Problem Set 12 (24.241 Symbolic Logic)

Due Saturday **Dec. 3rd** by **Noon** Eastern

Please scan and upload to Canvas as a pdf

Note: the last four questions are really straightforward and can be answered succinctly. The first two require more work. Unless noted otherwise, let '$\vdash$' stand for '$\vdash_{SND}$'

Question 0: if you worked with up to two classmates, please list their names!

1. In our inductive proof of the soundness of SND, prove the case where the sentence P_{k+1} is justified by Negation Elimination. [14*pts*]
   (this is the missing 'Case 10' in the *Logic Book*'s proof of soundness, p. 249).

2. Prove missing case (c) of Theorem 6.4.11 on p. 258 of the *Logic Book*, Chapter 6. i.e. If Γ^* is a maximally consistent set in SND and P and Q are two arbitrary wffs of SL, prove that $(P \vee Q) \in \Gamma^*$ if and only if either $P \in \Gamma^*$ or $Q \in \Gamma^*$. [28*pts*]
   – Note that you have to prove **BOTH directions** of this if-and-only-if statement!
   – In your proof, you will probably appeal to a schematic SND derivation, and you **MUST PROVIDE** this derivation (see bottom of p. 258 for examples)

3. Prove missing case (3) of Theorem 6.4.8 on p. 259-260 of the *Logic Book*. [20*pts*] i.e. Show that a sentence of the form $Q \vee R$ with k+1-many connectives is true on the truth value assignment $\mathbf{A}^*$ if and only if the sentence $Q \vee R$ belongs to a given maximally consistent set of sentences Γ^*. Where here "$\mathbf{A}^*$" is defined as the TVA that assigns True to every atomic sentence in Γ^* and False to every atomic sentence not in Γ^*.
   (Note that you will ultimately appeal to what you have just shown in the prior problem)

4. Let S be a sentence of SL and Γ an *infinite* set of SL sentences. Using the completeness and soundness theorems (but **NOT** compactness), prove the following:
   if $\Gamma \models S$, then there is some *finite* set $\Delta \subset \Gamma$ such that $\Delta \models S$. [14*pts*]

5. Using the soundness theorem, show that you cannot SND-derive two contradictory sentences R and $\sim R$ from just the atomic sentence B.
   (i.e. prove that the set $\{B\}$ is "consistent in SND".) [14*pts*]

6. Say that we add a new rule R^* to the rules of SND, to form a larger system SND*, equipped with its own single turnstile $\Gamma \vdash_{SND^*} S$ for "S is derivable from the set Γ using the rules of SND*".

   Say that there is a set Γ of sentences of SL and a sentence S of SL such that $\Gamma \vdash_{SND^*} S$ but $\Gamma \nvdash_{SND} S$ (i.e. we can derive S from Γ in SND* but not in SND).

   Prove that SND* is an unsound system of rules. [10*pts*]