

Sentential Tree Logic

LOGIC I

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Constructing the Root

Previously: $\Gamma \models \varphi$ if and only if $\Gamma, \neg\varphi \models \perp$.

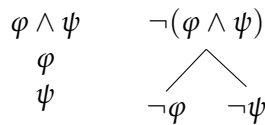
Proof: To show $\Gamma, \neg\varphi \models \perp$, we will show $\Gamma, \neg\varphi \vdash \perp$.

Resolution Rules

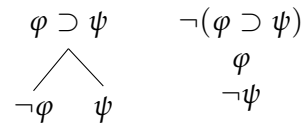
Root: A *root* is any finite sequence of SL sentences.

Tree: An SL *tree* consists of a root followed by any number of applications of the resolution rules given below:

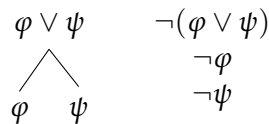
Conjunction



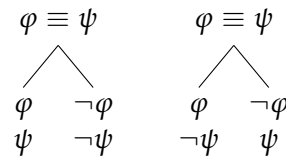
The Material Conditional



Disjunction



The Biconditional



Double Negation



Child: A *child* of φ in an SL tree is any sentence ψ immediately following φ .

Leaf: A *leaf* in an SL tree is any sentence which does not have a child.

Branch: A *branch* in an SL tree is any sequence of sentences beginning with the root of the tree and ending with a leaf of the tree where every sentence in the sequence besides the first is a child of its predecessor.

Note: Officially, an SL tree is an *ordered dyadic tree* of SL sentences where every sentence in the tree either belongs to the root, or results from resolving one of its ancestors.

Closure and Completion

Branch Closure: A branch in an SL tree is *closed* just in case it includes φ and $\neg\varphi$ for some SL sentence φ , and *open* otherwise.

Tree Closure: A tree is *closed* just in case every branch is closed, and *open* otherwise.

Resolvable: A sentence is *resolvable* in a branch just in case it has a resolution rule and the branch is open.

Resolved: A sentence is *resolved* in a branch just in case the resolution rule for that sentence has been applied in that branch.

Branch Completion: A branch is *complete* just in case every resolvable sentence in that branch has been resolved in that branch.

Tree Completion: A tree is *complete* if and only if every branch in the tree is complete.

Derivability

STL: $\Gamma \vdash \perp$ just in case there is a closed tree with root Γ .

Derivability: $\Gamma \vdash \varphi$ just in case $\Gamma, \neg\varphi \vdash \perp$.

Question 1: Why should we care about \vdash ?

Answer: So far, we shouldn't, but soon we will show that: $\Gamma \vdash \varphi$ iff $\Gamma \models \varphi$.

Suppose: Let's suppose $\Gamma \vdash \varphi$ iff $\Gamma \models \varphi$ for now.

Question 2: How can we determine whether Γ is satisfiable?

Answer: Show that $\Gamma \not\models \perp$.

Examples

1. Evaluate the following argument for validity:

$$\begin{array}{l} \neg R \supset \neg Q \\ P \wedge Q \\ \therefore P \wedge R \end{array}$$

2. Show that $A \vee B, B \supset C, A \equiv C \models C$.
3. Show that $(P \supset Q) \equiv (\neg Q \supset \neg P)$ is a tautology.
4. Show that $A \equiv \neg A$ is a contradiction.
5. Show that $\{P, P \supset Q, Q \supset \neg P\}$ is unsatisfiable.
6. Show that $\{P \supset Q, \neg P \vee \neg Q, Q \supset P\}$ is satisfiable.
7. Evaluate $P, P \supset Q, \neg Q \models A$.