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a. Two quantifiers

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- It's true iff **every pair** of objects α, β stand in the relation expressed by $\mathcal{B}xy$.
- $(\exists x)(\exists y)\,\mathcal{B}xy$ is a sentence:
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- So: $(\forall x)(\forall y)\, Axy$ does **not** symbolize "everyone admires everyone **else**." (To handle that, we'll need identity!)

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- A function f is **point-wise continuous** if

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11. Multiple quantifiers & The Identity Predicate

b. Multiple Determiners

“Determiner phrases” say what?

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- Determiner phrases: combine a determiner with a (possibily modified) noun:
- ‘all heroes’; ‘a cape’
- ‘some woman’; ‘the donkey’

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- What if your sentence contains more than one determiner phrase?
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- Think of determiner phrase as replaced with name or variable—result has one less determiner.
- When you're down to one determiner, apply known methods for single quantifiers.
- This results in formulas that express properties or relations, but themselves contain quantifiers.

Two separate determiner phrases

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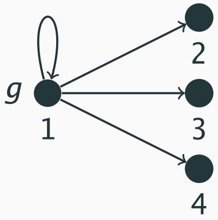
c. The identity predicate

Greta admires everyone (else)



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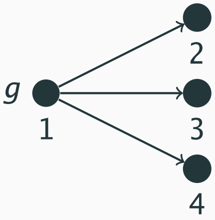

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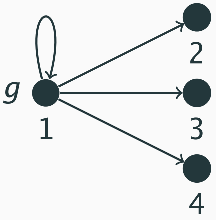

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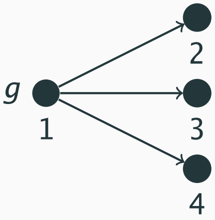

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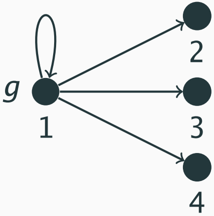

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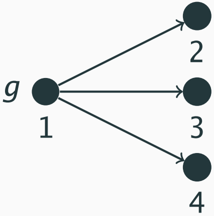

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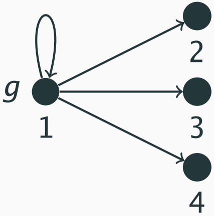

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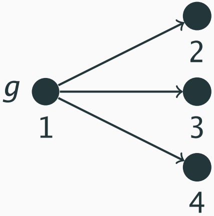

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$(\forall x)(\sim x = g \supset Agx)$

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- $x = y$ satisfied by all and only the pairs $\langle \alpha, \alpha \rangle$.
- $\sim x = y$ is satisfied by a pair $\langle \alpha, \beta \rangle$ iff α and β are different objects.

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- *Carnap* will not tolerate this nonsense! Take heed!

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- The closest quantifier (typically) determines whether you should use $\&$ or $\supset$:

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Everyone admires someone else vs. Someone admires everyone else

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- Content: No-one other than Greta is a hero, **AND** Greta is a hero:

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  $(\forall x)(Hx \supset x{=}g) \,\&\, Hg$

‘No-one other than’ vs. Singular “only”

- “**No-one other than Greta** is a hero”:

  $\sim(\exists x)(Hx \,\&\, \sim x{=}g)$

  $(\forall x)(Hx \supset x{=}g)$

- “**Only Greta** is a hero”:
- Content: No-one other than Greta is a hero, **AND** Greta is a hero:

  $(\forall x)(Hx \supset x{=}g) \,\&\, Hg$

  $(\forall x)(Hx \equiv x{=}g)$

Uniqueness

- Non-unique: "There is at least one hero":

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  - Or more succinctly: $(\exists x)(\forall y)(Hy \equiv x{=}y)$

11. Multiple quantifiers & The Identity Predicate

d. Numerical quantification

Numerical Quantification: n–many as ‘at least n’

- Cardinal numbers can be determiners:

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- Using QL, we can express the following kinds of sentences:
  - **At least** n people are ...
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  - **At most** n people are ...
- i.e. we can count on QL!

