

1. Symbolization in SL

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a. Symbolization keys and paraphrase

Symbolizing arguments

Argument 2 (Paraphrased)

Mandy enjoys skiing or Mandy enjoys hiking.

Not: Mandy enjoy hiking.

∴ Mandy enjoys skiing.

Symbolizing arguments

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Mandy enjoys skiing or Mandy enjoys hiking.

Not: Mandy enjoy hiking.

\therefore Mandy enjoys skiing.

Form of argument 2

S or H .

Not H .

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Symbolizing arguments

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Not: Mandy enjoy hiking.

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Form of argument 2

S or H .

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Symbolization of argument 2 in SL

$(S \vee H)$

$\sim H$

$\therefore S$

Symbolization keys

Definition

A symbolization key is a list that pairs **sentence letters** with the basic English sentences they represent.

For instance:

Symbolization key

S: Mandy enjoys skiing

H: Mandy enjoys hiking

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For instance:

B : Mandy enjoys skiing or hiking
is a **bad** choice of symbolization.

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- ▶ Two things to watch for: pronouns and coordination.
- ▶ Pronouns stand in for singular terms (e.g., names): explicitly replace pronouns by those names.
- ▶ “or”, “both ... and”, “neither ... nor” can connect sentences but also noun phrases and verb phrases: paraphrase them so that they connect sentences.

Example

If Mandy enjoys hiking, **she** also enjoys skiing.

Replace “she” by “Mandy”:

If [Mandy enjoys hiking] then [Mandy enjoys skiing].

Coordination of noun phrases

Example

Mandy and Sanjeev enjoy hiking.

Both [Mandy enjoys hiking] and [Sanjeev enjoys hiking].

Example

Sanjeev lives in **Erie or Chicago**.

Either [Sanjeev lives in Erie] or [Sanjeev lives in Chicago].

Exercise caution!

Good

Mandy and Sanjeev ate pizza.

Both [Mandy ate pizza] and [Sanjeev ate pizza].

Bad

Mandy and Sanjeev ate the whole pizza.

Both [Mandy ate the whole pizza] and [Sanjeev ate the whole pizza].

Coordination of verb phrases

Example

Mandy enjoys **skiing or hiking**.

Either [Mandy enjoys skiing] or [Mandy enjoys hiking].

Example

If Sanjeev enjoys **skiing and hiking**, he lives in Chicago.

If [Sanjeev enjoys skiing] and [Sanjeev enjoys hiking], then [Sanjeev lives in Chicago].

1. Symbolization in SL

b. Basic symbolization

Negation

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Mandy **doesn't** enjoy skiing.

It is not the case that [**Mandy enjoys skiing**].

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Mandy **doesn't** enjoy skiing.

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Even though Mandy lives in Erie, she enjoys hiking.

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Example

Even though Mandy lives in Erie, she enjoys hiking.
Both [**Mandy lives in Erie**] and [**Mandy enjoys hiking**].

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Both *E* and *H*.

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Sanjeev lives in Chicago **or** Erie.

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Ignore the suggestion that “either ... or ...” is exclusive. We'll always treat it as inclusive unless explicitly stated.

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- Mandy enjoys hiking **if** Sanjeev lives in Chicago.
- If [**Sanjeev lives in Chicago**] then [**Mandy enjoys hiking**].

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- **If C then H .**
- $(C \supset H)$

The parts of a conditional

- ▶ $(A \supset B)$ symbolizes:
 - “if A , B ”
 - “ B if A ” (note order is reversed!)
 - “ B provided A ”
- ▶ A is the **antecedent**: it symbolizes the condition that has to be met for the “then” part to apply. (Like a promise!)
- ▶ B is the **consequent**: it symbolizes what must be true (e.g. to keep your promise!) if the antecedent condition is true.

Example

Mandy doesn't enjoy hiking, **provided** Sanjeev lives in Chicago or Erie.

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If [Sanjeev lives in Chicago **or** Erie] then [Mandy **doesn't** enjoy hiking].

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If (either [**Sanjeev lives in Chicago**] or [**Sanjeev lives in Erie**]) then (it is not the case that [**Mandy enjoys hiking**]).

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If (either C or E) then (it is not the case that H).

$((C \vee E) \supset \sim H)$

1. Symbolization in SL

c. Conditionals

A logic puzzle

- ▶ Every card has a letter on one side and a number on the other side.
- ▶ You're a card inspector tasked with making sure that cards satisfy this quality standard:

If a card has an even number on one side, then it has a vowel on the other.

A logic puzzle

Which card(s) do you have to turn over to make sure that:

If a card has an even number on one side, then it has a vowel on the other.

