9. Semantics of QL

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a. Arguments and validity in QL

Validity of arguments

Valid?

Everyone is either good or evil.

Not everyone is a villain.

Only villains are evil.

$\therefore$ Some heroes are good.

Validity in QL

- Want to capture validity **in virtue of the meanings of the connectives and the quantifiers**
  (but ignoring the meanings of predicate symbols)
- So we want to ignore any restrictions the predicate symbols place on their **extensions**
- Hence: allow **any** extension in a potential counterexample
- An argument is **QL-valid** if there is **no interpretation** in which the premises are true and the conclusion false

Forms of arguments

Everyone is either good or evil.

Not everyone is a villain.

Only villains are evil.

∴ Some heroes are good.

$(\forall x)(Gx \vee Ex)$

$\sim(\forall x)\, Vx$

$(\forall x)(Ex \supset Vx)$

$\therefore\ (\exists x)(Hx \,\&\, Gx)$

(In)validity of arguments

$$(\forall x)(Gx \vee Ex)$$

$$\sim(\forall x)\, Vx$$

$$(\forall x)(Ex \supset Vx)$$

$$\therefore\ (\exists x)(Hx \,\&\, Gx)$$

Domain: the inner planets (Mecury, Venus, Mars, Earth)

Gx: x is smaller than Earth

Ex: x is inhabited

Vx: x has a moon

Hx: x has rings

9. Semantics of QL

b. Interpretations

Interpretations

- Domain: collection of objects (not empty)
- **Referents** for each name (which object it names)
- Properties of each object
  - **Extension** of each 1-place predicate symbol:
    the set of objects it applies to
- Relations between each pair of objects
  - **Extension** of each 2-place predicate symbol:
    all pairs of objects standing in that relation

Extensions

Domain: the inner planets
Gx: x is smaller than Earth
Ex: x is inhabited
Vx: x has a moon
Hx: x has rings

Domain: Mercury, Venus, Earth, Mars
Gx: Mercury, Venus, Mars
Ex: Earth
Vx: Earth, Mars
Hx: —

(In)validity of arguments

$$(\forall x)(Gx \vee Ex)$$
$$\sim(\forall x)\,Vx$$
$$(\forall x)(Ex \supset Vx)$$
$$\therefore\ (\exists x)(Hx \,\&\, Gx)$$

Domain: Mercury, Venus, Earth, Mars

Gx: Mercury, Venus, Mars

Ex: Earth

Vx: Earth, Mars

Hx: —

(In)validity of arguments

$(\forall x)(Gx \vee Ex)$

$\sim(\forall x)\, Vx$

$(\forall x)(Ex \supset Vx)$

$\therefore\ (\exists x)(Hx \,\&\, Gx)$

Domain: 1, 2, 3, 4

Gx: 1, 2, 4

Ex: 3

Vx: 3, 4

Hx: —

(In)validity of arguments

$$(\forall x)(Gx \vee Ex)$$

$$\sim(\forall x)\, Vx$$

$$(\forall x)(Ex \supset Vx)$$

$$\therefore\ (\exists x)(Hx \,\&\, Gx)$$

Domain: 1

Gx: 1

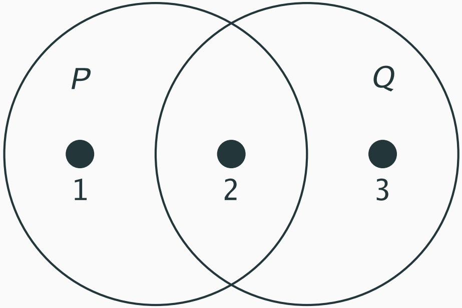
Ex: —

Vx: —

Hx: —

Extensions of predicates

Domain: 1, 2, 3
Px: 1, 2
Qx: 2, 3
Rx: —



$R = \varnothing$

(In)validity of arguments

$(\forall x)(Gx \vee Ex)$

$\sim(\forall x)\, Vx$

$(\forall x)(Ex \supset Vx)$

$\therefore\ (\exists x)(Hx \,\&\, Gx)$

Domain: 1, 2

Gx: 1

Ex: 2

Vx: 2

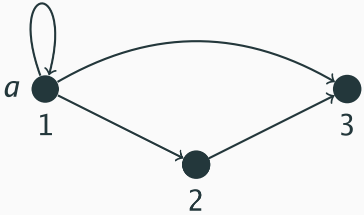
Hx: 2

Extensions of predicates

Domain: 1, 2, 3

a: 1

Axy: $\langle 1, 1\rangle$, $\langle 1, 2\rangle$, $\langle 1, 3\rangle$, $\langle 2, 3\rangle$



