overlay` but accepts two numbers—`x` and `y`—between two image arguments. It shifts the second image by `x` pixels to the right and `y` pixels down—all with respect to the first image's top-left corner; unsurprisingly, a negative `x` shifts the image to the left and a negative `y` up; and
- `overlay/align` is like `overlay` but accepts two strings that shift the anchor point(s) to other parts of the rectangles. There are nine different positions overall; experiment with all possibilities!

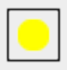
The *2htdp/image* library comes with many other primitive functions for combining images. As you get familiar with image processing, you will want to read up on those. For now, we introduce three more because they are important for creating animated scenes and images for games:

- `empty-scene` creates a rectangle of some given width and height;
- `place-image` places an image into a scene at a specified position. If the image doesn't fit into the given scene, it is appropriately cropped;
- `scene+line` consumes a scene, four numbers, and a color to draw a line into the given image. Experiment with it to see how it works.

arithmetic of numbers	arithmetic of images
`(+ 1 1) == 2`	`(overlay (square 4 "solid" "orange")`
	`(circle 6 "solid" "yellow"))`

```
                          ==
```

```
(+ 1 2) == 3        (underlay (circle 6 "solid" "yellow")
                              (square 4 "solid" "orange"))
                    ==
```

```
(+ 2 2) == 4        (place-image (circle 6 "solid" "yellow")
                                 10 10
                                 (empty-scene 20 20))
                    ==
```


```
     ...                              ...
```

Figure 10: Laws of image creation

The laws of arithmetic for images are analogous to those for numbers; see figure 10 for some examples and a comparison with numeric arithmetic. Again, no image gets destroyed or changed. Like `+`, these primitives just make up new images that combine the given ones in some manner.

Exercise 5. Use the *2htdp/image* library to create the image of a simple boat or tree. Make sure you can easily change the scale of the entire image. ▮

Exercise 6. Add the following line to the definitions area:

Copy and paste the image into your DrRacket.

```
(define cat
```

```
)
```

Create an expression that counts the number of pixels in the image. ▮

1.5 The Arithmetic of Booleans

We need one last kind of primitive data before we can design programs: Boolean values. There are only two kinds of *Boolean* values: `#true` and `#false`. Programs use Boolean values for representing decisions or the status of switches.

Computing with Boolean values is simple, too. In particular, BSL programs get away with three operations: `or`, `and`, and `not`. These operations are kind of like addition, multiplication, and negation for numbers. Of course, because there are only two Boolean values, it is actually possible to demonstrate how these functions work in **all** possible situations:

- `or` checks whether **any** of the given Boolean values is `#true`:

```
> (or #true #true)
#true
> (or #true #false)
#true
> (or #false #true)
#true
> (or #false #false)
#false
```

- `and` checks whether **all** of the given Boolean values are `#true`:

```
> (and #true #true)
#true
> (and #true #false)
#false
> (and #false #true)
#false
> (and #false #false)
#false
```

- and `not` always picks the Boolean that isn't given:

```
> (not #true)
#false
```

Unsurprisingly, or and and may be used with more than two expressions. Finally, there is more to or and and than these explanations suggest, but to explain the extra bit requires a second look at nested expressions.

Exercise 7. Boolean expressions can express some everyday problems. Suppose you want to decide whether today is an appropriate day to go to the mall. You go to the mall either if it is not sunny or if today is Friday (because that is when stores post new sales items).

Nadeem Hamid suggested this formulation of the exercise.

Here is how you could go about it using your new knowledge about Booleans. First add these two lines to the definitions area of DrRacket:

```
(define sunny #true)
(define friday #false)
```

Now create an expression that computes whether sunny is false or friday is true. So in this particular case, the answer is #false. (Why?)

See exercise 1 for how to create expressions in DrRacket. How many combinations of Booleans can you associate with sunny and friday? ▮

1.6 Mixing It Up with Booleans

One important use of Boolean values concerns calculations with different kinds of data. We know from the Prologue that BSL programs may name values via definitions. For example, we could start a program with

```
(define x 2)
```

and then compute its inverse:

```
(define inverse-of-x (/ 1 x))
```

This works fine, as long as we don't edit the program and change x to 0.

This is where Boolean values come in, in particular conditional calculations. First, the primitive function = determines whether two (or more) numbers are equal. If so, it produces #true, otherwise #false. Second, there is a kind of BSL expression that we haven't mentioned so far: the if expression. It uses the word "if" as if it were a primitive function; it isn't. The word "if" is followed by three expressions, separated by blank spaces (that includes tabs, line breaks, etc.). Naturally the entire expression is enclosed in parentheses. Here is an example:

```
(if (= x 0) 0 (/ 1 x))
```

This if expression contains (= x 0), 0, and (/ 1 x), three so-called *sub-expressions*. The evaluation of this expression proceeds in two steps:

1. The first expression is always evaluated. Its result must be a Boolean.

2. If the result of the first expression is #true, then the second expression is evaluated; otherwise the third one is. Whatever their results are, they are also the result of the entire if expression.

Right-click on the result and choose a different representation.

Given the definition of x above, you can experiment with if expressions in the interactions area:

```
> (if (= x 0) 0 (/ 1 x))
0.5
```

Using the laws of arithmetic, you can figure out the result yourself:

```
(if (= x 0) 0 (/ 1 x))
== ; because x stands for 2
(if (= 2 0) 0 (/ 1 2))
== ; 2 is not equal to 0, (= 2 0) is #false
(if #false 0 (/ 1 x))
(/ 1 2)
== ; normalize this to its decimal representation
0.5
```

In other words, DrRacket knows that x stands for 2 and that the latter is not equal to 0. Hence, (= x 0) produces the result #false, meaning if picks its third sub-expression to be evaluated.

Stop! Imagine you edit the definition so that it looks like this:

```
(define x 0)
```

What do you think

```
(if (= x 0) 0 (/ 1 x))
```

evaluates to in this context? Why? Show your calculation.

In addition to `=`, BSL provides a host of other comparison primitives. Explain what the following four comparison primitives determine about numbers: `<`, `<=`, `>`, `>=`.

Strings aren't compared with `=` and its relatives. Instead, you must use `string=?` or `string<=?` or `string>=?` if you ever need to compare strings. While it is obvious that `string=?` checks whether the two given strings are equal, the other two primitives are open to interpretation. Look up their documentation. Or, experiment, guess a general law, and then check in the documentation whether you guessed right.

You may wonder why it is ever necessary to compare strings with each other. So imagine a program that deals with traffic lights. It may use the strings `"green"`, `"yellow"`, and `"red"`. This kind of program may contain a fragment such as this:

```
(define current-color ...)
```

The dots in the definition of `current-color` aren't a part of the program, of course. Replace them with a string that refers to a color.

```
(define next-color
  (if (string=? "green" current-color) "yellow" ...))
```

It should be easy to imagine that this fragment deals with the computation that determines which light bulb is to be turned on next and which one should be turned off.

The next few chapters introduce better expressions than `if` to express conditional computations and, most importantly, systematic ways for designing them.

Exercise 8. Add the following line to the definitions area:

```
(define cat                )
```

Create a conditional expression that computes whether the image is tall or wide. An image should be labeled `"tall"` if its height is larger than or equal to its width; otherwise it is `"wide"`. See exercise 1 for how to create such expressions in DrRacket; as you experiment, replace the cat with a rectangle of your choice to ensure that you know the expected answer.

Now try the following modification. Create an expression that computes whether a picture is `"tall"`, `"wide"`, or `"square"`. ▌

1.7 Predicates: Know Thy Data

Remember the expression `(string-length 42)` and its result. Actually, the expression doesn't have a result, it signals an error. DrRacket lets you know about errors via red text in the interactions area and highlighting of the faulty expression (in the definitions area). This way of marking errors is particularly helpful when you use this expression (or its relatives) deeply nested within some other expression:

```
(* (+ (string-length 42) 1) pi)
```

Experiment with this expression by entering it both into DrRacket's interactions area and in the definitions area (and then click on *RUN*).

Of course, you really don't want such error-signaling expressions in your program. And usually, you don't make such obvious mistakes as using `42` as a string. It is quite common, however, that programs deal with variables that may stand for either a number or a string:

```
(define in ...)
```

```
(string-length in)
```

A variable such as `in` can be a placeholder for any value, including a number, and this value then shows up in the `string-length` expression.

One way to prevent such accidents is to use a *predicate*, which is a function that consumes a value and determines whether or not it belongs to some class of data. For example, the predicate `number?` determines whether the given value is a number or not:

```
> (number? 4)
#true
> (number? pi)
#true
> (number? #true)
#false
> (number? "fortytwo")
#false
```

As you see, the predicates produce Boolean values. Hence, when predicates are combined with conditional expressions, programs can protect expressions from misuse:

```
(define in ...)

(if (string? in) (string-length in) ...)
```

Every class of data that we introduced in this chapter comes with a predicate. Experiment with `number?`, `string?`, `image?`, and `boolean?` to ensure that you understand how they work.

In addition to predicates that distinguish different forms of data, programming languages also come with predicates that distinguish different kinds of numbers. In BSL, numbers are classified in two ways: by construction and by their exactness. Construction refers to the familiar sets of numbers: `integer?`, `rational?`, `real?`, and `complex?`, but many programming languages, including BSL, also choose to use finite approximations to well-known constants, which leads to somewhat surprising results with the `rational?` predicate:

```
> (rational? pi)
#true
```

As for exactness, we have mentioned the idea before. For now, experiment with `exact?` and `inexact?` to make sure they perform the checks that their names suggest. Later we are going to discuss the nature of numbers in some detail.

Put `(sqrt -1)` at the prompt in the interactions area and hit the "enter" key. Take a close look at the result. The result you see is the first so-called complex number anyone encounters. While your teacher may have told you that one doesn't compute the square root of negative numbers, the truth is that mathematicians and some programmers find it acceptable and useful to do so anyway. But don't worry: understanding complex numbers is not essential to being a program designer.

Exercise 9. Add the following line to the definitions area of DrRacket:

```
(define in ...)
```

Then create an expression that converts the value of `in` to a positive number. For a String, it determines how long the String is; for an Image, it uses the area; for a Number, it decrements the number by `1`, unless it is already `0` or negative; for `#true` it uses `10` and for `#false` `20`.

See exercise 1 for how to create expressions in DrRacket. ▌

Exercise 10. Now relax, eat, sleep, and then tackle the next chapter. ▌

2 Functions and Programs

As far as programming is concerned, "arithmetic" is half the game; the other half is "algebra." Of course, "algebra" relates to the school notion of algebra as little/much as the notion of "arithmetic" from the preceding chapter relates to arithmetic taught in grade-school arithmetic. Specifically, the algebra notions needed are variable, function definition, function application, and function composition. This chapter reacquaints you with these notions in a fun and accessible manner.

2.1 Functions

Programs are functions. Like functions, programs consume inputs and produce outputs. Unlike the functions you may know, programs work with a variety of data: numbers, strings, images, mixtures of all these, and so on. Furthermore, programs are triggered by events in the real world, and the outputs of programs affect the real world. For example, a spreadsheet program may react to an accountant's key presses by filling some cells with numbers, or the calendar program on a computer may launch a monthly payroll program on the last day of every month. Lastly, a program may not consume all of its input data at once, instead it may decide to process data in an incremental manner.

Definitions While many programming languages obscure the relationship between programs and functions, BSL brings it to the fore. Every BSL program consists of several definitions, usually followed by an expression that involves those definitions. There are two kinds of definitions:

- *constant definitions*, of the shape `(define Variable Expression)`, which we encountered in the preceding chapter; and
- *function definitions*, which come in many flavors, one of which we used in the Prologue.

Like expressions, function definitions in BSL come in a uniform shape:

```
(define (FunctionName Variable ... Variable)
  Expression)
```

That is, to define a function, we write down

- "`(define (`",
- the name of the function,
- followed by several variables, separated by space and ending in "`)`",
- and an expression followed by "`)`".

And that is all there is to it. Here are some small examples:

- `(define (f x) 1)`
- `(define (g x y) (+ 1 1))`
- `(define (h x y z) (+ (* 2 2) 3))`

Before we explain why these examples are silly, we need to explain what function definitions mean. Roughly speaking, a function definition introduces a new operation on data; put differently, it adds an operation to our vocabulary if we think of the primitive operations as the ones that are always available. Like a primitive function, a defined function consumes inputs. The number of variables determines how many inputs—also called *arguments* or *parameters*—a function consumes. Thus, `f` is a one-argument function, sometimes called a *unary* function. In contrast, `g` is a two-argument function, also dubbed *binary*, and `h` is a *ternary* or three-argument function. The expression—often referred to as the *function body*—determines the output.

The examples are silly because the expressions inside the functions do not involve the variables. Since variables are about inputs, not mentioning them in the expressions means that the function's output is independent of its input and therefore always the same. We don't need to write functions or programs if the output is always the same.

Variables aren't data; they represent data. For example, a constant definition such as

```
(define x 3)
```

says that `x` always stands for `3`. The variables in a *function header*, that is, the variables that follow the function name, are placeholders for **unknown** pieces of data, the inputs of the function. Mentioning a variable in the function body is the way to use these pieces of data when the function is applied and the values of the variables become known.

Consider the following fragment of a definition:

```
(define (ff a) ...)
```

Its function header is `(ff a)`, meaning `ff` consumes one piece of input, and the variable `a` is a placeholder for this input. Of course, at the time we define a function, we don't know what its input(s) will be. Indeed, the whole point of defining a function is that we can use the function many times on many different inputs.

Useful function bodies refer to the function parameters. A reference to a function parameter is really a reference to the piece of data that is the input to the function. If we complete the definition of `ff` like this

```
(define (ff a)
  (* 10 a))
```

we are saying that the output of a function is ten times its input. Presumably this function is going to be supplied with numbers as inputs because it makes no sense to multiply images or Boolean values or strings by `10`.

For now, the only remaining question is how a function obtains its inputs. And to this end, we turn to the notion of applying a function.

Applications A *function application* puts `define`d functions to work, and it looks just like the applications of a pre-defined operation:

- write "(",
- write down the name of a defined function `f`,
- write down as many arguments as `f` consumes, separated by space,
- and add ")" at the end.

With this bit of explanation, you can now experiment with functions in the interactions area just as we suggested you experiment with primitives to find out what they compute. The following three experiments, for example, confirm that `f` from above produces the same value no matter what input it is applied to:

```
> (f 1)
1
> (f "hello world")
1
> (f #true)
1
```

What does `(f (circle 3 "solid" "red"))` yield?

Remember to add `(require 2htdp/image)` to the definitions area.

See, even images as inputs don't change `f`'s behavior. But here is what happens when the function is applied to too few or too many arguments:

```
> (f)
f:expects 1 argument, found none
> (f 1 2 3 4 5)
f:expects only 1 argument, found 5
```

DrRacket signals an error that is just like those you see when you apply a primitive to the wrong number of arguments:

```
> (+)
+:expects at least 2 arguments, found none
```

Functions don't have to be applied at the prompt in the interactions area. It is perfectly acceptable to use function applications nested within other function applications:

```
> (+ (ff 3) 2)
32
> (* (ff 4) (+ (ff 3) 2))
1280
> (ff (ff 1))
100
```

Exercise 11. Define a function that consumes two numbers, *x* and *y*, and that computes the distance of point (*x*,*y*) to the origin.

In exercise 1 you developed the right-hand side of this function for concrete values of *x* and *y*. Now add a header. ▮

Exercise 12. Define the function `cvolume`, which accepts the length of a side of an equilateral cube and computes its volume. If you have time, consider defining `csurface`, too.

Hint An equilateral cube is a three-dimensional container bounded by six squares. You can determine the surface of a cube if you know that the square's area is its length multiplied by itself. Its volume is the length multiplied with the area of one of its squares. (Why?) ▮

Exercise 13. Define the function `string-first`, which extracts the first 1String from a **non-empty** string. ▮

Exercise 14. Define the function `string-last`, which extracts the last 1String from a non-empty string. ▮

Exercise 15. Define `==>`. The function consumes two Boolean values, call them `sunny` and `friday`. Its answer is `#true` if `sunny` is false or `friday` is true. **Note** Logicians call this Boolean operation *implication*, and they use the notation *sunny* => *friday* for this purpose. ▮

Exercise 16. Define the function `image-area`, which counts the number of pixels in a given image. See exercise 6 for ideas. ▮

Exercise 17. Define the function `image-classify`, which consumes an image and conditionally produces `"tall"` if the image is taller than wide, `"wide"` if it is wider than tall, or `"square"` if its width and height are the same. See exercise 8 for ideas. ▮

Exercise 18. Define the function `string-join`, which consumes two strings and appends them with `"_"` in between. See exercise 2 for ideas. ▌

Exercise 19. Define the function `string-insert`, which consumes a string `str` plus a number `i` and inserts `"_"` at the *i*th position of `str`. Assume `i` is a number between `0` and the length of the given string (inclusive). See exercise 3 for ideas. Ponder how `string-insert` copes with `""`. ▌

Exercise 20. Define the function `string-delete`, which consumes a string plus a number `i` and deletes the *i*th position from `str`. Assume `i` is a number between `0` (inclusive) and the length of the given string (exclusive). See exercise 4 for ideas. Can `string-delete` deal with empty strings? ▌

2.2 Computing

Function definitions and applications work in tandem. If you want to design programs, you must understand this collaboration because you need to imagine how DrRacket runs your programs and because you need to figure out **what** goes wrong **when** things go wrong—and they **will** go wrong.

While you may have seen this idea in an algebra course, we prefer to explain it our way. So here we go. Evaluating a function application proceeds in three steps: DrRacket determines the values of the argument expressions; it checks that the number of arguments and the number of function parameters are the same; if so, DrRacket computes the value of the body of the function, with all parameters replaced by the corresponding argument values. This last value is the value of the function application. This is a mouthful, so we need examples.

Here is a sample calculation for `f`:

```
(f (+ 1 1))
== ; DrRacket knows that (+ 1 1) == 2
(f 2)
== ; DrRacket replaced all occurrences of x with 2
1
```

That last equation is weird because `x` does not occur in the body of `f`. Therefore, replacing the occurrences of `x` with `2` in the function body produces `1`, which is the function body itself.

For `ff`, DrRacket performs a different kind of computation:

```
(ff (+ 1 1))
== ; DrRacket again knows that (+ 1 1) == 2
(ff 2)
== ; DrRacket replaces a with 2 in ff's body
(* 10 2)
== ; and from here, DrRacket uses plain arithmetic
20
```

The best point is that when you combine these laws of computation with those of arithmetic, you can pretty much predict the outcome of any program in BSL:

```
(+ (ff (+ 1 2)) 2)
== ; DrRacket knows that (+ 1 2) == 3
(+ (ff 3) 2)
== ; DrRacket replaces a with 3 in ff's body
(+ (* 10 3) 2)
== ; now DrRacket uses the laws of arithmetic
(+ 30 2)
==
32
```

Naturally, we can reuse the result of this computation in others:

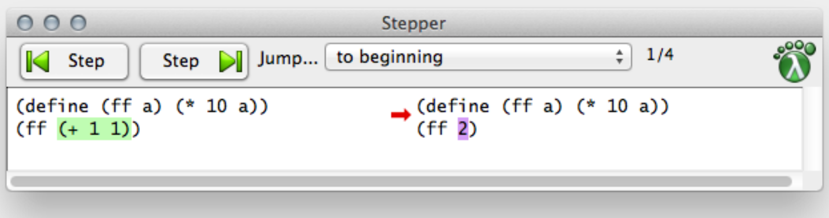
```
(* (ff 4) (+ (ff 3) 2))
== ; DrRacket substitutes 4 for a in ff's body
(* (* 10 4) (+ (ff 3) 2))
== ; DrRacket knows that (* 10 4) == 40
(* 40 (+ (ff 3) 2))
== ; now it uses the result of the above calculation
(* 40 32)
==
1280 ; because it is really just math
```

In sum, DrRacket is an incredibly fast algebra student; it knows all the laws of arithmetic and it is great at substitution. Even better, DrRacket cannot only determine the value of an expression; it can also show you **how** it does it. That is, it can show you step-by-step how to solve these algebra problems that ask you to determine the value of an expression.

Take a second look at the buttons that come with DrRacket. One of them looks like an "advance to next track" button on an audio player. If you click this button, the **stepper** window pops up and you can step through the evaluation of the program in the definitions area.

Enter the definition of `ff` into the definitions area. Add `(ff (+ 1 1))` at the bottom. Now click the *STEP*. The stepper window will show up; figure 11 shows what it looks like in version 6.2 of the software. At this point, you can use the forward and backward arrows to see all the computation steps that DrRacket uses to determine the value of an expression. Watch how the stepper performs the same calculations as we do.

Stop! Yes, you could have used DrRacket to solve some of your algebra homework. Experiment with the various options that the stepper offers.



```
(define (ff a) (* 10 a))
(ff (+ 1 1))
```

```
(define (ff a) (* 10 a))
(ff 2)
```

Figure 11: The DrRacket stepper

Exercise 21. Use DrRacket's stepper to evaluate `(ff (ff 1))` step-by-step. Also try `(+ (ff 1) (ff 1))`. Does DrRacket's stepper reuse the results of computations? ▌

At this point, you might think that you are back in an algebra course with all these computations involving uninteresting functions and numbers. Fortunately, this approach generalizes to **all** programs, including the interesting ones, in this book.

Let's start by looking at functions that process strings. Recall some of the laws of string arithmetic:

```
(string-append "hello" " " "world") == "hello world"
(string-append "bye" ", " "world") == "bye, world"
...
```

Now suppose we define a function that creates the opening of a letter:

```
(define (opening first-name last-name)
  (string-append "Dear " first-name ","))
```

When you apply this function to two strings, you get a letter opening:

```
> (opening "Matthew" "Fisler")
"Dear Matthew,"
```

More importantly, though, the laws of computing explain how DrRacket determines this result and how you can anticipate what DrRacket does:

```
(opening "Matthew" "Fisler")
==  ; DrRacket substitutes "Matthew" for first-name
(string-append "Dear " "Matthew" ",")
==
"Dear Matthew,"
```

Since `last-name` does not occur in the definition of `opening`, replacing it with `"Fisler"` has no effect.

The rest of the book introduces more forms of data. To explain operations on data, we always use laws like those of arithmetic in this book.

Eventually you will encounter imperative operations, which do not combine or extract values but **modify** them. To calculate with such operations, you will need to add some laws to those of arithmetic and substitution.

Exercise 22. Use DrRacket's stepper on this program fragment:

```
(define (distance-to-origin x y)
  (sqrt (+ (sqr x) (sqr y))))
(distance-to-origin 3 4)
```

Does the explanation match your intuition? ▌

Exercise 23. The first 1String in `"hello world"` is `"h"`. How does the following function compute this result?

```
(define (string-first s)
  (substring s 0 1))
```

Use the stepper to confirm your ideas. ▮

Exercise 24. Here is the definition of `==>`: y

```
(define (==> x y)
  (or (not x) y))
```

Use the stepper to determine the value of `(==> #true #false)`. ▮

Exercise 25. Take a look at this attempt to solve exercise 17:

```
(define (image-classify img)
  (cond
    [(>= (image-height img) (image-width img)) "tall"]
    [(= (image-height img) (image-width img)) "square"]
    [(<= (image-height img) (image-width img)) "wide"]))
```

Does stepping through an application suggest a fix? ▮

Exercise 26. What do you expect as the value of this program:

```
(define (string-insert s i)
  (string-append (substring s 0 i)
                 "_"
                 (substring s i)))

(string-insert "helloworld" 6)
```

Confirm your expectation with DrRacket and its stepper. ▮

2.3 Composing Functions

A program rarely consists of a single function definition. Typically, programs consist of a *main* definition and several other functions and turns the result of one function application into the input for another. In analogy to algebra, we call this way of defining functions *composition*, and we call these additional functions *auxiliary functions* or *helper functions*.

```
(define (letter fst lst signature-name)
  (string-append
    (opening fst)
    "\n\n"
    (body fst lst)
    "\n\n"
    (closing signature-name)))

(define (opening fst)
  (string-append "Dear " fst ","))

(define (body fst lst)
  (string-append
   "We have discovered that all people with the" "\n"
   "last name " lst " have won our lottery. So, " "\n"
   fst ", " "hurry and pick up your prize."))

(define (closing signature-name)
  (string-append
   "Sincerely,"
   "\n\n"
   signature-name
   "\n"))
```

Figure 12: A batch program

Consider the program of figure 12 for filling in letter templates. It consists of four functions. The first one is the main function, which produces a complete letter from the first and last name of the addressee plus a signature. The main function refers to three auxiliary functions to produce the three pieces of the letter—the opening, body, and signature—and composes the results in the correct order with `string-append`.

Stop! Enter these definitions into DrRacket's definitions area, click *RUN*, and evaluate these expressions in the interactions area:

```
> (letter "Matthew" "Fisler" "Felleisen")
"Dear Matthew,\n\nWe have discovered that ...\n"
> (letter "Kathi" "Felleisen" "Findler")
"Dear Kathi,\n\nWe have discovered that ...\n"
```

Aside The result is a long string that contains `"\n"`, which represents a new line when the string is **printed**. Now Add `(require 2htdp/batch-io)` to your program, which adds the function `write-file` to its repertoire; it allows you to **print** this string to the console:

Think of `'stdout` as a String for now.

```
> (write-file 'stdout (letter "Matt" "Fiss" "Fell"))
Dear Matt,

We have discovered that all people with the
last name Fiss have won our lottery. So,
Matt, hurry and pick up your prize.

Sincerely,
Fell
'stdout
```

Programs explains such batch programs in some depth. **End**

In general, when a problem refers to distinct tasks of computation, a program should consist of one function per task and a main function that puts it all together. We formulate this idea as a simple slogan:

> *Define one function per task.*

The advantage of following this slogan is that you get reasonably small functions, each of which is easy to comprehend and whose composition is easy to understand. Once you learn to design functions, you will recognize that getting small functions to work correctly is much easier than doing so with large ones. Better yet, if you ever need to change a part of the program due to some change to the problem statement, it tends to be much easier to find the relevant parts when it is organized as a collection of small functions as opposed to a large, monolithic block.

Here is a small illustration of this point with a sample problem:

> **Sample Problem** The owner of a monopolistic movie theater in a small town has complete freedom in setting ticket prices. The more he charges, the fewer people can afford tickets. The less he charges, the more it costs to run a show because attendance goes up. In a recent experiment the owner determined a relationship between the price of a ticket and average attendance.
>
> At a price of \$5.00 per ticket, 120 people attend a performance. For each 10-cent change in the ticket price, the average attendance changes by 15 people. That is, if the owner charges \$5.10, some 105 people attend on the average; if the price goes down to \$4.90, average attendance increases to 135. Let's translate this idea into a mathematical formula:
>
> $$avg.\ attendance = 120\ people - \frac{\$(change\ in\ price)}{\$0.10} \cdot 15\ people$$
>
> Stop! Explain the minus sign before you proceed.
>
> Unfortunately, the increased attendance also comes at an increased cost. Every performance comes at a fixed cost of \$180 to the owner plus a variable cost of \$0.04 per attendee.
>
> The owner would like to know the exact relationship between profit and ticket price in order to maximize the profit.

While the task is clear, how to go about it is not. All we can say at this point is that several quantities depend on each other.

When we are confronted with such a situation, it is best to tease out the various dependencies, one by one:

1. The problem statement specifies how the number of attendees depends on the ticket price. Computing this number is clearly a separate task and thus deserves its own function definition:

   ```
   (define (attendees ticket-price)
     (- 120 (* (- ticket-price 5.0) (/ 15 0.1))))
   ```

2. The *revenue* is exclusively generated by the sale of tickets, meaning it is exactly the product of ticket price and number of attendees:

   ```
   (define (revenue ticket-price)
     (* ticket-price (attendees ticket-price)))
   ```

3. The *cost* consists of two parts: a fixed part (\$180) and a variable part that depends on the number of attendees. Given

that the number of attendees is a function of the ticket price, a function for computing the cost of a show must also consume the ticket price so that it can reuse the `attendees` function:

```
(define (cost ticket-price)
  (+ 180 (* 0.04 (attendees ticket-price))))
```

4. Finally, *profit* is the difference between revenue and costs for some given ticket price:

```
(define (profit ticket-price)
  (- (revenue ticket-price)
     (cost ticket-price)))
```

The BSL definition of `profit` directly follows the suggestion of the informal problem description.

These four functions are all there is to the computation of the profit, and we can now use the profit function to determine a good ticket price.

Exercise 27. Our solution to the sample problem contains several constants in the middle of functions. As One Program, Many Definitions already points out, it is best to give names to such constants so that future readers understand where these numbers come from. Collect all definitions in DrRacket's definitions area and change them so that all magic numbers are refactored into constant definitions. ▮

Exercise 28. Determine the potential profit for these ticket prices: $1, $2, $3, $4, and $5. Which price maximizes the profit of the movie theater? Determine the best ticket price to a dime. ▮

Here is an alternative version of the same program, given as a single function definition:

```
(define (profit price)
  (- (* (+ 120
           (* (/ 15 0.1)
              (- 5.0 price)))
        price)
     (+ 180
        (* 0.04
           (+ 120
              (* (/ 15 0.1)
                 (- 5.0 price)))))))
```

Enter this definition into DrRacket and ensure that it produces the same results as the original version for $1, $2, $3, $4, and $5. A single look should suffice to show how much more difficult it is to comprehend this one function compared to the above four.

Exercise 29. After studying the costs of a show, the owner discovered several ways of lowering the cost. As a result of these improvements, there is no longer a fixed cost; a variable cost of $1.50 per attendee remains.

Modify both programs to reflect this change. When the programs are modified, test them again with ticket prices of $3, $4, and $5 and compare the results. ▮

2.4 Global Constants

As the Prologue already says, functions such as `profit` benefit from the use of global constants. Every programming language allows programmers to define constants. In BSL, such a definition has the following shape:

- write "`(define `",
- write down the name,
- followed by a space and an expression, and
- write down ")".

The name of a constant is a *global variable* while the definition is called a *constant definition*. We tend to call the expression in a constant definition the *right-hand side* of the definition.

Constant definitions introduce names for all forms of data: numbers, images, strings, and so on. Here are some simple examples:

```
; the current price of a movie ticket:
(define CURRENT-PRICE 5)

; useful to compute the area of a disk:
(define ALMOST-PI 3.14)
```

```
; a blank line:
(define NL "\n")

; an empty scene:
(define MT (empty-scene 100 100))
```

The first two are numeric constants, the last two are a string and an image. By convention, we use uppercase letters for global constants because it ensures that no matter how large the program is, the readers of our programs can easily distinguish such variables from others.

All functions in a program may refer to these global variables. A reference to a variable is just like using the corresponding constants. The advantage of using variable names instead of constants is that a single edit of a constant definition affects all uses. For example, we may wish to add digits to ALMOST-PI or enlarge an empty scene:

```
(define ALMOST-PI 3.14159)

; an empty scene:
(define MT (empty-scene 200 800))
```

Most of our sample definitions employ *literal constants* on the right-hand side, but the last one uses an expression. And indeed, a programmer can use arbitrary expressions to compute constants. Suppose a program needs to deal with an image of some size and its center:

```
(define WIDTH 100)
(define HEIGHT 200)

(define MID-WIDTH (/ WIDTH 2))
(define MID-HEIGHT (/ HEIGHT 2))
```

It can use two definitions with literal constants on the right-hand side and two *computed constants*, that is, variables whose values are not just literal constants but the results of computing the value of an expression.

Again, we state an imperative slogan:

For every constant mentioned in a problem statement, introduce one constant definition.