E

(1)

K

(2)

3

(3)

4

(4)

Another logic puzzle

- ▶ At an all-ages event where everyone has a drink
- ▶ You know how old some of the people are, and you can tell what some of them are drinking
- ▶ You're tasked with making sure that the following rule is followed:

If a person is drinking alcohol, then they are at least 21 years old.

Another logic puzzle

Which of these people do you have to check (age or drink) to ensure that:

If a person is drinking alcohol, then they must be at least 21 years old.

22
years

(1)

16
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Truth conditions of Material conditionals

If $\underbrace{\text{X is drinking alcohol}}_A$, then $\underbrace{\text{X is over 21}}_B$

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 - (or both)

Understanding material conditionals as promises

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- ▶ You can't break a promise whose conditions are not satisfied
- ▶ The material conditional is just like this: it is *vacuously* true whenever the antecedent is false
- ▶ You'll have to get used to this!
- ▶ Other conditionals (e.g. causal, subjunctive, counterfactual) are not *truth-functional* (so handling them is controversial!)

1. Symbolization in SL

d. “Only if” and “unless”

‘only if’ vs. a lonely ‘if’

- ▶ Sue drinks beer (A) **only if** she is over 21 (B)

$$A \supset B$$

‘only if’ vs. a lonely ‘if’

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- ▶ False if she's 25 (B), but drinks pop ($\sim A$).
- ▶ Not false if she's 16 and drinking beer.

Conditional recap: **only if** you can't remember!

- ▶ **Paraphrase** “ A only if B ” as “**if A then B** ”.

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 - “A **if** B” is $(B \supset A)$ (lonely if)

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Symbolize “A if and only if B” as $(A \equiv B)$.

Conditional Practice: only if vs. lonely if

Schematize the following sentence:

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*Jack will go to the store only if Susie goes to the store,
and Susie will go if Beatrice goes.*

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Symbolization Key:

- *J*: Jack will go to the store
- *S*: Susie goes to the store
- *B*: Beatrice goes to the store

Conditional Practice: only if vs. lonely if

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Symbolization Key:

- J : Jack will go to the store
- S : Susie goes to the store
- B : Beatrice goes to the store

Answer: $(J \supset S) \& (B \supset S)$

Unless (confusing unless you use a trick!)

Which of these people do you have to check (age or drink) to ensure that:

People are drinking pop unless they are over 21.

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Unless

$\underbrace{\text{X is drinking pop}}_A, \text{ unless } \underbrace{\text{X is over 21}}_B$

- “A unless B” can only be **false** if:

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- ▶ “A unless B” can be paraphrased and symbolized by:
 - “A if not B” ($\sim B \supset A$)
 - “either A or B” ($A \vee B$) [**Remember this one!!!**]

Trick for handling 'Unless'

Treat “**unless**” the same way you would treat “or”

Example

Mandy enjoys hiking unless Sanjeev lives in Chicago.

$(H \vee C)$

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- ▶ Since disjunction is symmetric, you won't have to remember order of atomic sentences if you symbolize 'unless' using 'or'.
- ▶ So you can't go wrong with this approach!

1. Symbolization in SL

e. More connectives

Biconditional: If and only if (' \equiv ') ---

Example

Mandy enjoys hiking **if and only if** she enjoys skiing.

Unlike the conditional, the biconditional is symmetric: the order of atomic sentences does not matter!

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$((S \supset H) \& (H \supset S))$

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Unlike the conditional, the biconditional is symmetric: the order of atomic sentences does not matter!

Exclusive or

Example

Mandy enjoys hiking or skiing but not both.

Example

Mandy enjoys hiking or skiing but not both.

[either H or S]

Example

Mandy enjoys hiking or skiing but **not both**.

[either H or S]

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Mandy enjoys hiking or skiing but not both.

[either H or S]

[it is not the case that [both H and S]].

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Mandy enjoys hiking or skiing **but** not both.

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[it is not the case that [both H and S]].

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Both [either H or S] and
[it is not the case that [both H and S]].

Exclusive or

Paraphrase sentences containing “either A or B but not both” using
“**both [either A or B] and**
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$((H \vee S) \& \sim(H \& S))$

An alternative: $(H \& \sim S) \vee (S \& \sim H)$

i.e. Either (H and not S) or (S and not H)

Neither ... nor ... ('nand')

Example

Mandy enjoys neither hiking nor skiing.

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$(\sim H \& \sim S)$

This connective has a special metalogical property that we might discuss at some point!

Mix & match

Example

Sarah lives in Chicago or Erie.

