0. What is logic?

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- Located on 9th floor, Dreyfoos Wing; turn RIGHT after entering Phil dept

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- Doing ample practice problems will be **essential**!

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- Let's say some words!

0. What is logic?

a. Arguments and validity

Causes vs. (normative) Reasons for belief

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- **Reasons for belief**: why you *ought* to believe something
  - Empirical data on cat attacks per capita
  - These are *normative* reasons for belief
  - Logic aims to characterize the structure of such reasons, when organized into *arguments*

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  - Such arguments might be descriptively pretty unpersuasive!
  - Comparative analysis of risk of blood clots from Janssen vaccine vs. risk of negative Covid health outcome
  - Comparative analysis of plane crashes vs. car crashes

An easy puzzle

Where does Sanjeev live?

Sanjeev lives in Chicago or in Erie.

Sanjeev doesn’t live in Erie.

An easy puzzle

Where does Sanjeev live?

Sanjeev lives in Chicago or in Erie.
Sanjeev doesn't live in Erie.

A: Obviously, in Chicago.

Arguments and sentences

Argument 1

Sanjeev lives in Chicago or in Erie.
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- The last sentence is called the **conclusion**.
- The others are called the **premises**.

Valid and invalid arguments

Argument 2

Mandy enjoys skiing or hiking (or both).
Mandy doesn't enjoy hiking.
$\therefore$ Mandy enjoys skiing.

Argument 3

Mandy enjoys skiing or hiking (or both).
Mandy enjoys skiing.
$\therefore$ Mandy doesn't enjoy hiking.

What's the difference?

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Definition

A **case** is some hypothetical scenario that makes each sentence in an argument either true or false.

Argument 2 is valid

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Mandy enjoys skiing or hiking.
Mandy doesn't enjoy hiking.
∴ Mandy enjoys skiing.

Argument 2 is **valid**: whenever the premises are true, the conclusion is also true.

Argument 3 is not valid

Argument 3

Mandy enjoys skiing or hiking.
Mandy enjoys skiing.
∴ Mandy doesn't enjoy hiking.

Argument 3 is **invalid**: there is a possible case where the premises are true and the conclusion isn't (Mandy enjoys both skiing and hiking).

A harder puzzle

Where does Sarah live?

Sarah lives in Chicago or Erie.
Amir lives in Chicago unless he enjoys hiking.
If Amir lives in Chicago, Sarah doesn't.
Neither Sarah nor Amir enjoy hiking.

0. What is logic?

b. Cases and determining validity

Validity

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Cases

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- E.g., imagine you have a friend, her name is Mandy, she loves hiking but hates skiing.
- That's a case where "Mandy enjoys hiking or skiing" is true.
- Some cases can be imagined even though they never happen IRL, e.g, "It is raining and the skies are clear."
- Some things you can't imagine, e.g., "There is a blizzard but there is no wind."

Determining validity

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Deductively Valid?

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All squirrels are rodents.
$\therefore$ Some squirrels have bushy tails.

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  - Imagine chinchillas still have bushy tails.

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- Imagine squirrels evolving so that they no longer have bushy tails. Then conclusion is false.
- But premises still true:
  - Imagine chinchillas still have bushy tails.
  - Imagine also that squirrels have not evolved too much—they are still rodents.

Valid?

All rodents have bushy tails.
All squirrels are rodents.
$\therefore$ All squirrels have bushy tails.

Valid?

All rodents have bushy tails.
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- If it were invalid, you'd have a case that makes the conclusion false: some squirrels without bushy tails.

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- If it were invalid, you'd have a case that makes the conclusion false: some squirrels without bushy tails.
- They would have to be rodents still (otherwise premise 2 false).
- And that would require that they have bushy tails (otherwise premise 1 false).

0. What is logic?

c. Other logical notions

Logical Consistency

Definition

Sentences are (logically) **consistent** if there is a case where they are all true.

- also called ‘jointly possible’ or ‘satisfiable’

Definition

Sentences are (logically) **inconsistent** if there is no case where they are all true.

- also called ‘jointly impossible’ or ‘unsatisfiable’

Consistent?

Some carnivores have bushy tails.
All carnivores are mammals.
No mammals have bushy tails.

Consistent?

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- No case makes them all true at the same time, so **inconsistent**.

Valid?

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$\therefore$ All birds are carnivores.

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- The premises cannot all be true in the same case, so inconsistent.
- So: no case makes all the premises true.
- So also: no case makes the premises true and the conclusion false.
- **Arguments with inconsistent premises are automatically valid**, regardless of what the conclusion is.