Exercise 30. Define constants for the price optimization program at the movie theater so that the price sensitivity of attendance (15 people for every 10 cents) becomes a computed constant. ▌

2.5 Programs

You are ready to create simple programs. From a coding perspective, a program is just a bunch of function and constant definitions. Usually one function is singled out as the "main" function, and this main function tends to compose others. From the perspective of launching a program, however, there are two distinct kinds:

- a *batch program* consumes all of its inputs at once and computes its result. Its main function is the composition of auxiliary functions, which may refer to additional auxiliary functions, and so on. When we launch a batch program, the operating system calls the main function on its inputs and waits for the program's output.
- an *interactive program* consumes some of its inputs, computes, produces some output, consumes more input, and so on. When an input shows up, we speak of an *event*, and we create interactive programs as *event-driven* programs. The main function of such an event-driven program uses an expression to describe which functions to call for which kinds of events. These functions are called *event handlers*.

  When we launch an interactive program, the main function informs the operating system of this description. As soon as input events happen, the operating system calls the matching event handler. Similarly, the operating system knows from the description when and how to present the results of these function calls as output.

This book focuses mostly on programs that interact via *graphical user interfaces (GUI)*; there are other kinds of interactive programs, and you will get to know those as you continue to study computer science.

Batch Programs As mentioned, a batch program consumes all of its inputs at once and computes the result from these inputs. Its main function expects some arguments, hands them to auxiliary functions, receives results from those, and composes these results into its own final answer.

Once programs are created, we want to use them. In DrRacket, we launch batch programs in the interactions area so that we can watch the program at work.

Programs are even more useful if they can retrieve the input from some file and deliver the output to some other file. Indeed, the name "batch program" dates to the early days of computing when a program read a file (or several files) from a batch of punch cards and placed the result in some other file(s), also a batch of cards. Conceptually, a batch program reads the input file(s) at once and also produces the result file(s) all at once.

We create such file-based batch programs with the *2htdp/batch-io* library, which adds two functions to our vocabulary (among others):

- `read-file`, which reads the content of an entire file as a string, and
- `write-file`, which creates a file from a given string.

These functions write strings to files and read strings from them:

Before you evaluate these expressions, save the definitions area in a file.

```
> (write-file "sample.dat" "212")
"sample.dat"
> (read-file "sample.dat")
"212"
```

After the first interaction the file named `"sample.dat"` contains

```
212
```

The result of `write-file` is an acknowledgment that it has placed the string in the file. If the file already exists, it replaces its content with the given string; otherwise, it creates a file and makes the given string its content. The second interaction, `(read-file "sample.dat")`, produces `"212"` because it turns the content of `"sample.dat"` into a String.

For pragmatic reasons, `write-file` also accepts `'stdout`, a special kind of token, as the first argument. It then displays the resulting file content in the current interactions area, for example:

```
> (write-file 'stdout "212\n")
212
'stdout
```

The names `'stdout` and `'stdin` are short for standard output device and standard input device, respectively.

By analogy, `read-file` accepts `'stdin` in lieu of a file name and then reads input from the current interactions area.

Let's illustrate the creation of a batch program with a simple example. Suppose we wish to create a program that converts a temperature measured on a Fahrenheit thermometer into a Celsius temperature. Don't worry, this question isn't a test about your physics knowledge; here is the conversion formula:

$$C = \frac{5}{9} \cdot (f - 32)$$

This book is not about memorizing facts, but we do expect you to know where to find them. Do you know where to find out how temperatures are converted?

Naturally, in this formula *f* is the Fahrenheit temperature and *C* is the Celsius temperature. While this formula might be good enough for a pre-algebra textbook, a mathematician or a programmer would write *C*(*f*) on the left side of the equation to remind readers that *f* is a given value and *C* is computed from *f*.

Translating this formula into BSL is straightforward:

```
(define (C f)
  (* 5/9 (- f 32)))
```

Recall that `5/9` is a number, a rational fraction to be precise, and that *C* depends on the given *f*, which is what the function notation expresses.

Launching this batch program in the interactions area works as usual:

```
> (C 32)
0
> (C 212)
100
> (C -40)
-40
```

But suppose we wish to use this function as part of a program that reads the Fahrenheit temperature from a file, converts this number into a Celsius temperature, and then creates another file that contains the result.

Once we have the conversion formula in BSL, creating the main function means composing `C` with existing primitive functions:

```
(define (convert in out)
  (write-file out
    (string-append
      (number->string
        (C
          (string->number
            (read-file in))))
        "\n")))
```

We call the main function `convert`. It consumes two file names: `in` for the file where the Fahrenheit temperature is found and `out` for where we want the Celsius result. A composition of five functions computes `convert`'s result. Let's step through `convert`'s body carefully:

1. `(read-file in)` retrieves the content of the named file as a string;

2. `string->number` turns this string into a number;

3. `C` interprets the number as a Fahrenheit temperature and converts it into a Celsius temperature;

4. `number->string` consumes this Celsius temperature and turns it into a string; and

5. `(write-file out ...)` places this string into the file named `out`.

This long list of steps might look overwhelming, and it doesn't even include the `string-append` part. Stop! Explain

```
(string-append ... "\n")
```

In contrast, the average function composition in a pre-algebra course involves two functions, possibly three. Keep in mind, though, that programs accomplish a real-world purpose while exercises in algebra merely illustrate the idea of function composition.

At this point, we can experiment with `convert`. To start with, we use `write-file` to create an input file for `convert`:

```
> (write-file "sample.dat" "212")
"sample.dat"
> (convert "sample.dat" 'stdout)
100
'stdout
> (convert "sample.dat" "out.dat")
"out.dat"
> (read-file "out.dat")
"100"
```

You can also create `"sample.dat"` with a file editor.

For the first interaction, we use `'stdout` so that we can view what `convert` outputs in DrRacket's interactions area. For the second one, `convert` is given the name `"out.dat"`. As expected, the call to `convert` returns this string; from the description of `write-file` we also know that it deposited a Fahrenheit temperature in the file. Here we read the content of this file with `read-file`, but you could also view it with a text editor.

In addition to running the batch program, it is also instructive to step through the computation. Make sure that the file `"sample.dat"` exists and contains just a number, then click the *STEP* button in DrRacket. Doing so opens another window in which you can peruse the computational process that the call to the main function of a batch program triggers. You will see that the process follows the above outline.

Exercise 31. Recall the `letter` program from Composing Functions. Here is how to launch the program and have it write its output to the interactions area:

```
> (write-file
   'stdout
   (letter "Matthew" "Fisler" "Felleisen"))
Dear Matthew,

We have discovered that all people with the
last name Fisler have won our lottery. So,
Matthew, hurry and pick up your prize.

Sincerely,
Felleisen
'stdout
```

Of course, programs are useful because you can launch them for many different inputs. Run `letter` on three inputs of your choice.

Here is a letter-writing batch program that reads names from three files and writes a letter to one:

```
(define (main in-fst in-lst in-signature out)
  (write-file out
              (letter (read-file in-fst)
                      (read-file in-lst)
                      (read-file in-signature))))
```

The function consumes four strings: the first three are the names of input files and the last one serves as an output file. It uses the first three to read one string each from the three named files, hands these strings to `letter`, and eventually writes the result of this function call into the file named by `out`, the fourth argument to `main`.

Create appropriate files, launch `main`, and check whether it delivers the expected letter in a given file. ▌

Interactive Programs Batch programs are a staple of business uses of computers, but the programs people encounter now are interactive. In this day and age, people mostly interact with desktop applications via a keyboard and a mouse. Furthermore, interactive programs can also react to computer-generated events, for example, clock ticks or the arrival of a message from some other computer.

Exercise 32. Most people no longer use desktop computers just to run applications but also employ cell phones, tablets, and their cars' information control screen. Soon people will use wearable computers in the form of intelligent glasses, clothes, and sports gear. In the somewhat more distant future, people may come with built-in bio computers that directly interact with body functions. Think of ten different forms of events that software applications on such computers will have to deal with. ▌

The purpose of this section is to introduce the mechanics of writing **interactive** BSL programs. Because many of the project-style examples in this book are interactive programs, we introduce the ideas slowly and carefully. You may wish to return to this section when you tackle some of the interactive programming projects; a second or third reading may clarify some of the advanced aspects of the mechanics.

By itself, a raw computer is a useless piece of physical equipment. It is called *hardware* because you can touch it. This equipment becomes useful once you install *software*, that is, a suite of programs. Usually the first piece of software to be installed on a computer is an *operating system*. It has the task of managing the computer for you, including connected devices such as the monitor, the keyboard, the mouse, the speakers, and so on. The way it works is that when a user presses a key on the keyboard, the operating system runs a function that processes keystrokes. We say that the keystroke is a *key event*, and the function is an *event handler*. In the same vein, the operating system runs an event handler for clock ticks, for mouse actions, and so on. Conversely, after an event handler is done with its work, the operating system may have to change the image on the screen, ring a bell, print a document, or perform a similar action. To accomplish these tasks, it also runs functions that translate the operating system's data into sounds, images, actions on the printer, and so on.

Naturally, different programs have different needs. One program may interpret keystrokes as signals to control a nuclear reactor; another passes them to a word processor. To make a general-purpose computer work on these radically different tasks, different programs install different event handlers. That is, a rocket-launching program uses one kind of function to deal with clock ticks while an oven's software uses a different kind.

Designing an interactive program requires a way to designate some function as the one that takes care of keyboard events, another function for dealing with clock ticks, a third one for presenting some data as an image, and so forth. It is the task of an interactive program's main function to communicate these designations to the operating system, that is, the software platform on which the program is launched.

DrRacket is a small operating system, and BSL is one of its programming languages. The latter comes with the *2htdp/universe* library, which provides `big-bang`, a mechanism for telling the operating system which function deals with which event. In addition, `big-bang` keeps track of the *state of the program*. To this end, it comes with one required sub-expression, whose value becomes the *initial state* of the program. Otherwise `big-bang` consists of one required clause and many optional clauses. The required `to-draw` clause tells DrRacket how to render the state of the program, including the initial one. Each of the other, optional clauses tells the operating system that a certain function takes care of a certain event. Taking care of an event in BSL means that the function consumes the state of the program and a description of the event, and that it produces the next state of the program. We therefore speak of the *current state* of the program.

Terminology In a sense, a `big-bang` expression describes how a program connects with a small segment of the world. This world might be a game that the program's users play, an animation that the user watches, or a text editor that the user employs to manipulate some notes. Programming language researchers therefore often say that `big-bang` is a description of a small world: its initial state, how states are transformed, how states are rendered, and how `big-bang` may determine other attributes of the current state. In this spirit, we also speak of the *state of the world* and even call `big-bang` programs *world programs*. **End**

Let's study this idea step-by-step, starting with this definition:

```
(define (number->square s)
  (square s "solid" "red"))
```

The function consumes a positive number and produces a solid red square of that size. After clicking *RUN*, experiment with the function, like this:

```
> (number->square 5)
```

```
> (number->square 10)
```

```
> (number->square 20)
```

It behaves like a batch program, consuming a number and producing an image, which DrRacket renders for you.

Now try the following big-bang expression in the interactions area:

```
> (big-bang 100 [to-draw number->square])
```

A separate window appears, and it displays a 100×100 red square. In addition, the DrRacket interactions area does not display another prompt; it is as if the program keeps running, and this is indeed the case. To stop the program, click on DrRacket's *STOP* button or the window's *CLOSE* button:

```
> (big-bang 100 [to-draw number->square])
100
```

When DrRacket stops the evaluation of a big-bang expression, it returns the current state, which in this case is just the initial state: 100.

Here is a more interesting big-bang expression:

```
> (big-bang 100
    [to-draw number->square]
    [on-tick sub1]
    [stop-when zero?])
```

This big-bang expression adds two optional clauses to the previous one: the on-tick clause tells DrRacket how to deal with clock ticks and the stop-when clause says when to stop the program. We read it as follows, starting with 100 as the initial state:

1. every time the clock ticks, subtract 1 from the current state;

2. then check whether zero? is true of the new state and if so, stop; and

3. every time an event handler returns a value, use number->square to render it as an image.

Now hit the "return" key and observe what happens. Eventually the evaluation of the expressions terminates and DrRacket displays 0.

The big-bang expression keeps track of the current state. Initially this state is 100. Every time the clock ticks, it calls the clock-tick handler and gets a new state. Hence, the state of big-bang changes as follows:

100, 99, 98, ..., 2, 1, 0

When the state's value becomes 0, the evaluation is done. For every other state—from 100 to 1—big-bang translates the state into an image, using number->square as the to-draw clause tells it to. Hence, the window displays a red square that shrinks from 100×100 pixels to 1×1 pixel over 100 clock ticks.

Let's add a clause for dealing with key events. First, we need a function that consumes the current state and a string that describes the key event and then returns a new state:

```
(define (reset s ke)
  100)
```

This function throws away its arguments and returns 100, which is the initial state of the big-bang expression we wish to modify. Second, we add an on-key clause to the big-bang expression:

```
> (big-bang 100
    [to-draw number->square]
    [on-tick sub1]
    [stop-when zero?]
    [on-key reset])
```

Stop! Explain what happens when you hit "return", count to 10, and finally press "a".

What you will see is that the red square shrinks at the rate of one pixel per clock tick. As soon as you press the "a" key, though, the red square reinflates to full size because reset is called on the current length of the square and "a" and returns 100. This number becomes big-bang's new state and number->square renders it as a full-sized red square.

In order to understand the evaluation of big-bang expressions in general, let's look at a schematic version:

```
(big-bang cw0
  [on-tick tock]
  [on-key ke-h]
  [on-mouse me-h]
```

```
   [to-draw render]
   [stop-when end?]
   ...)
```

This big-bang expression specifies three event handlers—tock, ke-h, and me-h—and a stop-when clause.

The evaluation of this big-bang expression starts with cw0, which is usually an expression. DrRacket, our operating system, installs the value of cw0 as the current state. It uses render to translate the current state into an image, which is then displayed in a separate window. Indeed, render is the **only** means for a big-bang expression to present data to the world.

Here is how events are processed:

- Every time the clock ticks, DrRacket applies tock to big-bang's current state and receives a value in response; big-bang treats this return value as the next current state.

- Every time a key is pressed, DrRacket applies ke-h to big-bang's current state and a string that represents the key; for example, pressing the "a" key is represented with "a" and the left arrow key with "left". When ke-h returns a value, big-bang treats it as the next current state.

- Every time a mouse enters the window, leaves it, moves, or is clicked, DrRacket applies me-h to big-bang's current state, the event's x- and y-coordinates, and a string that represents the kind of mouse event that happened; for example, clicking a mouse's button is represented with "button-down". When me-h returns a value, big-bang treats it as the next current state.

All events are processed in order; if two events seem to happen at the same time, DrRacket acts as a tie-breaker and arranges them in some order.

After an event is processed, big-bang uses both end? and render to check the current state:

- (end? cw) produces a Boolean value. If it is ##true, big-bang stops the computation immediately. Otherwise it proceeds.

- (render cw) is expected to produce an image and big-bang displays this image in a separate window.

current state	cw_0	cw_1	...
event	e_0	e_1	...
on clock tick	(tock cw_0)	(tock cw_1)	...
on keystroke	(ke-h cw_0 e_0)	(ke-h cw_1 e_1)	...
on mouse event	(me-h cw_0 e_0 ...)	(me-h cw_1 e_1 ...)	...
its image	(render cw_0)	(render cw_1)	...

Figure 13: How big-bang works

The table in figure 13 concisely summarizes this process. In the first row, it lists names for the current states. The second row enumerates names for the events that DrRacket encounters: e_0, e_1, and so on. Each e_i might be a clock tick, a key press, or a mouse event. The next three rows specify the result of dealing with the event:

- If e_0 is a clock tick, big-bang evaluates (tock cw_0) to produce cw_1.

- If e_0 is a key event, (ke-h cw_0 e_0) is evaluated and yields cw_1. The handler must be applied to the event itself because, in general, programs are going to react to each key differently.

- If e_0 is a mouse event, big-bang runs (me-h cw_0 e_0 ...) to get cw_1. The call is a sketch because a mouse event e_0 is really associated with several pieces of data—its nature and its coordinates—and we just wish to indicate that much.

- Finally, render turns the current state into an image, which is indicated by the last row. DrRacket displays these images in the separate window.

The column below cw_1 shows how cw_2 is generated, depending on what kind of event e_1 takes place.

Let's interpret this table with the specific sequence of events: the user presses the "a" key, then the clock ticks, and finally the user clicks the mouse to trigger a "button down" event at position (90,100). Then, in Racket notation,

1. cw1 is the result of (ke-h cw0 "a");

2. cw2 is the result of (tock cw1); and

3. cw3 is the result of (me-h cw2 90 100 "button-down").

We can actually express these three steps as a sequence of three definitions:

```
(define cw1 (ke-h cw0 "a"))
(define cw2 (tock cw1))
(define cw3 (me-h cw2 90 100 "button-down"))
```

Stop! How does big-bang display each of these three states?

Now let's consider a sequence of three clock ticks. In that case,

1. cw1 is the result of (tock cw0);

2. cw2 is the result of (tock cw1); and

3. cw3 is the result of (tock cw2).

Or, reformulated in BSL:

```
(define cw1 (tock cw0))
(define cw2 (tock cw1))
(define cw3 (tock cw2))
```

Indeed, we can also determine cw3 via a single expression:

```
(tock (tock (tock cw0)))
```

This determines the state that big-bang computes after three clock ticks. Stop! Reformulate the first sequence of events as an expression.

```
(define BACKGROUND (empty-scene 100 100))
(define DOT (circle 3 "solid" "red"))

(define (main y)
  (big-bang y
    [on-tick sub1]
    [stop-when zero?]
    [to-draw place-dot-at]
    [on-key stop]))

(define (place-dot-at y)
  (place-image DOT 50 y BACKGROUND))

(define (stop y ke)
  0)
```

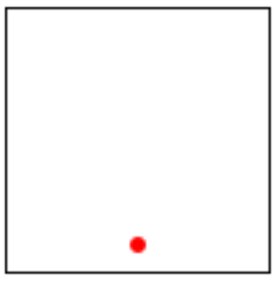
Figure 14: A first interactive program

In short, the sequence of events determines in which order big-bang conceptually traverses the above table of possible states to arrive at the current state for each time slot. Of course, big-bang does not touch the current state; it merely safeguards it and passes it to event handlers and other functions when needed.

From here, it is straightforward to define a first interactive program. See figure 14. The program consists of two constant definitions followed by three function definitions: main, which launches a big-bang interactive program; place-dot-at, which translates the current state into an image; and stop, which throws away its inputs and produces 0.

After clicking *RUN*, we can ask DrRacket to evaluate applications of these handler functions. This is one way to confirm their workings:

```
> (place-dot-at 89)
```



```
> (stop 89 "q")
0
```

Stop! Try now to understand how main reacts when you press a key.

One way to find out whether your conjecture is correct is to launch the main function on some reasonable number:

```
> (main 90)
```

Relax.

By now, you may feel that these first two chapters are overwhelming. They introduce many new concepts, including a new language, its vocabulary, its meaning, its idioms, a tool for writing down texts in this vocabulary, and a way of running these programs. Confronted with this plethora of ideas, you may wonder how one creates a program when presented with a problem statement. To answer this central question, the next chapter takes a step back and explicitly addresses the systematic design of programs. So take a breather and continue when ready.

3 How to Design Programs

The first few chapters of this book show that learning to program requires some mastery of many concepts. On the one hand, programming needs a language, a notation for communicating what we wish to compute. The languages for formulating programs are artificial constructions, though acquiring a programming language shares some elements with acquiring a natural language. Both come with vocabulary, grammar, and an understanding of what "phrases" mean.

On the other hand, it is critical to learn how to get from a problem statement to a program. We need to determine what is relevant in the problem statement and what can be ignored. We need to tease out what the program consumes, what it produces, and how it relates inputs to outputs. We have to know, or find out, whether the chosen language and its libraries provide certain basic operations for the data that our program is to process. If not, we might have to develop auxiliary functions that implement these operations. Finally, once we have a program, we must check whether it actually performs the intended computation. And this might reveal all kinds of errors, which we need to be able to understand and fix.

All this sounds rather complex, and you might wonder why we don't just muddle our way through, experimenting here and there, leaving well enough alone when the results look decent. This approach to programming, often dubbed "garage programming," is common and succeeds on many occasions; sometimes it is the launching pad for a start-up company. Nevertheless, the start-up cannot sell the results of the "garage effort" because only the original programmers and their friends can use them.

A good program comes with a short write-up that explains what it does, what inputs it expects, and what it produces. Ideally, it also comes with some assurance that it actually works. In the best circumstances, the program's connection to the problem statement is evident so that a small change to the problem statement is easy to translate into a small change to the program. Software engineers call this a "programming product."

All this extra work is necessary because programmers don't create programs for themselves. Programmers write programs for other programmers to read, and on occasion, people run these programs to get work done. Most programs are large, complex collections of collaborating functions, and nobody can write all these functions in a day. Programmers join projects, write code, leave projects; others take over their programs and work on them. Another difficulty is that the programmer's clients tend to change their mind about what problem they really want solved. They usually have it almost right, but more often than not, they get some details wrong. Worse, complex logical constructions such as programs almost always suffer from human errors; in short, programmers make mistakes. Eventually someone discovers these errors and programmers must fix them. They need to reread the programs from a month ago, a year ago, or twenty years ago and change them.

The word "other" also includes older versions of the programmer who usually cannot recall all the thinking that the younger version put into the production of the program.

Exercise 33. Research the "year 2000" problem. ▮

Here we present a design recipe that integrates a step-by-step process with a way of organizing programs around problem data. For the readers who don't like to stare at blank screens for a long time, this design recipe offers a way to make progress in a systematic manner. For those of you who teach others to design programs, the recipe is a device for diagnosing a novice's difficulties. For others, our recipe might be something that they can apply to other areas—say, medicine, journalism, or engineering. For those who wish to become real programmers, the design recipe also offers a way to understand and work on existing programs—though not all programmers use a method like this design recipe to come up with programs. The rest of this chapter is dedicated to the first baby steps into the world of the design recipe; the following chapters and parts refine and expand the recipe in one way or another.

3.1 Designing Functions

Information and Data The purpose of a program is to describe a computational process that consumes some information and produces new information. In this sense, a program is like the instructions a mathematics teacher gives to grade school students. Unlike a student, however, a program works with more than numbers: it calculates with navigation information, looks up a person's address, turns on switches, or inspects the state of a video game. All this information comes from a part of the real world—often called the program's *domain*—and the results of a program's computation represent more information in this domain.

Information plays a central role in our description. Think of information as facts about the program's domain. For a program that deals with a furniture catalog, a “table with five legs” or a “square table of two by two meters” are pieces of information. A game program deals with a different kind of domain, where “five” might refer to the number of pixels per clock tick that some object travels on its way from one part of the canvas to another. Or, a payroll program is likely to deal with “five deductions.”

For a program to process information, it must turn it into some form of *data* in the programming language; then it processes the data; and once it is finished, it turns the resulting data into information again. An interactive program may even intermingle these steps, acquiring more information from the world as needed and delivering information in between.

We use BSL and DrRacket so that you do **not** have to worry about the translation of information into data. In DrRacket’s BSL you can apply a function directly to data and observe what it produces. As a result, we avoid the serious chicken-and-egg problem of writing functions that convert information into data and vice versa. For simple kinds of information, designing such program pieces is trivial; for anything other than simple information, you need to know about parsing, for example, and **that** immediately requires a lot of expertise in program design.

Software engineers use the slogan *model-view-controller* (MVC) for the way BSL and DrRacket separate data processing from parsing information into data and turning data into information. Indeed, it is now accepted wisdom that well-engineered software systems enforce this separation, even though most introductory books still commingle them. Thus, working with BSL and DrRacket allows you to focus on the design of the core of programs, and, when you have enough experience with that, you can learn to design the information-data translation parts.

Here we use two preinstalled teachpacks to demonstrate the separation of data and information: `2htdp/batch-io` and `2htdp/universe`. Starting with this chapter, we develop design recipes for **batch** and **interactive** programs to give you an idea of how complete programs are designed. Do keep in mind that the libraries of full-fledged programming languages offer many more contexts for complete programs, and that you will need to adapt the design recipes appropriately.

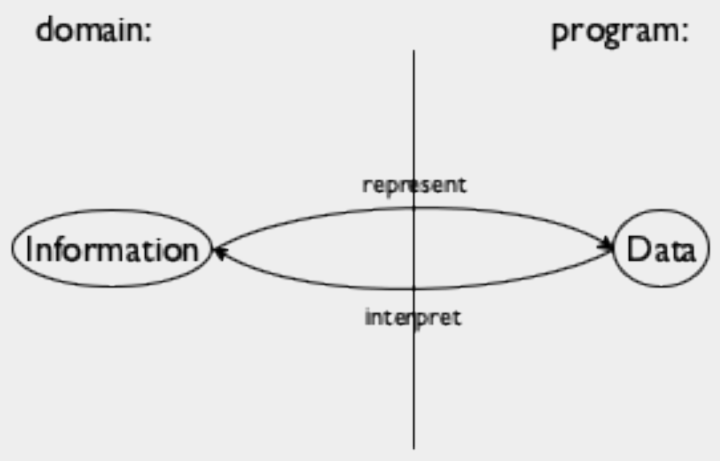


Figure 15: From information to data, and back

Given the central role of information and data, program design must start with the connection between them. Specifically, we, the programmers, must decide how to use our chosen programming language to *represent* the relevant pieces of information as data and how we should *interpret* data as information. Figure 15 explains this idea with an abstract diagram.

To make this idea concrete, let’s work through some examples. Suppose you are designing a program that consumes and produces information in the form of numbers. While choosing a representation is easy, an interpretation requires explaining what a number such as `42` denotes in the domain:

- `42` may refer to the number of pixels from the top margin in the domain of images;
- `42` may denote the number of pixels per clock tick that a simulation or game object moves;
- `42` may mean a temperature, on the Fahrenheit, Celsius, or Kelvin scale for the domain of physics;
- `42` may specify the size of some table if the domain of the program is a furniture catalog; or
- `42` could just count the number of characters in a string.

The key is to know how to go from numbers as information to numbers as data and vice versa.

Since this knowledge is so important for everyone who reads the program, we often write it down in the form of comments, which we call *data definitions*. A data definition serves two purposes. First, it names a collection of data—a *class*—using a meaningful word. Second, it informs readers how to create elements of this class and how to decide whether some arbitrary piece of data belongs to the collection.

Computing scientists use “class” to mean something like a “mathematical set.”

Here is a data definition for one of the above examples:

```
; A Temperature is a Number.
; interpretation represents Celsius degrees
```

The first line introduces the name of the data collection, Temperature, and tells us that the class consists of all Numbers. So, for example, if we ask whether `102` is a temperature, you can respond with "yes" because `102` is a number and all numbers are temperatures. Similarly, if we ask whether `"cold"` is a Temperature, you will say "no" because no string belongs to Temperature. And, if we asked you to make up a sample Temperature, you might come up with something like `-400`.

If you happen to know that the lowest possible temperature is approximately $-274°$ C, you may wonder whether it is possible to express this knowledge in a data definition. Since our data definitions are really just English descriptions of classes, you may indeed define the class of temperatures in a much more accurate manner than shown here. In this book, we use a stylized form of English for such data definitions, and the next chapter introduces the style for imposing constraints such as "larger than `-274`."

So far, you have encountered the names of four classes of data: Number, String, Image, and Boolean. With that, formulating a new data definition means nothing more than introducing a new name for an existing form of data, say, "temperature" for numbers. Even this limited knowledge, though, suffices to explain the outline of our design process.

The Design Process Once you understand how to represent input information as data and to interpret output data as information, the design of an individual function proceeds according to a straightforward process:

At this point, you may wish to reread the section on Systematic Program Design in the Preface, especially figure 1.

1. Express how you wish to represent information as data. A one-line comment suffices:

   ```
   ; We use numbers to represent centimeters.
   ```

   Formulate data definitions, like the one for Temperature, for the classes of data you consider critical for the success of your program.

2. Write down a signature, a statement of purpose, and a function header.

   A *function signature* is a comment that tells the readers of your design how many inputs your function consumes, from which classes they are drawn, and what kind of data it produces. Here are three examples for functions that respectively

   - consume one String and produce a Number:

     ```
     ; String -> Number
     ```

   - consume a Temperature and produce a String:

     ```
     ; Temperature -> String
     ```

     As this signature points out, introducing a data definition as an alias for an existing form of data makes it easy to read the intention behind signatures.

     Nevertheless, we recommend that you stay away from aliasing data definitions for now. A proliferation of such names can cause quite a lot of confusion. It takes practice to balance the need for new names and the readability of programs, and there are more important ideas to understand right now.

   - consume a Number, a String, and an Image:

     ```
     ; Number String Image -> Image
     ```

     Stop! What does this function produce?

   A *purpose statement* is a BSL comment that summarizes the purpose of the function in a single line. If you are ever in doubt about a purpose statement, write down the shortest possible answer to the question

   > *what does the function compute?*

   Every reader of your program should understand what your functions compute **without** having to read the function itself.

   A multi-function program should also come with a purpose statement. Indeed, good programmers write two purpose statements: one for the reader who may have to modify the code and another one for the person who wishes to use the program but not read it.

   Finally, a *header* is a simplistic function definition, also called a *stub*. Pick one variable name for each class of input in the signature; the body of the function can be any piece of data from the output class. These three function headers match the above three signatures:

   - `(define (f a-string) 0)`

- `(define (g n) "a")`
- `(define (h num str img) (empty-scene 100 100))`

Our parameter names reflect what kind of data the parameter represents. Sometimes, you may wish to use names that suggest the purpose of the parameter.

When you formulate a purpose statement, it is often useful to employ the parameter names to clarify what is computed. For example,

```
; Number String Image -> Image
; adds s to img,
; y pixels from the top and 10 from the left
(define (add-image y s img)
  (empty-scene 100 100))
```

At this point, you can click the *RUN* button and experiment with the function. Of course, the result is always the same value, which makes these experiments quite boring.

3. Illustrate the signature and the purpose statement with some functional examples. To construct a *functional example*, pick one piece of data from each input class from the signature and determine what you expect back.

   Suppose you are designing a function that computes the area of a square. Clearly this function consumes the length of the square's side, and that is best represented with a (positive) number. Assuming you have done the first process step according to the recipe, you add the examples between the purpose statement and the header and get this:

```
; Number -> Number
; computes the area of a square with side len
; given: 2, expect: 4
; given: 7, expect: 49
(define (area-of-square len) 0)
```

4. The next step is to take *inventory*, to understand what are the givens and what we need to compute. For the simple functions we are considering right now, we know that they are given data via parameters. While parameters are placeholders for values that we don't know yet, we do know that it is from this unknown data that the function must compute its result. To remind ourselves of this fact, we replace the function's body with a *template*.

   We owe the term "inventory" to Stephen Bloch.

   For now, the template contains just the parameters, so that the preceding example looks like this:

```
(define (area-of-square len)
   (... len ...))
```

   The dots remind you that this isn't a complete function, but a template, a suggestion for an organization.

   The templates of this section look boring. As soon as we introduce new forms of data, templates become interesting.

5. It is now time to *code*. In general, to code means to program, though often in the narrowest possible way, namely, to write executable expressions and function definitions.

   To us, coding means to replace the body of the function with an expression that attempts to compute from the pieces in the template what the purpose statement promises. Here is the complete definition for `area-of-square`:

```
; Number -> Number
; computes the area of a square with side len
; given: 2, expect: 4
; given: 7, expect: 49
(define (area-of-square len)
  (sqr len))
```

```
; Number String Image -> Image
; adds s to img, y pixels from top, 10 pixels to the left
; given:
;    5 for y,
;    "hello" for s, and
;    (empty-scene 100 100) for img
; expected:
;    (place-image (text "hello" 10 "red") 10 5 ...)
;    where ... is (empty-scene 100 100)
(define (add-image y s img)
  (place-image (text s 10 "red") 10 y img))
```

Figure 16: The completion of design step 5

To complete the `add-image` function takes a bit more work than that: see figure 16. In particular, the function needs to turn the given string `s` into an image, which is then placed into the given scene.