Amir lives in Chicago unless he enjoys hiking.

If Amir lives in Chicago, Sarah doesn't.

Neither Sarah nor Amir enjoy hiking.

∴ Sarah lives in Erie.

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Example

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$(C \vee E)$

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$(A \vee M)$

If Amir lives in Chicago, Sarah doesn't.

$(A \supset \sim C)$

Neither Sarah nor Amir enjoy hiking.

$(\sim S \& \sim M)$

\therefore Sarah lives in Erie.

$\therefore E.$

1. Symbolization in SL

f. Defining Formulae in SL

Will the real SL please stand up?

What we have said so far about Sentential Logic (SL):

- ▶ Uppercase letters, subscripts allowed: e.g. J , H , B , N_3

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- ▶ e.g. $((((A \& \sim(B \supset C \equiv D((($

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 - They end up being expressions that are true or false under an assignment of truth values to atomic sentence letters
- ▶ But how do we rigorously define the WFFs as a subset of the expressions?

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We define the well-formed formulae (wffs) *recursively*:

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 - $(\mathcal{P} \supset \mathcal{Q})$
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4. And that's all folks! (No other expressions of SL are wffs)

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- ▶ $(\sim(A \supset (\sim B \vee C)) \& ((A \equiv B) \supset C))$

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- ▶ If not, locate the two wffs in the scope of the outermost parentheses, and figure out which connective combines them
- ▶ Rinse and repeat: stop when you reach the atomic sentences
- ▶ $(\sim(A \supset (\sim B \vee C)) \& ((A \equiv B) \supset C))$
- ▶ $(\quad \mathcal{P} \quad \quad \& \quad \quad \mathcal{Q} \quad \quad)$

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- ▶ So technically, ' $\sim\mathcal{P}$ ' is not a sentence of SL! So we define a convention: ' $\sim\mathcal{P}$ ' abbreviates the result of concatenating the sentence \mathcal{P} with the negation symbol ' \sim '.

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- ▶ BOOM!
- ▶ Who knew that air-quotes (‘scare-quotes’?) could play such an important role in the foundations of logic?

1. Symbolization in SL

g. Use vs. 'Mention'

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 - Logic is a fascinating subject!

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 - We symbolize conjunction as '&' and disjunction using ' \vee '.

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 - **GOOD**: “ $P \& Q$ ” is a sentence in SL
 - We have to mention “ $P \& Q$ ” since it is not a sentence in English; it’s a sentence in SL. So we can’t *use* it in English.
- ▶ But psssst: we’ll often be lazy and won’t put quotes where we’re technically supposed to...

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 - "Obama" names Obama; "Obama" is a name.

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- ▶ Some more detailed glosses, in case one of these clicks:
- ▶ When we use a word, it does not have quotation marks
 - The thing referenced by the word is being *put to good use*
 - Barack Obama was the 44th president of the USA
- ▶ When we mention a word or string of symbols in a sentence, we put quotation marks around them to show that we are mentioning something:
 - “Obama” names the 44th president of the USA
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 - “ $p \& \sim q$ ” is a conjunction, whose second conjunct is a negation.

A Mistake on the Wiki!

- ▶ On a Wiki page discussing the geographical usage of 'soda' vs. 'pop' in the USA
- ▶ Sentence: *To a lesser extent soda is also fairly common further down the east coast in eastern Virginia, eastern Carolinas and coastal Florida. Here, soda is not too dominant but competes with multiple other terms*
- ▶ Cite your sources!

Use vs. 'Mention' PRACTICE!

Punctuate the following sentences with quotation-marks so as to make them true:

1. Hunt is the last name of the instructor for this course.

Use vs. 'Mention' PRACTICE!

Punctuate the following sentences with quotation-marks so as to make them true:

1. Hunt is the last name of the instructor for this course.
2. Hunt is a surname which begins with the letter H.

Use vs. 'Mention' PRACTICE!

Punctuate the following sentences with quotation-marks so as to make them true:

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3. I call my dog Ivy Ivy, but I never use Ivy's first name in addressing her.

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1. "Hunt" is the last name of the instructor for this course.

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Answers:

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2. "Hunt" is a surname which begins with the letter "H".

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Punctuate the following sentences with quotation-marks so as to make them true:

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Answers:

1. "Hunt" is the last name of the instructor for this course.
2. "Hunt" is a surname which begins with the letter "H".
3. I call my dog Ivy "Ivy", but I never use "Ivy's first name" in addressing her.