9. Semantics of QL

c. Truth of sentences of QL

Truth of sentences of QL

- Given an interpretation I ...
- An **atomic sentence** is true iff the referents of the constants are in the extension of the predicate:
  - Pa is true iff referent 'r' of a is in extension of P
  - Rab is true iff $\langle r, p \rangle$ is in extension of R (where r is referent of a, and p is referent of b)
- $\sim\mathcal{A}$ is true iff $\mathcal{A}$ is false
- $\mathcal{A} \vee \mathcal{B}$ is true iff at least one of $\mathcal{A}$, $\mathcal{B}$ is true
- $\mathcal{A} \,\&\, \mathcal{B}$ is true iff both $\mathcal{A}$, $\mathcal{B}$ are true
- $\mathcal{A} \supset \mathcal{B}$ is true iff $\mathcal{A}$ is false or $\mathcal{B}$ is true

Truth of quantified sentences

- $(\exists x)\,\mathcal{A}x$ is true iff $\mathcal{A}x$ is **satisfied** by **at least one** object in the domain
  - o satisfies $\mathcal{A}x$ iff $\mathcal{A}c$ is true in interpretation just like I, but with o as referent of c
- $(\forall x)\,\mathcal{A}x$ is true iff $\mathcal{A}x$ is **satisfied** by **every** object in the domain

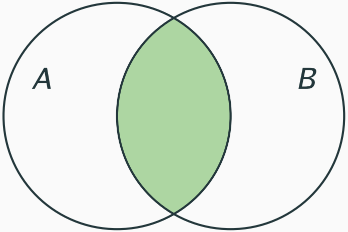
Truth of quantified sentences

- $(\exists x)(\mathcal{A}x \,\&\, \mathcal{B}x)$ is true iff some object satisfies '$\mathcal{A}x \,\&\, \mathcal{B}x$'
  - o satisfies '$\mathcal{A}x \,\&\, \mathcal{B}x$' iff it satisfies both $\mathcal{A}x$ and $\mathcal{B}x$
- $(\forall x)(\mathcal{A}x \supset \mathcal{B}x)$ is true iff **every** object satisfies '$\mathcal{A}x \supset \mathcal{B}x$'
  - o satisfies '$\mathcal{A}x \supset \mathcal{B}x$' iff either
    - o does not satisfy $\mathcal{A}x$ (vacuously true conditional)

      or
    - o does satisfy $\mathcal{B}x$

Making "Some As are Bs" true

- $(\exists x)(Ax \,\&\, Bx)$
- Extension of A and B must have something in common. (Filled area must contain at least one object)
- A and B can overlap, be equal, or be contained.
- Same situations make "No As are Bs" **false**.



Making “Some *A*s are *B*s” true

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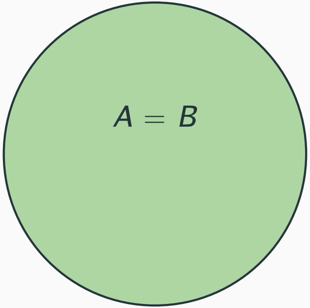

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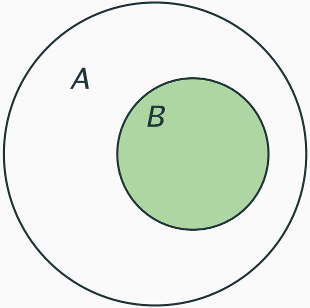