5. The last step of a proper design is to test the function on the examples that you worked out before. For now, testing works like this. Click the *RUN* button and enter function applications that match the examples in the interactions area:

```
> (area-of-square 2)
4
> (area-of-square 7)
49
```

The results must match the output that you expect; you must inspect each result and make sure it is equal to what is written down in the example portion of the design. If the result doesn't match the expected output, consider the following three possibilities:

a. You miscalculated and determined the wrong expected output for some of the examples.

b. Alternatively, the function definition computes the wrong result. When this is the case, you have a *logical error* in your program, also known as a *bug*.

c. Both the examples and the function definition are wrong.

When you do encounter a mismatch between expected results and actual values, we recommend that you first reassure yourself that the expected results are correct. If so, assume that the mistake is in the function definition. Otherwise, fix the example and then run the tests again. If you are still encountering problems, you may have encountered the third, somewhat rare, situation.

3.2 Finger Exercises: Functions

The first few of the following exercises are almost copies of those in Functions, though where the latter use the word "define" the exercises below use the word "design." What this difference means is that you should work through the design recipe to create these functions and your solutions should include all relevant pieces.

As the title of the section suggests, these exercises are practice exercises to help you internalize the process. Until the steps become second nature, never skip one because doing so leads to easily avoidable errors. There is plenty of room left in programming for complicated errors; we have no need to waste our time on silly ones.

Exercise 34. Design the function `string-first`, which extracts the first character from a non-empty string. Don't worry about empty strings. ▌

Exercise 35. Design the function `string-last`, which extracts the last character from a non-empty string. ▌

Exercise 36. Design the function `image-area`, which counts the number of pixels in a given image. ▌

Exercise 37. Design the function `string-rest`, which produces a string like the given one with the first character removed. ▌

Exercise 38. Design the function `string-remove-last`, which produces a string like the given one with the **last** character removed. ▌

3.3 Domain Knowledge

It is natural to wonder what knowledge it takes to code up the body of a function. A little bit of reflection tells you that this step demands an appropriate grasp of the domain of the program. Indeed, there are two forms of such *domain knowledge*:

1. Knowledge from external domains, such as mathematics, music, biology, civil engineering, art, and so on. Because programmers cannot know all of the application domains of computing, they must be prepared to understand the language of a variety of application areas so that they can discuss problems with domain experts. Mathematics is at the intersection of many, but not all, domains. Hence, programmers must often pick up new languages as they work through problems with domain experts.

2. Knowledge about the library functions in the chosen programming language. When your task is to translate a mathematical formula involving the tangent function, you need to know or guess that your chosen language comes with a function such as BSL's `tan`. When your task involves graphics, you will benefit from understanding the possibilities of the *2htdp/image* library.

Since you can never predict the area you will be working in, or which programming language you will have to use, it is imperative that you have a solid understanding of the full possibilities of whatever computer languages are around and suitable. Otherwise some domain expert with half-baked programming knowledge will take over your job.

You can recognize problems that demand domain knowledge from the data definitions that you work out. As long as the data definitions use classes that exist in the chosen programming language, the definition of the function body (and program) mostly relies on expertise in the domain. Later, when we introduce complex forms of data, the design of functions demands computer science knowledge.

3.4 From Functions to Programs

Not all programs consist of a single function definition. Some require several functions; many also use constant definitions. No matter what, it is always important to design every function systematically, though global constants as well as auxiliary functions change the design process a bit.

When you have defined global constants, your functions may use them to compute results. To remind yourself of their existence, you may wish to add these constants to your templates; after all, they belong to the inventory of things that may contribute to the function definition.

Multi-function programs come about because interactive programs automatically need functions that handle key and mouse events, functions that render the state as music, and possibly more. Even batch programs may require several different functions because they perform several separate tasks. Sometimes the problem statement itself suggests these tasks; other times you will discover the need for auxiliary functions as you are in the middle of designing some function.

For these reasons, we recommend keeping around a list of needed functions or a *wish list*. Each entry on a wish list should consist of three things: a meaningful name for the function, a signature, and a purpose statement. For the design of a batch program, put the main function on the wish list and start designing it. For the design of an interactive program, you can put the event handlers, the `stop-when` function, and the scene-rendering function on the list. As long as the list isn't empty, pick a wish and design the function. If you discover during the design that you need another function, put it on the list. When the list is empty, you are done.

We owe the term "wish list" to John Stone.

3.5 On Testing

Testing quickly becomes a labor-intensive chore. While it is easy to run small programs in the interactions area, doing so requires a lot of mechanical labor and intricate inspections. As programmers grow their systems, they wish to conduct many tests. Soon this labor becomes overwhelming, and programmers start to neglect it. At the same time, testing is the first tool for discovering and preventing basic flaws. Sloppy testing quickly leads to buggy functions—that is, functions with hidden problems—and buggy functions retard projects, often in multiple ways.

Hence, it is critical to mechanize tests instead of performing them manually. Like many programming languages, BSL includes a testing facility, and DrRacket is aware of this facility. To introduce this testing facility, we take a second look at the function that converts temperatures in Fahrenheit to Celsius temperatures from Programs. Here is the definition:

```
; Number -> Number
; converts Fahrenheit temperatures to Celsius
; given 32, expect 0
; given 212, expect 100
; given -40, expect -40
(define (f2c f)
  (* 5/9 (- f 32)))
```

Testing the function's examples calls for three computations and three comparisons between two numbers each. You can formulate these tests and add them to the definitions area in DrRacket:

```
(check-expect (f2c -40) -40)
(check-expect (f2c 32) 0)
(check-expect (f2c 212) 100)
```

When you now click the *RUN* button, you see a report from BSL that the program passed all three tests—and you have nothing else to do.

In addition to getting tests to run automatically, the `check-expect` forms show another advantage when tests fail. To see how this works, change one of the above tests so that the result is wrong, for example

```
(check-expect (f2c -40) 40)
```

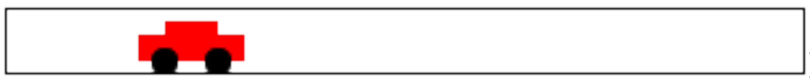
When you now click the *RUN* button, an additional window pops up. The window's text explains that one of three tests failed. For the failed test, the window displays three pieces: the computed value, the result of the function call (`-40`); the expected value (`40`); and a hyperlink to the text of the failed test case.

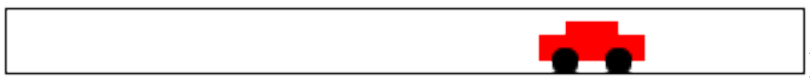
```
; Number -> Number
; converts Fahrenheit temperatures to Celsius temperatures

(check-expect (f2c -40) -40)
(check-expect (f2c 32) 0)
(check-expect (f2c 212) 100)

(define (f2c f)
  (* 5/9 (- f 32)))
```

Figure 17: Testing in BSL

You can place `check-expect` specifications above or below the function definitions that they test. When you click *RUN*, DrRacket collects all `check-expect` specifications and evaluates them **after** all function definitions have been added to the "vocabulary" of operations. Figure 17 shows how to exploit this freedom to combine the example and test step. Instead of writing down the examples as comments, you can translate them directly into tests. When you're all done with the design of the function, clicking *RUN* performs the test. And if you ever change the function for some reason, the next click retests the function.

Last but not least, `check-expect` also works for images. That is, you can test image-producing functions. Say you wish to design the function `render`, which places the image of a car, dubbed `CAR`, into a background scene, named `BACKGROUND`. For the design of this function, you may formulate the tests such as the following:

```
(check-expect (render 50)
```


```
              )
(check-expect (render 200)
```


```
              )
```

Alternatively, you could write them like this:

For additional ways of formulating tests, see intermezzo 1.

```
(check-expect (render 50)
              (place-image CAR 50 Y-CAR BACKGROUND))
(check-expect (render 200)
              (place-image CAR 200 Y-CAR BACKGROUND))
```

This alternative approach helps you figure out how to express the function body and is therefore preferable. One way to develop such expressions is to experiment in the interactions area.

Because it is so useful to have DrRacket conduct the tests and not to check everything yourself manually, we immediately switch to this style of testing for the rest of the book. This form of testing is dubbed *unit testing*, and BSL's unit-testing framework is especially tuned for novice programmers. One day you will switch to some other programming language; one of your first tasks will be to figure out its unit-testing framework.

3.6 Designing World Programs

While the previous chapter introduces the *2htdp/universe* library in an ad hoc way, this section demonstrates how the design recipe also helps you create world programs systematically. It starts with a brief summary of the *2htdp/universe* library based on data definitions and function signatures. Then it spells out the design recipe for world programs.

The teachpack expects that a programmer develops a data definition that represents the state of the world and a function `render` that knows how to create an image for every possible state of the world. Depending on the needs of the program, the programmer must then design functions that respond to clock ticks, keystrokes, and mouse events. Finally, an interactive program may need to stop when its current world belongs to a sub-class of states; `end?` recognizes these *final states*. Figure 18 spells out this idea in a schematic and simplified way.

```
; WorldState: data that represents the state of the world (cw)

; WorldState -> Image
; when needed, big-bang obtains the image of the current
; state of the world by evaluating (render cw)
(define (render ws) ...)

; WorldState -> WorldState
; for each tick of the clock, big-bang obtains the next
; state of the world from (clock-tick-handler cw)
```

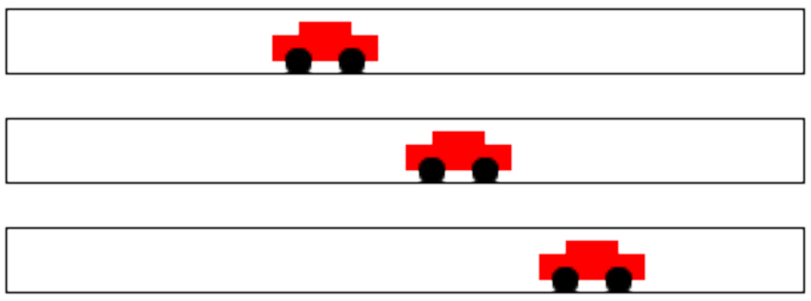
```
(define (clock-tick-handler cw) ...)

; WorldState String -> WorldState
; for each keystroke, big-bang obtains the next state
; from (keystroke-handler cw ke); ke represents the key
(define (keystroke-handler cw ke) ...)

; WorldState Number Number String -> WorldState
; for each mouse gesture, big-bang obtains the next state
; from (mouse-event-handler cw x y me) where x and y are
; the coordinates of the event and me is its description
(define (mouse-event-handler cw x y me) ...)

; WorldState -> Boolean
; after each event, big-bang evaluates (end? cw)
(define (end? cw) ...)
```

Figure 18: The wish list for designing world programs

Assuming that you have a rudimentary understanding of the workings of `big-bang`, you can focus on the truly important problem of designing world programs. Let's construct a concrete example for the following design recipe:

> **Sample Problem** Design a program that moves a car from left to right on the world canvas, three pixels per clock tick.

For this problem statement, it is easy to imagine scenes for the domain:



In this book, we often refer to the domain of an interactive `big-bang` program as a "world," and we speak of designing "world programs."

The design recipe for world programs, like the one for functions, is a tool for systematically moving from a problem statement to a working program. It consists of three big steps and one small one:

1. For all those properties of the world that remain the same over time and are needed to render it as an Image, introduce constants. In BSL, we specify such constants via definitions. For the purpose of world programs, we distinguish between two kinds of constants:

   a. "Physical" constants describe general attributes of objects in the world, such as the speed or velocity of an object, its color, its height, its width, its radius, and so forth. Of course these constants don't really refer to physical facts, but many are analogous to physical aspects of the real world.

      In the context of our sample problem, the radius of the car's wheels and the distance between the wheels are such "physical" constants:

      ```
      (define WIDTH-OF-WORLD 200)

      (define WHEEL-RADIUS 5)
      (define WHEEL-DISTANCE (* WHEEL-RADIUS 5))
      ```

      Note how the second constant is computed from the first.

   b. Graphical constants are images of objects in the world. The program composes them into images that represent the complete state of the world.

      Here are graphical constants for wheel images of our sample car:

      ```
      (define WHEEL
        (circle WHEEL-RADIUS "solid" "black"))
      ```

      We suggest you experiment in DrRacket's interactions area to develop such graphical constants.

      ```
      (define SPACE
        (rectangle ... WHEEL-RADIUS ... "white"))
      (define BOTH-WHEELS
      ```

```
  (beside WHEEL SPACE WHEEL))
```

Graphical constants are usually computed, and the computations tend to involve physical constants and other images.

It is good practice to annotate constant definitions with a comment that explains what they mean.

2. Those properties that change over time—in reaction to clock ticks, keystrokes, or mouse actions—give rise to the current state of the world. Your task is to develop a data representation for all possible states of the world. The development results in a data definition, which comes with a comment that tells readers how to represent world information as data and how to interpret data as information about the world.

Choose simple forms of data to represent the state of the world.

For the running example, it is the car's distance from the left margin that changes over time. While the distance to the right margin changes, too, it is obvious that we need only one or the other to create an image. A distance is measured in numbers, so the following is an adequate data definition:

```
; A WorldState is a Number.
; interpretation the number of pixels between
; the left border of the scene and the car
```

An alternative is to count the number of clock ticks that have passed and to use this number as the state of the world. We leave this design variant as an exercise.

3. Once you have a data representation for the state of the world, you need to design a number of functions so that you can form a valid `big-bang` expression.

To start with, you need a function that maps any given state into an image so that `big-bang` can render the sequence of states as images:

```
; render
```

Next you need to decide which kind of events should change which aspects of the world state. Depending on your decisions, you need to design some or all of the following three functions:

```
; clock-tick-handler
; keystroke-handler
; mouse-event-handler
```

Finally, if the problem statement suggests that the program should stop if the world has certain properties, you must design

```
; end?
```

For the generic signatures and purpose statements of these functions, see figure 18. Adapt these generic purpose statements to the particular problems you solve so that readers know what they compute.

In short, the desire to design an interactive program automatically creates several initial entries for your wish list. Work them off one by one and you get a complete world program.

Let's work through this step for the sample program. While `big-bang` dictates that we must design a rendering function, we still need to figure out whether we want any event-handling functions. Since the car is supposed to move from left to right, we definitely need a function that deals with clock ticks. Thus, we get this wish list:

```
; WorldState -> Image
; places the image of the car x pixels from
; the left margin of the BACKGROUND image
(define (render x)
  BACKGROUND)

; WorldState -> WorldState
; adds 3 to x to move the car right
(define (tock x)
  x)
```

Note how we tailored the purpose statements to the problem at hand, with an understanding of how `big-bang` will use these functions.

4. Finally, you need a `main` function. Unlike all other functions, a `main` function for world programs doesn't demand design or testing. Its sole reason for existing is that you can **launch** your world program conveniently from DrRacket's interactions area.

The one decision you must make concerns `main`'s arguments. For our sample problem, we opt for one argument: the initial state of the world. Here we go:

```
; WorldState -> WorldState
; launches the program from some initial state
(define (main ws)
   (big-bang ws
     [on-tick tock]
     [to-draw render]))
```

Hence, you can launch this interactive program with

```
> (main 13)
```

to watch the car start at 13 pixels from the left margin. It will stop when you close big-bang's window. Remember that big-bang returns the current state of the world when the evaluation stops.

Naturally, you don't have to use the name "WorldState" for the class of data that represents the states of the world. Any name will do as long as you use it consistently for the signatures of the event-handling functions. Also, you don't have to use the names tock, end?, or render. You may name these functions whatever you like, as long as you use the same names when you write down the clauses of the big-bang expression. Lastly, you may have noticed that you may list the clauses of a big-bang expression in any order as long as you list the initial state first.

Let's now work through the rest of the program design process, using the design recipe for functions and other design concepts spelled out so far.

Exercise 39. Good programmers ensure that an image such as CAR can be enlarged or reduced via a single change to a constant definition. We started the development of our car image with a single plain definition:

```
(define WHEEL-RADIUS 5)
```

Good programmers establish a single point of control for all aspects of their programs, not just the graphical constants. Several chapters deal with this issue.

The definition of WHEEL-DISTANCE is based on the wheel's radius. Hence, changing WHEEL-RADIUS from 5 to 10 doubles the size of the car image. This kind of program organization is dubbed *single point of control*, and good design employs this idea as much as possible.

Develop your favorite image of an automobile so that WHEEL-RADIUS remains the single point of control. ▮

The next entry on the wish list is the clock tick handling function:

```
; WorldState -> WorldState
; moves the car by 3 pixels for every clock tick
(define (tock ws) ws)
```

Since the state of the world represents the distance between the left margin of the canvas and the car, and since the car moves at three pixels per clock tick, a concise purpose statement combines these two facts into one. This also makes it easy to create examples and to define the function:

```
; WorldState -> WorldState
; moves the car by 3 pixels for every clock tick
; examples:
;   given: 20, expect 23
;   given: 78, expect 81
(define (tock ws)
  (+ ws 3))
```

The last design step calls for confirmation that the examples work as expected. So we click the *RUN* button and evaluate these expressions:

```
> (tock 20)
23
> (tock 78)
81
```

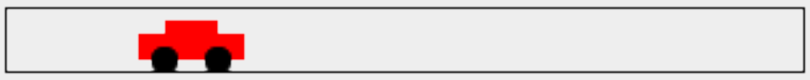
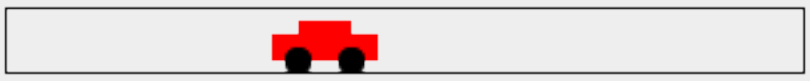
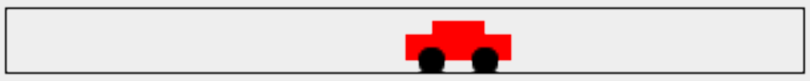
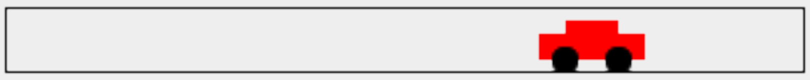
Since the results are as expected, the design of tock is finished.

Exercise 40. Formulate the examples as BSL tests, that is, using the check-expect form. Introduce a mistake. Re-run the tests. ▮

Our second entry on the wish list specifies a function that translates the state of the world into an image:

```
; WorldState -> Image
; places the car into the BACKGROUND scene,
; according to the given world state
(define (render ws)
  BACKGROUND)
```

To make examples for a rendering function, we suggest arranging a table like the upper half of figure 19. It lists the given world states and the desired scenes. For your first few rendering functions, you may wish to draw these images by hand.

ws	its image
50	
100	
150	
200	

ws	an expression
50	`(place-image CAR 50 Y-CAR BACKGROUND)`
100	`(place-image CAR 100 Y-CAR BACKGROUND)`
150	`(place-image CAR 150 Y-CAR BACKGROUND)`
200	`(place-image CAR 200 Y-CAR BACKGROUND)`

Figure 19: Examples for a moving car program

Even though this kind of image table is intuitive and explains what the running function is going to display—a moving car—it does not explain **how** the function creates this result. To get from here to there, we recommend writing down expressions like those in the lower half of figure 19 that create the images in the table. The capitalized names refer to the obvious constants: the image of a car, its fixed y-coordinate, and the background scene, which is currently empty.

This extended table suggests a pattern for the formula that goes into the body of the `render` function:

```
; WorldState -> Image
; places the car into the BACKGROUND scene,
; according to the given world state
 (define (render ws)
   (place-image CAR ws Y-CAR BACKGROUND))
```

And that is mostly all there is to designing a simple world program.

Exercise 41. Finish the sample problem and get the program to run. That is, assuming that you have solved exercise 39, define the constants `BACKGROUND` and `Y-CAR`. Then assemble all the function definitions, including their tests. When your program runs to your satisfaction, add a tree to the scenery. We used

```
(define tree
  (underlay/xy (circle 10 "solid" "green")
               9 15
               (rectangle 2 20 "solid" "brown")))
```

to create a tree-like shape. Also add a clause to the `big-bang` expression that stops the animation when the car has disappeared on the right side. ▌

After settling on an initial data representation for world states, a careful programmer may have to revisit this fundamental design decision during the rest of the design process. For example, the data definition for the sample problem represents the car as a point. But (the image of) the car isn't just a mathematical point without width and height. Hence, the interpretation statement—the number of pixels from the left margin—is an ambiguous statement. Does this statement measure the distance between the left margin and the left end of the car? Its center point? Or even its right end? We ignored this issue here and leave it to BSL's image primitives to make the decision for us. If you don't like the result, revisit the data definition above and modify it or its interpretation statement to suit your taste.

Exercise 42. Modify the interpretation of the sample data definition so that a state denotes the x-coordinate of the right-most edge of the car. ▌

Exercise 43. Let's work through the same problem statement with a time-based data definition:

```
; An AnimationState is a Number.
; interpretation the number of clock ticks
; since the animation started
```

Like the original data definition, this one also equates the states of the world with the class of numbers. Its interpretation, however, explains that the number means something entirely different.

Design the functions `tock` and `render`. Then develop a `big-bang` expression so that once again you get an animation of a car traveling from left to right across the world's canvas.

How do you think this program relates to animate from Prologue: How to Program?

Use the data definition to design a program that moves the car according to a sine wave. (Don't try to drive like that.) ▮

We end the section with an illustration of mouse event handling, which also illustrates the advantages that a separation of view and model provide.

Dealing with mouse movements is occasionally tricky because they aren't exactly what they seems to be. For a first idea of why that is, read On Mice and Keys.

Suppose we wish to allow people to move the car through "hyperspace":

> **Sample Problem** Design a program that moves a car across the world canvas, from left to right, at the rate of three pixels per clock tick. **If the mouse is clicked anywhere on the canvas, the car is placed at the x-coordinate of that click.**

The bold part is the expansion of the sample problem from above.

When we are confronted with a modified problem, we use the design process to guide us to the necessary changes. If used properly, this process naturally determines what we need to add to our existing program to cope with the expansion of the problem statement. So here we go:

1. There are no new properties, meaning we do not need new constants.

2. The program is still concerned with just one property that changes over time, the x-coordinate of the car. Hence, the data representation suffices.

3. The revised problem statement calls for a mouse-event handler, without giving up on the clock-based movement of the car. Hence, we state an appropriate wish:

   ```
   ; WorldState Number Number String -> WorldState
   ; places the car at x-mouse
   ; if the given me is "button-down"
   (define (hyper x-position-of-car x-mouse y-mouse me)
     x-position-of-car)
   ```

4. Lastly, we need to modify `main` to take care of mouse events. All this requires is the addition of an `on-mouse` clause that defers to the new entry on our wish list:

   ```
   (define (main ws)
      (big-bang ws
        [on-tick tock]
        [on-mouse hyper]
        [to-draw render]))
   ```

   After all, the modified problem calls for dealing with mouse clicks and everything else remains the same.

The rest is a mere matter of designing one more function, and for that we use the design recipe for functions.

An entry on the wish list covers the first two steps of the design recipe for functions. Hence, our next step is to develop some functional examples:

```
; WorldState Number Number String -> WorldState
; places the car at x-mouse
; if the given me is "button-down"
; given: 21 10 20 "enter"
; wanted: 21
; given: 42 10 20 "button-down"
; wanted: 10
; given: 42 10 20 "move"
; wanted: 42
(define (hyper x-position-of-car x-mouse y-mouse me)
  x-position-of-car)
```

The examples say that if the string argument is equal to `"button-down"`, the function returns `x-mouse`; otherwise it returns `x-position-of-car`.

Exercise 44. Formulate the examples as BSL tests. Click *RUN* and watch them fail. ▮

To complete the function definition, we must appeal to your fond memories from Prologue: How to Program, specifically memories about the conditional form.

Using `cond`, `hyper` is a two-line definition:

In the next chapter, we explain designing with cond in detail.

```
; WorldState Number Number String -> WorldState
; places the car at x-mouse
; if the given me is "button-down"
(define (hyper x-position-of-car x-mouse y-mouse me)
  (cond
    [(string=? "button-down" me) x-mouse]
    [else x-position-of-car]))
```

If you solved exercise 44, rerun the program and watch all tests succeed. Assuming the tests do succeed, evaluate

```
(main 1)
```

in DrRacket's interactions area and transport your car through hyperspace.

You may wonder why this program modification is so straightforward. There are really two reasons. First, this book and its software strictly separate the data that a program tracks—the *model*—and the image that it shows—the *view*. In particular, functions that deal with events have nothing to do with how the state is rendered. If we wish to modify how a state is rendered, we can focus on the function specified in a `to-draw` clause. Second, the design recipes for programs and functions organize programs in the right way. If anything changes in a problem statement, following the design recipe a second time naturally points out where the original problem solution has to change. While this may look obvious for the simple kind of problems we are dealing with now, it is critical for the kind of problems that programmers encounter in the real world.

3.7 Virtual Pet Worlds

This exercise section introduces the first two elements of a virtual pet game. It starts with just a display of a cat that keeps walking across the canvas. Of course, all the walking makes the cat unhappy and its unhappiness shows. As with all pets, you can try petting, which helps some, or you can try feeding, which helps a lot more.

So let's start with an image of our favorite cat:

```
(define cat1
```

```
)
```

Copy the cat image and paste it into DrRacket, then give the image a name with `define`, just like above.

Exercise 45. Design a "virtual cat" world program that continuously moves the cat from left to right. Let's call it `cat-prog` and let's assume it consumes the starting position of the cat. Furthermore, make the cat move three pixels per clock tick. Whenever the cat disappears on the right, it reappears on the left. You may wish to read up on the `modulo` function. ▮

Exercise 46. Improve the cat animation with a slightly different image:

```
(define cat2
```

```
)
```

Adjust the rendering function from exercise 45 so that it uses one cat image or the other based on whether the x-coordinate is odd. Read up on `odd?` in the HelpDesk, and use a `cond` expression to select cat images. ▮

Exercise 47. Design a world program that maintains and displays a "happiness gauge." Let's call it `gauge-prog`, and let's agree that the program consumes the maximum level of happiness. The gauge display starts with the maximum score, and with each clock tick, happiness decreases by `-0.1`; it never falls below `0`, the minimum happiness score. Every time the down arrow key is pressed, happiness increases by `1/5`; every time the up arrow is pressed, happiness jumps by `1/3`.

To show the level of happiness, we use a scene with a solid, red rectangle with a black frame. For a happiness level of 0, the red bar should be gone; for the maximum happiness level of 100, the bar should go all the way across the scene.

Note When you know enough, we will explain how to combine the gauge program with the solution of exercise 45.

Then we will be able to help the cat because as long as you ignore it, it becomes less happy. If you pet the cat, it becomes happier. If you feed the cat, it becomes much, much happier. So you can see why you want to know a lot more about designing world programs than these first three chapters can tell you. ▮

4 Intervals, Enumerations, and Itemizations

At the moment, you have four choices to represent information as data: numbers, strings, images, and Boolean values. For many problems this is enough, but there are many more for which these four collections of data in BSL (or other programming languages) don't suffice. Actual designers need additional ways of representing information as data.

At a minimum, good programmers must learn to design programs with restrictions on these built-in collections. One way to restrict is to enumerate a bunch of elements from a collection and to say that these are the only ones that are going to be used for some problem. Enumerating elements works only when there is a finite number of them. To accommodate collections with "infinitely" many elements, we introduce intervals, which are collections of elements that satisfy a specific property.

Infinite may just mean "so large that enumerating the elements is entirely impractical."

Defining enumerations and intervals means distinguishing among different kinds of elements. To distinguish in code requires conditional functions, that is, functions that choose different ways of computing results depending on the value of some argument. Both Many Ways to Compute and Mixing It Up with Booleans illustrate with examples of how to write such functions. Neither section uses design, however. Both just present some new construct in your favorite programming language (that's BSL), and offer some examples on how to use it.

In this chapter, we discuss a general design for enumerations and intervals, new forms of data descriptions. We start with a second look at the `cond` expression. Then we go through three different kinds of data descriptions: enumerations, intervals, and itemizations. An enumeration lists every single piece of data that belongs to it, while an interval specifies a range of data. The last one, itemizations, mixes the first two, specifying ranges in one clause of its definition and specific pieces of data in another. The chapter ends with the general design strategy for such situations.

4.1 Programming with Conditionals

Recall the brief introduction to conditional expressions in Prologue: How to Program. Since `cond` is the most complicated expression form in this book, let's take a close look at its general shape:

```
(cond
  [ConditionExpression1 ResultExpression1]
  [ConditionExpression2 ResultExpression2]
  ...

  [ConditionExpressionN ResultExpressionN])
```

Brackets make `cond` lines stand out. It is fine to use `( ... )` in place of `[ ... ]`.

A `cond` expression starts with `(cond`, its *keyword*, and ends in `)`. Following the keyword, a programmer writes as many *`cond` lines* as needed; each `cond` line consists of **two** expressions, enclosed in opening and closing brackets: `[` and `]`.

A `cond` line is also known as a *cond clause*.

Here is a function definition that uses a conditional expression:

```
(define (next traffic-light-state)
  (cond
    [(string=? "red" traffic-light-state) "green"]
    [(string=? "green" traffic-light-state) "yellow"]
    [(string=? "yellow" traffic-light-state) "red"]))
```

Like the mathematical example in Prologue: How to Program, this example illustrates the convenience of using `cond` expressions. In many problem contexts, a function must distinguish several different situations. With a `cond` expression, you can use one line per possibility and thus remind the reader of the code for the different situations from the problem statement.

A note on pragmatics: Contrast `cond` expressions with `if` expressions from Mixing It Up with Booleans. The latter distinguish one situation from all others. As such, `if` expressions are much less suited for multi-situation contexts; they are best used when all we wish to say is "one or the other." We therefore **always** use `cond` for situations when we wish to remind the reader of our code that some distinct situations come directly from data definitions. For other pieces of code, we use whatever construct is most convenient.

When the conditions get too complex in a cond expression, you occasionally wish to say something like "in all other cases." For these kinds of problems, cond expressions permit the use of the else keyword for the very last cond line:

```
(cond
  [ConditionExpression1 ResultExpression1]
  [ConditionExpression2 ResultExpression2]
  ...
  [else DefaultResultExpression])
```

If you make the mistake of using else in some other cond line, BSL in DrRacket signals an error:

```
> (cond
    [(> x 0) 10]
    [else 20]
    [(< x 10) 30])
cond:found an else clause that isn't the last clause in its cond expression
```

That is, BSL rejects grammatically incorrect phrases because it makes no sense to figure out what such a phrase might mean.

Imagine designing a function that, as part of a game-playing program, computes some award at the end of the game. Here is its header:

```
; A PositiveNumber is a Number greater than/equal to 0.

; PositiveNumber -> String
; computes the reward level from the given score s
```

And here are two variants for a side-by-side comparison:

```
(define (reward s)        (define (reward s)
  (cond                     (cond
    [(<= 0 s 10)              [(<= 0 s 10)
     "bronze"]                 "bronze"]
    [(and (< 10 s)            [(and (< 10 s)
          (<= s 20))                (<= s 20))
     "silver"]                 "silver"]
    [(< 20 s)                 [else
     "gold"]))                 "gold"]))
```

The variant on the left uses a cond with three full-fledged conditions; on the right, the function comes with an else clause. To formulate the last condition for the function on the left, you must calculate that (< 20 s) holds because

- s is in PositiveNumber
- (<= 0 s 10) is #false
- (and (< 10 s) (<= s 20)) evaluates to #false as well.

While the calculation looks simple in this case, it is easy to make small mistakes and to introduce bugs into your program. It is therefore better to formulate the function definition as shown on the right, **if** you know that you want the exact opposite—called the *complement*—of all previous conditions in a cond.