1. Symbolization in SL

h. Ambiguity

Types of ambiguity

- ▶ Lexical ambiguity: one word—many meanings
e.g., “bank”, “crane”
- ▶ Syntactic ambiguity: one sentence—many readings
e.g.,
 - “Flying planes can be dangerous” (Chomsky)
 - “One morning I shot an elephant in my pajamas.
How he got in my pajamas, I don’t know.” (Groucho Marx)

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Ambiguity of $\&$ and \vee

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The man who was hanged by a comma



- ▶ Sir Roger Casement (1864–1916)
- ▶ British consul to Congo and Peru
- ▶ Tried to recruit Irish revolutionaries in Germany during WWI
- ▶ Tried for treason

Treason Act of 1351 (20th century court added the comma!)

ITEM, Whereas divers Opinions have been before this Time in what Case Treason shall be said, and in what not; the King, at the Request of the Lords and of the Commons, hath made a Declaration in the Manner as hereafter followeth, that is to say; When a Man doth compass or imagine the Death of our Lord the King, or of our Lady his Queen or of their eldest Son and Heir; or if a Man do violate the King's Companion, or the King's eldest Daughter unmarried, or the Wife of the King's eldest Son and Heir; or **if a Man** do levy War against our Lord the King in his Realm, or **be adherent to the King's Enemies in his Realm, giving to them Aid and Comfort in the Realm or elsewhere**, and thereof be probably attainted of open Deed by the People of their Condition: . . . And it is to be understood, that in the Cases above rehearsed, that ought to be judged Treason which extends to our Lord the King, and his Royal Majesty: . . .

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- A: Casement was adherent to the King's enemies in the realm. (false)

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► Symbolization key:

A: Casement was adherent to the King's enemies in the realm. (false)

G: Casement gave aid and comfort to the King's enemies in the realm. (false)

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- With the comma, 'treason' now defined as:

- $$(A \vee B) \vee (G \vee H) \quad [\text{and } B \text{ is true}]$$

1. Symbolization in SL

i. Practice Problems!

Validity, Paraphrasing, Symbolizing

Classify the following arguments as sound, valid but unsound, or invalid. Next, paraphrase the argument in logical form. Finally, symbolize the argument in SL.

1. Detroit will win the Super Bowl this year if they didn't make the playoffs. Detroit didn't make the playoffs. Therefore: Detroit will win the Super Bowl this year.

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3. Cleveland is a city in Ohio. Pittsburgh is a city in Ohio. Therefore: there are at least two cities in Ohio.

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3. Cleveland is a city in Ohio. Pittsburgh is a city in Ohio. Therefore: there are at least two cities in Ohio.
4. Either Toledo is in Michigan or the Upper Peninsula is in Michigan. Toledo is not in Michigan. Therefore: the Upper Peninsula is in Michigan.

Solution to Question 1:

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$$(\sim P) \supset D$$
$$\sim P$$
$$\therefore D$$

More Complex Symbolizations

Paraphrase the following English sentences using our logical symbolism. Any condensed clauses should be expanded into complete sentences. Identify as well the main connective and practice symbolizing with atomic sentences.

1. If they either drain the swamp and reopen the road or dredge the harbor, they will provide the uplanders with a market and themselves with a bustling trade.
2. Unless his new novel does very well and gets him a large advance, Malone will take a position in a college writing program or he will take out a second mortgage and not sell his car.
3. If Germany annexes Austria, Czechoslovakia will be militarily defensible only if France honors her treaty obligations and arranges for the transit of Soviet troops across Poland or Romania.

Drain the Swamp!

Sentence: If they either drain the swamp and reopen the road or dredge the harbor, they will provide the uplanders with a market and themselves with a bustling trade.

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Sentence: *If they either drain the swamp and reopen the road or dredge the harbor, they will provide the uplanders with a market and themselves with a bustling trade.*

- Paraphrase: **IF** [(they drain the swamp and they reopen the road) or (they dredge the harbor)] **THEN** (they will provide the uplanders with a market and they will provide themselves with a bustling trade).

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- ▶ Alternative paraphrase using connectives: [(they drain the swamp & they reopen the road) \vee (they dredge the harbor)] \supset (they will provide the uplanders with a market & they will provide themselves with a bustling trade)].

Drain the Swamp (still draining...)

Paraphrase: **IF** [(they drain the swamp and they reopen the road) or (they dredge the harbor)] **THEN** (they will provide the uplanders with a market and they will provide themselves with a bustling trade).

Symbolization Key:

S: they drain the swamp

R: they reopen the road

H: they dredge the harbor

M: they will provide the uplanders with a market

T: they will provide themselves with a bustling trade

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Symbolization-answer: $((S \& R) \vee (H)) \supset (M \& T)$