0. What is logic?

But first: What is this class?

See the syllabus!!! See Canvas page! Enroll in **Carnap** page!

- Download textbook: *forallX Fall 2022 MIT edition* (updated throughout semester, but hopefully not too drastically!)
- Problem Sets typically due Fridays 5pm
- Most problem sets submitted online via **Carnap**
- Office hours could change based on when various reading groups are scheduled
- Located on 9th floor, Dreyfoos Wing; turn RIGHT after entering Phil dept

And second: Why come to class?

- We’ll cover the essential content, in a digestible manner (helpful if you find it hard to make time to read)
- We’ll do practice problems! Often geared toward homework and exam questions
- You might gain some ‘insider information’

- This is a **skills-based** course:
  the goal is to learn how to **do** logic, rather than to memorize or regurgitate facts about logic.
- Doing ample practice problems will be **essential**!

Names and Quick Intros

- Ideally you'll feel completely comfortable in this class
- It'll probably be easier to focus if you feel comfortable
- I find it helps to say some words to feel comfortable!
- Let's say some words!

0. What is logic?

a. Arguments and validity

Causes vs. (normative) Reasons for belief

- **Causes of belief**: why you actually believe something
  - Attacked by cat at age 8: causes you to believe ‘cats are dangerous’
  - These are *descriptive* reasons for belief
  - Often these are not rationally compelling reasons
- **Reasons for belief**: why you *ought* to believe something
  - Empirical data on cat attacks per capita
  - These are *normative* reasons for belief
  - Logic aims to characterize the structure of such reasons, when organized into *arguments*

Rhetorically vs. Logically Good Arguments

- **Rhetorically good argument**: argument that actually persuades listeners
  - Might be *descriptively good*, while being logically awful
  - Might rely solely on emotional tricks or rhetoric
  - Anecdotes: “my friend got a blood clot from the Johnson & Johnson Covid vaccine. Therefore, avoid this vaccine!”
  - A plane recently crashed: I better drive!
- **Logically good arguments**: arguments that *ought* to persuade listeners, if they were rational
  - Such arguments might be descriptively pretty unpersuasive!
  - Comparative analysis of risk of blood clots from Janssen vaccine vs. risk of negative Covid health outcome
  - Comparative analysis of plane crashes vs. car crashes

An easy puzzle

Where does Sanjeev live?

Sanjeev lives in Chicago or in Erie.
Sanjeev doesn’t live in Erie.

A: Obviously, in Chicago.

Arguments and sentences

Argument 1

Sanjeev lives in Chicago or in Erie.
Sanjeev doesn't live in Erie.
Therefore, Sanjeev lives in Chicago.

- Such an argument consists of (declarative) **sentences**.
- Declarative sentences are the kinds that can be **true** or **false**.
- "Therefore" ($\therefore$) indicates that the last sentence (supposedly) **follows from** the first two.
- The last sentence is called the **conclusion**.
- The others are called the **premises**.

Valid and invalid arguments

Argument 2

Mandy enjoys skiing or hiking (or both).
Mandy doesn't enjoy hiking.
∴ Mandy enjoys skiing.

Argument 3

Mandy enjoys skiing or hiking (or both).
Mandy enjoys skiing.
∴ Mandy doesn't enjoy hiking.

What's the difference?

(Deductive) Validity

Definition

An argument is (deductively) **valid** if there is no case where all its premises are true and the conclusion is false.

Definition

An argument is **invalid** if there is at least one case where all its premises are true and the conclusion is false (i.e., if it is not valid).

Definition

A **case** is some hypothetical scenario that makes each sentence in an argument either true or false.

Argument 2 is valid

Argument 2

Mandy enjoys skiing or hiking.
Mandy doesn't enjoy hiking.
$\therefore$ Mandy enjoys skiing.

Argument 2 is **valid**: whenever the premises are true, the conclusion is also true.

Argument 3 is not valid

Argument 3

Mandy enjoys skiing or hiking.
Mandy enjoys skiing.
∴ Mandy doesn't enjoy hiking.

Argument 3 is **invalid**: there is a possible case where the premises are true and the conclusion isn't (Mandy enjoys both skiing and hiking).

A harder puzzle

Where does Sarah live?

Sarah lives in Chicago or Erie.

Amir lives in Chicago unless he enjoys hiking.

If Amir lives in Chicago, Sarah doesn’t.

Neither Sarah nor Amir enjoy hiking.

0. What is logic?

b. Cases and determining validity

Validity

Definition

An argument is **valid** if there is no case where all its premises are true and the conclusion is false.

