Homework #4

注意：沒有交代過程或過程不清將不予計分或扣分

1. Consider the problem

$$\text{Maximize} \quad \mathbf{x}^T Q \mathbf{x}$$

$$\text{Subject to} \quad \mathbf{x}^T P \mathbf{x} = 1$$

where $Q = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$. Use the Lagrange condition and the second-order conditions to derive the maximizer.

2. Consider the problem

$$\text{Minimize} \quad 2x_1 + 3x_2 - 4, \quad x_1, x_2 \in \mathbb{R}$$
$$\text{Subject to} \quad x_1 x_2 = 6$$

a. Use Lagrange's theorem to find all possible local minimizers and maximizes.

b. Use the second-order sufficient conditions to specify which points are strict local minimizers and which are strict local maximizers.

c. Are the points in part b global minimizers or maximizers? Explain.

3. Consider the problem

$$\text{Minimize} \quad x_2 - (x_1 - 2)^3 + 3$$

$$\text{Subject to} \quad x_2 \geq 1$$

where x_1 and x_2 are real variables. Answer each of the following questions making sure that you give complete reasoning for your answers.

a. Write down the KKT condition for the problem, and find all points that satisfy the condition. Check whether or not each point is regular.

b. Determine whether or not the point(s) in part a satisfy the second-order necessary condition.

c. Determine whether or not the point(s) in part b satisfy the second-order sufficient condition.

4. Consider the following problem

$$\text{Minimize} \quad x_1 x_2$$

$$\text{Subject to} \quad \begin{array}{l} x_1 + x_2 \geq 3 \\ x_2 \geq x_1 \end{array}$$

a. Write down the KKT condition for the problem, and find all points that satisfy the condition. Check whether or not each point is regular.

b. Determine whether or not the point(s) in part a satisfy the second-order necessary condition.

c. Determine whether or not the point(s) in part b satisfy the second-order sufficient condition.

5. Consider the following problem

$$\text{Minimize } (x_1-2)^2+x_2-2$$

$$\text{Subject to } \begin{array}{l} x_2-x_1=2 \\ x_1+x_2\le 2 \end{array}$$

a. Write down the KKT condition for the problem, and find all points that satisfy the condition. Check whether or not each point is regular.

b. Determine whether or not the point(s) in part a satisfy the second-order necessary condition.

c. Determine whether or not the point(s) in part b satisfy the second-order sufficient condition.

6. Utilize a projected gradient algorithm to iteratively search the minimizer of the following optimization problem:

$$\text{Minimize } \frac{1}{2}(x_1^2+x_2^2)$$

$$\text{Subject to } x_1+x_2=1$$

Use fixed step size $\alpha_k=0.5$ and initial point $\mathbf{x}^{(0)}=\begin{bmatrix}1 & 0\end{bmatrix}^T$. Perform four iterations to arrive at $\mathbf{x}^{(4)}$.

7. Consider the problem

$$\text{Minimize } \frac{1}{2}(x_1^2+2x_2^2)$$

$$\text{Subject to } x_1^2+x_2^2=1$$

Apply a fixed-step-size projected gradient algorithm to this problem by **properly** select an initial point $\mathbf{x}^{(0)}$, step size α, and projection operator. Perform four iterations to arrive at $\mathbf{x}^{(4)}$.