Regimentation

LOGIC I
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From Last Time...

Definitions: Here is slightly different take on the same definitions:

Well-Formed Sentences: The set WFSS of $\mathcal{L}^{\text{PL}}$ is the smallest set to satisfy:

- φ is a wfs of $\mathcal{L}^{\text{PL}}$ if φ is a sentence letter of $\mathcal{L}^{\text{PL}}$;
- $\neg\varphi$ is a wfs of $\mathcal{L}^{\text{PL}}$ if φ is a wfs of $\mathcal{L}^{\text{PL}}$;
- $(\varphi \wedge \psi)$ is a wff of $\mathcal{L}^{\text{PL}}$ if φ and ψ are wfss of $\mathcal{L}^{\text{PL}}$;
- $(\varphi \vee \psi)$ is a wff of $\mathcal{L}^{\text{PL}}$ if φ and ψ are wfss of $\mathcal{L}^{\text{PL}}$;
- $(\varphi \rightarrow \psi)$ is a wff of $\mathcal{L}^{\text{PL}}$ if φ and ψ are wfss of $\mathcal{L}^{\text{PL}}$;
- $(\varphi \leftrightarrow \psi)$ is a wff of $\mathcal{L}^{\text{PL}}$ if φ and ψ are wfss of $\mathcal{L}^{\text{PL}}$.

Semantics: For an interpretation $\mathcal{I}$, a VALUATION function $\mathcal{V}_{\mathcal{I}}$ is the smallest function to assign truth-values to every sentence of SL that satisfies the semantic clauses:

- $\mathcal{V}_{\mathcal{I}}(\varphi) = \mathcal{I}(\varphi)$ if φ is a sentence letter of $\mathcal{L}^{\text{PL}}$.
- $\mathcal{V}_{\mathcal{I}}(\neg\varphi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 0$ (i.e., $\mathcal{V}_{\mathcal{I}}(\varphi) \neq 1$).
- $\mathcal{V}_{\mathcal{I}}(\varphi \wedge \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ and $\mathcal{V}_{\mathcal{I}}(\psi) = 1$.
- $\mathcal{V}_{\mathcal{I}}(\varphi \vee \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ or $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ (or both).
- $\mathcal{V}_{\mathcal{I}}(\varphi \rightarrow \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 0$ or $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ (or both).
- $\mathcal{V}_{\mathcal{I}}(\varphi \leftrightarrow \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = \mathcal{V}_{\mathcal{I}}(\psi)$.

Observe: Observe the symmetry between the above.

Recall: The hierarchy of sentences from before...

Complexity

Complexity: $\texttt{Comp}(\varphi)$ is the smallest function to satisfy all of the following conditions for all wfss φ and ψ of $\mathcal{L}^{\text{PL}}$:

- $\texttt{Comp}(\varphi) = 0$ if φ is a sentence letter;
- $\texttt{Comp}(\neg\varphi) = \texttt{Comp}(\varphi) + 1$;
- $\texttt{Comp}(\varphi \wedge \psi) = \texttt{Comp}(\varphi) + \texttt{Comp}(\psi) + 1$;
- ...

Question: Do we need to include corner quotes?

Validity

$\mathcal{L}^{\text{PL}}$ Validity: An argument in $\mathcal{L}^{\text{PL}}$ is *valid* iff its conclusion is a logical consequence of its premises.

English Validity: An argument in English is *valid* iff it has a (faithful) regimentation (in some language) that is valid.

- Note the imprecision here; there is no avoiding this.

Soundness: An argument is *sound* iff it is valid and has true premises (on an interpretation we care about, probably the intended interpretation).

Examples

Rain

1. If it is raining on a week day, Sam took his car.
2. Kate borrowed Sam's car only if Sam did not take it.
3. Kate borrowed Sam's car just in case she visited her parents.
4. It is raining and Kate visited her parents.
5. Either it is not a week day or it is not raining.

Task 2: Regiment this argument and construct its truth table.

Observe: This argument can be adequately regimented and evaluate in SL.

Negation

Uninitiated

A1. If Sam attended the gathering, then he has been initiated.
A2. Sam is uninitiated.
A3. Sam did not attend the gathering.

Observe: Being uninitiated is the same as not being initiated.

Uninvited

B1. Arden is not invited.
B2. Arden is uninvited.

Observe: Arden can fail to be invited without being uninvited.

Question: What about the converse?

Disjunction

Party

C1. If Adi or James make it to the party, Isa will be happy.
C2. If Adi and James make it to the party, Isa will be happy.

Observe: This argument suggests an inclusive reading of 'or'.

Race

D1. Sasha won the 100 meter dash.
D2. Josh won the high jump.
D3. Either Sasha won the 100 meter dash or Josh won the high jump

Observe: We could strengthen the conclusion.

Vault

E1. If Kin uses the remote, the trunk will open.
E2. If Yu tries the handle, the trunk will open.
E3. If Kin uses the remote and Yu tries the handle, the trunk won't open.
E4. If Kin uses the remote or Yu tries the handle, the trunk will open.

Observe: We cannot regiment the conclusion with inclusive-'or'.

Question: Can we salvage the validity of this argument?

Conjunction

Exam

F1. Henry failed and Megan passed.
F2. Megan passed and Henry failed.

Observe: Perfectly adequate and valid regimentation.

Gym

G1. Kate took a shower and went to the gym.
G2. Kate went to the gym and took a shower.

Observe: Conjunction in English can track temporal order.

Question: How can we capture the invalidity of this argument in $\mathcal{L}^{\mathrm{PL}}$?