Tautology (logically necessary)

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- It's not the case that I am standing and that I am not standing. ('*Law of Non-Contradiction*')

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- $\Rightarrow$ **Arguments with tautologies as conclusions are automatically valid**, regardless of what the premises are.

Logical equivalence

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Definition

Two sentences are (logically) **equivalent** if there is no case where one is true and the other is false.

- What can you say about an argument where one of the premises is equivalent to the conclusion? Is it automatically valid?
- Can you have two equivalent sentences that are inconsistent?

0. What is logic?

d. What are we going to learn? And why?

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- Logic investigates what makes the first argument **valid** and the second **invalid**.

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- Meta-logical properties of logical systems

Plan for the course

- Sentential Logic (SL), aka Truth–functional logic

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  - Expressive completeness, normal forms

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  - Formal proofs make mechanical **proof checking** and **proof search** possible.

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    - Formal specification and verification (of programs, of hardware designs)
    - Theoretical computer science (theory of computational complexity, semantics of programming languages)

0. What is logic?

e. Symbolization and SL

Validity in virtue of form

Argument 1

Sanjeev lives in Chicago or Erie.

Sanjeev doesn't live in Erie.

∴ Sanjeev lives in Chicago.

Argument 2

Mandy enjoys skiing or hiking.

Mandy doesn't enjoy hiking.

∴ Mandy enjoys skiing.

Validity in virtue of form

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$\therefore$ Sanjeev lives in Chicago.

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Form of arguments 1 & 2

X or Y.

Not Y.

$\therefore X$.

Some valid argument forms

Disjunctive syllogism

X or Y.

Not Y.

$\therefore X$.

Modus ponens

If X then Y.

X.

$\therefore Y$.

Hypothetical syllogism

If X then Y.

If Y then Z.

$\therefore$ If X then Z.

Symbolizing arguments

Symbolization key

S: Mandy enjoys skiing

H: Mandy enjoys hiking

Argument 2

Mandy enjoys skiing or Mandy enjoys hiking.	$(S \vee H)$
Not: Mandy enjoy hiking.	$\sim H$
$\therefore$ Mandy enjoys skiing.	$\therefore S$

The language of SL

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This can get complicated, e.g.:

"Mandy enjoys skiing or hiking, and if she lives in Erie, she doesn't enjoy both."

$$((S \vee H) \,\&\, (E \supset \sim(S \,\&\, H)))$$

0. What is logic?

f. Bonus: Some History of Logic!

The beginnings

"The Philosopher"

- Rules of debate & rhetoric

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- All ungulates have hooves.
  No fish have hooves.
  ∴ No fish are ungulates.

The middle ages

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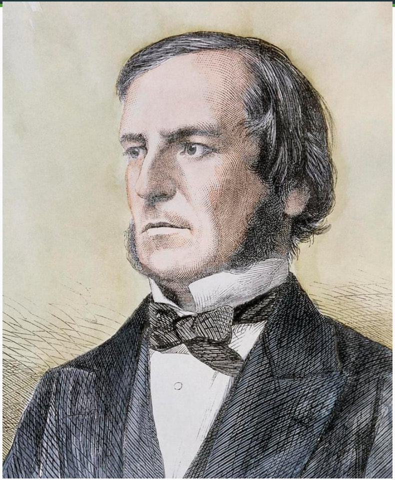

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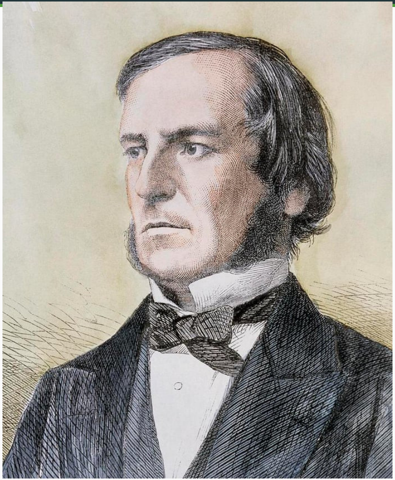

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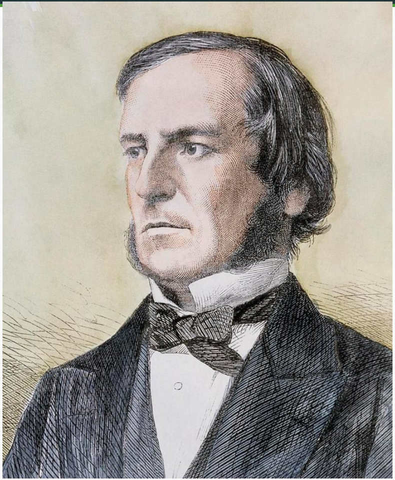

