Theta Quadrant Lateral Line to Display Intersection Calculation

The vertical (Z-axis) intersections are straightforward, requiring only a weighted average calculation. This document focuses on drawing a straight line across the curved path of the display.

To find the intersection of the display and the line connecting two rectangular points, (x_{start}, y_{start}) and (x_{end}, y_{end}), I found parametric line equarions for each. Here are the parametric equations:

Line Segment:

$x = x_{\text{start}} + (x_{\text{end}} - x_{\text{start}}) * t$

$y = y_{\text{start}} + (y_{\text{end}} - y_{\text{start}}) * t$

Display:

$x = \text{Cos}[\theta] * s$

$y = \text{Sin}[\theta] * s$

Setting x = x and solving for s:

$$\text{Solve}[x_{\text{start}} + (x_{\text{end}} - x_{\text{start}}) * t == \text{Cos}[\theta] * s, s]$$

$$\{\{s \to \text{Sec}[\theta]\ (t\, x_{\text{end}} + x_{\text{start}} - t\, x_{\text{start}})\}\}$$

Now that I have found unknown s, I can set y=y.

$$y_{\text{start}} + (y_{\text{end}} - y_{\text{start}}) * t == \text{Sin}[\theta] * s$$

I can insert s into this equation, then solve for t.

$$\text{Solve}[y_{\text{start}} + (y_{\text{end}} - y_{\text{start}}) * t == \text{Sin}[\theta] * \text{Sec}[\theta]\ (t\, x_{\text{end}} + x_{\text{start}} - t\, x_{\text{start}}), t]$$

$$\left\{\left\{t \to \frac{y_{\text{start}} - x_{\text{start}} \text{Tan}[\theta]}{-y_{\text{end}} + y_{\text{start}} + x_{\text{end}} \text{Tan}[\theta] - x_{\text{start}} \text{Tan}[\theta]}\right\}\right\}$$

Inserting t into the x parametric equation for the line segment, I can solve for x.

$$\text{FullSimplify}\left[x == x_{\text{start}} + (x_{\text{end}} - x_{\text{start}}) * \frac{y_{\text{start}} - x_{\text{start}} \text{Tan}[\theta]}{-y_{\text{end}} + y_{\text{start}} + x_{\text{end}} \text{Tan}[\theta] - x_{\text{start}} \text{Tan}[\theta]}\right]$$

$$x == \frac{x_{\text{start}}\, y_{\text{end}} - x_{\text{end}}\, y_{\text{start}}}{y_{\text{end}} - y_{\text{start}} + (-x_{\text{end}} + x_{\text{start}}) \text{Tan}[\theta]}$$

Now that I have the x value of the intersection of the display and the line segment, all I have to do is convert that to polar form to get r. I don't need the y value! The conversion equation is $r = \frac{x}{\text{Cos}[\theta]}$. x will be plugged into this equation.

$$\text{FullSimplify}\left[r == \frac{\frac{x_{\text{start}}\, y_{\text{end}} - x_{\text{end}}\, y_{\text{start}}}{y_{\text{end}} - y_{\text{start}} + (-x_{\text{end}} + x_{\text{start}}) \text{Tan}[\theta]}}{\text{Cos}[\theta]}\right]$$

$$r == \frac{\text{Sec}[\theta]\ (x_{\text{start}}\, y_{\text{end}} - x_{\text{end}}\, y_{\text{start}})}{y_{\text{end}} - y_{\text{start}} + (-x_{\text{end}} + x_{\text{start}}) \text{Tan}[\theta]}$$

I also wanted to find an equation for if endpoints are stored in polar form. This was done very easily with a rectangular to polar conversion on the previous equation.

$$x_{start} := r_{start}\,\mathrm{Cos}[\theta_{start}]$$
$$x_{end} := r_{end}\,\mathrm{Cos}[\theta_{end}]$$
$$y_{start} := r_{start}\,\mathrm{Sin}[\theta_{start}]$$
$$y_{end} := r_{end}\,\mathrm{Sin}[\theta_{end}]$$
$$\mathrm{FullSimplify}\left[r == \frac{\mathrm{Sec}[\theta]\,(x_{start}\,y_{end} - x_{end}\,y_{start})}{y_{end} - y_{start} + (-x_{end} + x_{start})\,\mathrm{Tan}[\theta]}\right]$$

$$r == \frac{\mathrm{Sin}[\theta_{end} - \theta_{start}]\,r_{end}\,r_{start}}{-\mathrm{Sin}[\theta - \theta_{end}]\,r_{end} + \mathrm{Sin}[\theta - \theta_{start}]\,r_{start}}$$

The rectangular form requires Sec[*θ*] and Tan[*θ*] lookup tables, whereas the polar form requires only Sin[*θ*]. But in general, they are of about equal complexity.