Theta Quadrant r & Z Calculation

Equation of the display plane, with respect to θ.

$$\vec{n} == [\mathrm{Sin}[\theta], \mathrm{Cos}[\theta], 0]$$
$$p_0 == (0, 0, 0)$$

$$x\,\mathrm{Sin}[\theta] + y\,\mathrm{Cos}[\theta] == 0$$

Parametric equations of the 3D line in rectangular coordinates.

$$x == x_{start} + (x_{end} - x_{start})\, t$$
$$y == y_{start} + (y_{end} - y_{start})\, t$$
$$z == z_{start} + (z_{end} - z_{start})\, t$$

Combining the plane and line equations together, solving for t.

$$\mathrm{Solve}[(x_{start} + (x_{end} - x_{start})\, t)\, \mathrm{Sin}[\theta] + (y_{start} + (y_{end} - y_{start})\, t)\, \mathrm{Cos}[\theta] == 0,\ t]$$

$$\left\{\left\{t \to \frac{-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start}}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}\right\}\right\}$$

Inserting t to find point (x,y,z) at θ.

$$x == x_{start} + (x_{end} - x_{start}) * \frac{-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start}}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}$$

$$y == y_{start} + (y_{end} - y_{start}) * \frac{-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start}}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}$$

$$z == z_{start} + (z_{end} - z_{start}) * \frac{-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start}}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}$$

$$x == x_{start} + \frac{(x_{end} - x_{start})\,(-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start})}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}$$

$$y == y_{start} + \frac{(y_{end} - y_{start})\,(-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start})}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}}$$

$$z == \frac{(-\mathrm{Sin}[\theta]\, x_{start} - \mathrm{Cos}[\theta]\, y_{start})\,(z_{end} - z_{start})}{\mathrm{Sin}[\theta]\, x_{end} - \mathrm{Sin}[\theta]\, x_{start} + \mathrm{Cos}[\theta]\, y_{end} - \mathrm{Cos}[\theta]\, y_{start}} + z_{start}$$

Switching to cylindrical system.
Input: x = r Cos[θ]. y = r Sin[θ]. z = z.
Output: $r = \frac{y}{\text{Sin}[\theta]}$**. z = z.**

$$x_{start} := r_{start}\,\text{Cos}[\theta_{start}]$$
$$x_{end} := r_{end}\,\text{Cos}[\theta_{end}]$$
$$y_{start} := r_{start}\,\text{Sin}[\theta_{start}]$$
$$y_{end} := r_{end}\,\text{Sin}[\theta_{end}]$$
$$z_{start} := z_{start}$$
$$z_{end} := z_{end}$$

$$\text{FullSimplify}\left[r == \frac{y_{start} + \frac{(y_{end}-y_{start})\left(-\text{Sin}[\theta]\,x_{start}-\text{Cos}[\theta]\,y_{start}\right)}{\text{Sin}[\theta]\,x_{end}-\text{Sin}[\theta]\,x_{start}+\text{Cos}[\theta]\,y_{end}-\text{Cos}[\theta]\,y_{start}}}{\text{Sin}[\theta]}\right]$$

$$\text{FullSimplify}\left[z == \frac{(\text{Sin}[\theta]\,x_{start} + \text{Cos}[\theta]\,y_{start})\,(z_{end} - z_{start})}{\text{Sin}[\theta]\,(-x_{end} + x_{start}) + \text{Cos}[\theta]\,(-y_{end} + y_{start})} + z_{start}\right]$$

$$r == -\frac{\text{Sin}[\theta_{end} - \theta_{start}]\,r_{end}\,r_{start}}{\text{Sin}[\theta + \theta_{end}]\,r_{end} - \text{Sin}[\theta + \theta_{start}]\,r_{start}}$$

$$z + \frac{\text{Sin}[\theta + \theta_{end}]\,r_{end}\,(z_{end} - z_{start})}{\text{Sin}[\theta + \theta_{end}]\,r_{end} - \text{Sin}[\theta + \theta_{start}]\,r_{start}} == z_{end}$$

Notice the exclusive use of the sine function.

Extracting parts that are independent of θ and thus only need to be calculated and stored when the program is first run.

$$\text{NUM} := \text{Sin}[\theta_{end} - \theta_{start}]\,r_{end}\,r_{start}$$
$$\Delta Z := z_{end} - z_{start}$$

Extracting parts that are used multiple times during on-the-fly calculation. Trig modification to ensure 0° ≤ a ≤ 360° in Sin[a].

$$\text{V1} := \text{Sin}[\theta - \theta_{end} + 360\,°]\,r_{end}$$
$$\text{V2} := \text{Sin}[\theta - \theta_{start}]\,r_{start} - \text{V1}$$

Using these new variables to find r and z.

$$r == \frac{\text{NUM}}{\text{V2}}$$

$$z == z_{end} + \frac{\text{V1} * \Delta Z}{\text{V2}}$$

$$r == \frac{\text{Sin}[\theta_{end} - \theta_{start}]\,r_{end}\,r_{start}}{-\text{Sin}[\theta - \theta_{end}]\,r_{end} + \text{Sin}[\theta - \theta_{start}]\,r_{start}}$$

$$z == z_{end} + \frac{\text{Sin}[\theta - \theta_{end}]\,r_{end}\,(z_{end} - z_{start})}{-\text{Sin}[\theta - \theta_{end}]\,r_{end} + \text{Sin}[\theta - \theta_{start}]\,r_{start}}$$