Definition

An argument is **invalid** if there is at least one case where all its premises are true and the conclusion is false (i.e., if it is not valid).

Cases

Definition

A **case** is some hypothetical scenario that makes each sentence in an argument either true or false.

- E.g., imagine you have a friend, her name is Mandy, she loves hiking but hates skiing.
- That's a case where "Mandy enjoys hiking or skiing" is true.
- Some cases can be imagined even though they never happen IRL, e.g, "It is raining and the skies are clear."
- Some things you can't imagine, e.g., "There is a blizzard but there is no wind."

Determining validity

- Imagine a case where the conclusion is false.
- Are the premises true? You’re done: invalid.
- Otherwise, change or expand the case to make them true (without making the conclusion also true).
- Can’t? (Probably) valid.

OR

- Imagine a case where all premises are true.
- Is the conclusion false? You’re done: invalid.
- Otherwise, change or expand the case to make it false (without making the premises false).
- Can’t? (Probably) valid.

Deductively Valid?

Some rodents have bushy tails.
All squirrels are rodents.
∴ Some squirrels have bushy tails.

- Imagine squirrels evolving so that they no longer have bushy tails. Then conclusion is false.
- But premises still true:
  - Imagine chinchillas still have bushy tails.
  - Imagine also that squirrels have not evolved too much—they are still rodents.

Valid?

All rodents have bushy tails.
All squirrels are rodents.
∴ All squirrels have bushy tails.

- If it were invalid, you'd have a case that makes the conclusion false: some squirrels without bushy tails.
- They would have to be rodents still (otherwise premise 2 false).
- And that would require that they have bushy tails (otherwise premise 1 false).

0. What is logic?

c. Other logical notions

Logical Consistency

Definition

Sentences are (logically) **consistent** if there is a case where they are all true.

- also called ‘jointly possible’ or ‘satisfiable’

Definition

Sentences are (logically) **inconsistent** if there is no case where they are all true.

- also called ‘jointly impossible’ or ‘unsatisfiable’

Consistent?

Some carnivores have bushy tails.
All carnivores are mammals.
No mammals have bushy tails.

- No case makes them all true at the same time, so **inconsistent**.

Valid?

Some carnivores have bushy tails.
All carnivores are mammals.
No mammals have bushy tails.
∴ All birds are carnivores.

- The premises cannot all be true in the same case, so inconsistent.
- So: no case makes all the premises true.
- So also: no case makes the premises true and the conclusion false.
- **Arguments with inconsistent premises are automatically valid**, regardless of what the conclusion is.

Tautology (logically necessary)

Definition

A sentence is a **tautology** if there is no case where it is false.

- also called a 'necessary truth' or 'truth-functionally true'

- If it's snowing, then it's snowing.
- Every fawn is a deer.
- The number 5 is prime.
- It's not the case that I am standing and that I am not standing.
  ('*Law of Non-Contradiction*')

Tautology

What can you say about an argument where the conclusion is a tautology?

- If the conclusion is a tautology, there is no case where it is false.
- So there is no case where both (1) it is false and (2) the premises of the argument are all true.
- $\Rightarrow$ **Arguments with tautologies as conclusions are automatically valid**, regardless of what the premises are.

Logical equivalence

Definition

Two sentences are (logically) **equivalent** if there is no case where one is true and the other is false.

- What can you say about an argument where one of the premises is equivalent to the conclusion? Is it automatically valid?
- Can you have two equivalent sentences that are inconsistent?

0. What is logic?

d. What are we going to learn? And why?

What is logic?

- **Logic is the science of what follows from what.**
- Sometimes a conclusion follows from the premises, sometimes it does not:
  - Mandy lives in Chicago.
    Everyone who lives in Chicago likes hiking.
    $\therefore$ Mandy likes hiking.
  - Mandy lives in Chicago.
    Everyone who likes hiking lives in Chicago.
    $\therefore$ Mandy likes hiking.
- Logic investigates what makes the first argument **valid** and the second **invalid**.

What is formal logic?