4.2 Computing Conditionally

From reading the Many Ways to Compute and Mixing It Up with Booleans, you roughly know how DrRacket evaluates conditional expressions. Let's go over the idea a bit more precisely for cond expressions. Take another look at this definition:

```
(define (reward s)
  (cond
    [(<= 0 s 10) "bronze"]
    [(and (< 10 s) (<= s 20)) "silver"]
    [else "gold"]))
```

This function consumes a numeric score—a positive number—and produces a color.

Just looking at the cond expression, you cannot predict which of the three cond clauses is going to be used. And that is the point of a function. The function deals with many different inputs, for example, 2, 3, 7, 18, 29. For each of these inputs, it may have to proceed in a different manner. Differentiating among the varying classes of inputs is the purpose of the cond expression.

Take, for example

```
(reward 3)
```

You know that DrRacket replaces function applications with the function's body after substituting the argument for the parameter. Hence,

```
(reward 3) ; say “equals”
==
(cond
  [(<= 0 3 10) "bronze"]
  [(and (< 10 3) (<= 3 20)) "silver"]
  [else "gold"])
```

At this point, DrRacket evaluates one condition at a time. For the **first** one to evaluate to `#true`, it continues with the result expression:

```
(reward 3)
==
(cond
  [(<= 0 3 10) "bronze"]
  [(and (< 10 3) (<= 3 20)) "silver"]
  [else "gold"])
==
(cond
  [#true "bronze"]
  [(and (< 10 3) (<= 3 20)) "silver"]
  [else "gold"])
==
"bronze"
```

Here the first condition holds because `3` is between `0` and `10`.

Here is a second example:

```
(reward 21)
==
(cond
  [(<= 0 21 10) "bronze"]
  [(and (< 10 21) (<= 21 20)) "silver"]
  [else "gold"])
==
(cond
  [#false "bronze"]
  [(and (< 10 21) (<= 21 20)) "silver"]
  [else "gold"])
==
(cond
  [(and (< 10 21) (<= 21 20)) "silver"]
  [else "gold"])
```

Note how the first condition evaluated to `#false` this time, and as mentioned in Many Ways to Compute the entire `cond` clause is dropped. The rest of the calculation proceeds as expected:

```
(cond
  [(and (< 10 21) (<= 21 20)) "silver"]
  [else "gold"])
==
(cond
  [(and #true (<= 21 20)) "silver"]
  [else "gold"])
==
(cond
  [(and #true #false) "silver"]
  [else "gold"])
==
(cond
  [#false "silver"]
  [else "gold"])
==
(cond
  [else "gold"])
  == "gold"
```

Like the first condition, the second one also evaluates to `#false` and thus the calculation proceeds to the third `cond` line. The `else` tells DrRacket to replace the entire `cond` expression with the answer from this clause.

Exercise 48. Enter the definition of `reward` followed by `(reward 18)` into the definitions area of DrRacket and use the stepper to find out **how** DrRacket evaluates applications of the function. ▮

Exercise 49. A `cond` expression is really just an expression and may therefore show up in the middle of another expression:

```
(- 200 (cond [(> y 200) 0] [else y]))
```

Use the stepper to evaluate the expression for `y` as `100` and `210`.

```
(define WIDTH  100)
(define HEIGHT  60)
(define MTSCN  (empty-scene WIDTH HEIGHT))

(define ROCKET      )
(define ROCKET-CENTER-TO-TOP
  (- HEIGHT (/ (image-height ROCKET) 2)))

(define (create-rocket-scene.v5 h)
  (cond
    [(<= h ROCKET-CENTER-TO-TOP)
     (place-image ROCKET 50 h MTSCN)]
    [(> h ROCKET-CENTER-TO-TOP)
     (place-image ROCKET 50 ROCKET-CENTER-TO-TOP MTSCN)]))
```

Figure 20: Recall from One Program, Many Definitions

Nesting `cond` expressions can eliminate common expressions. Consider the function for launching a rocket, repeated in figure 20. Both branches of the `cond` expression have the same shape except as indicated with `...`:

```
(place-image ROCKET X ... MTSCN)
```

Reformulate `create-rocket-scene.v5` to use a `nested` expression; the resulting function mentions `place-image` only once. ▮

4.3 Enumerations

Not all strings represent mouse events. If you looked in HelpDesk when the last section introduced the `on-mouse` clause for `big-bang`, you found out that only six strings are used to notify programs of mouse events:

```
; A MouseEvt is one of these Strings:
; – "button-down"
; – "button-up"
; – "drag"
; – "move"
; – "enter"
; – "leave"
```

The interpretation of these strings is quite obvious. One of the first two strings shows up when the computer user clicks the mouse button or releases it. In contrast, the third and fourth are about moving the mouse and possibly holding down the mouse button at the same time. Finally, the last two strings represent the events of a mouse moving over the edge of the canvas: either going into the canvas from the outside or exiting the canvas.

More importantly, the data definition for representing mouse events as strings looks quite different from the data definitions we have seen so far. It is called an *enumeration*, and it is a data representation in which every possibility is listed. It should not come as a surprise that enumerations are common. Here is a simple one:

```
; A TrafficLight is one of the following Strings:
; – "red"
; – "green"
; – "yellow"
; interpretation the three strings represent the three
; possible states that a traffic light may assume
```

It is a simplistic representation of the states that a traffic light can take on. Unlike others, this data definition also uses a slightly different phrase to explain what the term TrafficLight means, but this is an inessential difference.

We call it "simplistic" because it does not include the "off" state, the "blinking red" state, or the "blinking yellow" state.

Programming with enumerations is mostly straightforward. When a function's input is a class of data whose description spells out its elements on a case-by-case basis, the function should distinguish just those cases and compute the result on a per-case basis. For example, if you wanted to define a function that computes the next state of a traffic light, given the current state as an element of TrafficLight, you would come up with a definition like this one:

```
; TrafficLight -> TrafficLight
; yields the next state given current state s
(check-expect (traffic-light-next "red") "green")
(define (traffic-light-next s)
  (cond
    [(string=? "red" s) "green"]
    [(string=? "green" s) "yellow"]
    [(string=? "yellow" s) "red"]))
```

Because the data definition for TrafficLight consists of three distinct elements, the `traffic-light-next` function naturally distinguishes between three different cases. For each case, the result expression is just another string, the one that corresponds to the next case.

Exercise 50. If you copy and paste the above function definition into the definitions area of DrRacket and click *RUN*, DrRacket highlights two of the three `cond` lines. This coloring tells you that your test cases do not cover the full `cond`itional. Add enough tests to make DrRacket happy. ▌

Exercise 51. Design a `big-bang` program that simulates a traffic light for a given duration. The program renders the state of a traffic light as a solid circle of the appropriate color, and it changes state on every clock tick. What is the most appropriate initial state? Ask your engineering friends. ▌

The main idea of an enumeration is that it defines a collection of data as a **finite** number of pieces of data. Each item explicitly spells out which piece of data belongs to the class of data that we are defining. Usually, the piece of data is just shown as is; on some occasions, the item of an enumeration is an English sentence that describes a finite number of elements of pieces of data with a single phrase.

Here is an important example:

```
; A 1String is a String of length 1,
; including
; – "\\" (the backslash),
; – " " (the space bar),
; – "\t" (tab),
; – "\r" (return), and
; – "\b" (backspace).
; interpretation represents keys on the keyboard
```

You know that such a data definition is proper if you can describe all of its elements with a BSL test. In the case of 1String, you can find out whether some string `s` belongs to the collection with

```
(= (string-length s) 1)
```

An alternative way to check that you have succeeded is to enumerate all the members of the collection of data that you wish to describe:

```
; A 1String is one of:
; – "q"
; – "w"
; – "e"
; – "r"
; ...
; – "\t"
; – "\r"
; – "\b"
```

If you look at your keyboard, you find ←, ↑, and similar labels. Our chosen programming language, BSL, uses its own data definition to represent this information. Here is an excerpt:

You know where to find the full definition.

```
; A KeyEvent is one of:
; – 1String
; – "left"
; – "right"
; – "up"
```

```
 ; – ...
```

The first item in this enumeration describes the same bunch of strings that 1String describes. The clauses that follow enumerate strings for special key events, such as pressing one of the four arrow keys or releasing a key.

At this point, we can actually design a key-event handler systematically. Here is a sketch:

```
; WorldState KeyEvent -> ...
(define (handle-key-events w ke)
  (cond
    [(= (string-length ke) 1) ...]
    [(string=? "left" ke) ...]
    [(string=? "right" ke) ...]
    [(string=? "up" ke) ...]
    [(string=? "down" ke) ...]
    ...))
```

This event-handling function uses a cond expression, and for each line in the enumeration of the data definition, there is one cond line. The condition in the first cond line identifies the KeyEvents identified in the first line of the enumeration, the second cond clause corresponds to the second data enumeration line, and so on.

```
; A Position is a Number.
; interpretation distance between the left margin and the ball

; Position KeyEvent -> Position
; computes the next location of the ball

(check-expect (keh 13 "left") 8)
(check-expect (keh 13 "right") 18)
(check-expect (keh 13 "a") 13)
(define (keh p k)              (define (keh p k)
  (cond                          (cond
    [(= (string-length k) 1)
     p]
    [(string=? "left" k)           [(string=? "left" k)
     (- p 5)]                       (- p 5)]
    [(string=? "right" k)          [(string=? "right" k)
     (+ p 5)]                       (+ p 5)]
    [else p]))                     [else p]))
```

Figure 21: Conditional functions and special enumerations

When programs rely on data definitions that come with the chosen programming language (such as BSL) or its libraries (such as the *2htdp/universe* library), it is common that they use only a part of the enumeration. To illustrate this point, let us look at a representative problem.

> **Sample Problem** Design a key-event handler that moves a red dot left or right on a horizontal line in response to pressing the left and right arrow keys.

Figure 21 presents **two** solutions to this problem. The function on the left is organized according to the basic idea of using one cond clause per line in the data definition of the input, KeyEvent. In contrast, the right-hand side displays a version that uses the three essential lines: two for the keys that matter and one for everything else. The reordering is appropriate because only two of the cond-lines are relevant, and they can be cleanly separated from other lines. Naturally, this kind of rearrangement is done **after** the function is designed properly.

4.4 Intervals

Imagine yourself responding to the following sample design task:

> **Sample Problem** Design a program that simulates the descent of a UFO.

After a bit of thinking, you could come up with something like figure 22. Stop! Study the definitions and replace the dots before you read on.

```
; A WorldState is a Number.
; interpretation number of pixels between the top and the UFO

(define WIDTH 300) ; distances in terms of pixels
(define HEIGHT 100)
(define CLOSE (/ HEIGHT 3))
```

```
(define MTSCN (empty-scene WIDTH HEIGHT))
(define UFO (overlay (circle 10 "solid" "green") ...))

; WorldState -> WorldState
(define (main y0)
  (big-bang y0
     [on-tick nxt]
     [to-draw render]))

; WorldState -> WorldState
; computes next location of UFO
(check-expect (nxt 11) 14)
(define (nxt y)
  (+ y 3))

; WorldState -> Image
; places UFO at given height into the center of MTSCN
(check-expect (render 11) (place-image UFO ... 11 MTSCN))
(define (render y)
  (place-image UFO ... y MTSCN))
```

Figure 22: UFO, descending

Before you release this "game" program, however, you may wish to add the display of the status line to the canvas:

> **Sample Problem** Add a status line. It says “descending” when the UFO’s height is above one third of the height of the canvas. It switches to `"closing in"` below that. And finally, when the UFO has reached the bottom of the canvas, the status notifies the player that the UFO has “landed.”

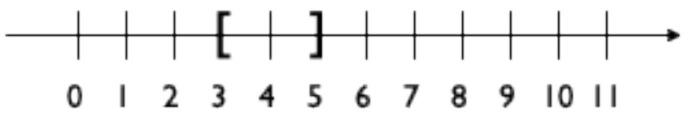
You are free to use appropriate colors for the status line.

In this case, we don’t have a finite enumeration of distinct elements or distinct sub-classes of data. After all, conceptually, the interval between `0` and `HEIGHT` (for some number greater than `0`) contains an infinite number of numbers and a large number of integers. Therefore we use intervals to superimpose some organization on the generic data definition, which just uses “numbers” to describe the class of coordinates.

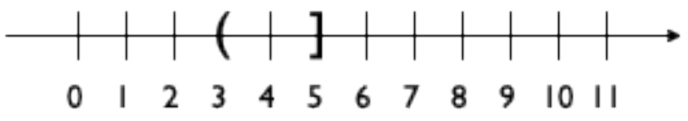
An *interval* is a description of a class of numbers via boundaries. The simplest interval has two boundaries: left and right. If the left boundary is to be included in the interval, we say it is *closed* on the left. Similarly, a right-closed interval includes its right boundary. Finally, if an interval does not include a boundary, it is said to be *open* at that boundary.

Pictures of, and notations for, intervals use brackets for closed boundaries and parentheses for open boundaries. Here are four such intervals:

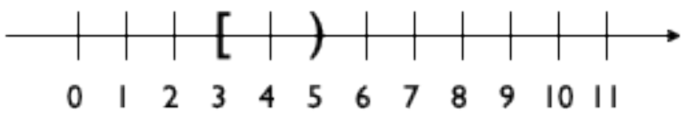
- [3,5] is a closed interval:



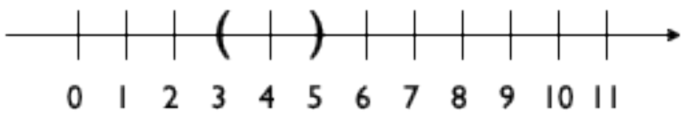
- (3,5] is a left-open interval:



- [3,5) is a right-open interval:



- and (3,5) is an open interval:

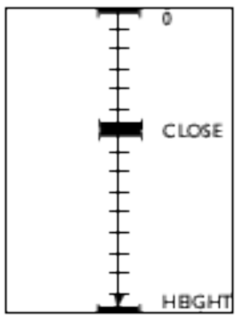


Exercise 52. Which integers are contained in the four intervals above? ▌

The interval concept helps us formulate a data definition that captures the revised problem statement better than the “numbers”-based definition:

```
; A WorldState falls into one of three intervals:
; – between 0 and CLOSE
; – between CLOSE and HEIGHT
; – below HEIGHT
```

Specifically, there are three intervals, which we may picture as follows:



What you see is the standard number line, turned vertical and broken into intervals. Each interval starts with an angular downward-pointing bracket (▬) and ends with an upward-pointing bracket (▬). The picture identifies three intervals in this manner:

- the upper interval goes from `0` to `CLOSE`;
- the middle one starts at `CLOSE` and reaches `HEIGHT`; and
- the lower, invisible interval is just a single line at `HEIGHT`.

On a plain number line, the last interval starts at `HEIGHT` and goes on forever.

Visualizing the data definition in this manner helps with the design of functions in two ways. First, it immediately suggests how to pick examples. Clearly we want the function to work inside of all the intervals, and we want the function to work properly at the ends of each interval. Second, the image tells us that we need to formulate a condition that determines whether or not some "point" is within one of the intervals.

Putting the two together also raises a question, namely, how exactly the function deals with the end points. In the context of our example, two points on the number line belong to two intervals: `CLOSE` belongs to both the upper interval and the middle one, while `HEIGHT` seems to fall into both the middle one and the lowest one. Such overlaps usually cause problems for programs, and they ought to be avoided.

BSL functions avoid them naturally due to the way `cond` expressions are evaluated. Consider this natural organization of a function that consumes elements of `WorldState`:

```
; WorldState -> WorldState
(define (f y)
  (cond
    [(<= 0 y CLOSE) ...]
    [(<= CLOSE y HEIGHT) ...]
    [(>= y HEIGHT) ...]))
```

The three `cond` lines correspond to the three intervals. Each condition identifies those values of `y` that are in between the limits of the intervals. Due to the way `cond` lines are checked one by one, however, a `y` value of `CLOSE` makes BSL pick the first `cond` line, and a `y` value of `HEIGHT` triggers the evaluation of the second *`ResultExpression`*.

If we wanted to make this choice obvious and immediate for every reader of our code, we would use different conditions:

```
; WorldState -> WorldState
(define (g y)
  (cond
    [(<= 0 y CLOSE) ...]
    [(and (< CLOSE y) (<= y HEIGHT)) ...]
    [(> y HEIGHT) ...]))
```

Note how the second `cond` line uses `and` to combine a strictly-less check with a less-than-or-equal check instead of `f`'s `<=` with three arguments.

```
; WorldState -> Image
; adds a status line to the scene created by render

(check-expect (render/status 10)
              (place-image (text "descending" 11 "green")
                           10 10
                           (render 10)))

(define (render/status y)
  (cond
    [(<= 0 y CLOSE)
     (place-image (text "descending" 11 "green")
                  10 10
```

```
                    (render y))]
      [(and (< CLOSE y) (<= y HEIGHT))
       (place-image (text "closing in" 11 "orange")
                    10 10
                    (render y))]
      [(> y HEIGHT)
       (place-image (text "landed" 11 "red")
                    10 10
                    (render y))]))
```

Figure 23: Rendering with a status line

Given all that, we can complete the definition of the function that adds the requested status line to our UFO animation; see figure 23 for the complete definition. The function uses a `cond` expression to distinguish the three intervals. In each `cond` clause, the *`ResultExpression`* uses `render` (from figure 22) to create the image with the descending UFO and then places an appropriate text at position (10,10) with `place-image`.

To run this version, you need to change `main` from figure 22 a bit:

```
; WorldState -> WorldState
(define (main y0)
  (big-bang y0
            [on-tick nxt]
            [to-draw render/status]))
```

One aspect of this function definition might disturb you, and to clarify why, let's refine the sample problem from above just a tiny bit:

> **Sample Problem** Add a status line, **positioned at (20,20)**, that says "descending" when the UFO's height is above one third of the height of the canvas. ...

This could be the response of a client who has watched your animation for a first time.

```
; WorldState -> Image
; adds a status line to the scene created by render

(check-expect (render/status 42)
              (place-image (text "closing in" 11 "orange")
                           20 20
                           (render 42)))

(define (render/status y)
  (place-image
    (cond
      [(<= 0 y CLOSE)
       (text "descending" 11 "green")]
      [(and (< CLOSE y) (<= y HEIGHT))
       (text "closing in" 11 "orange")]
      [(> y HEIGHT)
       (text "landed" 11 "red")])
    20 20
    (render y)))
```

Figure 24: Rendering with a status line, revised

At this point, you have no choice but to change the function `render/status` at **six** distinct places because you have three copies of one external piece of information: the location of the status line. To avoid multiple changes for a single element, programmers try to avoid copies. You have two choices to fix this problem. The first one is to use constant definitions, which you might recall from early chapters. The second one is to think of the `cond` expression as an expression that may appear anywhere in a function, including in the middle of some other expression; see figure 24 and compare with figure 23. In this revised definition of `render/status`, the `cond` expression is the first argument to `place-image`. As you can see, its result is always a `text` image that is placed at position (20,20) into the image created by `(render y)`.

4.5 Itemizations

An interval distinguishes different sub-classes of numbers, which, in principle, is an infinitely large class. An enumeration spells out item for item the useful elements of an existing class of data. Some data definitions need to include elements from both. They use *itemizations*, which generalize intervals and enumerations. They allow the combination of any already-defined data classes with each other and with individual pieces of data.

Consider the following example, a rewrite of an important data definition from Enumerations:

```
; A KeyEvent is one of:
; – 1String
; – "left"
; – "right"
; – "up"
; – ...
```

In this case, the `KeyEvent` data definition refers to the 1String data definition. Since functions that deal with KeyEvents often deal with 1Strings separately from the rest and do so with auxiliary functions, we now have a convenient way to express signatures for these functions, too.

The description of the `string->number` primitive employs the idea of an itemization in a sophisticated way. Its signature is

```
; String -> NorF
; converts the given string into a number;
; produces #false if impossible
(define (string->number s) (... s ...))
```

meaning that the result signature names a simple class of data:

```
; An NorF is one of:
; – #false
; – a Number
```

This itemization combines one piece of data (`#false`) with a large, and distinct, class of data (Number).

Now imagine a function that consumes the result of `string->number` and adds `3`, dealing with `#false` as if it were `0`:

```
; NorF -> Number
; adds 3 to the given number; 3 otherwise
(check-expect (add3 #false) 3)
(check-expect (add3 0.12) 3.12)
(define (add3 x)
  (cond
    [(false? x) 3]
    [else (+ x 3)]))
```

As above, the function's body consists of a `cond` expression with as many clauses as there are items in the enumeration of the data definition. The first `cond` clause recognizes when the function is applied to `#false`; the corresponding result is `3` as requested. The second clause is about numbers and adds `3` as required.

Let's study a somewhat more purposeful design task:

> **Sample Problem** Design a program that launches a rocket when the user of your program presses the space bar. The program first displays the rocket sitting at the bottom of the canvas. Once launched, it moves upward at three pixels per clock tick.

This revised version suggests a representation with two classes of states:

```
; An LR (short for launching rocket) is one of:
; – "resting"
; – NonnegativeNumber
; interpretation "resting" represents a grounded rocket
; a number denotes the height of a rocket in flight
```

While the interpretation of `"resting"` is obvious, the interpretation of numbers is ambiguous in its notion of height:

1. the word "height" could refer to the distance between the ground and the rocket's point of reference, say, its center; or

2. it could mean the distance between the top of the canvas and the reference point.

Either one works fine. The second one uses the conventional computer meaning of the word "height." It is thus slightly more convenient for functions that translate the state of the world into an image, and we therefore choose to interpret the number in that spirit.

To drive home this choice, exercise 57 below asks you to solve the exercises of this section using the first interpretation of height.

Exercise 53. The design recipe for world programs demands that you translate information into data and vice versa to ensure a complete understanding of the data definition. It's best to draw some world scenarios and to represent them with data and, conversely, to pick some data examples and to draw pictures that match them. Do so for the LR definition, including at least `HEIGHT` and `0` as examples. ■

In reality, rocket launches come with countdowns:

> **Sample Problem** Design a program that launches a rocket when the user presses the space bar. At that point, the simulation starts a countdown for three ticks, before it displays the scenery of a rising rocket. The rocket should move upward at a rate of three pixels per clock tick.

Following the program design recipe, we first collect constants:

```
(define HEIGHT 300) ; distances in pixels
(define WIDTH  100)
(define YDELTA 3)

(define BACKG  (empty-scene WIDTH HEIGHT))
(define ROCKET (rectangle 5 30 "solid" "red"))

(define CENTER (/ (image-height ROCKET) 2))
```

While `WIDTH` and `HEIGHT` describe the dimensions of the canvas and the background scene, `YDELTA` describes how fast the rocket moves along the y-axis, as specified in the problem statement. The `CENTER` constant is the **computed** center of the rocket.

Next we turn to the development of a data definition. This revision of the problem clearly calls for three distinct sub-classes of states:

```
; An LRCD (for launching rocket countdown) is one of:
; – "resting"
; – a Number between -3 and -1
; – a NonnegativeNumber
; interpretation a grounded rocket, in countdown mode,
; a number denotes the number of pixels between the
; top of the canvas and the rocket (its height)
```

The second, new sub-class of data—three negative numbers—represents the world after the user pressed the space bar and before the rocket lifts off.

At this point, we write down our wish list for a function that renders states as images and for any event-handling functions that we may need:

```
; LRCD -> Image
; renders the state as a resting or flying rocket
(define (show x)
  BACKG)

; LRCD KeyEvent -> LRCD
; starts the countdown when space bar is pressed,
; if the rocket is still resting
(define (launch x ke)
  x)

; LRCD -> LRCD
; raises the rocket by YDELTA,
;  if it is moving already
(define (fly x)
  x)
```

Remember that the design recipe for world programs dictates these signatures, though the choice of names for the data collection and the event handlers are ours. Also, we have specialized the purpose statements to fit our problem statement.

From here, we use the design recipe for functions to create complete definitions for all three of them, starting with examples for the first one:

```
(check-expect
 (show "resting")
 (place-image ROCKET 10 HEIGHT BACKG))

(check-expect
 (show -2)
 (place-image (text "-2" 20 "red")
              10 (* 3/4 WIDTH)
```

```
               (place-image ROCKET 10 HEIGHT BACKG)))

 (check-expect
  (show 53)
  (place-image ROCKET 10 53 BACKG))
```

As before in this chapter, we make one test per sub-class in the data definition. The first example shows the resting state, the second the middle of a countdown, and the last one the rocket in flight. Furthermore, we express the expected values as expressions that draw appropriate images. We used DrRacket's interactions area to create these images; what would **you** do?

A close look at the examples reveals that making examples also means making choices. Nothing in the problem statement actually demands how exactly the rocket is displayed before it is launched, but doing so is natural. Similarly, nothing says to display a number during the countdown, yet it adds a nice touch. Lastly, if you solved exercise 53 you also know that `0` and `HEIGHT` are special points for the third clause of the data definition.

In general, intervals deserve special attention when you make up examples, that is, they deserve at least three kinds of examples: one from each end and another one from inside. Since the second sub-class of LRCD is a (finite) interval and the third one is a half-open interval, let's take a look at their end points:

- Clearly, `(show -3)` and `(show -1)` must produce images like the one for `(show -2)`. After all, the rocket still rests on the ground, even if the countdown numbers differ.
- The case for `(show HEIGHT)` is different. According to our agreement, the value `HEIGHT` represents the state when the rocket has just been launched. Pictorially this means the rocket is still resting on the ground. Based on the last test case above, here is the test case that expresses this insight:

  ```
  (check-expect
   (show HEIGHT)
   (place-image ROCKET 10 HEIGHT BACKG))
  ```

  Except that if you evaluate the "expected value" expression by itself in DrRacket's interactions area, you see that the rocket is halfway underground. This shouldn't be the case, of course, meaning that we need to adjust this test case and the above:

  ```
  (check-expect
   (show HEIGHT)
   (place-image ROCKET 10 (- HEIGHT CENTER) BACKG))

  (check-expect
   (show 53)
   (place-image ROCKET 10 (- 53 CENTER) BACKG))
  ```
- Finally, determine the result you now expect from `(show 0)`. It is a simple but revealing exercise.

Following the precedents in this chapter, `show` uses a `cond` expression to deal with the three clauses of the data definition:

```
 (define (show x)
   (cond
     [(string? x) ...]
     [(<= -3 x -1) ...]
     [(>= x 0) ...]))
```

Each clause identifies the corresponding sub-class with a precise condition: `(string? x)` picks the first sub-class, which consists of just one element, the string `"resting"`; `(<= -3 x -1)` completely describes the second sub-class of data; and `(>= x 0)` is a test for all non-negative numbers.

Exercise 54. Why would it be **incorrect** to use `(string=? "resting" x)` as the first condition in `show`? Conversely, formulate a completely accurate condition, that is, a Boolean expression that evaluates to `#true` precisely when `x` belongs to the first sub-class of LRCD. ▌

Combining the examples and the above skeleton of the `show` function yields a complete definition in a reasonably straightforward manner:

```
 (define (show x)
   (cond
     [(string? x)
      (place-image ROCKET 10 (- HEIGHT CENTER) BACKG)]
     [(<= -3 x -1)
      (place-image (text (number->string x) 20 "red")
                   10 (* 3/4 WIDTH)
                   (place-image ROCKET
```

```
                                 10 (- HEIGHT CENTER)
                                 BACKG))]
    [(>= x 0)
     (place-image ROCKET 10 (- x CENTER) BACKG)]))
```

Indeed, this way of defining functions is highly effective and is an essential element of the full-fledged design approach in this book.

Exercise 55. Take another look at `show`. It contains three instances of an expression with the approximate shape:

```
(place-image ROCKET 10 (- ... CENTER) BACKG)
```

This expression appears three times in the function: twice to draw a resting rocket and once to draw a flying rocket. Define an auxiliary function that performs this work and thus shorten `show`. Why is this a good idea? You may wish to reread Prologue: How to Program. ▌

Let's move on to the second function, which deals with the key event to launch the rocket. We have its header material, so we formulate examples as tests:

```
(check-expect (launch "resting" " ") -3)
(check-expect (launch "resting" "a") "resting")
(check-expect (launch -3 " ") -3)
(check-expect (launch -1 " ") -1)
(check-expect (launch 33 " ") 33)
(check-expect (launch 33 "a") 33)
```

An inspection of these six examples shows that the first two are about the first sub-class of LRCD, the third and fourth concern the countdown, and the last two are about key events when the rocket is already in the air.

Since writing down the sketch of a `cond` expression worked well for the design of the `show` function, we do it again:

```
(define (launch x ke)
  (cond
    [(string? x) ...]
    [(<= -3 x -1) ...]
    [(>= x 0) ...]))
```

Looking back at the examples suggests that nothing changes when the world is in a state that is represented by the second or third sub-class of data. Meaning, `launch` should produce `x` when this happens:

```
(define (launch x ke)
  (cond
    [(string? x) ...]
    [(<= -3 x -1) x]
    [(>= x 0) x]))
```

Finally, the first example identifies the exact case when the `launch` function produces a new world state:

```
(define (launch x ke)
  (cond
    [(string? x) (if (string=? " " ke) -3 x)]
    [(<= -3 x -1) x]
    [(>= x 0) x]))
```

Specifically, when the state of the world is `"resting"` and the user presses the space bar, the function starts the countdown with `-3`.

Copy the code into the definitions area of DrRacket and ensure that the above definitions work. At that point, you may wish to add a function for running the program:

```
; LRCD -> LRCD
(define (main1 s)
  (big-bang s
    [to-draw show]
    [on-key launch]))
```

This function does **not** specify what to do when the clock ticks; after all, we haven't designed `fly` yet. Still, with `main1` it is possible to run this incomplete version of the program and to check that you can start the countdown. What would you provide as the argument in a call to `main1`?

```
; LRCD -> LRCD
; raises the rocket by YDELTA if it is moving already
```

```
(check-expect (fly "resting") "resting")
(check-expect (fly -3) -2)
(check-expect (fly -2) -1)
(check-expect (fly -1) HEIGHT)
(check-expect (fly 10) (- 10 YDELTA))
(check-expect (fly 22) (- 22 YDELTA))

(define (fly x)
  (cond
    [(string? x) x]
    [(<= -3 x -1) (if (= x -1) HEIGHT (+ x 1))]
    [(>= x 0) (- x YDELTA)]))
```

Figure 25: Launching a countdown and a liftoff

The design of fly—the clock-tick handler—proceeds just like the design of the preceding two functions, and figure 25 displays the result of the design process. Once again the key is to cover the space of possible input data with a goodly bunch of examples, especially for the two intervals. These examples ensure that the countdown and the transition from the countdown to the liftoff work properly.

Exercise 56. Define main2 so that you can launch the rocket and watch it lift off. Read up on the on-tick clause to determine the length of one tick and how to change it.

If you watch the entire launch, you will notice that once the rocket reaches the top something curious happens. Explain. Add a stop-when clause to main2 so that the simulation of the liftoff stops gracefully when the rocket is out of sight. ▮

The solution of exercise 56 yields a complete, working program, but one that behaves a bit strangely. Experienced programmers tell you that using negative numbers to represent the countdown phase is too “brittle.” The next chapter introduces the means to provide a good data definition for this problem. Before we go there, however, the next section spells out in detail how to design programs that consume data described by itemizations.

Exercise 57. Recall that the word “height” forced us to choose one of two possible interpretations. Now that you have solved the exercises in this section, solve them again using the first interpretation of the word. Compare and contrast the solutions. ▮

4.6 Designing with Itemizations

What the preceding three sections have clarified is that the design of functions can—and must—exploit the organization of the data definition. Specifically, if a data definition singles out certain pieces of data or specifies ranges of data, then the creation of examples and the organization of the function reflect these cases and ranges.

