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[24.241]: Regarding Existential Completeness

1 message

Josh Hunt <joshhunt@mit.edu> Thu, Dec 8, 2022 at 6:45 PM
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Hello those of you in class yesterday,

I write to you *in earnest* regarding existential completeness. On my run today, I ended up reflecting some more on our discussion, and I can happily confirm that the second example provided in class *does* demonstrate the need for our condition (iii) in building up $\Gamma*$ (recall: condition iii enforces adding a substitution instance whenever we add an existential sentence).

Without this condition (iii), we can form a maximally QND-consistent set that is not existentially complete. Start with (∃x)~Fx. Its substitution instances have the form ~F[c/x], e.g. ~Fd. As noted yesterday, we will hit the `enemies' of each substitution instance before we hit the corresponding substitution instance. E.g., Fb occurs earlier than ~Fb (the enemy always has one less symbol than its corresponding substitution instance, so its enumeration index has two fewer digits).

Hence, following just conditions (i) and (ii), we would form a maximally QND-consistent set that contains (∃x)~Fx and every enemy instance Fc, for all constants c, but not a single substitution instance ~F[c/x] of our existential sentence (∃x)~Fx.

There was a later worry that this set would necessarily contain (∀x)Fx. But this is not so: as a simple derivation shows, adding (∀x)Fx to a set containing (∃x)~Fx results in QND-inconsistency (this is a good exercise).

And note that we can't appeal to the membership lemma clause (f) to argue that this set would contain (∀x)Fx (in virtue of containing all of its substitution instances). This is because we used existential-completeness to prove clause (f). So the membership lemma would fail for this set.

So basically, we need to enforce condition (iii)—or something like it—to ensure existential completeness, which we need to prove the membership lemma. And it is through the membership lemma that we go from syntactic consistency to being able to construct a model that satisfies everything in our set.

Thanks again for asking about this and discussing it! I at least feel more existentially complete for it.

Cheers,
-Josh

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"And the thought is always consoling that, often in philosophy, it is more instructive to travel than to get anywhere." -- Crispin Wright