Midterm Review

Logic I
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Derivable Schemata

Contraposition: $\varphi \supset \psi \vdash \neg\psi \supset \neg\varphi$.

Hypothetical Syllogism: $\varphi \supset \psi, \psi \supset \chi \vdash \varphi \supset \chi$.

Disjunctive Syllogism: $\varphi \vee \psi, \neg\varphi \vdash \psi$.

∨-Conditional: $\varphi \supset \psi \dashv\vdash \neg\varphi \vee \psi$.

¬-Conditional: $\neg(\varphi \supset \psi) \dashv\vdash \varphi \wedge \neg\psi$.

Conditional Weakening: $\psi \vdash \varphi \supset \psi$.

Double Negation: $\neg\neg\varphi \dashv\vdash \varphi$.

∧-De Morgan's: $\neg(\varphi \wedge \psi) \dashv\vdash \neg\varphi \vee \neg\psi$.

∨-De Morgan's: $\neg(\varphi \vee \psi) \dashv\vdash \neg\varphi \wedge \neg\psi$.

Modus Tollens: $\varphi \supset \psi, \neg\psi \vdash \neg\varphi$.

Regimentation

Complete the following tasks for arguments (A) and (B):

Task 1: Write a symbolization key and regiment the argument.

Task 2: Determine if the argument is valid.

Task 3: Provide an SD proof if valid, and a countermodel otherwise.

(A) If Dorothy plays the piano in the morning, then Roger wakes up cranky. Dorothy plays piano in the morning unless she is distracted. So if Roger does not wake up cranky, then Dorothy must be distracted.

(B) If Cam remembered to do his chores, then things are clean but not neat. Cam forgot only if things are neat but not clean. Therefore, things are clean just in case they are not neat.

Lemmas

Lemma 1: Every satisfiable branch $\mathcal{B}$ in an SL tree X is open.

Lemma 2: If X is an SL tree with a satisfiable branch $\mathcal{B}$, then any tree X' which is the result of resolving a sentence in $\mathcal{B}$ has a satisfiable branch $\mathcal{B}'$.

Lemma 3: Every SL tree with a satisfiable root has a satisfiable branch.

Lemma 4: Every SL tree X has a finite number of branches.

Lemma 5: For any SL tree X with root Γ and $\varphi \in [X]$, there is an SL tree Y with root Γ where $\texttt{Res}(Y) < \texttt{Res}(X)$.

Lemma 6: For any tree X with root Γ, there is a complete tree X' with root Γ.

Lemma 7: Every complete open branch in an SL tree is satisfiable.

Soundness

1. Assume Γ is satisfiable.
2. Let X be an SL tree with root Γ.
3. So X has a satisfiable branch $\mathcal{B}$ by *Lemma 3*.
4. So $\mathcal{B}$ is open by *Lemma 1*.
5. So X is not closed.
6. More generally, there is no closed SL tree with root Γ.
7. By contraposition, QED.

Completeness

1. Assume there is no closed tree with root Γ.
2. Roots are trees, and so Γ has a complete tree X by Lemma 6.
3. So X is a complete open tree with a complete open branch $\mathcal{B}$.
4. By *Lemma 7*, $\mathcal{B}$ is satisfiable, and so Γ is satisfiable.
5. By contraposition, if $\Gamma \vDash \bot$, then $\Gamma \vdash \bot$.