In this section, we refine the design recipe of From Functions to Programs so that you can proceed in a systematic manner when you encounter problems concerning functions that consume itemizations, including enumerations and intervals. To keep the explanation grounded, we illustrate the six design steps with the following, somewhat simplistic, example:

> **Sample Problem** The state of Tax Land has created a three-stage sales tax to cope with its budget deficit. Inexpensive items, those costing less than $1,000, are not taxed. Luxury items, with a price of more than $10,000, are taxed at the rate of eight percent (8.00%). Everything in between comes with a five percent (5.00%) markup.
>
> Design a function for a cash register that, given the price of an item, computes the sales tax.

Keep this problem in mind as we revise the steps of the design recipe:

1. When the problem statement distinguishes different classes of input information, you need carefully formulated data definitions.

   A data definition must use distinct *clauses* for each sub-class of data or in some cases just individual pieces of data. Each clause specifies a data representation for a particular sub-class of information. The key is that each sub-class of data is distinct from every other class, so that our function can proceed by analyzing disjoint cases.

   Our sample problem deals with prices and taxes, which are usually positive numbers. It also clearly distinguishes three ranges:

   ```
   ; A Price falls into one of three intervals:
   ; — 0 through 1000
   ; — 1000 through 10000
   ; — 10000 and above.
   ; interpretation the price of an item
   ```

   Do you understand how these ranges relate to the original problem?

2. As far as the signature, purpose statement, and function header are concerned, you proceed as before.

Here is the material for our running example:

```
; Price -> Number
; computes the amount of tax charged for p
(define (sales-tax p) 0)
```

Developers in the real world do not use plain numbers in the chosen programming language for representing amounts of money. See intermezzo 4 for some problems with numbers.

3. For functional examples, however, it is imperative that you pick at least one example from each sub-class in the data definition. Also, if a sub-class is a finite range, be sure to pick examples from the boundaries of the range and from its interior.

Since our sample data definition involves three distinct intervals, let's pick all boundary examples and one price from inside each interval and determine the amount of tax for each: `0`, `537`, `1000`, `1282`, `10000`, and `12017`.

Stop! Try to calculate the tax for each of these prices.

Here is our first attempt, with rounded tax amounts:

0	537	1000	1282	10000	12017
0	0	????	64	?????	961

The question marks point out that the problem statement uses the vague phrase "those costing less than $1,000" and "more than $10,000" to specify the tax table. While a programmer may jump to the conclusion that these words mean "strictly less" or "strictly more," the lawmakers may have meant to say "less than or equal to" or "more than or equal to," respectively. Being skeptical, we decide here that Tax Land legislators always want more money to spend, so the tax rate for $1,000 is 5% and the rate for $10,000 is 8%. A programmer at a tax company would have to ask a tax-law specialist.

Now that we have figured out how the boundaries are to be interpreted in the domain, we could refine the data definition. We trust you can do this on your own.

Before we go, let's turn some of the examples into test cases:

```
(check-expect (sales-tax 537) 0)
(check-expect (sales-tax 1000) (* 0.05 1000))
(check-expect (sales-tax 12017) (* 0.08 12017))
```

Take a close look. Instead of just writing down the expected result, we write down how to compute the expected result. This makes it easier later to formulate the function definition.

Stop! Write down the remaining test cases. Think about why you may need more test cases than sub-classes in the data definition.

4. The biggest novelty is the conditional template. In general,

the template mirrors the organization of sub-classes with a `cond`.

This slogan means two concrete things. First, the function's body must be a conditional expression with as many clauses as there are distinct sub-classes in the data definition. If the data definition mentions three distinct sub-classes of input data, you need three `cond` clauses; if it has seventeen sub-classes, the `cond` expression contains seventeen clauses. Second, you must formulate one condition expression per `cond` clause. Each expression involves the function parameter and identifies one of the sub-classes of data in the data definition:

```
(define (sales-tax p)
  (cond
    [(and (<= 0 p) (< p 1000)) ...]
    [(and (<= 1000 p) (< p 10000)) ...]
    [(>= p 10000) ...]))
```

5. When you have finished the template, you are ready to define the function. Given that the function body already contains a schematic `cond` expression, it is natural to start from the various `cond` lines. For each `cond` line, you may assume that the input parameter meets the condition and so you exploit the corresponding test cases. To formulate the corresponding result expression, you write down the computation for this example as an expression that involves the function parameter. Ignore all other possible kinds of input data when you work on one line; the other `cond` clauses take care of those.

```
(define (sales-tax p)
  (cond
    [(and (<= 0 p) (< p 1000)) 0]
    [(and (<= 1000 p) (< p 10000)) (* 0.05 p)]
    [(>= p 10000) (* 0.08 p)]))
```

5. Finally, run the tests and ensure that they cover all `cond` clauses.

   What do you do when one of your test cases fails? Review the end of Designing Functions concerning test failures.

Exercise 58. Introduce constant definitions that separate the intervals for low prices and luxury prices from the others so that the legislators in Tax Land can easily raise the taxes even more. ▮

4.7 Finite State Worlds

With the design knowledge in this chapter, you can develop a complete simulation of American traffic lights. When such a light is green and it is time to stop the traffic, the light turns yellow, and, after that, it turns red. When the light is red and it is time to get the traffic going, the light simply switches to green.

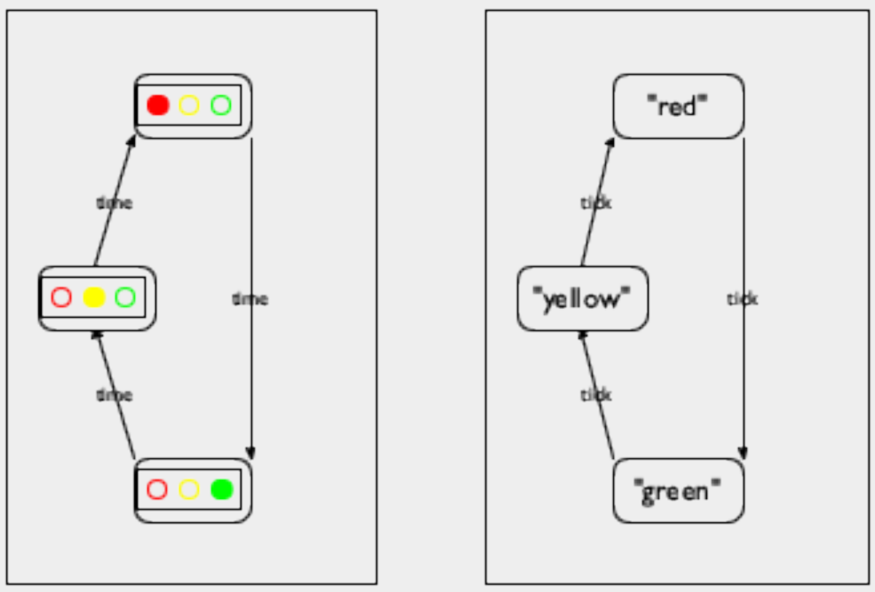


Figure 26: How a traffic light functions

The left-hand side of Figure 26 summarizes this description as a *state transition diagram*. Such a diagram consists of *states* and arrows that connect these states. Each state depicts a traffic light in one particular configuration: red, yellow, or green. Each arrow shows how the world can change, from which state it can *transition* to another state. Our sample diagram contains three arrows, because there are three possible ways in which the traffic light can change. Labels on the arrows indicate the reason for changes; a traffic light transitions from one state to another as time passes.

In many situations, state transition diagrams have only a finite number of states and arrows. Computer scientists call such diagrams *finite state machines* (FSM), also known as *finite state automata* (FSA). Despite their simplicity, FSMs/FSAs play an important role in computer science.

To create a world program for an FSA, we must first pick a data representation for the possible "states of the world," which, according to Designing World Programs, represents those aspects of the world that may change in some ways as opposed to those that remain the same. In the case of our traffic light, what changes is the color of the light, that is, which bulb is turned on. The size of the bulbs, their arrangement (horizontal or vertical), and other aspects don't change. Since there are only three states, we reuse the string-based data definition of TrafficLight from above.

The right-hand side of figure 26 is a diagrammatic interpretation of the TrafficLight data definition. Like the diagram in figure 26, it consists of three states, arranged in such a way that it is easy to view each data element as a representation of a concrete configuration. Also, the arrows are now labeled with **tick** to suggest that our world program uses the passing of time as the trigger that changes the state of the traffic light. If we wanted to simulate a manually operated light, we might choose transitions based on keystrokes.

Now that we know how to represent the states of our world, how to go from one to the next, and that the state changes at every tick of the clock, we can write down the signature, a purpose statement, and a stub for the two functions we must design:

```
; TrafficLight -> TrafficLight
; yields the next state, given current state cs
(define (tl-next cs) cs)

; TrafficLight -> Image
; renders the current state cs as an image
(define (tl-render current-state)
  (empty-scene 90 30))
```

Preceding sections use the names `render` and `next` to name the functions that translate a state of the world into an image and that deal with clock ticks. Here we prefix these names with some syllable that suggests to which world the functions belong. Because the specific functions have appeared before, we leave them as exercises.

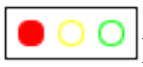
Exercise 59. Finish the design of a world program that simulates the traffic light FSA. Here is the main function:

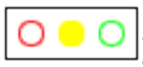
```
; TrafficLight -> TrafficLight
; simulates a clock-based American traffic light
(define (traffic-light-simulation initial-state)
  (big-bang initial-state
    [to-draw tl-render]
    [on-tick tl-next 1]))
```

The function's argument is the initial state for the `big-bang` expression, which tells DrRacket to redraw the state of the world with `tl-render` and to handle clock ticks with `tl-next`. Also note that it informs the computer that the clock should tick once per second.

Complete the design of `tl-render` and `tl-next`. Start with copying TrafficLight, `tl-next`, and `tl-render` into DrRacket's definitions area.

Here are some test cases for the design of the latter:

```
(check-expect (tl-render "red") <image>)
```


```
(check-expect (tl-render "yellow") <image>)
```


Your function may use these images directly. If you decide to create images with the functions from the *2htdp/image* library, design an auxiliary function for creating the image of a one-color bulb. Then read up on the `place-image` function, which can place bulbs into a background scene. ▮

Exercise 60. An alternative data representation for a traffic light program may use numbers instead of strings:

```
; An N-TrafficLight is one of:
; – 0 interpretation the traffic light shows red
; – 1 interpretation the traffic light shows green
; – 2 interpretation the traffic light shows yellow
```

It greatly simplifies the definition of `tl-next`:

```
; N-TrafficLight -> N-TrafficLight
; yields the next state, given current state cs
(define (tl-next-numeric cs) (modulo (+ cs 1) 3))
```

Reformulate `tl-next`'s tests for `tl-next-numeric`.

Does the `tl-next` function convey its intention more clearly than the `tl-next-numeric` function? If so, why? If not, why not? ▮

Exercise 61. As From Functions to Programs says, programs must define constants and use names instead of actual constants. In this spirit, a data definition for traffic lights must use constants, too:

```
(define RED 0)
(define GREEN 1)
(define YELLOW 2)

; An S-TrafficLight is one of:
; – RED
; – GREEN
; – YELLOW
```

This form of data definition is what a seasoned designer would use.

If the names are chosen properly, the data definition does not need an interpretation statement.

```
; S-TrafficLight -> S-TrafficLight
; yields the next state, given current state cs

(check-expect (tl-next- ... RED) YELLOW)
(check-expect (tl-next- ... YELLOW) GREEN)

(define (tl-next-numeric cs)      (define (tl-next-symbolic cs)
  (modulo (+ cs 1) 3))              (cond
                                      [(equal? cs RED) GREEN]
                                      [(equal? cs GREEN) YELLOW]
                                      [(equal? cs YELLOW) RED]))
```

Figure 27: A symbolic traffic light

Figure 27 displays two different functions that switch the state of a traffic light in a simulation program. Which of the two is properly designed using the recipe for itemization? Which of the two continues to work if you change the constants to the following

```
(define RED "red")
(define GREEN "green")
(define YELLOW "yellow")
```

Does this help you answer the questions?

Aside The `equal?` function in figure 27 compares two arbitrary values, regardless of what these values are. Equality is a complicated topic in the world of programming. **End** ▮

Here is another finite state problem that introduces a few additional complications:

> **Sample Problem** Design a world program that simulates the working of a door with an automatic door closer. If this kind of door is locked, you can unlock it with a key. An unlocked door is closed, but someone pushing at the door opens it. Once the person has passed through the door and lets go, the automatic door takes over and closes the door again. When a door is closed, it can be locked again.

To tease out the essential elements, we again draw a transition diagram; see the left-hand side of figure 28. Like the traffic light, the door has three distinct states: locked, closed, and open. Locking and unlocking are the activities that cause the door to transition from the locked to the closed state and vice versa. As for opening an unlocked door, we say that one needs to **push** the door open. The remaining transition is unlike the others because it doesn't require any activities by anyone or anything else. Instead, the door closes automatically over time. The corresponding transition arrow is labeled with *time* to emphasize this.

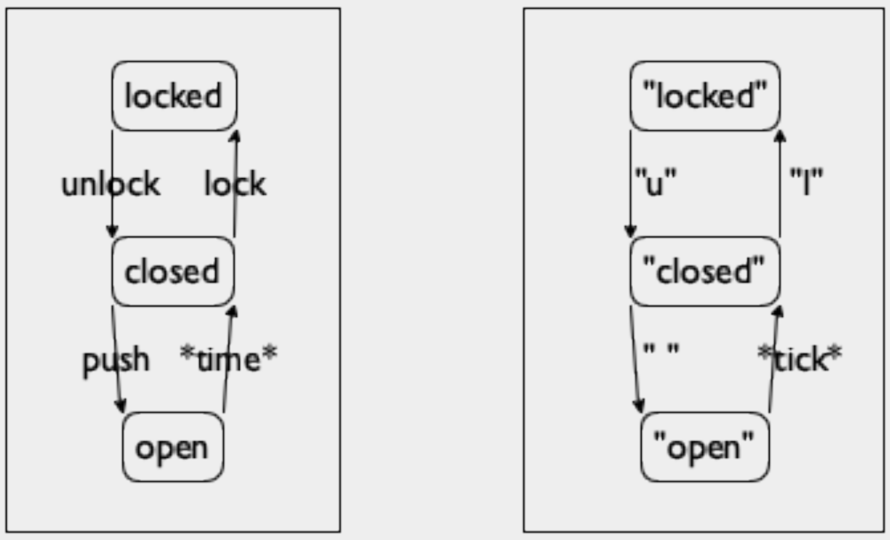


Figure 28: A transition diagram for a door with an automatic closer

Following our recipe, we start with a translation of the three real-world states into BSL data:

```
(define LOCKED "locked")    ; A DoorState is one of:
(define CLOSED "closed")    ; – LOCKED
(define OPEN "open")        ; – CLOSED
                            ; – OPEN
```

We also keep in mind the lesson of exercise 61, namely, that it is best to define symbolic constants and formulate data definitions in terms of such constants.

The next step of a world design demands that we translate the chosen actions in our domain—the arrows in the left-hand diagram—into interactions with the computer that the *2htdp/universe* library can deal with. Our pictorial representation of the door's states and transitions, specifically the arrow from `open` to `closed`, suggests the use of clock ticks. For the other arrows, we could use either key presses or mouse clicks. Let's use three keystrokes: `"u"` for unlocking the door, `"l"` for locking it, and the space bar `" "` for pushing it open. The right-hand-side diagram of figure 28 expresses these choices graphically; it translates the state-machine diagram from the world of information into the world of data in BSL.

Once we have decided to use the passing of time for one action and key presses for the others, we must design functions that render the current state of the world—represented as DoorState—and that transform it into the next state of the world. And that, of course, amounts to a wish list of `big-bang` functions:

- `door-closer`, which closes the door during one tick;
- `door-action`, which acts on it in response to pressing a key; and
- `door-render`, which translates the current state into an image.

Stop! Formulate appropriate signatures.

We start with `door-closer`. Since `door-closer` acts as the `on-tick` handler, we get its signature from our choice of DoorState as the collection of world states:

```
; DoorState -> DoorState
; closes an open door over the period of one tick
(define (door-closer state-of-door) state-of-door)
```

Making up examples is trivial when the world can only be in one of three states. Here we use a table to express the basic idea, just like in some of the mathematical examples given above:

given state	desired state
LOCKED	LOCKED
CLOSED	CLOSED
OPEN	CLOSED

Stop! Express these examples as BSL tests.

The template step demands a conditional with three clauses:

```
(define (door-closer state-of-door)
  (cond
    [(string=? LOCKED state-of-door) ...]
    [(string=? CLOSED state-of-door) ...]
    [(string=? OPEN state-of-door) ...]))
```

and the process of turning this template into a function definition is dictated by the examples:

```
(define (door-closer state-of-door)
  (cond
    [(string=? LOCKED state-of-door) LOCKED]
    [(string=? CLOSED state-of-door) CLOSED]
    [(string=? OPEN state-of-door) CLOSED]))
```

Don't forget to run your tests.

The second function, `door-action`, takes care of the remaining three arrows of the diagram. Functions that deal with keyboard events consume both a world and a key event, meaning the signature is as follows:

```
; DoorState KeyEvent -> DoorState
; turns key event k into an action on state s
(define (door-action s k)
  s)
```

We once again present the examples in tabular form:

given state	given key event	desired state
LOCKED	"u"	CLOSED
CLOSED	"l"	LOCKED
CLOSED	" "	OPEN
OPEN	—	OPEN

The examples combine information from our drawing with the choices we made about mapping actions to keyboard events. Unlike the table of examples for traffic light, this table is incomplete. Think of some other examples; then consider why our table suffices.

From here, it is straightforward to create a complete design:

```
(check-expect (door-action LOCKED "u") CLOSED)
(check-expect (door-action CLOSED "l") LOCKED)
(check-expect (door-action CLOSED " ") OPEN)
(check-expect (door-action OPEN "a") OPEN)
(check-expect (door-action CLOSED "a") CLOSED)

(define (door-action s k)
  (cond
    [(and (string=? LOCKED s) (string=? "u" k))
     CLOSED]
    [(and (string=? CLOSED s) (string=? "l" k))
     LOCKED]
    [(and (string=? CLOSED s) (string=? " " k))
     OPEN]
    [else s]))
```

Note the use of `and` to combine two conditions: one concerning the current state of the door and the other concerning the given key event.

Lastly, we need to render the state of the world as a scene:

```
; DoorState -> Image
; translates the state s into a large text image
(check-expect (door-render CLOSED)
              (text CLOSED 40 "red"))
(define (door-render s)
  (text s 40 "red"))
```

This simplistic function uses large text. Here is how we run it all:

```
; DoorState -> DoorState
; simulates a door with an automatic door closer
(define (door-simulation initial-state)
  (big-bang initial-state
    [on-tick door-closer]
    [on-key door-action]
    [to-draw door-render]))
```

Now it is time for you to collect the pieces and run them in DrRacket to see whether it all works.

Exercise 62. During a door simulation the "open" state is barely visible. Modify `door-simulation` so that the clock ticks once every three seconds. Rerun the simulation. ▮

5 Adding Structure

Suppose you want to design a world program that simulates a ball bouncing back and forth on a straight vertical line between the floor and ceiling of some imaginary, perfect room. Assume that it always moves two pixels per clock tick. If you follow the design recipe, your first goal is to develop a data representation for what changes over time. Here, the ball's position and its direction change over time, but that's **two** values while `big-bang` keeps track of just one. Thus the question arises how one piece of data can represent two changing quantities of information.

Mathematicians know tricks that "merge" two numbers into a single number such that it is possible to retrieve the original ones. Programmers consider these kinds of tricks evil because they obscure a program's true intentions.

Here is another scenario that raises the same question. Your cell phone is mostly a few million lines of code wrapped in plastic. Among other things, it administrates your contacts. Each contact comes with a name, a phone number, an email address, and perhaps some other information. When you have lots of contacts, each single contact is best represented as a single piece of data; otherwise the various pieces could get mixed up by accident.

Because of such programming problems, every programming language provides some mechanism to combine several pieces of data into a single piece of *compound data* and ways to retrieve the constituent values when needed. This chapter introduces BSL's mechanics, so-called structure type definitions, and how to design programs that work on compound data.

5.1 From Positions to `posn` Structures

A position on a world canvas is uniquely identified by two pieces of data: the distance from the left margin and the distance from the top margin. The first is called an *x-coordinate* and the second one is the *y-coordinate*.

DrRacket, which is basically a BSL program, represents such positions with `posn` structures. A `posn` structure combines two numbers into a single value. We can create a `posn` structure with the operation `make-posn`, which consumes two numbers and makes a `posn`. For example,

```
(make-posn 3 4)
```

is an expression that creates a `posn` structure whose x-coordinate is `3` and whose y-coordinate is `4`.

A `posn` structure has the same status as a number or a Boolean or a string. In particular, both primitive operations and functions may consume and produce structures. Also, a program can name a `posn` structure:

```
(define one-posn (make-posn 8 6))
```

Stop! Describe `one-posn` in terms of coordinates.

Before doing anything else, let's take a look at the laws of computation for `posn` structures. That way, we can both create functions that process `posn` structures and predict what they compute.

5.2 Computing with `posn`s

While functions and the laws of functions are completely familiar from pre-algebra, `posn` structures appear to be a new idea. Then again, the concept of a `posn` ought to look like the Cartesian points or positions in the plane you may have encountered before.

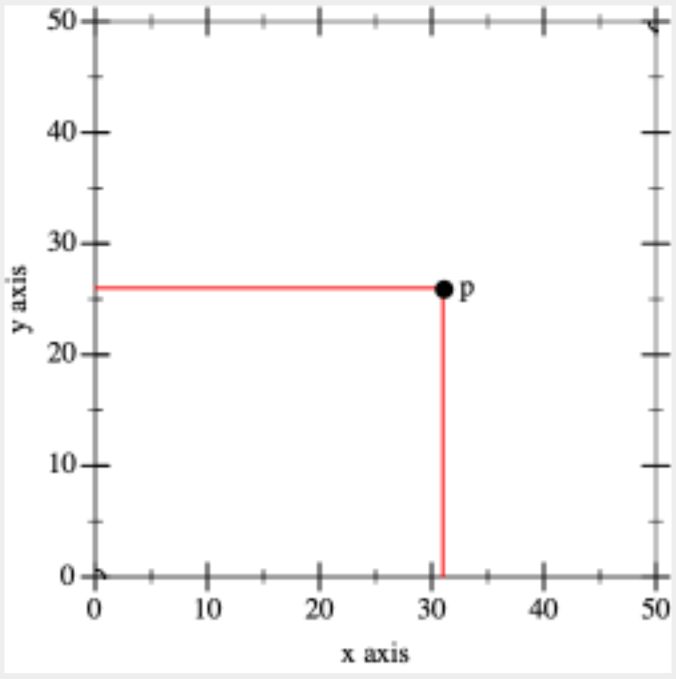


Figure 29: A Cartesian point

Selecting a Cartesian point's pieces is also a familiar process. For example, when a teacher says, "take a look at the graph of figure 29 and tell me what p_x and p_y are," you are likely to answer 31 and 26, respectively, because you know that you need to read off the values where the vertical and horizontal lines that radiate out from p hit the axes.

We thank Neil Toronto for the `plot` library.

We can express this idea in BSL. Assume you add

```
(define p (make-posn 31 26))
```

to the definitions area, click *RUN*, and perform these interactions:

```
> (posn-x p)
31
> (posn-y p)
26
```

Defining `p` is like marking the point in a Cartesian plane; using `posn-x` and `posn-y` is like subscripting `p` with indexes: p_x and p_y.

Computationally speaking, `posn` structures come with two equations:

```
  (posn-x (make-posn x0 y0))  ==  x0
  (posn-y (make-posn x0 y0))  ==  y0
```

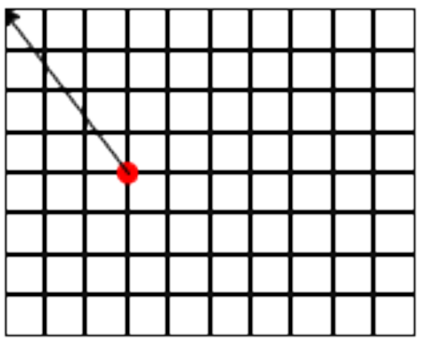
DrRacket uses these equations during computations. Here is an example of a computation involving `posn` structures:

```
(posn-x p)
== ; DrRacket replaces p with (make-posn 31 26)
(posn-x (make-posn 31 26))
== ; DrRacket uses the law for posn-x
31
```

Stop! Confirm the second interaction above with your own computation. Also use DrRacket's stepper to double-check.

5.3 Programming with `posn`

Now consider designing a function that computes the distance of some location to the origin of the canvas:



The picture clarifies that “distance” means the length of the most direct path—“as the crow flies”—from the designated point to the top-left corner of the canvas.

Here are the purpose statement and the header:

```
; computes the distance of ap to the origin
(define (distance-to-0 ap)
  0)
```

The key is that `distance-to-0` consumes a single value, some `posn`. It produces a single value, the distance of the location to the origin.

In order to make up examples, we need to know how to compute this distance. For points with `0` as one of the coordinates, the result is the other coordinate:

```
(check-expect (distance-to-0 (make-posn 0 5)) 5)
(check-expect (distance-to-0 (make-posn 7 0)) 7)
```

For the general case, we could try to figure out the formula on our own, or we may recall the formula from our geometry courses. As you know, this is domain knowledge that you might have, but in case you don’t we supply it; after all, this domain knowledge isn’t computer science. So, here is the distance formula for (*x*,*y*) again:

$$\sqrt{x^2 + y^2}$$

Given this formula, we can easily make up some more functional examples:

```
(check-expect (distance-to-0 (make-posn 3 4)) 5)
(check-expect (distance-to-0 (make-posn 8 6)) 10)
(check-expect (distance-to-0 (make-posn 5 12)) 13)
```

Just in case you’re wondering, we rigged the examples so that the results would be easy to figure out. This isn’t the case for all `posn` structures.

Stop! Plug the x- and y-coordinates from the examples into the formula. Confirm the expected results for all five examples.

Next we can turn our attention to the definition of the function. The examples imply that the design of `distance-to-0` does not need to distinguish between different situations; it always just computes the distance from the x- and y-coordinates inside the given `posn` structure. But the function must select these coordinates from the given `posn` structure. And for that, it uses the `posn-x` and `posn-y` primitives. Specifically, the function needs to compute `(posn-x ap)` and `(posn-y ap)` because `ap` is the name of the given, unknown `posn` structure:

```
(define (distance-to-0 ap)
  (... (posn-x ap) ...
   ... (posn-y ap) ...))
```

Using this template and the examples, the rest is straightforward:

```
(define (distance-to-0 ap)
  (sqrt
    (+ (sqr (posn-x ap))
       (sqr (posn-y ap)))))
```

The function squares `(posn-x ap)` and `(posn-y ap)`, which represent the x- and y-coordinates, sums up the results, and takes the square root. With DrRacket, we can also quickly check that our new function produces the proper results for our examples.

Exercise 63. Evaluate the following expressions:

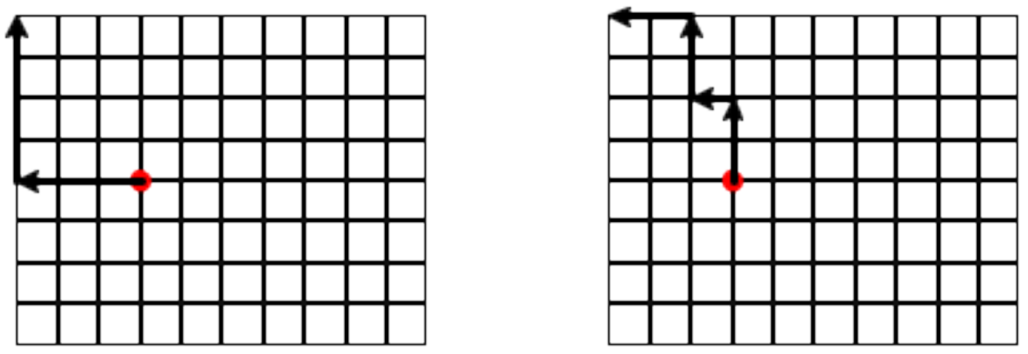
- `(distance-to-0 (make-posn 3 4))`

- `(distance-to-0 (make-posn 6 (* 2 4)))`

- `(+ (distance-to-0 (make-posn 12 5)) 10)`

by hand. Show all steps. Assume that `sqr` performs its computation in a single step. Check the results with DrRacket’s stepper. ▮

Exercise 64. The Manhattan distance of a point to the origin considers a path that follows the rectangular grid of streets found in Manhattan. Here are two examples:



The left one shows a “direct” strategy, going as far left as needed, followed by as many upward steps as needed. In comparison, the right one shows a “random walk” strategy, going some blocks leftward, some upward, and so on until the destination—here, the origin—is reached.

Stop! Does it matter which strategy you follow?

Design the function `manhattan-distance`, which measures the Manhattan distance of the given `posn` to the origin. ▌

5.4 Defining Structure Types

Unlike numbers or Boolean values, structures such as `posn` usually don’t come with a programming language. Only the mechanism to define structure types is provided; the rest is left up to the programmer. This is also true for BSL.

A *structure type definition* is another form of definition, distinct from constant and function definitions. Here is how the creator of DrRacket defined the `posn` structure type in BSL:

```
(define-struct posn [x y])
```

In general, a structure type definition has this shape:

```
(define-struct StructureName [FieldName ...])
```

The use of brackets in a structure type definition is a convention, not a necessity. It makes the field names stand out. Replacing brackets with parentheses is perfectly acceptable.

The keyword `define-struct` signals the introduction of a new structure type. It is followed by the name of the structure. The third part of a structure type definition is a sequence of names enclosed in brackets; these names are the *fields*.

A structure type definition actually defines functions. But, unlike an ordinary function definition, **a structure type definition defines many functions** simultaneously. Specifically, it defines three kinds of functions:

- one *constructor*, a function that creates *structure instances*. It takes as many values as there are fields; as mentioned, *structure* is short for structure instance. The phrase *structure type* is a generic name for the collection of all possible instances;
- one *selector* per field, which extracts the value of the field from a structure instance; and
- one *structure predicate*, which, like ordinary predicates, distinguishes instances from all other kinds of values.

A program can use these as if they were functions or built-in primitives.

Curiously, a structure type definition makes up names for the various new operations it creates. For the name of the constructor, it prefixes the structure name with “make-” and for the names of the selectors it postfixes the structure name with the field names. Finally, the predicate is just the structure name with “?” added, pronounced “huh” when read aloud.

This naming convention looks complicated and perhaps even confusing. But, with a little bit of practice, you’ll get the hang of it. It also explains the functions that come with `posn` structures: `make-posn` is the constructor, `posn-x` and `posn-y` are selectors. While we haven’t encountered `posn?` yet, we now know that it exists; the next chapter explains the role of these predicates in detail.

Exercise 65. Take a look at the following structure type definitions:

- `(define-struct movie [title producer year])`
- `(define-struct person [name hair eyes phone])`
- `(define-struct pet [name number])`
- `(define-struct CD [artist title price])`
- `(define-struct sweater [material size producer])`

Write down the names of the functions (constructors, selectors, and predicates) that each introduces. ▌

Enough with posn structures for a while. Let's look at a structure type definition that we might use to keep track of contacts such as those in your cell phone:

```
(define-struct entry [name phone email])
```

Here are the names of the functions that this definition introduces:

- make-entry, which consumes three values and constructs an instance of entry;
- entry-name, entry-phone, and entry-email, which consume one instance of entry and select one of the three field values; and
- entry?, the predicate.

Since each entry combines three values, the expression

```
(make-entry "Al Abe" "666-7771" "lee@x.me")
```

creates an entry structure with "Al Abe" in the name field, "666-7771" in the phone field, and "lee@x.me" in the email field.

