Derivation Rules of Natural Deduction (System SND)

Conjunction Introduction (&I)

$$\begin{array}{l|ll} m & \Phi & \\ n & \Psi & \\ & \Phi \,\&\, \Psi & :m,\, n\ \&\mathrm{I} \\ & \Psi \,\&\, \Phi & :m,\, n\ \&\mathrm{I} \end{array}$$

Conjunction Elimination (&E)

$$\begin{array}{l|ll} m & \Phi \,\&\, \Psi & \\ & \Phi & :m\ \&\mathrm{E} \\ & \Psi & :m\ \&\mathrm{E} \end{array}$$

Conditional Introduction ($\supset$I)

$$\begin{array}{l|l|ll} m & & \underline{\Phi} & :\text{AS for } \supset\mathrm{I} \\ n & & \Psi & \\ & \multicolumn{2}{l}{\Phi \supset \Psi} & :m\text{–}n \supset\mathrm{I} \end{array}$$

Conditional Elimination ($\supset$E)

$$\begin{array}{l|ll} m & \Phi \supset \Psi & \\ n & \Phi & \\ & \Psi & :m,\, n \supset\mathrm{E} \end{array}$$

Negation Introduction ($\sim$I)

$$\begin{array}{l|l|ll} m & & \underline{\Phi} & :\text{AS for } \sim\mathrm{I} \\ n & & \Psi & \\ o & & \sim\Psi & \\ & \multicolumn{2}{l}{\sim\Phi} & :m\text{–}o \sim\mathrm{I} \end{array}$$

Negation Elimination ($\sim$E)

$$\begin{array}{l|l|ll} m & & \underline{\sim\Phi} & :\text{AS for } \sim\mathrm{E} \\ n & & \Psi & \\ & & \vdots & \\ o & & \sim\Psi & \\ & \multicolumn{2}{l}{\Phi} & :m\text{–}o \sim\mathrm{E} \end{array}$$

Disjunction Introduction ($\vee$I)

$$\begin{array}{l|ll} m & \Phi & \\ & \Phi \vee \Psi & :m \vee\mathrm{I} \\ & \Psi \vee \Phi & :m \vee\mathrm{I} \end{array}$$

Disjunction Elimination ($\vee$E)

$$\begin{array}{l|l|ll} m & \multicolumn{2}{l}{\Phi \vee \Psi} & \\ i & & \underline{\Phi} & :\text{AS for } \vee\mathrm{E} \\ j & & \Omega & \\ & \multicolumn{2}{l}{--} & \\ k & & \underline{\Psi} & :\text{AS for } \vee\mathrm{E} \\ l & & \Omega & \\ & \multicolumn{2}{l}{\Omega} & :m,\, i\text{–}j,\, k\text{–}l \vee\mathrm{E} \end{array}$$

Biconditional Introduction ($\equiv$I)

$$\begin{array}{l|l|ll} i & & \underline{\Phi} & :\text{AS for } \equiv\mathrm{I} \\ j & & \Psi & \\ & \multicolumn{2}{l}{--} & \\ k & & \underline{\Psi} & :\text{AS for } \equiv\mathrm{I} \\ l & & \Phi & \\ & \multicolumn{2}{l}{\Phi \equiv \Psi} & :i\text{–}j,\, k\text{–}l \equiv\mathrm{I} \end{array}$$

Biconditional Elimination ($\equiv$E)

$$\begin{array}{l|ll} m & \Phi \equiv \Psi & \\ n & \Psi & \\ & \Phi & :m,\, n \equiv\mathrm{E} \end{array}$$