Truth Tables

LOGIC I
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Truth Functions

Previously: For an interpretation $\mathcal{I}$, a VALUATION function $\mathcal{V}_{\mathcal{I}}$ is the smallest function to assign truth-values to every sentence of SL that satisfies the semantic clauses:

(A) $\mathcal{V}_{\mathcal{I}}(\varphi) = \mathcal{I}(\varphi)$ iff φ is a sentence letter of SL.

($\neg$) $\mathcal{V}_{\mathcal{I}}(\neg\varphi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 0$ (i.e., $\mathcal{V}_{\mathcal{I}}(\varphi) \neq 1$).

($\wedge$) $\mathcal{V}_{\mathcal{I}}(\varphi \wedge \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ and $\mathcal{V}_{\mathcal{I}}(\psi) = 1$.

($\vee$) $\mathcal{V}_{\mathcal{I}}(\varphi \vee \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ or $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ (or both).

($\supset$) $\mathcal{V}_{\mathcal{I}}(\varphi \supset \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = 0$ or $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ (or both).

($\equiv$) $\mathcal{V}_{\mathcal{I}}(\varphi \equiv \psi) = 1$ iff $\mathcal{V}_{\mathcal{I}}(\varphi) = \mathcal{V}_{\mathcal{I}}(\psi)$.

Truth Tables: Use the semantics to fill out the CHARACTERISTIC TRUTH TABLES given below:

φ	$\neg\varphi$
1	0
0	1

φ	ψ	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \supset \psi$	$\varphi \equiv \psi$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Sentential Operators: The connectives are SENTENTIAL OPERATORS which map sentences to sentences.

Truth Functional: The connectives express truth-functions:
$\mathcal{V}_{\mathcal{I}}(\neg\varphi) = 1 - \mathcal{V}_{\mathcal{I}}(\varphi)$;
$\mathcal{V}_{\mathcal{I}}(\varphi \wedge \psi) = \mathcal{V}_{\mathcal{I}}(\varphi) \times \mathcal{V}_{\mathcal{I}}(\psi)$.

HOMEWORK: Given an interpretation $\mathcal{I}$, specify the truth-values of $\varphi \vee \psi$, $\varphi \supset \psi$, and $\varphi \equiv \psi$ as a function of the truth-values of φ and ψ in a similar fashion as above.

Task 1: How many unary/binary truth-functions are there?

Adequacy: Given the expressive limitations of SL, what should we hope to be able to adequately regiment?

Examples

Complex Arguments

Rain

(1) If it is raining on a week day, Sam took his car.

(2) Kate borrowed Sam's car only if Sam did not take it.

(3) Kate borrowed Sam's car just in case she visited her parents.

(3) It is raining and Kate visited her parents.

∴ It is not a week day.

Task 2: Regiment this argument and construct its truth table.
Observe: This argument can be adequately regimented and evaluate in SL.

Conjunction

Gym

(1) Kate took a shower and went to the gym.

∴ Kate went to the gym and took a shower.

Task 3: Regiment this argument and construct its truth table.
Observe: Conjunction in English can track temporal order.
Question: How can we capture the invalidity of this argument in SL?

Disjunction

Vault

(1) If Kin uses the remote, the trunk will open.

(2) If Adi tries the handle, the trunk will open.

(3) If Kin uses the remote and Adi tries the handle, the trunk won't open.

∴ If Kin uses the remote or Adi tries the handle, the trunk will open.

Task 4: Regiment this argument and construct its truth table.
Observe: We cannot regiment the conclusion with inclusive-'or'.
Question: Can we salvage the validity of this argument?

The Material Conditional

Roses

(1) Sugar is sweet.

∴ The roses are only red if sugar is sweet.

Task 5: Regiment this argument and construct its truth table.

Observe: The locution 'only if' appears to assert something stronger than $\supset$.

Vacation

(1) Casey is not on vacation.

∴ If Casey is on vacation, then he is in Paris.

Crimson

(1) Mary doesn't like the ball unless it is crimson.

(2) Mary likes the ball.

∴ If the ball is blue, then Mary likes it.

The Biconditional

Rectangle

(1) The room is a square.

(2) The room is a rectangle.

∴ The room is a square if and only if it is a rectangle.

Work

(1) Kin isn't a professor.

(2) Sue isn't a chef.

∴ Kin is a professor just in case Sue is a chef.

Applications

Objection: The semantics for SL is not good for anything.

Response: SL is perfect for necessary claims (like in mathematics), as well as sentences where we only care about their truth-value as opposed to their modal profile or subject-matter.