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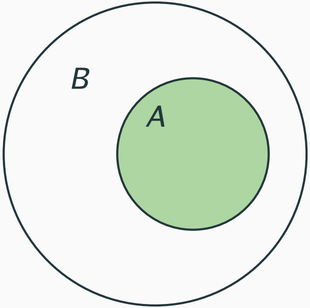
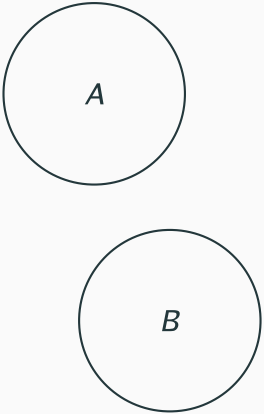


Making "Some *A*s are *B*s" false

- $\sim(\exists x)\,(Ax \,\&\, Bx)$
- Extension of *A* and *B* must have nothing in common.
- *A* and *B* don't overlap, or one or both empty.
- Same situations make "No *A*s are *B*s" **true**.



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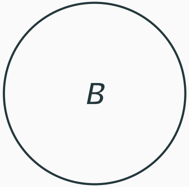

$A = \varnothing$

Making “Some *A*s are *B*s” false

- $\sim(\exists x)\,(Ax \,\&\, Bx)$
- Extension of *A* and *B* must have nothing in common.
- *A* and *B* don’t overlap, or one or both empty.
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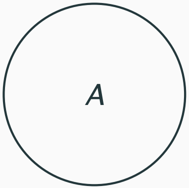
$B = \varnothing$



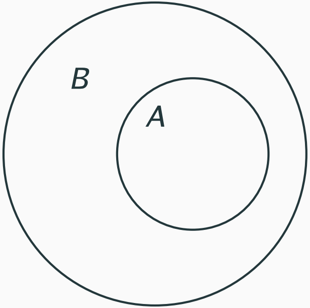
Making "Some As are Bs" false

- $\sim(\exists x)\,(Ax \,\&\, Bx)$
- Extension of A and B must have nothing in common.
- A and B don't overlap, or one or both empty.
- Same situations make "No As are Bs" **true**.

$$A = B = \varnothing$$

Making “All As are Bs” true

- $(\forall x)(Ax \supset Bx)$
- Extension of A must be contained in extension of B.
- Extensions of A and B can be the same.
- Extension of A can be empty.
- Same situations make ...
  - “Only Bs are As” **true**.
  - “Some As are not Bs” **false**.



Making "All *A*s are *B*s" true

- $(\forall x)\,(Ax \supset Bx)$
- Extension of A must be contained in extension of B.
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- Same situations make ...
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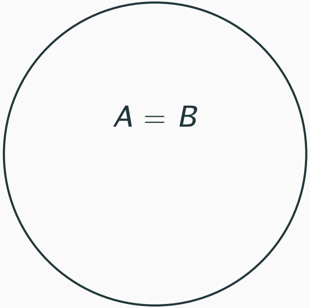

Making "All *A*s are *B*s" true

- $(\forall x)(Ax \supset Bx)$
- Extension of *A* must be contained in extension of *B*.
- Extensions of *A* and *B* can be the same.
- Extension of *A* can be empty.
- Same situations make ...
  - "Only *B*s are *A*s" **true**.
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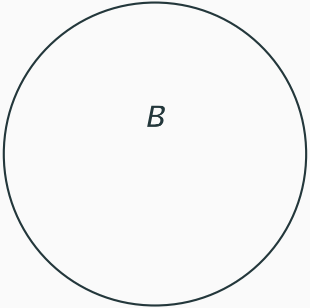
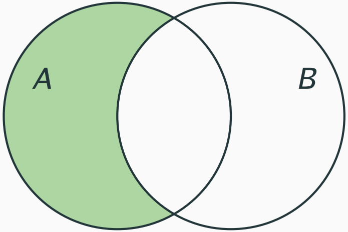


$A = \varnothing$

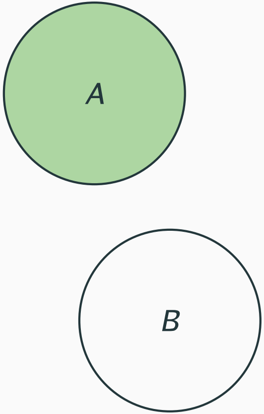
Making "All *A*s are *B*s" false

- $(\forall x)(Ax \supset Bx)$
- Extension of *A* must contain something not in *B*.
- Extensions of *A* cannot be empty, but *B* may be empty.
- Same situations make ...
  - "Only *B*s are *A*s" **false**.
  - "Some *A*s are not *B*s" **true**.