Exercise 66. Revisit the structure type definitions of exercise 65. Make sensible guesses as to what kind of values go with which fields. Then create at least one instance per structure type definition. ▮

Every structure type definition introduces a new kind of structure, distinct from all others. Programmers want this kind of expressive power because they wish to convey an **intention** with the structure name. Wherever a structure is created, selected, or tested, the text of the program explicitly reminds the reader of this intention. If it weren't for these future readers of code, programmers could use one structure definition for structures with one field, another for structures with two fields, a third for structures with three, and so on.

In this context, let's study another programming problem:

> **Sample Problem** Develop a structure type definition for a program that deals with "bouncing balls," briefly mentioned at the very beginning of this chapter. The ball's location is a single number, namely the distance of pixels from the top. Its constant *speed* is the number of pixels it moves per clock tick. Its *velocity* is the speed **plus** the direction in which it moves.

Since the ball moves along a straight, vertical line, a number is a perfectly adequate data representation for its velocity:

- A positive number means the ball moves down.
- A negative number means it moves up.

We can use this domain knowledge to formulate a structure type definition:

```
(define-struct ball [location velocity])
```

Both fields are going to contain numbers, so (make-ball 10 -3) is a good data example. It represents a ball that is 10 pixels from the top and moves up at 3 pixels per clock tick.

Notice how, in principle, a ball structure merely combines two numbers, just like a posn structure. When a program contains the expression (ball-velocity a-ball), it immediately conveys that this program deals with the representation of a ball and its velocity. In contrast, if the program used posn structures instead, (posn-y a-ball) might mislead a reader of the code into thinking that the expression is about a y-coordinate.

Exercise 67. Here is another way to represent bouncing balls:

```
(define SPEED 3)
(define-struct balld [location direction])
(make-balld 10 "up")
```

Interpret this code fragment and create other instances of balld. ▮

Since structures are values, just like numbers or Booleans or strings, it makes sense that one instance of a structure occurs inside another instance. Consider game objects. Unlike bouncing balls, such objects don't always move along vertical lines. Instead, they move in some "oblique" manner across the canvas. Describing both the location and the velocity of a ball moving across a 2-dimensional world canvas demands two numbers: one per direction. For the location part, the two numbers represent the x- and y-coordinates. Velocity describes the *changes* in the horizontal and vertical direction; in other words, these "change numbers" must be added to the respective coordinates to find out where the object will be next.

It is physics that tells you to add an object's velocity to its location to obtain its next location. Developers need to learn whom to ask about which domain.

Clearly, posn structures can represent locations. For the velocities, we define the vel structure type:

```
(define-struct vel [deltax deltay])
```

It comes with two fields: `deltax` and `deltay`. The word “delta” is commonly used to speak of change when it comes to simulations of physical activities, and the x and y parts indicate which axis is concerned.

Now we can use instances of `ball` to combine a `posn` structure with a `vel` structure to represent balls that move in straight lines but not necessarily along only vertical (or horizontal) lines:

```
(define ball1
  (make-ball (make-posn 30 40) (make-vel -10 5)))
```

One way to interpret this instance is to think of a ball that is `30` pixels from the left and `40` pixels from the top. It moves `10` pixels toward the left per clock tick, because subtracting 10 pixels from the x-coordinate brings it closer to the left. As for the vertical direction, the ball drops at `5` pixels per clock tick, because adding positive numbers to a y-coordinate increases the distance from the top.

Exercise 68. An alternative to the nested data representation of balls uses four fields to keep track of the four properties:

```
(define-struct ballf [x y deltax deltay])
```

Programmers call this a *flat representation*. Create an instance of `ballf` that has the same interpretation as `ball1`. ▮

Yet another alternative is to use *complex numbers*. If you know about them, contemplate a data representation that uses them for both location and velocity. For example, in BSL, `4-3i` is a complex number and could be used to represent the location or velocity (4,-3).

For a second example of nested structures, let’s briefly look at the example of contact lists. Many cell phones support contact lists that allow several phone numbers per name: one for a home line, one for the office, and one for a cell phone number. For phone numbers, we wish to include both the area code and the local number. Since this nests the information, it’s best to create a nested data representation, too:

```
(define-struct centry [name home office cell])

(define-struct phone [area number])

(make-centry "Shriram Fisler"
             (make-phone 207 "363-2421")
             (make-phone 101 "776-1099")
             (make-phone 208 "112-9981"))
```

The intention here is that an entry on a contact list has four fields: a name and three phone records. The latter are represented with instance of `phone`, which separates the area code from the local phone number.

In sum, nesting information is natural. The best way to represent such information with data is to mirror the nesting with nested structure instances. Doing so makes it easy to interpret the data in the application domain of the program, and it is also straightforward to go from examples of information to data. Of course, it is really the task of data definitions to specify how to go back and forth between information and data. Before we study data definitions for structure type definitions, however, we first take a systematic look at computing with, and thinking about, structures.

5.5 Computing with Structures

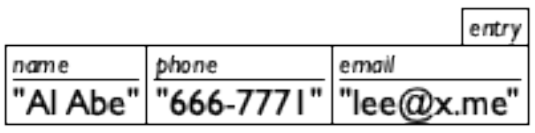
Structure types generalize Cartesian points in two ways. First, a structure type may specify an arbitrary number of fields: zero, one, two, three, and so forth. Second, structure types name fields, they don’t number them. This helps programmers read code because it is much easier to remember that a family name is available in a field called `last-name` than in the 7th field.

Most programming languages also support structure-like data that use numeric field names.

In the same spirit, computing with structure instances generalizes the manipulation of Cartesian points. To appreciate this idea, let us first look at a diagrammatic way to think about structure instances as lockboxes with as many compartments as there are fields. Here is a representation of

```
(define pl (make-entry "Al Abe" "666-7771" "lee@x.me"))
```

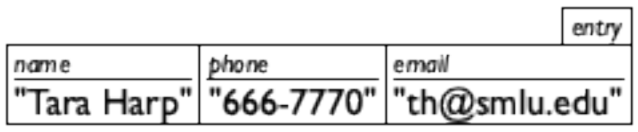
as such a diagram:



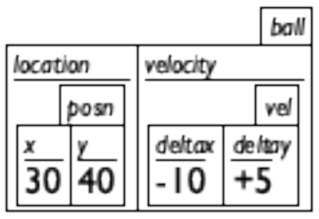
The box’s italicized label identifies it as an instance of a specific structure type; each compartment is labeled, too. Here is another instance:

```
(make-entry "Tara Harp" "666-7770" "th@smlu.edu")
```

corresponds to a similar box diagram, though the content differs:



Not surprisingly, nested structure instances have a diagram of boxes nested in boxes. Thus, `ball1` from above is equivalent to this diagram:



In this case, the outer box contains two boxes, one per field.

Exercise 69. Draw box representations for the solution of exercise 65. ▮

In the context of this imagery, a selector is like a key. It opens a specific compartment for a certain kind of box and thus enables the holder to extract its content. Hence, applying `entry-name` to `pl` from above yields a string:

```
> (entry-name pl)
"Al Abe"
```

But `entry-name` applied to a `posn` structure signals an error:

```
> (entry-name (make-posn 42 5))
entry-name:expects an entry, given (posn 42 5)
```

If a compartment contains a box, it might be necessary to use two selectors in a row to get to the desired number:

```
> (ball-velocity ball1)
(make-vel -10 5)
```

Applying `ball-velocity` to `ball1` extracts the value of the velocity field, which is an instance of `vel`. To get to the velocity along the x axis, we apply a selector to the result of the first selection:

```
> (vel-deltax (ball-velocity ball1))
-10
```

Since the inner expression extracts the velocity from `ball1`, the outer expression extracts the value of the `deltax` field, which in this case is `-10`.

The interactions also show that structure instances are values. DrRacket prints them exactly as entered, just like for plain values such as numbers:

```
> (make-vel -10 5)
(make-vel -10 5)
> (make-entry "Tara Harp" "666-7770" "th@smlu.edu")
(make-entry "Tara Harp" "666-7770" "th@smlu.edu")
> (make-centry
    "Shriram Fisler"
    (make-phone 207 "363-2421")
    (make-phone 101 "776-1099")
    (make-phone 208 "112-9981"))
(make-centry ...)
```

Stop! Try this last interaction at home, so you see the proper result.

Generally speaking, a structure type definition not only creates new functions and new ways to create values, but it also adds new laws of computation to DrRacket's knowledge. These laws generalize those for `posn` structures in Computing with `posn`s, and they are best understood by example.

When DrRacket encounters a structure type definition with two fields,

```
(define-struct ball [location velocity])
```

it introduces two laws, one per selector:

```
(ball-location (make-ball l0 v0))  ==  l0
(ball-velocity (make-ball l0 v0))  ==  v0
```

For different structure type definitions, it introduces analogous laws. Thus,

```
(define-struct vel [deltax deltay])
```

DrRacket adds these two laws to its knowledge:

```
  (vel-deltax (make-vel dx0 dy0))  ==  dx0
  (vel-deltay (make-vel dx0 dy0))  ==  dy0
```

Using these laws, we can now explain the interaction from above:

```
(vel-deltax (ball-velocity ball1))
== ; DrRacket replaces ball1 with its value
(vel-deltax
  (ball-velocity
    (make-ball (make-posn 30 40) (make-vel -10 5))))
== ; DrRacket uses the law for ball-velocity
(vel-deltax (make-vel -10 5))
== ; DrRacket uses the law for vel-deltax
-10
```

Exercise 70. Spell out the laws for these structure type definitions:

```
(define-struct centry [name home office cell])
(define-struct phone [area number])
```

Use DrRacket's stepper to confirm `101` as the value of this expression:

```
(phone-area
 (centry-office
  (make-centry "Shriram Fisler"
    (make-phone 207 "363-2421")
    (make-phone 101 "776-1099")
    (make-phone 208 "112-9981"))))  ""
```

Predicates are the last idea that we must discuss to understand structure type definitions. As mentioned, every structure type definition introduces one new predicate. DrRacket uses these predicates to discover whether a selector is applied to the proper kind of value; the next chapter explains this idea in detail. Here we just want to convey that these predicates are just like the predicates from "arithmetic." While `number?` recognizes numbers and `string?` recognizes strings, predicates such as `posn?` and `entry?` recognize `posn` structures and `entry` structures. We can confirm our ideas of how they work with experiments in the interactions area. Assume that the definitions area contains these definitions:

```
(define ap (make-posn 7 0))

(define pl (make-entry "Al Abe" "666-7771" "lee@x.me"))
```

If `posn?` is a predicate that distinguishes `posn`s from all other values, we should expect that it yields `#false` for numbers and `#true` for `ap`:

```
> (posn? ap)
#true
> (posn? 42)
#false
> (posn? #true)
#false
> (posn? (make-posn 3 4))
#true
```

Similarly, `entry?` distinguishes `entry` structures from all other values:

```
> (entry? pl)
#true
> (entry? 42)
#false
> (entry? #true)
#false
```

In general, a predicate recognizes exactly those values constructed with a constructor of the same name. Intermezzo 1: Beginning Student Language explains this law in detail, and it also collects the laws of computing for BSL in one place.

Exercise 71. Place the following into DrRacket's definitions area:

```
; distances in terms of pixels:
(define HEIGHT 200)
```

```
(define MIDDLE (quotient HEIGHT 2))
(define WIDTH  400)
(define CENTER (quotient WIDTH 2))

(define-struct game [left-player right-player ball])

(define game0
  (make-game MIDDLE MIDDLE (make-posn CENTER CENTER)))
```

Click *RUN* and evaluate the following expressions:

```
(game-ball game0)
(posn? (game-ball game0))
(game-left-player game0)
```

Explain the results with step-by-step computations. Double-check your computations with DrRacket's stepper. ▮

5.6 Programming with Structures

Proper programming calls for data definitions. With the introduction of structure type definitions, data definitions become interesting. Remember that a data definition provides a way of representing information into data and interpreting that data as information. For structure types, this calls for a description of what kind of data goes into which field. For some structure type definitions, formulating such descriptions is easy and obvious:

```
(define-struct posn [x y])
; A Posn is a structure:
;   (make-posn Number Number)
; interpretation a point x pixels from left, y from top
```

It doesn't make any sense to use other kinds of data to create a `posn`. Similarly, all fields of `entry`—our structure type definition for entries on a contact list—are clearly supposed to be strings, according to our usage in the preceding section:

```
(define-struct entry [name phone email])
; An Entry is a structure:
;   (make-entry String String String)
; interpretation a contact's name, phone#, and email
```

For both `posn` and `entry`, a reader can easily interpret instances of these structures in the application domain.

Contrast this simplicity with the structure type definition for `ball`, which obviously allows at least two distinct interpretations:

```
(define-struct ball [location velocity])
; A Ball-1d is a structure:
;   (make-ball Number Number)
; interpretation 1 distance to top and velocity
; interpretation 2 distance to left and velocity
```

Whichever one we use in a program, we must stick to it consistently. As Defining Structure Types shows, however, it is also possible to use `ball` structures in an entirely different manner:

```
; A Ball-2d is a structure:
;   (make-ball Posn Vel)
; interpretation a 2-dimensional position and velocity

(define-struct vel [deltax deltay])
; A Vel is a structure:
;   (make-vel Number Number)
; interpretation (make-vel dx dy) means a velocity of
; dx pixels [per tick] along the horizontal and
; dy pixels [per tick] along the vertical direction
```

Here we name a second collection of data, Ball-2d, distinct from Ball-1d, to describe data representations for balls that move in straight lines across a world canvas. In short, it is possible to use **one and the same** structure type in **two different ways**. Of course, within one program, it is best to stick to one and only one use; otherwise you are setting yourself up for problems.

Also, Ball-2d refers to another one of our data definitions, namely, the one for Vel. While all other data definitions have thus far referred to built-in data collections (Number, Boolean, String), it is perfectly acceptable, and indeed common, that one of your data definitions refers to another.

Exercise 72. Formulate a data definition for the above `phone` structure type definition that accommodates the given examples.

Next formulate a data definition for phone numbers using this structure type definition:

```
(define-struct phone# [area switch num])
```

Historically, the first three digits make up the area code, the next three the code for the phone switch (exchange) of your neighborhood, and the last four the phone with respect to the neighborhood. Describe the content of the three fields as precisely as possible with intervals. ▌

At this point, you might be wondering what data definitions really mean. This question, and its answer, is the topic of the next section. For now, we indicate how to use data definitions for program design.

Here is a problem statement to set up some context:

> **Sample Problem** Your team is designing an interactive game program that moves a red dot across a 100×100 canvas and allows players to use the mouse to reset the dot. Here is how far you got together:
>
> ```
> (define MTS (empty-scene 100 100))
> (define DOT (circle 3 "solid" "red"))
>
> ; A Posn represents the state of the world.
>
> ; Posn -> Posn
> (define (main p0)
>   (big-bang p0
>     [on-tick x+]
>     [on-mouse reset-dot]
>     [to-draw scene+dot]))
> ```
>
> Your task is to design `scene+dot`, the function that adds a red dot to the empty canvas at the specified position.

The problem context dictates the signature of your function:

```
; Posn -> Image
; adds a red spot to MTS at p
(define (scene+dot p) MTS)
```

Adding a purpose statement is straightforward. As Designing Functions mentions, it uses the function's parameter to express what the function computes.

Now we work out a couple of examples and formulate them as tests:

```
(check-expect (scene+dot (make-posn 10 20))
              (place-image DOT 10 20 MTS))
(check-expect (scene+dot (make-posn 88 73))
              (place-image DOT 88 73 MTS))
```

Given that the function consumes a Posn, we know that the function can extract the values of the `x` and `y` fields:

```
(define (scene+dot p)
  (... (posn-x p) ... (posn-y p) ...))
```

Once we see these additional pieces in the body of the function, the rest of the definition is straightforward. Using `place-image`, the function puts `DOT` into `MTS` at the coordinates contained in `p`:

```
(define (scene+dot p)
  (place-image DOT (posn-x p) (posn-y p) MTS))
```

A function may produce structures. Let's resume our sample problem from above because it includes just such a task:

> **Sample Problem** A colleague is asked to define `x+`, a function that consumes a Posn and increases the x-coordinate by `3`.

Recall that the `x+` function handles clock ticks.

We can adapt the first few steps of the design of `scene+dot`:

```
; Posn -> Posn
; increases the x-coordinate of p by 3
(check-expect (x+ (make-posn 10 0)) (make-posn 13 0))
(define (x+ p)
  (... (posn-x p) ... (posn-y p) ...))
```

The signature, the purpose, and the example all come out of the problem statement. Instead of a header—a function with a default result—our sketch contains the two selector expressions for Posns. After all, the information for the result must come from the inputs, and the input is a structure that contains two values.

Finishing the definition is easy now. Since the desired result is a Posn, the function uses `make-posn` to combine the pieces:

```
(define (x+ p)
  (make-posn (+ (posn-x p) 3) (posn-y p)))
```

Exercise 73. Design the function `posn-up-x`, which consumes a Posn `p` and a Number `n`. It produces a Posn like `p` with `n` in the `x` field.

A neat observation is that we can define `x+` using `posn-up-x`:

```
(define (x+ p)
  (posn-up-x p (+ (posn-x p) 3)))
```

Note Functions such as `posn-up-x` are often called *updaters* or *functional setters*. They are extremely useful when you write large programs. ▮

A function may also produce instances from atomic data. While `make-posn` is a built-in primitive that does so, our running problem provides another fitting illustration:

> **Sample Problem** Another colleague is tasked to design `reset-dot`, a function that resets the dot when the mouse is clicked.

To tackle this problem, you need to recall from Designing World Programs that mouse-event handlers consume four values: the current state of the world, the x- and y-coordinates of the mouse click, and a MouseEvt.

By adding the knowledge from the sample problem to the program design recipe, we get a signature, a purpose statement, and a header:

```
; Posn Number Number MouseEvt -> Posn
; for mouse clicks, (make-posn x y); otherwise p
(define (reset-dot p x y me) p)
```

Examples for mouse-event handlers need a Posn, two numbers, and a MouseEvt, which is just a special kind of String. A mouse click, for example, is represented with one of two strings: `"button-down"` and `"button-up"`. The first one signals that a user clicked the mouse button, the latter signals its release. With this in mind, here are two examples, which you may wish to study and interpret:

```
(check-expect
  (reset-dot (make-posn 10 20) 29 31 "button-down")
  (make-posn 29 31))
(check-expect
  (reset-dot (make-posn 10 20) 29 31 "button-up")
  (make-posn 10 20))
```

Although the function consumes only atomic forms of data, its purpose statement and the examples suggest that it differentiates between two kinds of MouseEvts: `"button-down"` and all others. Such a case split suggests a `cond` expression:

```
(define (reset-dot p x y me)
  (cond
    [(mouse=? "button-down" me) (... p ... x y ...)]
    [else (... p ... x y ...)]))
```

Following the design recipe, this skeleton mentions the parameters to remind you of what data is available.

The rest is straightforward again because the purpose statement itself dictates what the function computes in each of the two cases:

```
(define (reset-dot p x y me)
  (cond
    [(mouse=? me "button-down") (make-posn x y)]
    [else p]))
```

As above, we could have mentioned that `make-posn` creates instances of Posn, but you know this and we don't need to remind you constantly.

Exercise 74. Copy all relevant constant and function definitions to DrRacket's definitions area. Add the tests and make sure they pass. Then run the program and use the mouse to place the red dot. ▮

Many programs deal with nested structures. We illustrate this point with another small excerpt from a world program:

Sample Problem Your team is designing a game program that keeps track of an object that moves across the canvas at changing speed. The chosen data representation requires two data definitions:

Remember, it's about physics.

```
(define-struct ufo [loc vel])
; A UFO is a structure:
;   (make-ufo Posn Vel)
; interpretation (make-ufo p v) is at location
; p moving at velocity v
```

It is your task to develop `ufo-move-1`. The function computes the location of a given UFO after one clock tick passes.

Let us start with some examples that explore the data definitions a bit:

```
(define v1 (make-vel 8 -3))
(define v2 (make-vel -5 -3))
```

The order of these definitions matters. See Intermezzo 1: Beginning Student Language.

```
(define p1 (make-posn 22 80))
(define p2 (make-posn 30 77))

(define u1 (make-ufo p1 v1))
(define u2 (make-ufo p1 v2))
(define u3 (make-ufo p2 v1))
(define u4 (make-ufo p2 v2))
```

The first four are elements of Vel and Posn. The last four combine the first four in all possible combinations.

Next we write down a signature, a purpose, some examples, and a function header:

```
; UFO -> UFO
; determines where u moves in one clock tick;
; leaves the velocity as is

(check-expect (ufo-move-1 u1) u3)
(check-expect (ufo-move-1 u2)
              (make-ufo (make-posn 17 77) v2))

(define (ufo-move-1 u) u)
```

For the function examples, we use the data examples **and** our domain knowledge of positions and velocities. Specifically, we know that a vehicle that is moving north at 60 miles per hour and west at 10 miles per hour is going to end up 60 miles north from its starting point and 10 miles west after one hour of driving. After two hours, it will be 120 miles north from the starting point and 20 miles to its west.

As always, a function that consumes a structure instance can (and probably must) extract information from the structure to compute its result. So once again we add selector expressions to the function definition:

```
(define (ufo-move-1 u)
  (... (ufo-loc u) ... (ufo-vel u) ...))
```

Note The selector expressions raise the question whether we need to refine this sketch even more. After all, the two expressions extract instances of Posn and Vel, respectively. These two are also structure instances, and we could extract values from them in turn. Here is what the resulting skeleton would look like:

```
; UFO -> UFO
(define (ufo-move-1 u)
  (... (posn-x (ufo-loc u)) ...
   ... (posn-y (ufo-loc u)) ...
   ... (vel-deltax (ufo-vel u)) ...
   ... (vel-deltay (ufo-vel u)) ...))
```

Doing so obviously makes the sketch look quite complex, however. For truly realistic programs, following this idea to its logical end would create incredibly complex program outlines. More generally, *If a function deals with nested structures, develop one function per level of nesting.*

In the second part of the book, this guideline becomes even more important and we refine it a bit. **End**

Here we focus on how to combine the given Posn and the given Vel in order to obtain the next location of the UFO—because that's what our physics knowledge tells us. Specifically, it says to "add" the two together, where "adding" can't mean the operation we usually apply to numbers. So let us imagine that we have a function for adding a Vel to a Posn:

```
; Posn Vel -> Posn
; adds v to p
(define (posn+ p v) p)
```

Writing down the signature, purpose, and header like this is a legitimate way of programming. It is called "making a wish" and is a part of "making a wish list" as described in From Functions to Programs.

The key is to make wishes in such a way that we can complete the function that we are working on. In this manner, we can split difficult programming tasks into different tasks, a technique that helps us solve problems in reasonably small steps. For the sample problem, we get a complete definition for `ufo-move-1`:

```
(define (ufo-move-1 u)
  (make-ufo (posn+ (ufo-loc u) (ufo-vel u))
            (ufo-vel u)))
```

Because `ufo-move-1` and `posn+` are complete definitions, we can even click *RUN*, which checks that DrRacket doesn't complain about grammatical problems with our work so far. Naturally, the tests fail because `posn+` is just a wish, not the function we need.

Now it is time to focus on `posn+`. We have completed the first two steps of the design (data definitions, signature/purpose/header), so we must create examples. One easy way to create functional examples for a "wish" is to use the examples for the original function and to turn them into examples for the new function:

In geometry, the operation corresponding to `posn+` is called a *translation*.

```
(check-expect (posn+ p1 v1) p2)
(check-expect (posn+ p1 v2) (make-posn 17 77))
```

For this problem, we know that `(ufo-move-1 (make-ufo p1 v1))` is to produce `p2`. At the same time, we know that `ufo-move-1` applies `posn+` to `p1` and `v1`, implying that `posn+` must produce `p2` for these inputs. Stop! Check our manual calculations to ensure that you are following what we are doing.

We are now able to add selector expressions to our design sketch:

```
(define (posn+ p v)
  (... (posn-x p) ... (posn-y p) ...
   ... (vel-deltax v) ... (vel-deltay v) ...))
```

Because `posn+` consumes instances of Posn and Vel and because each piece of data is an instance of a two-field structure, we get four expressions. In contrast to the nested selector expressions from above, these are simple applications of a selector to a parameter.

If we remind ourselves what these four expressions represent, or if we recall how we computed the desired results from the two structures, our completion of the definition of `posn+` is straightforward:

```
(define (posn+ p v)
  (make-posn (+ (posn-x p) (vel-deltax v))
             (+ (posn-y p) (vel-deltay v))))
```

The first step is to add the velocity in the horizontal direction to the x-coordinate and the velocity in the vertical direction to the y-coordinate. This yields two expressions, one per new coordinate. With `make-posn` we can combine them into a single Posn again.

Exercise 75. Enter these definitions and their test cases into the definitions area of DrRacket and make sure they work. This is the first time that you have dealt with a "wish," and you need to make sure you understand how the two functions work together. ▮

5.7 The Universe of Data

Every language comes with a universe of data. This data represents information from and about the external world; it is what programs manipulate. This universe of data is a collection that not only contains all built-in data but also any piece of data that any program may ever create.

Remember that mathematicians call data

The left side of figure 30 shows one way to imagine the universe of BSL. Since there are infinitely many numbers and strings, the collection of all data is infinite. We indicate "infinity" in the figure with "...", but a real definition would have to avoid this imprecision.

collections or data classes *sets*.

Neither programs nor individual functions in programs deal with the entire universe of data. It is the purpose of a data definition to describe parts of this universe and to name these parts so that we can refer to them concisely. Put differently, a named data definition is a description of a collection of data, and that name is usable in other data definitions and in function signatures. In a function signature, the name specifies what data a function will deal with and, implicitly, which part of the universe of data it won't deal with.

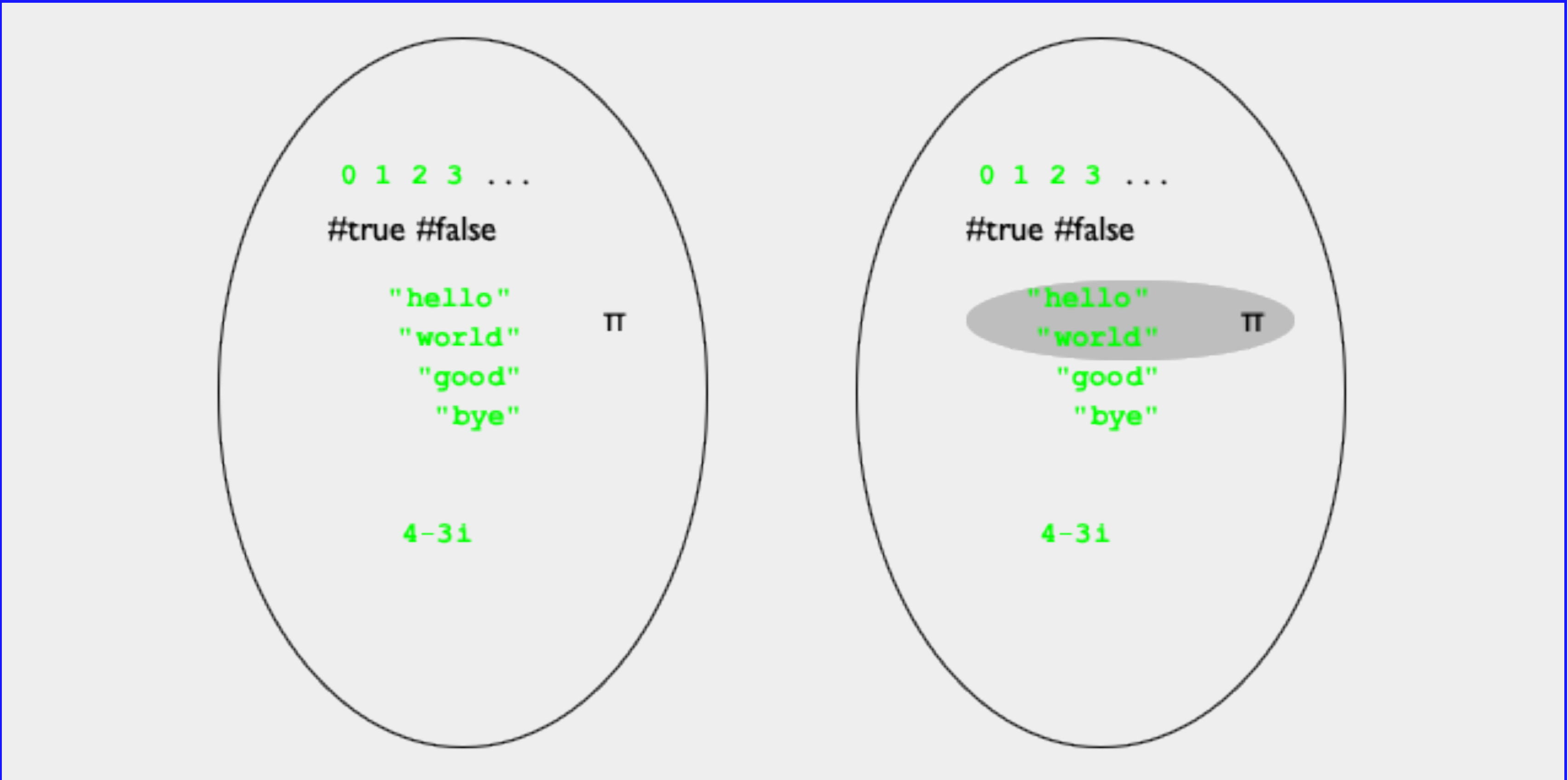


Figure 30: The universe of data

Practically, the data definitions of the first four chapters restrict built-in collections of data. They do so via an explicit or implicit itemization of all included values. For example, the region shaded with gray on the right side in figure 30 depicts the following data definition:

```
; A BS is one of:
; – "hello",
; – "world", or
; – pi.
```

While this particular data definition looks silly, note the stylized mix of English and BSL that is used. Its meaning is precise and unambiguous, clarifying exactly which elements belong to BS and which don't.

The definition of structure types completely revised the picture. When a programmer defines a structure type, the universe expands with all possible structure instances. For example, the addition of `posn` means that instances of `posn` with all possible values in the two fields appear. The middle bubble in figure 31 depicts the addition of these values, including such seeming nonsense as `(make-posn "hello" 0)` and `(make-posn (make-posn 0 1) 2)`. And yes, some of these instances of `posn` make no sense to us. But, a BSL program may construct any of them.