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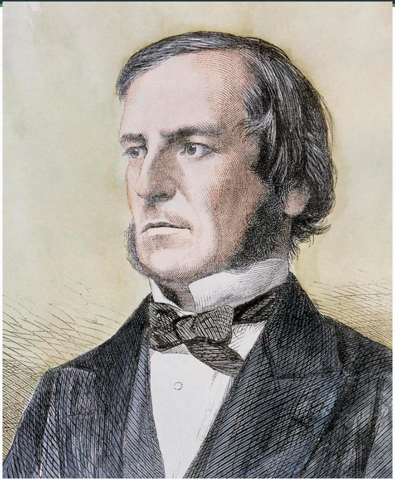

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- John Venn (1834-1923)

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- Ernst Schröder (1841-1902)

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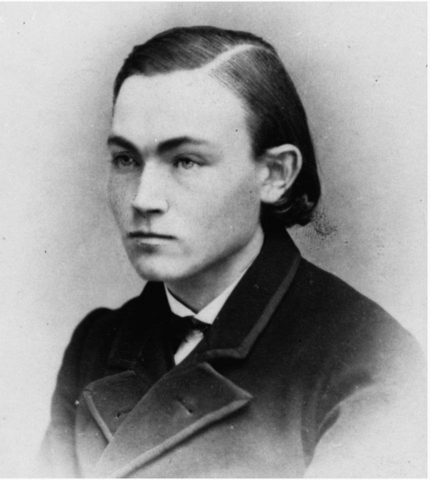

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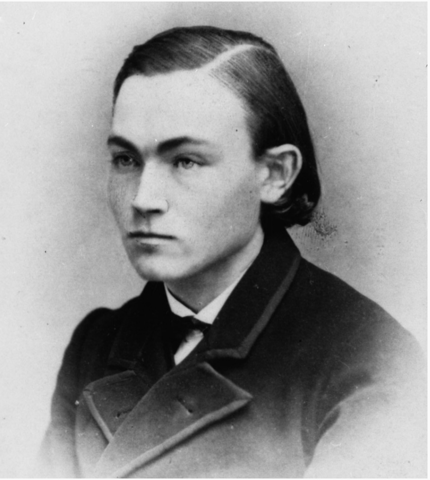

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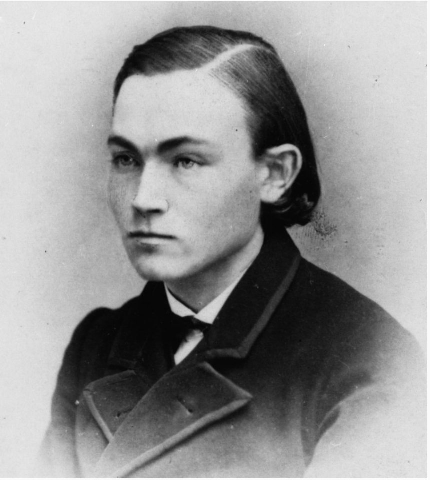

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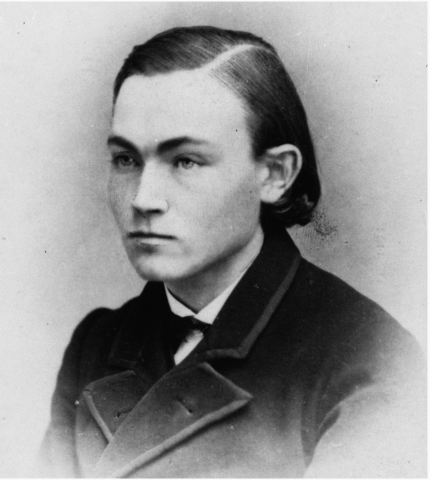

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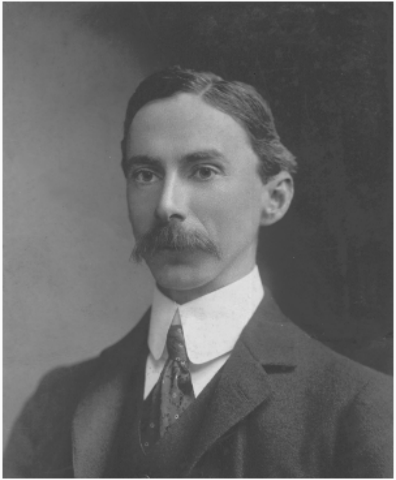