Making "All As are Bs" false

- $(\forall x)(Ax \supset Bx)$
- Extension of A must contain something not in B.
- Extensions of A cannot be empty, but B may be empty.
- Same situations make . . .
  - "Only Bs are As" **false**.
  - "Some As are not Bs" **true**.



Making "All As are Bs" false

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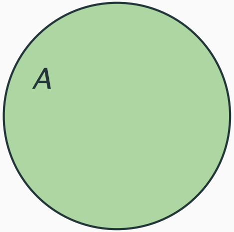


$B = \varnothing$

9. Semantics of QL

d. Testing for validity

Arguments involving quantifiers

1. If an action x is morally wrong then A is blameworthy for freely doing x.

2. If x is rationally optimal (there is no action which A has reason to think there is more reason for A to do), then A is not blameworthy for freely doing x.

3. Therefore, if x is morally wrong, then x is not rationally optimal. (Principle of moral categoricity.)

–John Skorupski, *Ethical Explorations*, 2000 (link)

Symbolizing Skorupski

1. If an action x is morally wrong then A is blameworthy for freely doing x.
2. If x is rationally optimal, then A is not blameworthy for freely doing x.
3. Therefore, if x is morally wrong, then x is not rationally optimal.

Domain: actions

Wx: x is morally wrong

Bx: A is blameworthy for freely doing x

Ox: x is rationally optimal

$$(\forall x)(Wx \supset Bx)$$

$$(\forall x)(Ox \supset \sim Bx)$$

$$\therefore (\forall x)(Wx \supset \sim Ox)$$

Symbolizing Skorupski

Domain: actions

Wx: x is morally wrong

Bx: A is blameworthy for freely doing x

Ox: x is rationally optimal

$(\forall x)(Wx \supset Bx)$

$(\forall x)(Ox \supset \sim Bx)$

$\therefore (\forall x)(Wx \supset \sim Ox)$

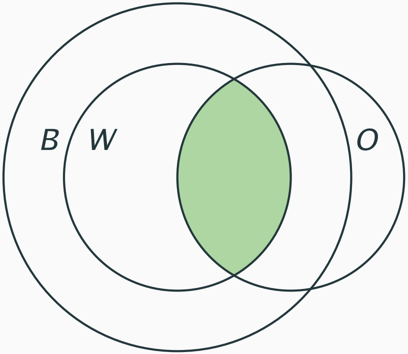
All Ws are Bs

No Os are Bs (iff No Bs are Os)