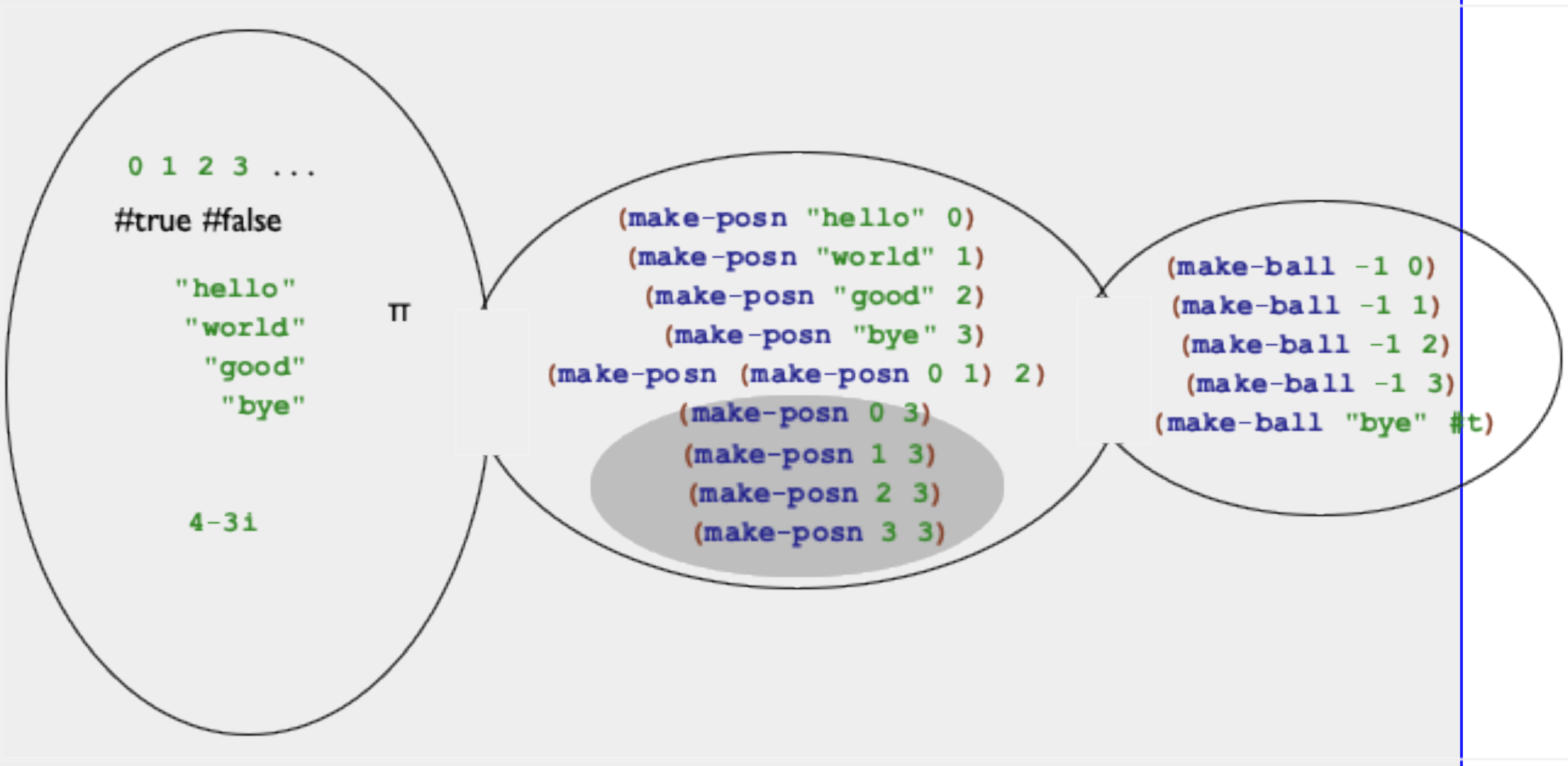


Figure 31: Adding structure to a universe

Adding yet another structure type definition mixes and matches everything again. Say we add the definition for `ball`, also with two fields. As the third bubble in figure 31 shows, this addition creates instances of `ball` that contain numbers, `posn` structures, and so on, as well as instances of `posn` that contain instances of `ball`. Try it out in DrRacket! Add

```
(define-struct ball [location velocity])
```

to the definitions area, hit *RUN*, and create some structure instances.

As far as the pragmatics of data definitions is concerned, a data definition for structure types describes large collections of data via combinations of existing data definitions with instances. When we write

```
; Posn is (make-posn Number Number)
```

we are describing an infinite number of possible instances of `posn`. Like above, the data definitions use combinations of natural language, data collections defined elsewhere, and data constructors. Nothing else should show up in a data definition at the moment.

A data definition for structures specifies a new collection of data made up of those instances to be used by our functions. For example, the data definition for Posns identifies the region shaded in gray in the center bubble of the universe in figure 31, which includes all those `posn` structures whose two fields contain numbers. At the same time, it is perfectly possible to construct an instance of `posn` that doesn't satisfy the requirement that both fields contain numbers:

```
(make-posn (make-posn 1 1) "hello")
```

This structure contains a `posn` in the `x` field and a string in the `y` field.

Exercise 76. Formulate data definitions for the following structure type definitions:

- `(define-struct movie [title producer year])`
- `(define-struct person [name hair eyes phone])`
- `(define-struct pet [name number])`
- `(define-struct CD [artist title price])`
- `(define-struct sweater [material size producer])`

Make sensible assumptions as to what kind of values go into each field. ▮

Exercise 77. Provide a structure type definition and a data definition for representing points in time since midnight. A point in time consists of three numbers: hours, minutes, and seconds. ▮

Exercise 78. Provide a structure type and a data definition for representing three-letter words. A word consists of lowercase letters, represented with the 1Strings `"a"` through `"z"` plus `#false`. **Note** This exercise is a part of the design of a hangman game; see exercise 396. ▮

Programmers not only write data definitions, they also read them in order to understand programs, to expand the kind of data they can deal with, to eliminate errors, and so on. We read a data definition to understand how to create data that belongs to the designated collection and to determine whether some piece of data belongs to some specified class.

Since data definitions play such a central and important role in the design process, it is often best to illustrate data definitions with examples just like we illustrate the behavior of functions with examples. And indeed, creating data examples from a data definition is straightforward:

- for a built-in collection of data (number, string, Boolean, images), choose your favorite examples;

  Note On occasion, people use descriptive names to qualify built-in data collections, such as `NegativeNumber` or `OneLetterString`. They are no replacement for a well-written data definition. **End**

- for an enumeration, use several of the items of the enumeration;

- for intervals, use the end points (if they are included) and at least one interior point;

- for itemizations, deal with each part separately; and

- for data definitions for structures, follow the natural language description; that is, use the constructor and pick an example from the data collection named for each field.

That's all there is to constructing examples from data definitions for most of this book, though data definitions are going to become much more complex than what you have seen so far.

Exercise 79. Create examples for the following data definitions:

-
  ```
  ; A Color is one of:
  ; — "white"
  ; — "yellow"
  ; — "orange"
  ; — "green"
  ; — "red"
  ; — "blue"
  ; — "black"
  ```

  Note DrRacket recognizes many more strings as colors. **End**

-
  ```
  ; H is a Number between 0 and 100.
  ; interpretation represents a happiness value
  ```

-
  ```
  (define-struct person [fstname lstname male?])
  ; A Person is a structure:
  ;   (make-person String String Boolean)
  ```

  Is it a good idea to use a field name that looks like the name of a predicate?

-
  ```
  (define-struct dog [owner name age happiness])
  ; A Dog is a structure:
  ;   (make-dog Person String PositiveInteger H)
  ```

  Add an interpretation to this data definition, too.

-
  ```
  ; A Weapon is one of:
  ; — #false
  ; — Posn
  ; interpretation #false means the missile hasn't
  ; been fired yet; a Posn means it is in flight
  ```

The last definition is an unusual itemization, combining built-in data with a structure type. The next chapter deals with such definitions in depth. ▮

5.8 Designing with Structures

The introduction of structure types reinforces the need for all six steps in the design recipe. It no longer suffices to rely on built-in data collections to represent information; it is now clear that programmers must create data definitions for all but the simplest problems.

This section adds a design recipe, illustrating it with the following:

> **Sample Problem** Design a function that computes the distance of objects in a 3-dimensional space to the origin.

Here we go:

1. When a problem calls for the representation of pieces of information that belong together or describe a natural whole, you need a structure type definition. It requires as many fields as there are relevant properties. An instance of

this structure type corresponds to the whole, and the values in the fields correspond to its attributes.

A data definition for a structure type introduces a name for the collection of instances that are legitimate. Furthermore, it must describe which kind of data goes with which field. **Use only names of built-in data collections or previously defined data definitions.**

In the end, we (and others) must be able to use the data definition to create sample structure instances. Otherwise, something is wrong with our data definition. To ensure that we can create instances, our data definitions should come with **data examples**.

Here is how we apply this idea to the sample problem:

```
(define-struct r3 [x y z])
; An R3 is a structure:
;   (make-r3 Number Number Number)

(define ex1 (make-r3 1 2 13))
(define ex2 (make-r3 -1 0 3))
```

The structure type definition introduces a new kind of structure, `r3`, and the data definition introduces R3 as the name for all instances of `r3` that contain only numbers.

2. You still need a signature, a purpose statement, and a function header but they remain the same. Stop! Do it for the sample problem.

3. Use the examples from the first step to create functional examples. For each field associated with intervals or enumerations, make sure to pick end points and intermediate points to create functional examples. We expect you to continue working on the sample problem.

4. A function that consumes structures usually—though not always—extracts the values from the various fields in the structure. To remind yourself of this possibility, add a selector for each field to the templates for such functions.

   Here is what we have for the sample problem:

```
; R3 -> Number
; determines the distance of p to the origin
(define (r3-distance-to-0 p)
  (... (r3-x p) ... (r3-y p) ... (r3-z p) ...))
```

   You may want to write down next to each selector expression what kind of data it extracts from the given structure; you can find this information in the data definition. Stop! Just do it!

5. Use the selector expressions from the template when you define the function. Keep in mind that you may not need some of them.

6. Test. Test as soon as the function header is written. Test until all expressions have been covered. Test again when you make changes.

Finish the sample problem. If you cannot remember the distance of a 3-dimensional point to the origin, look it up in a geometry book.

There you will find a formula such as $\sqrt{x^2 + y^2 + z^2}$.

Exercise 80. Create templates for functions that consume instances of the following structure types:

- `(define-struct movie [title director year])`
- `(define-struct pet [name number])`
- `(define-struct CD [artist title price])`
- `(define-struct sweater [material size color])`

No, you do not need data definitions for this task. ▌

Exercise 81. Design the function `time->seconds`, which consumes instances of time structures (see exercise 77) and produces the number of seconds that have passed since midnight. For example, if you are representing 12 hours, 30 minutes, and 2 seconds with one of these structures and if you then apply `time->seconds` to this instance, the correct result is `45002`. ▌

Exercise 82. Design the function `compare-word`. The function consumes two three-letter words (see exercise 78). It produces a word that indicates where the given ones agree and disagree. The function retains the content of the structure fields if the two agree; otherwise it places `#false` in the field of the resulting word. **Hint** The exercises mentions two tasks: the comparison of words and the comparison of “letters.” ▌

5.9 Structure in the World

When a world program must track two independent pieces of information, we must use a collection of structures to represent the world state data. One field keeps track of one piece of information and the other field the second piece of information. Naturally, if the domain world contains more than two independent pieces of information, the structure type definition must specify as many fields as there are distinct pieces of information.

Consider a space invader game that consists of a UFO and a tank. The UFO descends along a straight vertical line and a tank moves horizontally at the bottom of a scene. If both objects move at known constant speeds, all that's needed to describe these two objects is one piece of information per object: the y-coordinate for the UFO and the x-coordinate for the tank. Putting those together requires a structure with two fields:

```
(define-struct space-game [ufo tank])
```

We leave it to you to formulate an adequate data definition for this structure type definition, including an interpretation. Ponder the hyphen in the name of the structure. BSL really allows the use of all kinds of characters in the names of variables, functions, structures, and field names. What are the selector names for this structure? The name of the predicate?

Every time we say "piece of information," we don't necessarily mean a single number or a single word. A piece of information may itself combine several pieces of information. Creating a data representation for that kind of information naturally leads to nested structures.

Let's add a modicum of spice to our imaginary space invader game. A UFO that descends only along a vertical line is boring. To turn this idea into an interesting game where the tank attacks the UFO with some weapon, the UFO must be able to descend in nontrivial lines, perhaps jumping randomly. An implementation of this idea means that we need two coordinates to describe the location of the UFO, so that our revised data definition for the space game becomes:

```
; A SpaceGame is a structure:
;   (make-space-game Posn Number).
; interpretation (make-space-game (make-posn ux uy) tx)
; describes a configuration where the UFO is
; at (ux,uy) and the tank's x-coordinate is tx
```

Understanding what kind of data representations are needed for world programs takes practice. The following two sections introduce several reasonably complex problem statements. Solve them before moving on to the kind of games that you might like to design on your own.

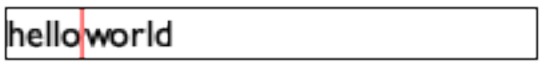
5.10 A Graphical Editor

To program in BSL, you open DrRacket, type on the keyboard, and watch text appear. Pressing the left arrow on the keyboard moves the cursor to the left; pressing the backspace (or delete) key erases a single letter to the left of the cursor—if there is a letter to start with.

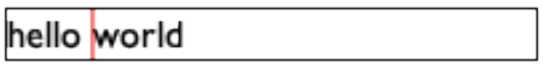
This process is called "editing," though its precise name should be "text editing of programs" because we will use "editing" for a more demanding task than typing on a keyboard. When you write and revise other kinds of documents, say, an English assignment, you are likely to use other software applications, called word processors, though computer scientists dub all of them editors or even graphical editors.

You are now in a position to design a world program that acts as a one-line editor for plain text. Editing here includes entering letters and somehow changing the already existing text, including the deletion and the insertion of letters. This implies some notion of position within the text. People call this position a *cursor*; most graphical editors display it in such a way that it can easily be spotted.

Take a look at the following editor configuration:


helloworld

Someone might have entered the text "helloworld" and hit the left arrow key five times, causing the cursor to move from the end of the text to the position between "o" and "w." Pressing the space bar would now cause the editor to change its display as follows:


hello world

In short, the action inserts " " and places the cursor between it and "w."

Given this much, an editor must track two pieces of information:

1. the text entered so far, and
2. the current location of the cursor.

And this suggests a structure type with two fields.

We can imagine several different ways of going from the information to data and back. For example, one field in the structure may contain the entire text entered, and the other the number of characters between the first character (counting from the left) and the cursor. Another data representation is to use two strings in the two fields: the part of the text to the left of the cursor and the part of the text to its right. Here is our preferred approach to representing the state of an editor:

```
(define-struct editor [pre post])
; An Editor is a structure:
;   (make-editor String String)
; interpretation (make-editor s t) describes an editor
; whose visible text is (string-append s t) with
; the cursor displayed between s and t
```

Solve the next few exercises based on this data representation.

Exercise 83. Design the function `render`, which consumes an Editor and produces an image.

The purpose of the function is to render the text within an empty scene of 200×20 pixels. For the cursor, use a 1×20 red rectangle and for the strings, black text of size 16.

Develop the image for a sample string in DrRacket's interactions area. We started with this expression:

```
(overlay/align "left" "center"
               (text "hello world" 11 "black")
               (empty-scene 200 20))
```

You may wish to read up on `beside`, `above`, and such functions. When you are happy with the looks of the image, use the expression as a test and as a guide to the design of `render`. ▌

Exercise 84. Design `edit`. The function consumes two inputs, an editor `ed` and a KeyEvent `ke`, and it produces another editor. Its task is to add a single-character KeyEvent `ke` to the end of the *pre* field of `ed`, unless `ke` denotes the backspace (`"\b"`) key. In that case, it deletes the character immediately to the left of the cursor (if there are any). The function ignores the tab key (`"\t"`) and the return key (`"\r"`).

The function pays attention to only two KeyEvents longer than one letter: `"left"` and `"right"`. The left arrow moves the cursor one character to the left (if any), and the right arrow moves it one character to the right (if any). All other such KeyEvents are ignored.

Develop a goodly number of examples for `edit`, paying attention to special cases. When we solved this exercise, we created 20 examples and turned all of them into tests.

Hint Think of this function as consuming KeyEvents, a collection that is specified as an enumeration. It uses auxiliary functions to deal with the Editor structure. Keep a wish list handy; you will need to design additional functions for most of these auxiliary functions, such as `string-first`, `string-rest`, `string-last`, and `string-remove-last`. If you haven't done so, solve the exercises in Functions. ▌

Exercise 85. Define the function `run`. Given the `pre` field of an editor, it launches an interactive editor, using `render` and `edit` from the preceding two exercises for the `to-draw` and `on-key` clauses, respectively. ▌

Exercise 86. Notice that if you type a lot, your editor program does not display all of the text. Instead the text is cut off at the right margin. Modify your function `edit` from exercise 84 so that it ignores a keystroke if adding it to the end of the `pre` field would mean the rendered text is too wide for your canvas. ▌

Exercise 87. Develop a data representation for an editor based on our first idea, using a string and an index. Then solve the preceding exercises again. **Retrace the design recipe. Hint** if you haven't done so, solve the exercises in Functions.

Note on Design Choices The exercise is a first study of making design choices. It shows that the very first design choice concerns the data representation. Making the right choice requires planning ahead and weighing the complexity of each. Of course, getting good at this is a question of gaining experience. ▌

5.11 More Virtual Pets

In this section we continue our virtual zoo project from Virtual Pet Worlds. Specifically, the goal of the exercise is to combine the cat world program with the program for managing its happiness gauge. When the combined program runs, you see the cat walking across the canvas, and, with each step, its happiness goes down. The only way to make the cat happy is to feed it (down arrow) or to pet it (up arrow). Finally, the goal of the last exercise in this section is to create another virtual, happy pet.

Exercise 88. Define a structure type that keeps track of the cat's x-coordinate and its happiness. Then formulate a data definition for cats, dubbed *VCat*, including an interpretation. ▌

Exercise 89. Design the `happy-cat` world program, which manages a walking cat and its happiness level. Let's assume that the cat starts out with perfect happiness.

Hints (1) Reuse the functions from the world programs in Virtual Pet Worlds. (2) Use structure type from the preceding exercise to represent the state of the world. ▮

Exercise 90. Modify the `happy-cat` program from the preceding exercises so that it stops whenever the cat's happiness falls to `0`. ▮

Exercise 91. Extend your structure type definition and data definition from exercise 88 to include a direction field. Adjust your `happy-cat` program so that the cat moves in the specified direction. The program should move the cat in the current direction, and it should turn the cat around when it reaches either end of the scene. ▮

```
(define cham           )
```

The above drawing of a chameleon is a **transparent** image. To insert it into DrRacket, insert it with the "Insert Image" menu item. Using this instruction preserves the transparency of the drawing's pixels.

When a partly transparent image is combined with a colored shape, say a rectangle, the image takes on the underlying color. In the chameleon drawing, it is actually the inside of the animal that is transparent; the area outside is solid white. Try out this expression in your DrRacket:

```
(define background
  (rectangle (image-width cham)
             (image-height cham)
             "solid"
             "red"))

(overlay cham background)
```

Exercise 92. Design the `cham` program, which has the chameleon continuously walking across the canvas from left to right. When it reaches the right end of the canvas, it disappears and immediately reappears on the left. Like the cat, the chameleon gets hungry from all the walking, and, as time passes by, this hunger expresses itself as unhappiness.

For managing the chameleon's happiness gauge, you may reuse the happiness gauge from the virtual cat. To make the chameleon happy, you feed it (down arrow, two points only); petting isn't allowed. Of course, like all chameleons, ours can change color, too: `"r"` turns it red, `"b"` blue, and `"g"` green. Add the chameleon world program to the virtual cat game and reuse functions from the latter when possible.

Start with a data definition, *VCham*, for representing chameleons. ▮

Exercise 93. Copy your solution to exercise 92 and modify the copy so that the chameleon walks across a tricolor background. Our solution uses these colors:

```
(define BACKGROUND
  (beside (empty-scene WIDTH HEIGHT "green")
          (empty-scene WIDTH HEIGHT "white")
          (empty-scene WIDTH HEIGHT "red")))
```

Have some Italian pizza when you're done.

but you may use any colors. Observe how the chameleon changes colors to blend in as it crosses the border between two colors.

Note When you watch the animation carefully, you see the chameleon riding on a white rectangle. If you know how to use image editing software, modify the picture so that the white rectangle is invisible. Then the chameleon will really blend in. ▮

The preceding two chapters introduce two ways of formulating data definitions. Those that employ itemization (enumeration and intervals) are used to create small collections from large ones. Those that use structures combine multiple collections. Since the development of data representations is the starting point for proper program design, it cannot surprise you that programmers frequently want to itemize data definitions that involve structures or to use structures to combine itemized data.

Recall the imaginary space invader game from Structure in the World in the preceding chapter. Thus far, it involves one UFO, descending from space, and one tank on the ground, moving horizontally. Our data representation uses a structure with two fields: one for the data representation of the UFO and another one for the data representation of the tank. Naturally, players will want a tank that can fire off a missile. All of a sudden, we can think of a second kind of state that contains three independently moving objects: the UFO, the tank, and the missile. Thus we have two distinct structures: one for representing two independently moving objects and another one for the third. Since a world state may now be one of these two structures, it is natural to use an itemization to describe all possible states:

1. the state of the world is a structure with **two** fields, or

2. the state of the world is a structure with **three** fields.

As far as our domain is concerned—the actual game—the first kind of state represents the time before the tank has launched its sole missile and the second one the time after the missile has been fired.

No worries, the next part of the book is about firing as many missiles as you want, without reloading.

This chapter introduces the basic idea of itemizing data definitions that involve structures. Because we have all the other ingredients we need, we start straight with itemizing structures. After that, we discuss some examples, including world programs that benefit from our new power. The last section is about errors in programming.

6.1 Designing with Itemizations, Again

Let's start with a refined problem statement for our space invader game from Programming with Structures.

> **Sample Problem** Design a game program using the *2htdp/universe* library for playing a simple space invader game. The player is in control of a tank (a small rectangle) that must defend our planet (the bottom of the canvas) from a UFO (see Intervals for one possibility) that descends from the top of the canvas to the bottom. In order to stop the UFO from landing, the player may fire a single missile (a triangle smaller than the tank) by hitting the space bar. In response, the missile emerges from the tank. If the UFO collides with the missile, the player wins; otherwise the UFO lands and the player loses.
>
> Here are some details concerning the three game objects and their movements. First, the tank moves a constant speed along the bottom of the canvas, though the player may use the left arrow key and the right arrow key to change directions. Second, the UFO descends at a constant velocity but makes small random jumps to the left or right. Third, once fired, the missile ascends along a straight vertical line at a constant speed at least twice as fast as the UFO descends. Finally, the UFO and the missile collide if their reference points are close enough, for whatever you think "close enough" means.

The following two subsections use this sample problem as a running example, so study it well and solve the following exercise before you continue. Doing so will help you understand the problem in enough depth.

Exercise 94. Draw some sketches of what the game scenery looks like at various stages. Use the sketches to determine the constant and the variable pieces of the game. For the former, develop physical and graphical constants that describe the dimensions of the world (canvas) and its objects. Also develop some background scenery. Finally, create your initial scene from the constants for the tank, the UFO, and the background. ▮

Defining Itemizations The first step in our design recipe calls for the development of data definitions. One purpose of a data definition is to describe the construction of data that represents the state of the world; another is to describe all possible pieces of data that the event-handing functions of the world program may consume. Since we haven't seen itemizations that include structures, this first subsection introduces this idea. While this probably won't surprise you, pay close attention.

As argued in the introduction to this chapter, the space invader game with a missile-firing tank requires a data representation for two different kinds of game states. We choose two structure type definitions:

```
(define-struct aim [ufo tank])
(define-struct fired [ufo tank missile])
```

For this space invader game, we could get away with one structure type definition of three fields where the third field contains #false until the missile is fired, and a Posn for the missile's coordinates thereafter. See below.

The first one is for the time period when the player is trying to get the tank in position for a shot, and the second one is for representing states after the missile is fired. Before we can formulate a data definition for the complete game state, however, we need data representations for the tank, the UFO, and the missile.

Assuming constant definitions for such physical constants as `WIDTH` and `HEIGHT`, which are the subject of exercise 94, we formulate the data definitions like this:

```
; A UFO is a Posn.
; interpretation (make-posn x y) is the UFO's location
; (using the top-down, left-to-right convention)

(define-struct tank [loc vel])
; A Tank is a structure:
;   (make-tank Number Number).
; interpretation (make-tank x dx) specifies the position:
; (x, HEIGHT) and the tank's speed: dx pixels/tick

; A Missile is a Posn.
; interpretation (make-posn x y) is the missile's place
```

Each of these data definitions describes nothing but a structure, either a newly defined one, `tank`, or a built-in data collection, Posn. Concerning the latter, it may surprise you a little bit that Posns are used to represent two distinct aspects of the world. Then again, we have used numbers (and strings and Boolean values) to represent many different kinds of information in the real world, so reusing a collection of structures such as `Posn` isn't a big deal.

Now we are in a position to formulate the data definitions for the state of the space invader game:

```
; A SIGS is one of:
; – (make-aim UFO Tank)
; – (make-fired UFO Tank Missile)
; interpretation represents the complete state of a
; space invader game
```

The shape of the data definition is that of an itemization. Each clause, however, describes the content of a structure type, just like the data definition for structure types we have seen so far. Still, this data definition shows that not every data definition comes with exactly one structure type definition; here one data definition involves two distinct structure type definitions.

The meaning of such a data definition is also straightforward. It introduces the name SIGS for the collection of all those structure instances that you can create according to the definition. So let us create some:

- Here is an instance that describes the tank maneuvering into position to fire the missile:

  ```
  (make-aim (make-posn 20 10) (make-tank 28 -3))
  ```

- This one is just like the previous one but the missile has been fired:

  ```
  (make-fired (make-posn 20 10)
              (make-tank 28 -3)
              (make-posn 28 (- HEIGHT TANK-HEIGHT)))
  ```

  Of course, the capitalized names refer to the physical constants that you defined.

- Finally, here is one where the missile is about to collide with the UFO:

  ```
  (make-fired (make-posn 20 100)
              (make-tank 100 3)
              (make-posn 22 103))
  ```

  This example assumes that the canvas is more than 100 pixels tall.

Notice that the first instance of SIGS is generated according to the first clause of the data definition, and the second and third follow the second clause. Naturally the numbers in each field depend on your choices for global game constants.

Exercise 95. Explain why the three instances are generated according to the first or second clause of the data definition. ▮

Exercise 96. Sketch how each of the three game states could be rendered assuming a 200×200 canvas. ▮

The Design Recipe With a new way of formulating data definitions comes an inspection of the design recipe. This chapter introduces a way to combine two or more means of describing data, and the revised design recipe reflects this, especially the first step:

1. When do you need this new way of defining data? You already know that the need for itemizations is due to distinctions among different classes of information in the problem statement. Similarly, the need for structure-based data definitions is due to the demand to group several different pieces of information.

   An itemization of different forms of data—including collections of structures—is required when your problem statement distinguishes different kinds of information and when at least some of these pieces of information consist of several different pieces.

One thing to keep in mind is that data definitions may refer to other data definitions. Hence, if a particular clause in a data definition looks overly complex, it is acceptable to write down a separate data definition for this clause and refer to this auxiliary definition.

And, as always, formulate data examples using the data definitions.

2. The second step remains the same. Formulate a function signature that mentions only the names of defined or built-in data collections, add a purpose statement, and create a function header.

3. Nothing changes for the third step. You still need to formulate functional examples that illustrate the purpose statement from the second step, and you still need one example per item in the itemization.

4. The development of the template now exploits two different dimensions: the itemization itself and the use of structures in its clauses.

   By the first, the body of the template consists of a `cond` expression that has as many `cond` clauses as the itemizations has items. Furthermore, you must add a condition to each `cond` clause that identifies the sub-class of data in the corresponding item.

   By the second, if an item deals with a structure, the template contains the selector expressions—in the `cond` clause that deals with the sub-class of data described in the item.

   When you choose to describe the data with a separate data definition, however, you do **not** add selector expressions. Instead, you create a template for the separate data definition to the task at hand and refer to that template with a function call. The latter indicates that this sub-class of data is being processed separately.

   Before going through the work of developing a template, briefly reflect on the nature of the function. If the problem statement suggests that there are several tasks to be performed, it is likely that a composition of several, separately designed functions is needed instead of a template. In that case, skip the template step.

5. Fill the gaps in the template. The more complex you make your data definitions, the more complex this step becomes. The good news is that this design recipe can help in many situations.

   If you are stuck, fill the easy cases first and use default values for the others. While this makes some of the test cases fail, you are making progress and you can visualize this progress.

   If you are stuck on some cases of the itemization, analyze the examples that correspond to those cases. Determine what the pieces of the template compute from the given inputs. Then consider how to combine these pieces (plus some constants) to compute the desired output. Keep in mind that you might need an auxiliary function.

   Also, if your template "calls" another template because the data definitions refer to each other, assume that the other function delivers what its purpose statement and its examples promise—even if this other function's definition isn't finished yet.

6. Test. If tests fail, determine what's wrong: the function, the tests, or both. Go back to the appropriate step.

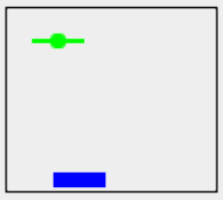
Go back to Designing Functions, reread the description of the simple design recipe, and compare it to this revision.

Let's illustrate the design recipe with the design of a rendering function for the sample problem at the beginning of this section. Recall that a `big-bang` expression needs such a rendering function to turn the state of the world into an image after every clock tick, mouse click, or keystroke.

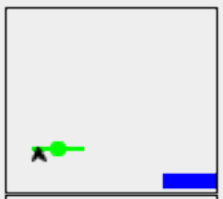
The signature of this rendering function says that it maps an element of the state-of-the-world class to the class of Images:

```
; SIGS -> Image
; adds TANK, UFO, and possibly MISSILE to
; the BACKGROUND scene
(define (si-render s) BACKGROUND)
```

Here `TANK`, `UFO`, `MISSILE`, and `BACKGROUND` are the requested image constants from exercise 94. Recall that this signature is just an instance of the general signature for rendering functions, which always consume the collections of world states and produce some image.

s	(si-render s)
`(make-aim (make-posn 10 20) (make-tank 28 -3))`	

```
(make-fired
  (make-posn 20 100)
  (make-tank 100 3)
  (make-posn 22 103))
```



```
(make-fired
  (make-posn 10 20)
  (make-tank 28 -3)
  (make-posn 32 (- HEIGHT TANK-HEIGHT 10)))
```

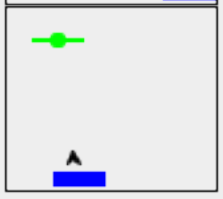


Figure 32: Rendering space invader game states, by example

Since the itemization in the data definition consists of two items, let's make three examples, using the data examples from above. See figure 32. Unlike the function tables found in mathematics books, this table is rendered vertically. The left column contains sample inputs for our desired function; the right column lists the corresponding desired results. As you can see, we used the data examples from the first step of the design recipe, and they cover both items of the itemization.

Next we turn to the development of the template, the most important step of the design process. First, we know that the body of `si-render` must be a `cond` expression with two `cond` clauses. Following the design recipe, the two conditions are `(aim? s)` and `(fired? s)`, and they distinguish the two possible kinds of data that `si-render` may consume:

```
(define (si-render s)
  (cond
    [(aim? s) ...]
    [(fired? s) ...]))
```

Second, we add selector expressions to every `cond` clause that deals with structures. In this case, both clauses concern the processing of structures: `aim` and `fired`. The former comes with two fields and thus requires two selector expressions for the first `cond` clause, and the latter kind of structures consists of three values and thus demands three selector expressions:

```
(define (si-render s)
  (cond
    [(aim? s) (... (aim-tank s) ... (aim-ufo s) ...)]
    [(fired? s) (... (fired-tank s) ... (fired-ufo s)
                 ... (fired-missile s) ...)]))
```

The template contains nearly everything we need to finish our task. To complete the definition, we figure out for each `cond` line how to combine the values we have in order to compute the expected result. Beyond the pieces of the input, we may also use globally defined constants, for example, `BACKGROUND`, which is obviously of help here; primitive or built-in operations; and, if all else fails, wish-list functions, that is, we describe functions we wish we had.

