Midterm Exam

注意：1. 所有題目至少提供程式碼與程式執行結果 (含中間過程)
2. 請使用 MATLAB 程式語言

1. Consider the function
$$f(x) = x^4 - 10x^3 + 40x^2 - 50x$$
Modify the codes of **golden section search** in the lecture notes to find the value of x that minimizes f over the range of [0, 2]. Locate this value of x to within the range of 0.005.

2. Modify the codes of **Newton's method** in the lecture notes to find the intersection of $y_1 = (x-1)^2 - 1$ and $y_2 = \cos(2x)$ in the interval $0 \le x \le 3$.
3. Modify the codes of **conjugate gradient method** in the lecture notes to find the minimizer of
$$f(x_1, x_2) = 4x_1^2 + x_1x_2 + 3x_2^2 + 2x_1 + x_2 + 1$$
4. Use the following code to generate data pairs (t_i, y_i):
t=0:0.25:10; y=3*sin(2t+0.5)+rand(1,21);
We wish to fit the data pairs with a sinusoid $y = A\sin(\omega t + \phi)$ with proper choices of A, ω, and ϕ. Modify the codes of **Levenberg-Marquardt algorithm** in the lecture notes to solve the nonlinear least square problem.
5. Consider the following constrained optimization problem:
$$\text{Minimize} \quad e^{x_1}(4x_1^2 + 4x_1x_2 + 2x_2^2 + 2x_2 + 1)$$
$$\text{Subject to} \quad x_1x_2 - x_1 - x_2 \le -1.5$$
$$-x_1x_2 - 10 \le 0$$
Using solver-based approach in MATLAB optimization toolbox and select **fmincon** slover, write code to find the minimizer of the problem. Supply the solver with analytic derivative/gradient information of both objective and constraint functions to improve the accuracy and efficiency of the results.
6. Consider the following linear program:
$$\text{Maximize} \quad x_1 + 2x_2$$
$$\text{Subject to} \quad \begin{aligned} -2x_1 + x_2 + x_3 &= 2 \\ -x_1 + 2x_2 + x_4 &= 7 \\ x_1 + x_5 &= 3 \\ x_i \ge 0,\ i &= 1,2,3,4,5 \end{aligned}$$
Using both problem-based and solver-based approaches in MATLAB optimization toolbox, wirite code to find the minimizer of the problem.