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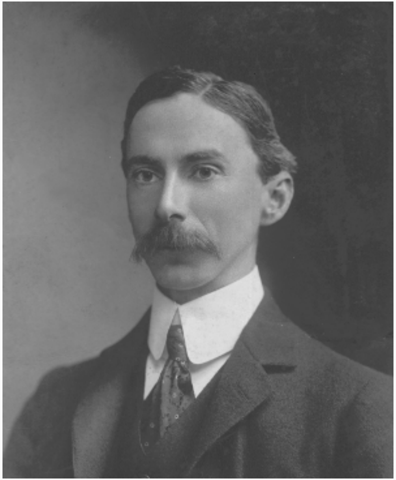

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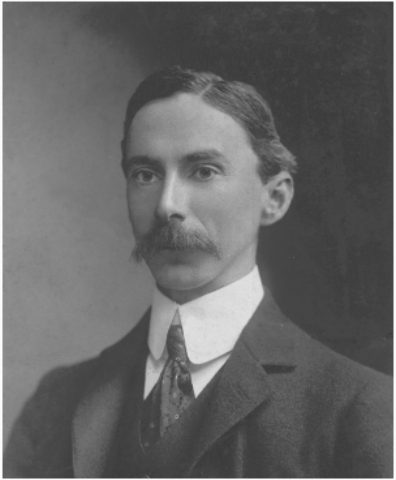

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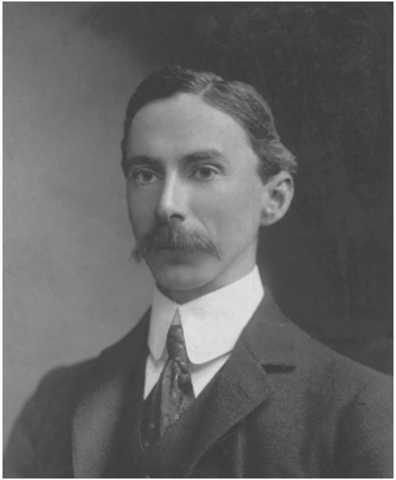

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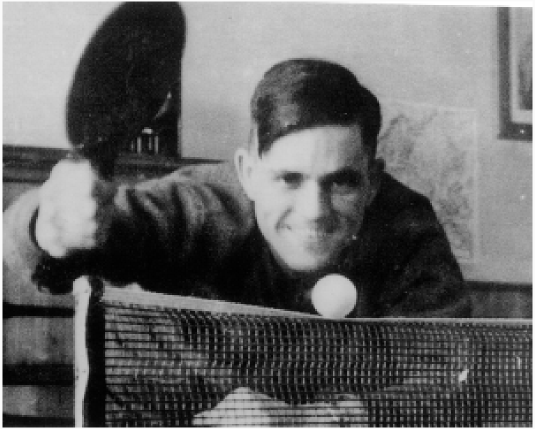
Modern logic: Gerhard Gentzen



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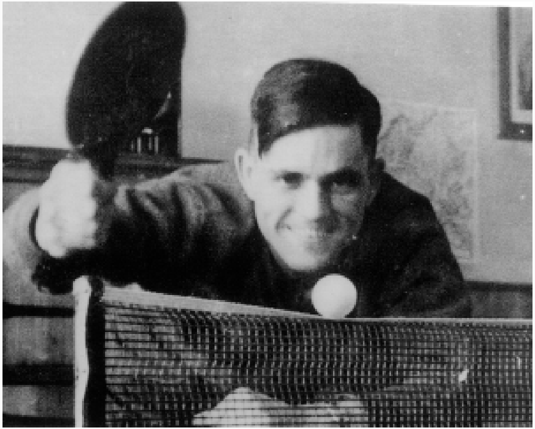

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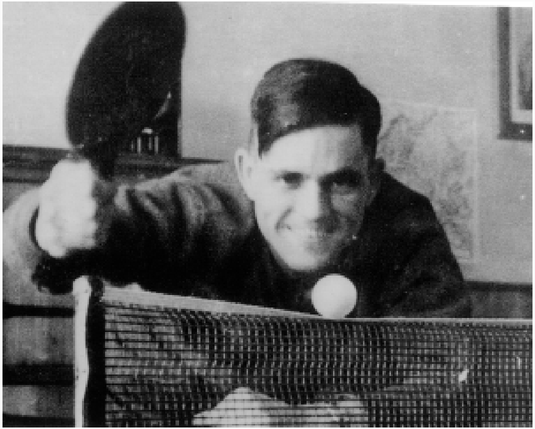

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Barcan Marcus!

- Extend logic with operators for “possible” and “necessary”

Modern logic: modal logic

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- Pioneered by philosophers, now used by computer scientists

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