- Studies logical properties of **formal languages** (SL and QL, not English).
  - Logical consequence (what follows from what?)
  - Logical consistency (when do sentences contradict one another?)
- Expressive power (what can be expressed in a given formal language, and how?)
- Formal models (mathematical structures described by formal language)
- Inference and proof systems (how can it be proved that something follows from something else?)
- Meta-logical properties of logical systems

Plan for the course

- Sentential Logic (SL), aka Truth-functional logic
  - Symbolization in the formal language of SL (H, $\vee$, &, $\supset$, $\sim$)
  - Testing for validity: truth-tables and TREES!
  - Proofs in natural deduction
- Quantifier Logic (QL), aka First-order logic
  - More fine-grained symbolization ($E(m, h)$, $\forall$ ‘every’, $\exists$ ‘some’, $=$)
  - Semantics: interpretations
  - Proofs in natural deduction
- Metalogic (sprinkled in Unit 1 as well!):
  - Soundness & Completeness of our deduction systems
  - Compactness
  - Expressive completeness, normal forms

What is logic good for? (Philosophy)

- Logic originates in philosophy (e.g. Aristotle); traditionally considered a sub-discipline of philosophy.
- Valid arguments are critical in philosophical research.
- Formal tools of logic are useful for making various philosophical notions precise, e.g.,
  - Possibility and necessity
  - Time
  - Composition and parthood (mereology)
  - Moral obligation and permissibility
  - Belief and knowledge
- Logic applies to the semantics of natural language (philosophy of language, linguistics).

What is logic good for? (Mathematics)

- Formal logic was developed largely in a quest for the foundations of mathematics (19th century).
- Logical systems provide precise foundational framework for mathematics:
  - Axiomatic systems (e.g, geometry)
  - Algebraic structures (e.g., groups)
  - Set theory (e.g, Zermelo–Fraenkel with Choice)
- Precision
  - Formal language makes claims more precise.
  - Formal structures can point to alternatives, unveil gaps in proofs.
  - Formal proof systems make proofs rigorous.
  - Formal proofs make mechanical **proof checking** and **proof search** possible.

What is logic good for? (Computer Science)

- Computer science deals with lots of formal languages.
- Logic is a good example of how to set up and use formal languages.
- ‘Logic : Computer Science’ = ‘Calculus : Natural Science’
- Applications of logical systems in CS are numerous:
  - Combinational logic circuits
  - Database query languages
  - Logic programming
  - Knowledge representation
  - Automated reasoning
  - Formal specification and verification (of programs, of hardware designs)
  - Theoretical computer science (theory of computational complexity, semantics of programming languages)

0. What is logic?

e. Symbolization and SL

Validity in virtue of form

Argument 1

Sanjeev lives in Chicago or Erie.
Sanjeev doesn’t live in Erie.
$\therefore$ Sanjeev lives in Chicago.

Argument 2

Mandy enjoys skiing or hiking.
Mandy doesn’t enjoy hiking.
$\therefore$ Mandy enjoys skiing.

Form of arguments 1 & 2

X or Y.
Not Y.
$\therefore X$.

Some valid argument forms

Disjunctive syllogism

X or Y.

Not Y.

$\therefore\ X$.

Modus ponens

If X then Y.

X.

$\therefore\ Y$.

Hypothetical syllogism

If X then Y.

If Y then Z.

$\therefore$ If X then Z.

Symbolizing arguments

Symbolization key

S: Mandy enjoys skiing
H: Mandy enjoys hiking

Argument 2

Mandy enjoys skiing or Mandy enjoys hiking.	$(S \vee H)$
Not: Mandy enjoy hiking.	$\sim H$
$\therefore$ Mandy enjoys skiing.	$\therefore S$

The language of SL

- CAPITAL **Sentence letters**, such as 'H' and 'S', to symbolize *atomic sentences* (e.g. 'Mandy likes hiking')
- **Connectives**, to indicate how atomic sentences are connected

$\vee$	either ... or ...	['inclusive or']
&	both ... and ...	
$\supset$	if ... then ...	['material conditional']
$\sim$	not ...	[it is not the case that]

This can get complicated, e.g.:

"Mandy enjoys skiing or hiking, and if she lives in Erie, she doesn't enjoy both."

$$((S \vee H) \mathbin{\&} (E \supset {\sim}(S \mathbin{\&} H)))$$

0. What is logic?

f. Bonus: Some History of Logic!

The beginnings

"The Philosopher"

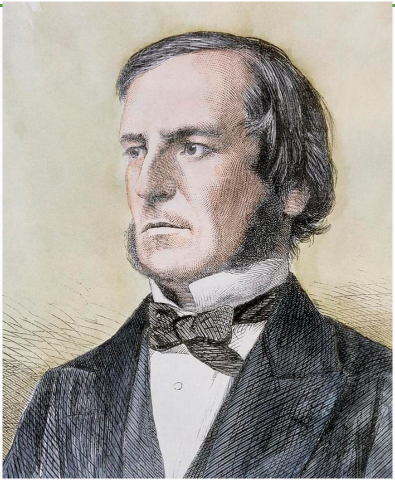
- Rules of debate & rhetoric
- Ancient India: Gautama, **Nyāya Sūtras** (600 BCE–200 CE)
- Ancient Greece: Aristotle (384-322 BCE)
- Cataloged valid arguments ("syllogisms"), e.g.,
- All ungulates have hooves.
  No fish have hooves.
  ∴ No fish are ungulates.

