What is Logic?

Logic I
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Definitions

Proposition:	A PROPOSITION is a way for things to be which is either true or false.
Declarative Sentence:	A DECLARATIVE SENTENCE is a grammatical string of symbols which, on an interpretation, expresses a proposition that is either true or false.
Argument:	An ARGUMENT is a finite sequence of declarative sentences where the final sentence is the CONCLUSION and the preceding sentences are the PREMISES.

Examples

Snow

(1) It's snowing.
$\therefore$ John drove to work.

This argument may be compelling, but not certain.

Red

(1) The ball is crimson.
$\therefore$ The ball is red.

This argument provides certainty, but not on all interpretations.

Museum

(1) Kate is either at home or at the Museum.
(2) Kate is not at home.
$\therefore$ Kate is at the Museum.

This argument's certainty is independent of the interpretation.

Informal Validity

Question 1: What goes wrong if we assume the premises but deny the conclusion in *Red* and *Museum*?

Answer: Nature of 'crimson' and 'red' *vs.* meaning of 'or' and 'not'.

Informal Semantics: Give informal semantics for disjunction and negation.

Complex Sentences: Observe that complex sentences in *Museum* are composed from simpler sentences *via* 'not' and 'or'.

Atomic Sentences: A declarative sentence is ATOMIC just in case it is not composed of simpler declarative sentences.

Task 1: Identify atomic sentences in *Museum*.

Informal Interpretation: Let an INFORMAL INTERPRETATION assign every atomic sentence of English to exactly one TRUTH-VALUE 1 or 0.

Informal Validity: An argument in English is INFORMALLY VALID just in case its conclusion is true in every informal interpretation in which its premises are true.

Task 2: Use semantics to show that *Museum* is informally valid.

Formal Languages

Problem 1: There is no set of all atomic sentences of English, and so no clear notion of an informal interpretation of English.

Suggestion: Could choose some large set of atomic English sentences, but this would be arbitrary and hard to specify.

Solution 1: We will *regiment* English arguments in a formal language that is both general and easy to specify precisely.

Sentential Logic: The SENTENCES of SL are composed of sentence letters $A, B, C, \ldots$ and sentential operators $\neg, \vee, \wedge, \supset$, and $\equiv$.

Task 3: Regiment *Museum* in SL: $A \vee B, \neg A \vDash B$.

Task 4: Provide semantic clauses for SL.

Interpretation: An INTERPRETATION $\mathcal{I}$ of the sentences of SL assigns exactly one truth-value (1 or 0) to each sentence letter.

Disjunction: $\mathcal{I}(\varphi \vee \psi) = 1$ just in case $\mathcal{I}(\varphi) = 1$ or $\mathcal{I}(\psi) = 1$ (or both).

Negation: $\mathcal{I}(\neg\varphi) = 1$ just in case $\mathcal{I}(\varphi) = 0$.

Logical Validity: An argument in SL is LOGICALLY VALID just in case its conclusion is true in every interpretation in which its premises are true.

Task 5: Show that *Museum* is logically valid.

Logical Form

Picasso

(1) The painting is either a Picasso or a counterfeit and illegally traded.

(2) The painting is not a Picasso.

$\therefore$ The painting is a counterfeit and illegally traded.

This argument is also logically valid.

Question 2: How does this argument relate to *Museum*?

Task 6: Regiment *Picasso* in SL: $A \vee (B \wedge C), \neg A \vDash B \wedge C$.

Logical Form: Both arguments are instances of $\varphi \vee \psi, \neg\varphi \vDash \psi$ which is a logically valid argument form.

Question 3: How many logically valid argument forms are there, and how could we hope to describe this space?

Suggestion: Logical validity in SL describes the space of logically valid arguments, where the logically valid argument forms are patterns in this space.

Problem 2: SL cannot regiment all logically valid arguments.

Socrates: All men are mortal, Socrates is a man $\vDash$ Socrates is mortal.

Solution 2: Rather, logical validity in SL provides a partial answer, where we may extend the language to provide a broader description of logical validity, e.g. QL.

Proof Theory

Model Theory: We have characterized logical reasoning in terms of truth-preservation across a space of interpretations of the formal language by providing elements of a model theoretic semantics for SL.

Task 7: Can we make *Snow* and *Red* logically valid?

Syntactic Account: Another approach focuses entirely on syntax, using rules to specify which inferences are deductively valid given the meanings of the logical constants.

Metalogic: Amazingly, these two strategies coincide for both SL and QL, and we will prove these important results later in this course.

Neutrality: These methods accommodate reasoning about anything whatsoever, though not all logical constants are equally well understood.