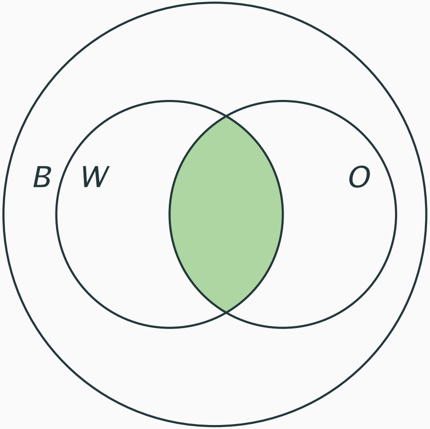
∴ No Ws are Os

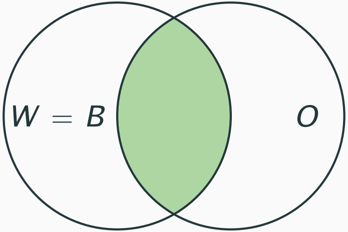
Determining validity

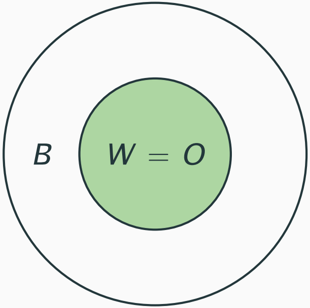
- Make conclusion $(\forall x)(Wx \supset \sim Ox)$ false.
- Make $(\exists x)(Wx \,\&\, Ox)$ true.
- Make $(\forall x)(Wx \supset Bx)$ true.
- $(\exists x)(Ox \,\&\, Bx)$ is now forced to be true.
- So, $(\forall x)(Ox \supset \sim Bx)$ is false.
- But those are not the only possibilities!



Other configurations







9. Semantics of QL

e. Semantic notions in QL

Semantics notions in QL

- $\mathcal{P}_1, \ldots, \mathcal{P}_n \vDash \mathcal{Q}$ if no interpretation makes all of $\mathcal{P}_1, \ldots, \mathcal{P}_n$ true and $\mathcal{Q}$ false.
- $\mathcal{P}$ is a **validity** ($\vDash \mathcal{P}$) if it is true in every interpretation.
- $\mathcal{P}$ and $\mathcal{Q}$ are **equivalent in QL** if no interpretation makes one true but the other false.
- $\mathcal{P}_1, \ldots, \mathcal{P}_n$ are **jointly satisfiable in QL** if some interpretation makes all of them true

Using interpretations

- By providing one suitable interpretation we **can** show that...
  - an argument is **not valid** in QL
  - a sentence is **not a validity** in QL
  - two sentences are **not equivalent** in QL
  - some sentences **are satisfiable** in QL
- But we **cannot** show using any number of interpretations that...
  - an argument **is valid** in QL
  - a sentence **is a validity** in QL
  - two sentences **are equivalent** in QL
  - some sentences **are not satisfiable** in QL

Examples

- $(\forall x)(Ax \vee Bx)$ and $(\forall x)\, Ax \vee (\forall x)\, Bx$ are not equivalent.
- $(\forall x)(Ax \supset Bx), (\forall x)(Ax \supset \sim Bx)$ are jointly satisfiable.
- $(\forall x)(\sim Ax \supset Bx), (\exists x)(Bx \,\&\, Cxb) \not\vDash (\exists x)(\sim Ax \,\&\, Cxb)$.
- $\not\vDash (\exists x)\, Aax \supset (\exists x)\, Axx$.

Test solutions on this week's practice problems!

9. Semantics of QL

f. Arguing about interpretations

Arguing about Interpretations

- No interpretation(s) can show that an argument is valid.
- That's because there is no way to inspect all possible interpretations.
- But we can show that arguments are valid, by:
  - a formal proof (a future topic!)
  - an informal argument
- The informal argument makes use of the **truth conditions** for sentences of QL.
- Analogous to arguing about valuations in SL.

Example

$$((\forall x)\mathcal{A}x \lor (\forall x)\mathcal{B}x) \vDash (\forall x)(\mathcal{A}x \lor \mathcal{B}x)$$

- Suppose an interpretation makes premise $(\forall x)\mathcal{A}x \lor (\forall x)\mathcal{B}x$ true.
- By truth conditions for $\lor$, it makes either $(\forall x)\mathcal{A}x$ or $(\forall x)\mathcal{B}x$ true.
- Suppose it's the first, i.e., $(\forall x)\mathcal{A}x$ is true.
  - By truth conditions for $\forall$, every object in the domain satisfies $\mathcal{A}x$.
  - By the truth conditions for $\lor$, every object satisfies $\mathcal{A}x \lor \mathcal{B}x$
  - So, by the truth conditions for $\forall$, $(\forall x)(\mathcal{A}x \lor \mathcal{B}x)$ is true.
- Suppose it's the second, i.e., $(\forall x)\mathcal{B}x$ is true: Similarly...
- These are the only possibilities: so any interpretation that makes the premise true must also make the conclusion true.