The middle ages

Avicenna!

- Ibn Sīnā (Avicenna) (980-1037): worked on like *everything*
- Pierre Abelard (1079-1142)
- William Ockham (1285-1347)
- Jean Buridan (1301-1358)

Mathematical logic



Boole!

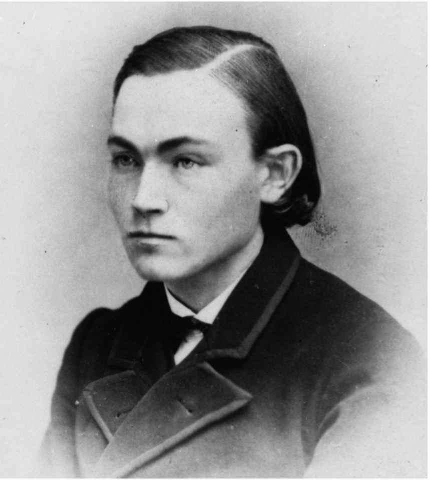
- Augustus De Morgan (1806-1871)
- George Boole (1815-1864) [self–taught, and so can you!]
- Charles Lutwidge Dodgson (aka Lewis Caroll) (1832-1898) [thank him for the **trees!**]
- John Venn (1834-1923)

Modern logic: Peirce at al

Ladd Franklin!

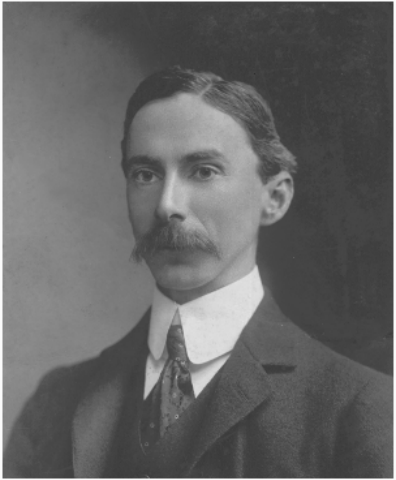
- Charles Sanders Peirce (1839-1914): a Cambridge native!
- Christine Ladd Franklin (1847-1930)
- Ernst Schröder (1841-1902)

Modern logic: Gottlob Frege



- 1848-1925
- Predicates and quantifiers
- Plan to turn all of math into theorems of logic alone
- Did Frege plagarize ideas from the Stoics???

Modern logic: Bertrand Russell



- 1870-1972
- Showed Frege's system contradictory (1902)
- Fixed it (*Principia mathematica* 1910-13, 3 vols.)
- Plan to turn all of math into theorems of logic alone

Modern logic: David Hilbert

- 1862-1943
- Combined Russell's and Schröder's systems
- First modern logic textbook
- Plan to turn all of math into consequences of a single set of premises

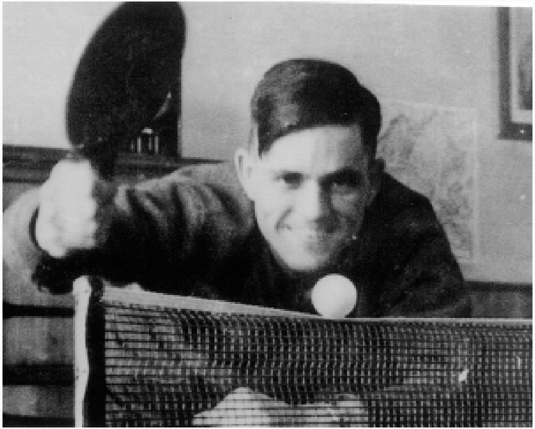
Modern logic: Kurt Gödel

- 1906-1978
- Showed that every valid argument has a proof
- Showed that Frege/Russell's and Hilbert's plans can't work

Modern logic: Alan Turing

- 1912–1954
- Showed that unlike SL, QL has no decision procedure
- Invented Turing machines ("father of computer science")

Modern logic: Gerhard Gentzen



Ping pong let's GOOOOOOO!

- 1909-1945
- Invented natural deduction
- Founded theory of proofs

Modern logic: modal logic

Barcan Marcus!

- Extend logic with operators for "possible" and "necessary"
- Pioneered by philosophers, now used by computer scientists
- Rudolf Carnap, Saul Kripke, Ruth Barcan Marcus