Consider the first `cond` clause, where we have a data representation of a tank, that is, `(aim-tank s)`, and the data representation of a UFO, that is, `(aim-ufo s)`. From the first example in figure 32, we know that we need to add the two objects to the background scenery. In addition, the design recipe suggests that if these pieces of data come with their own data definition, we are to consider defining helper (auxiliary) functions and to use those to compute the result:

```
... (tank-render (aim-tank s)
                 (ufo-render (aim-ufo s) BACKGROUND))
```

Here `tank-render` and `ufo-render` are wish-list functions:

```
; Tank Image -> Image
; adds t to the given image im
(define (tank-render t im) im)

; UFO Image -> Image
; adds u to the given image im
(define (ufo-render u im) im)
```

```
; SIGS -> Image
; renders the given game state on top of BACKGROUND
; for examples see figure 32
(define (si-render s)
  (cond
    [(aim? s)
     (tank-render (aim-tank s)
                  (ufo-render (aim-ufo s) BACKGROUND))]
    [(fired? s)
```

```
     (tank-render
       (fired-tank s)
       (ufo-render (fired-ufo s)
                   (missile-render (fired-missile s)
                                   BACKGROUND)))]))
```

Figure 33: The complete rendering function

With a bit of analogy, we can deal with the second cond clause in the same way. Figure 33 shows the complete definition. Best of all, we can immediately reuse our wish-list functions, tank-render and ufo-render, and all we need to add is a function for including a missile in the scene. The appropriate wish-list entry is:

```
; Missile Image -> Image
; adds m to the given image im
(define (missile-render m im) im)
```

As above, the comment describes in sufficient detail what we want.

Exercise 97. Design the functions tank-render, ufo-render, and missile-render. Compare this expression:

```
(tank-render
  (fired-tank s)
  (ufo-render (fired-ufo s)
              (missile-render (fired-missile s)
                              BACKGROUND)))
```

with this one:

```
(ufo-render
  (fired-ufo s)
  (tank-render (fired-tank s)
               (missile-render (fired-missile s)
                               BACKGROUND)))
```

When do the two expressions produce the same result? ▮

Exercise 98. Design the function si-game-over? for use as the stop-when handler. The game stops if the UFO lands or if the missile hits the UFO. For both conditions, we recommend that you check for proximity of one object to another.

The stop-when clause allows for an optional second sub-expression, namely a function that renders the final state of the game. Design si-render-final and use it as the second part for your stop-when clause in the main function of exercise 100. ▮

Exercise 99. Design si-move. This function is called for every clock tick to determine to which position the objects move now. Accordingly, it consumes an element of SIGS and produces another one.

Moving the tank and the missile (if any) is relatively straightforward. They move in straight lines at a constant speed. Moving the UFO calls for small random jumps to the left or the right. Since you have never dealt with functions that create random numbers, the rest of this exercise is a longish hint on how to deal with this issue.

BSL comes with a function that creates random numbers. Introducing this function illustrates why the signatures and purpose statements play such an important role during the design. Here is the relevant material for the function you need:

```
; Number -> Number
; produces a number in the interval [0,n),
; possibly a different one each time it is called
(define (random n) ...)
```

Since the signature and purpose statement precisely describe what a function computes, you can now experiment with random in DrRacket's interactions area. Stop! Do so!

If random produces different numbers (almost) every time it is called, testing functions that use random is difficult. To start with, separate si-move and its proper functionality into two parts:

The idea that you must use random is BSL knowledge, not a part of the design skills you must acquire, which is why we provide this hint. Also, random is the first and only BSL primitive that is not a mathematical function. Functions in programming are inspired by mathematical functions, but they are not identical concepts.

```
(define (si-move w)
  (si-move-proper w (random ...)))

; SIGS Number -> SIGS
; moves the space-invader objects predictably by delta
(define (si-move-proper w delta)
  w)
```

With this definition you separate the creation of a random number from the act of moving the game objects. While `random` may produce different results every time it is called, `si-move-proper` can be tested on specific numeric inputs and is thus guaranteed to return the same result when given the same inputs. In short, most of the code remains testable.

Instead of calling `random` directly, you may wish to design a function that creates a random x-coordinate for the UFO. Consider using `check-random` from BSL's testing framework to test such a function. ▌

Exercise 100. Design the function `si-control`, which plays the role of the key-event handler. As such, it consumes a game state and a KeyEvent and produces a new game state. It reacts to three different keys:

- pressing the left arrow ensures that the tank moves left;
- pressing the right arrow ensures that the tank moves right; and
- pressing the space bar fires the missile if it hasn't been launched yet.

Once you have this function, you can define the `si-main` function, which uses `big-bang` to spawn the game-playing window. Enjoy! ▌

```
; SIGS.v2 -> Image
; renders the given game state on top of BACKGROUND
(define (si-render.v2 s)
  (tank-render
    (sigs-tank s)
    (ufo-render (sigs-ufo s)
                (missile-render.v2 (sigs-missile s)
                                   BACKGROUND))))
```

Figure 34: Rendering game states again

Data representations are rarely unique. For example, we could use a single structure type to represent the states of a space invader game:

```
(define-struct sigs [ufo tank missile])
; A SIGS.v2 (short for SIGS version 2) is a structure:
;   (make-sigs UFO Tank MissileOrNot)
; interpretation represents the complete state of a
; space invader game

; A MissileOrNot is one of:
; – #false
; – Posn
; interpretation#false means the missile is in the tank;
; Posn says the missile is at that location
```

Unlike the first data representation for game states, this second version does not distinguish between before and after the missile launch. Instead, each state contains some data about the missile though this piece of data may just be `#false`, indicating that the missile hasn't been fired yet.

As a result, the functions for this second data representation of states differ from the functions for the first one. In particular, functions that consume an element of SIGS.v2 do not use a `cond` expression because there is only one kind of element in the collection. In terms of design approach, the design recipe for structures from Designing with Structures suffices. Figure 34 shows the result of designing the rendering function for this data representation.

In contrast, the design of functions using MissileOrNot requires the recipe from this section. Let's look at the design of `missile-render.v2`, whose job it is to add a missile to an image. Here is the header material:

```
; MissileOrNot Image -> Image
; adds an image of missile m to scene s
(define (missile-render.v2 m s)
  s)
```

As for examples, we must consider at least two cases: one when `m` is `#false` and another one when `m` is a Posn. In the first case, the missile hasn't been fired, which means that no image of a missile is to be added to the given scene. In the second case, the missile's position is specified and that is where the image of the missile must show up. Figure 35 demonstrates the workings of the function with two distinct scenarios.

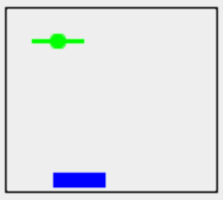
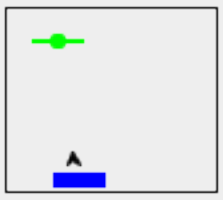
m	(missile-render.v2 m s)
#false	
(make-posn 32 (- HEIGHT TANK-HEIGHT 10))	

Figure 35: Rendering the space invader games, with tanks

Exercise 101. Turn the examples in figure 35 into test cases. ▮

Now we are ready to develop the template. Because the data definition for the major argument (`m`) is an itemization with two items, the function body is likely to consist of a `cond` expression with two clauses:

```
(define (missile-render.v2 m s)
  (cond
    [(boolean? m) ...]
    [(posn? m) ...]))
```

Following the data definition again, the first `cond` clause checks whether `m` is a Boolean value and the second one checks whether it is an element of Posn. And, if someone were to accidentally apply `missile-render.v2` to `#true` and to some image, the function would use the first `cond` clause to compute the result. We have more to say on such errors below.

The second template step requests selector expressions in all those `cond` clauses that deal with structures. In our example, this is true for the second clause, and the selector expressions extract the x- and y-coordinates from the given Posn:

```
(define (missile-render.v2 m s)
  (cond
    [(boolean? m) ...]
    [(posn? m) (... (posn-x m) ... (posn-y m) ...)]))
```

Compare this template with the one for `si-render` above. The data definition for the latter deals with two distinct structure types, and therefore the function template for `si-render` contains selector expressions in both `cond` clauses. The data definition for MissileOrNot, however, mixes items that are plain values with items that describe structures. Both kinds of definitions are perfectly fine; the key for you is to follow the recipe and to find a code organization that matches the data definition.

Here is the complete function definition:

```
(define (missile-render.v2 m s)
  (cond
    [(boolean? m) s]
    [(posn? m)
     (place-image MISSILE (posn-x m) (posn-y m) s)]))
```

Doing this step-by-step, you first work on the easy clauses; in this function that's the first one. Since it says the missile hasn't been fired, the function returns the given `s`. For the second clause, you need to remember that `(posn-x m)` and `(posn-y m)` select the coordinates for the image of the missile. This function must add `MISSILE` to `s`, so you have to figure out the best combination of primitive operations and your own functions to combine the four values. The choice of this combining operation is precisely where your creative insight as a programmer comes into play.

Exercise 102. Design all other functions that are needed to complete the game for this second data definition. ▮

Exercise 103. Develop a data representation for the following four kinds of zoo animals:

- **spiders**, whose relevant attributes are the number of remaining legs (we assume that spiders can lose legs in accidents) and the space they need in case of transport;
- **elephants**, whose only attributes are the space they need in case of transport;
- **boa constrictors**, whose attributes include length and girth; and

- **armadillos**, for which you must determine appropriate attributes, including one that determines the space needed for transport.

Develop a template for functions that consume zoo animals.

Design the `fits?` function, which consumes a zoo animal and a description of a cage. It determines whether the cage's volume is large enough for the animal. ▮

Exercise 104. Your home town manages a fleet of vehicles: automobiles, vans, buses, and SUVs. Develop a data representation for vehicles. The representation of each vehicle must describe the number of passengers that it can carry, its license plate number, and its fuel consumption (miles per gallon). Develop a template for functions that consume vehicles. ▮

Exercise 105. Some program contains the following data definition:

```
; A Coordinate is one of:
; – a NegativeNumber
; interpretation on the y axis, distance from top
; – a PositiveNumber
; interpretation on the x axis, distance from left
; – a Posn
; interpretation an ordinary Cartesian point
```

Make up at least two data examples per clause in the data definition. For each of the examples, explain its meaning with a sketch of a canvas. ▮

6.2 Mixing Up Worlds

This section suggests several design problems for world program, starting with simple extension exercises concerning our virtual pets.

Exercise 106. In More Virtual Pets we discussed the creation of virtual pets that come with happiness gauges. One of the virtual pets is a cat; the other one, a chameleon. Each program is dedicated to a single pet, however.

Design the `cat-cham` world program. Given both a location and an animal, it walks the latter across the canvas, starting from the given location. Here is the chosen data representation for animals:

```
; A VAnimal is either
; – a VCat
; – a VCham
```

where *VCat* and *VCham* are your data definitions from exercises 88 and 92.

Given that VAnimal is the collection of world states, you need to design

- a rendering function from VAnimal to Image;
- a function for handling clock ticks, from VAnimal to VAnimal; and
- a function for dealing with key events so that you can feed and pet and colorize your animal—as applicable.

It remains impossible to change the color of a cat or to pet a chameleon. ▮

Exercise 107. Design the `cham-and-cat` program, which deals with both a virtual cat **and** a virtual chameleon. You need a data definition for a "zoo" containing both animals and functions for dealing with it.

The problem statement leaves open how keys manipulate the two animals. Here are two possible interpretations:

1. Each key event goes to both animals.
2. Each key event applies to only one of the two animals.

   For this alternative, you need a data representation that specifies a *focus* animal, that is, the animal that can currently be manipulated. To switch focus, have the key-handling function interpret `"k"` for "kitty" and `"l"` for lizard. Once a player hits `"k"`, the following keystrokes apply to the cat only—until the player hits `"l"`.

Choose one of the alternatives and design the appropriate program. ▮

Exercise 108. In its default state, a pedestrian crossing light shows an orange person standing on a red background. When it is time to allow the pedestrian to cross the street, the light receives a signal and switches to a green, walking person. This phase lasts for 10 seconds. After that the light displays the digits `9`, `8`, ..., `0` with odd numbers colored orange and even numbers colored green. When the countdown reaches 0, the light switches back to its default state.

Design a world program that implements such a pedestrian traffic light. The light switches from its default state when you press the space bar on your keyboard. All other transitions must be reactions to clock ticks. You may wish to use the following images

or you can make up your own stick figures with the image library. ▮

Exercise 109. Design a world program that recognizes a pattern in a sequence of KeyEvents. Initially the program shows a 100 by 100 white rectangle. Once your program has encountered the first desired letter, it displays a yellow rectangle of the same size. After encountering the final letter, the color of the rectangle turns green. If any “bad” key event occurs, the program displays a red rectangle.

```
           conventional                 defined abbreviations
; ExpectsToSee.v1 is one of:        ; ExpectsToSee.v2 is one of:
; – "start, expect an 'a'"          ; – AA
; – "expect 'b', 'c', or 'd'"       ; – BB
; – "finished"                      ; – DD
; – "error, illegal key"            ; – ER

                                    (define AA "start, ...")
                                    (define BB "expect ...")
                                    (define DD "finished")
                                    (define ER "error, ...")
```

Figure 36: Two ways of writing a data definition for FSMs

The specific sequences that your program looks for start with "a", followed by an arbitrarily long mix of "b" and "c", and ended by a "d". Clearly, "acbd" is one example of an acceptable string; two others are "ad" and "abcbbbcd". Of course, "da", "aa", or "d" do not match.

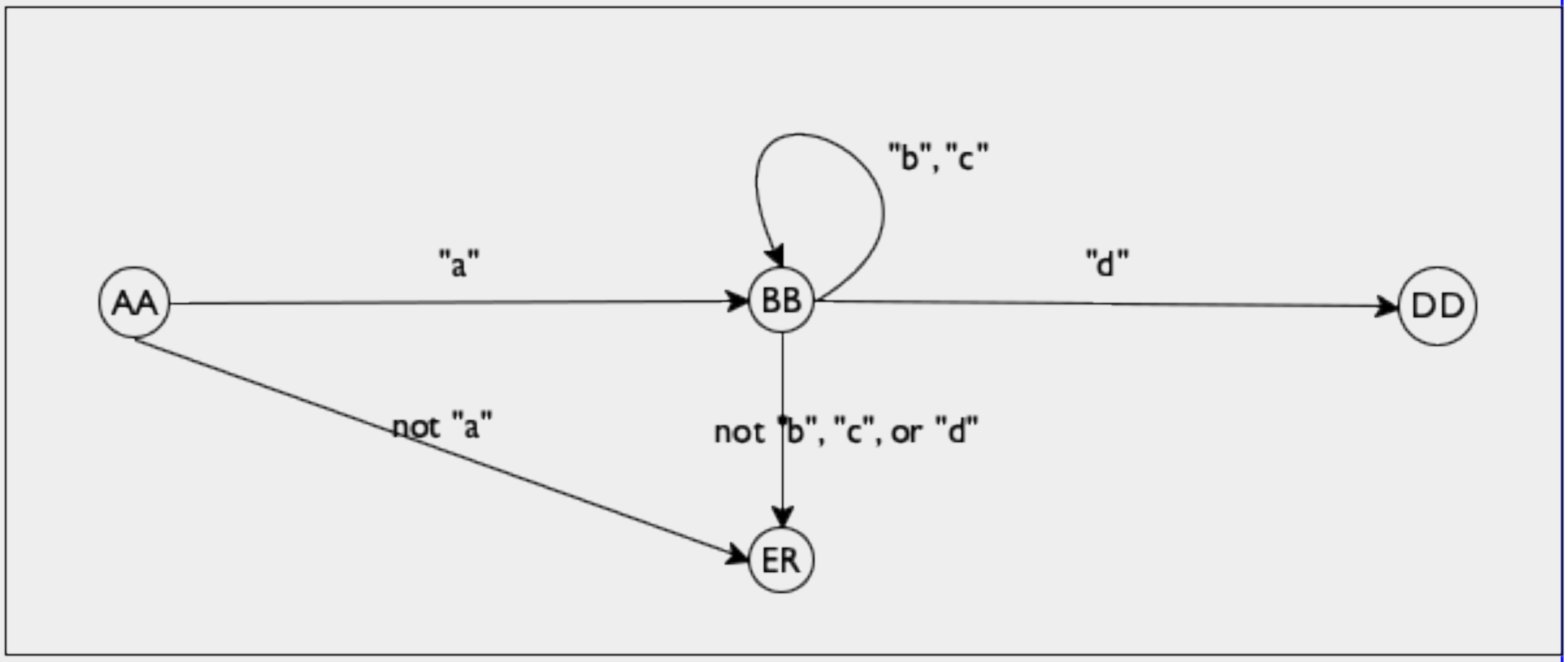


Figure 37: A finite state machine as a diagram

Hint Your solution implements a finite state machine (FSM), an idea introduced in Finite State Worlds as one design principle behind world programs. As the name says, an FSM program may be in one of a finite number of states. The first state is called an *initial state*. Each key event causes the machine to reconsider its current state; it may transition to the same state or to another one. When your program recognizes a proper sequence of key events, it transitions to a *final state*.

For a sequence-recognition problem, states typically represent the letters that the machine expects to see next; see figure 36 for a data definition. Take a look at the last state, which says an illegal input has been encountered. Figure 37 shows how to think of these states and their relationships in a diagrammatic manner. Each node corresponds to one of the four finite states; each arrow specifies which KeyEvent causes the program to transition from one state to another.

The data definition on the right uses the naming technique introduced in exercise 61.

History In the 1950s, Stephen C. Kleene, whom we would call a computer scientist, invented *regular expressions* as a notation for the problem of recognizing text patterns. For the above problem, Kleene would write

```
a (b|c)* d
```

which means `a` followed by `b` or `c` arbitrarily often until `d` is encountered. ▮

6.3 Input Errors

One central point of this chapter concerns the role of predicates. They are critical when you must design functions that process mixes of data. Such mixes come up naturally when your problem statement mentions many different kinds of information, but they also come up when you hand your functions and programs to others. After all, you know and respect your data definitions and function signatures. You never know, however, what your friends and colleagues do, and you especially don't know how someone without knowledge of BSL and programming uses your programs. This section therefore presents one way of protecting programs from inappropriate inputs.

It is a form of self-delusion to expect that we always respect our own function signatures. Calling a function on the wrong kind of data happens to the best of us. While many languages are like BSL and expect programmers to check signatures on their own, others do so automatically at the cost of some additional complexity.

Let's demonstrate this point with a simple program, a function for computing the area of a disk:

```
; Number -> Number
; computes the area of a disk with radius r
(define (area-of-disk r)
  (* 3.14 (* r r)))
```

Our friends may wish to use this function for their geometry homework. Unfortunately, when our friends use this function, they may accidentally apply it to a string rather than a number. When that happens, the function stops the program execution with a mysterious error message:

```
> (area-of-disk "my-disk")
*:expects a number as 1st argument, given "my-disk"
```

With predicates, you can prevent this kind of cryptic error message and signal an informative error of your own choice.

```
; Any BSL value is one of:
; – Number
; – Boolean
; – String
; – Image
; – (make-posn Any Any)
; ...
; – (make-tank Any Any)
; ...
```

Figure 38: The universe of BSL data

Specifically, we can define checked versions of our functions, when we wish to hand them to our friends. Because our friends may not know much BSL, we must expect that they apply this *checked function* to arbitrary BSL values: numbers, strings, images, Posns, and so on. Although we cannot anticipate which structure types will be defined in BSL, we know the rough shape of the data definition for the collection of all BSL values. Figure 38 displays this shape of this data definition. As discussed in The Universe of Data, the data definition for Any is open-ended because every structure type definition adds new instances. These instances may contain Any values again, which implies that the data definition of Any must refer to itself—a scary thought at first.

Based on this itemization, the template for a checked function has the following rough shape:

```
; Any -> ???
(define (checked-f v)
  (cond
    [(number? v) ...]
    [(boolean? v) ...]
    [(string? v) ...]
    [(image? v) ...]
    [(posn? v) (...(posn-x v) ... (posn-y v) ...)]
    ...
    ; which selectors are needed in the next clause?
    [(tank? v) ...]
    ...))
```

Of course, nobody can list all clauses of this definition; fortunately, that's not necessary. What we do know is that for all those values in the class of values for which the original function is defined, the checked version must produce the same results; for all others, it must signal an error.

Concretely, our sample function `checked-area-of-disk` consumes an arbitrary BSL value and uses `area-of-disk` to compute the area of a disk if the input is a number. It must stop with an error message otherwise; in BSL we use the function `error` to accomplish this. The error function consumes a string and stops the program:

```
(error "area-of-disk: number expected")
```

Hence the rough definition of `checked-area-of-disk` looks like this:

```
(define MESSAGE "area-of-disk: number expected")

(define (checked-area-of-disk v)
  (cond
    [(number? v) (area-of-disk v)]
    [(boolean? v) (error MESSAGE)]
    [(string? v) (error MESSAGE)]
    [(image? v) (error MESSAGE)]
    [(posn? v) (error MESSAGE)]
    ...
    [(tank? v) (error MESSAGE)]
    ...))
```

The use of `else` helps us finish this definition in the natural way:

```
; Any -> Number
; computes the area of a disk with radius v,
; if v is a number
(define (checked-area-of-disk v)
  (cond
    [(number? v) (area-of-disk v)]
    [else (error "area-of-disk: number expected")]))
```

And just to make sure we get what we want, let's experiment:

```
> (checked-area-of-disk "my-disk")
area-of-disk:number expected
```

Writing checked functions is important if we distribute our programs for others to use. Designing programs that work properly, however, is far more important. This book focuses on the design process for proper program design, and, to do this without distraction, we agree that we always adhere to data definitions and signatures. At least, we almost always do so, and on rare occasions we may ask you to design checked versions of a function or a program.

Exercise 110. A checked version of `area-of-disk` can also enforce that the arguments to the function are positive numbers, not just arbitrary numbers. Modify `checked-area-of-disk` in this way. ▮

Exercise 111. Take a look at these definitions:

```
(define-struct vec [x y])
; A vec is
;   (make-vec PositiveNumber PositiveNumber)
; interpretation represents a velocity vector
```

Develop the function `checked-make-vec`, which is to be understood as a checked version of the primitive operation `make-vec`. It ensures that the arguments to `make-vec` are positive numbers. In other words, `checked-make-vec` enforces our informal data definition. ▮

Predicates You might wonder how you can design your own predicates. After all, checked functions really seem to have this general shape:

```
; Any -> ...
; checks that a is a proper input for function g
(define (checked-g a)
  (cond
    [(XYZ? a) (g a)]
    [else (error "g: bad input")]))
```

where `g` itself is defined like this:

```
; XYZ -> ...
(define (g some-x) ...)
```

We assume that there is a data definition labeled *XYZ*, and that `(XYZ? a)` produces `#true` when `a` is an element of XYZ and `#false` otherwise.

For `area-of-disk`, which consumes Numbers, `number?` is clearly the appropriate predicate. In contrast, for some functions like `missile-render` from above, we clearly need to define our own predicate because MissileOrNot is a made-up, not a built-in, data collection. So let us design a predicate for MissileOrNot.

We recall the signature for predicates:

```
; Any -> Boolean
; is a an element of the MissileOrNot collection
(define (missile-or-not? a) #false)
```

It is a good practice to use questions as purpose statements for predicates, because applying a predicate is like asking a question about a value. The question mark "?" at the end of the name also reinforces this idea; some people may tack on "huh" to pronounce the name of such functions.

Making up examples is also straightforward:

```
(check-expect (missile-or-not? #false) #true)
(check-expect (missile-or-not? (make-posn 9 2)) #true)
(check-expect (missile-or-not? "yellow") #false)
```

The first two examples recall that every element of MissileOrNot is either `#false` or some Posn. The third test says that strings aren't elements of the collection. Here are three more tests:

```
(check-expect (missile-or-not? #true) #false)
(check-expect (missile-or-not? 10) #false)
(check-expect (missile-or-not? empty-image) #false)
```

Explain the expected answers!

Since predicates consume all possible BSL values, their templates are just like the templates for `checked-f`. Stop! Find the template and take a second look before you read on.

As with checked functions, a predicate doesn't need all possible `cond` lines. Only those that might produce `#true` are required:

```
(define (missile-or-not? v)
  (cond
    [(boolean? v) ...]
    [(posn? v) (... (posn-x v) ... (posn-y v) ...)]
    [else #false]))
```

All other cases are summarized via an `else` line that produces `#false`.

Given the template, the definition of `missile-or-not?` is a simple matter of thinking through each case:

```
(define (missile-or-not? v)
  (cond
    [(boolean? v) (boolean=? #false v)]
    [(posn? v) #true]
    [else #false]))
```

Only `#false` is a legitimate MissileOrNot; `#true` isn't. We express this idea with `(boolean=? #false v)`, but `(false? v)` would also do:

```
(define (missile-or-not? v)
  (cond
    [(false? v) #true]
    [(posn? v) #true]
    [else #false]))
```

Naturally all elements of `Posn` are also members of MissileOrNot, which explains the `#true` in the second line.

Exercise 112. Reformulate the predicate now using an `or` expression. ▌

Exercise 113. Design predicates for the following data definitions from the preceding section: SIGS, Coordinate (exercise 105), and VAnimal. ▌

To wrap up, let us mention `key-event?` and `mouse-event?` as two important predicates that you may wish to use in your world programs. They check the expected property, but you should check out their documentation to make sure you understand what they compute.

6.4 Checking the World

In a world program, many things can go wrong. Although we just agreed to trust that our functions are always applied to the proper kind of data, in a world program we may juggle too many things at once to place that much trust in ourselves. When we design a world program that takes care of clock ticks, mouse clicks, keystrokes, and rendering, it is just too easy to get one of those interplays wrong. Of course, going wrong doesn't mean that BSL recognizes the mistake immediately. For example, one of our functions may produce a result that isn't quite an element of your data representation for world states. At the same time, big-bang accepts this piece of data and holds on to it, until the next event takes place. It is only when the following event handler receives this inappropriate piece of data that the program may fail. But it may get worse because even the second and third and fourth event-handling step may actually cope with inappropriate state values, and it all blows up much later in the process.

To help with this kind of problem, big-bang comes with an optional check-with clause that accepts a predicate for world states. If, for example, we chose to represent all world states with Number, we could express this fact easily like this:

```
(define (main s0)
  (big-bang s0 ... [check-with number?] ...))
```

As soon as any event-handling function produces something other than a number, the world stops with an appropriate error message.

A check-with clause is even more useful when the data definition is not just a class of data with a built-in predicate like number? but something subtle such as this interval definition:

```
; A UnitWorld is a number
;   between 0 (inclusive) and 1 (exclusive).
```

In that case you want to formulate a predicate for this interval:

```
; Any -> Boolean
; is x between 0 (inclusive) and 1 (exclusive)

(check-expect (between-0-and-1? "a") #false)
(check-expect (between-0-and-1? 1.2) #false)
(check-expect (between-0-and-1? 0.2) #true)
(check-expect (between-0-and-1? 0.0) #true)
(check-expect (between-0-and-1? 1.0) #false)

(define (between-0-and-1? x)
  (and (number? x) (<= 0 x) (< x 1)))
```

With this predicate you can now monitor every single transition in your world program:

```
(define (main s0)
  (big-bang s0
            ...
            [check-with between-0-and-1?]
            ...))
```

If any of the world-producing handlers creates a number outside of the interval, or worse, a non-numeric-value, our program discovers this mistake immediately and gives us a chance to fix the mistake.

Exercise 114. Use the predicates from exercise 113 to check the space invader world program, the virtual pet program (exercise 106), and the editor program (A Graphical Editor). ▮

6.5 Equality Predicates

An *equality predicate* is a function that compares two elements of the same collection of data. Recall the definition of TrafficLight, which is the collection of three strings: "red", "green", and "yellow". Here is one way to define the light=? function:

```
; TrafficLight TrafficLight -> Boolean
; are the two (states of) traffic lights equal

(check-expect (light=? "red" "red") #true)
(check-expect (light=? "red" "green") #false)
(check-expect (light=? "green" "green") #true)
(check-expect (light=? "yellow" "yellow") #true)

(define (light=? a-value another-value)
  (string=? a-value another-value))
```

When we click *RUN*, all tests succeed, but unfortunately other interactions reveal conflicts with our intentions:

```
> (light=? "salad" "greens")
#false
> (light=? "beans" 10)
string=?:expects a string as 2nd argument, given 10
```

Compare these interactions with other, built-in equality predicates:

```
> (boolean=? "#true" 10)
boolean=?:expects a boolean as 1st argument, given "#true"
```

Try `(string=? 10 #true)` and `(= 20 "help")` on your own. All of them signal an error about being applied to the wrong kind of argument.

A checked version of `light=?` enforces that both arguments belong to TrafficLight; if not, it signals an error like those that built-in equality predicates issue. We call the predicate for `TrafficLight` `light?` for brevity:

The case of characters matters; `"red"` is different from `"Red"` or `"RED"`.

```
; Any -> Boolean
; is the given value an element of TrafficLight
(define (light? x)
  (cond
    [(string? x) (or (string=? "red" x)
                     (string=? "green" x)
                     (string=? "yellow" x))]
    [else #false]))
```

Now we can wrap up the revision of `light=?` by just following our original analysis. First, the function determines that the two inputs are elements of TrafficLight; if not it uses `error` to signal the mistake:

```
(define MESSAGE
  "traffic light expected, given some other value")

; Any Any -> Boolean
; are the two values elements of TrafficLight and,
; if so, are they equal

(check-expect (light=? "red" "red") #true)
(check-expect (light=? "red" "green") #false)
(check-expect (light=? "green" "green") #true)
(check-expect (light=? "yellow" "yellow") #true)

(define (light=? a-value another-value)
  (if (and (light? a-value) (light? another-value))
      (string=? a-value another-value)
      (error MESSAGE)))
```

Exercise 115. Revise `light=?` so that the error message specifies which of the two arguments isn't an element of TrafficLight. ▮

While it is unlikely that your programs will use `light=?`, they ought to use `key=?` and `mouse=?`, two equality predicates that we briefly mentioned at the end of the last subsection. Naturally, `key=?` is an operation for comparing two KeyEvents; similarly, `mouse=?` compares two MouseEvts. While both kinds of events are represented as strings, it is important to realize that not all strings represent key events or mouse events.

We recommend using `key=?` in key-event handlers and `mouse=?` in mouse-event handlers from now on. The use of `key=?` in a key-event handler ensures that the function really compares strings that represent key events and not arbitrary strings. As soon as, say, the function is accidentally applied to `"hello\n world"`, `key=?` signals an error and thus informs us that something is wrong.

7 Summary

In this first part of the book, you learned a bunch of simple but important lessons. Here is a summary:

1. A **good programmer** designs programs. A bad programmer tinkers until the program seems to work.

2. The **design recipe** has two dimensions. One concerns the process of design, that is, the sequence of steps to be taken. The other explains how the chosen data representation influences the design process.

3. Every well-designed program consists of many constant definitions, structure type definitions, data definitions, and function definitions. For **batch programs**, one function is the "main" function, and it typically composes several other functions to perform its computation. For **interactive programs**, the `big-bang` function plays the role of the main function; it specifies the initial state of the program, an image-producing output function, and at most three event handlers: one for clock ticks, one for mouse clicks, and one for key events. In both kinds of programs, function

definitions are presented "top down," starting with the main function, followed by those functions mentioned in the main function, and so on.

4. Like all programming languages, *Beginning Student Language* comes with **a vocabulary and a grammar**. Programmers must be able to determine the **meaning** of each sentence in a language so that they can anticipate how the program performs its computation when given an input. The following intermezzo explains this idea in detail.

5. Programming languages, including BSL, come with a rich set of libraries so that programmers don't have to reinvent the wheel all the time. A programmer should become comfortable with the functions that a library provides, especially their signatures and purpose statements. Doing so simplifies life.

6. A programmer must get to know the "tools" that a chosen programming language offers. These tools are either part of the language—such as `cond` or `max`—or they are "imported" from a library. In this spirit, make sure you understand the following terms: **structure type** definition, **function** definition, **constant** definition, **structure instance**, **data definition**, `big-bang`, and **event-handling function**.