At least n

- At least 1 hero is inspiring:

$$(\exists x)(Hx \,\&\, Ix)$$

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- At least 3 heroes are inspiring:

$$(\exists x)(\exists y)(\exists z)\Big((\sim x{=}y \,\&\, (\sim y = z \,\&\, \sim x = z)) \,\&\, \\ ((Hx \,\&\, Ix) \,\&\, ((Hy \,\&\, Iy) \,\&\, (Hz \,\&\, Iz)))\Big)$$

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- There are at least n As, i.e. "$(\exists^{\geq n} x)\, Ax$":

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- Note: must state that **every pair** of variables is different, e.g.,

$$(\exists x_1)(\exists x_2)(\exists x_3)((\sim x_1 = x_2 \;\&\; \sim x_2 = x_3) \;\&\; (Hx_1 \;\&\; (Hx_2 \;\&\; Hx_3)))$$

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    - Take extension of Hx to be: $1, 2$
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    - Both "$\sim 1 = 2$" and "$\sim 2 = 3$" are true.

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  - Then 1 can play role of x_1 and x_3, 2 role of x_2.
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- At least n Bs are Cs: substitute '$Bx \,\&\, Cx$' for 'Ax':

$$(\exists^{\geq n} x)(Bx \,\&\, Cx)$$

Exactly one (i.e. Uniqueness; see above)

- There is exactly one hero:

$$(\exists x)(Hx \,\&\, {\sim}(\exists y)(Hy \,\&\, {\sim}x{=}y))$$

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- In general: "g has property A **uniquely**":

$$Ag \,\&\, (\forall y)(Ay \supset g{=}y)$$

$$\text{or just:} \qquad (\forall y)(Ay \equiv g{=}y)$$

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- $\sim(\exists^{\geq (n+1)} x)\, Ax$ is equivalent to:

$$\begin{aligned} &(\forall x_1) \ldots (\forall x_{n+1})((Ax_1 \,\&\, \ldots \,\&\, Ax_{n+1}) \supset \\ &\quad (x_1 = x_2 \vee (x_1 = x_3 \vee \cdots \vee (x_1 = x_{n+1} \vee \\ &\quad\quad (x_2 = x_3 \vee \cdots \vee (x_2 = x_{n+1} \vee \\ &\quad\quad\quad \ddots \\ &\quad\quad\quad x_n = x_{n+1}) \ldots))) \end{aligned}$$

11. Multiple quantifiers & The Identity Predicate

e. Both ‘both’ and ‘neither’

Schematizing ‘Both’

- “Both heroes inspire”: this means that
  There are **exactly 2** heroes, and both inspire:

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- Note: “Both heroes inspire” implies “There are exactly two inspiring heroes”, but not vice versa!
- e.g. if there are exactly two inspiring heros and one (or more) not-inspiring hero(s)

Schematizing ‘Neither’

- “Neither hero inspires”: this means that

Schematizing ‘Neither’

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  There are **exactly 2** heroes, and neither of them inspires:

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11. Multiple quantifiers & The Identity Predicate

f. ‘The’ Definite Description

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$$(\exists x)(Ax \,\&\, Wx\ell \,\&\, (\forall y)((Ay \,\&\, Wy\ell) \supset x{=}y) \,\&\, Sx)$$

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- Compare:
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  2. There is exactly one inspiring hero:

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- (2) can be true without (1), but not vice versa.
- (Namely when there is exactly one inspiring hero, but also a non-inspiring hero.)
- So (1) entails (2), but not vice versa.

Strawson's analysis (presuppositional theories)

- According to Russell, "The hero wears a cape" is **false** if there is no hero, or if there is more than one.

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- Consider: "the present King of France is bald."
- P. F. Strawson disagrees with these truth conditions.
  Rather, we only succeed in making a statement if there is a unique hero (or a unique king of France).

Strawson's analysis (presuppositional theories)

- According to Russell, "The hero wears a cape" is **false** if there is no hero, or if there is more than one.
- Consider: "the present King of France is bald."
- P. F. Strawson disagrees with these truth conditions. Rather, we only succeed in making a statement if there is a unique hero (or a unique king of France).
- "There is a unique hero" is not part of what is **said** by a definite description, but is only presupposed.

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- So plural possessives are NOT definite descriptions.

11. Multiple quantifiers & The Identity Predicate

g. Using quantifiers to express properties

Our symbolization key

Domain: people alive in 2022 and items of clothing

a: Autumn

g: Greta

Px: $\_\_\_\_\__x$ is a person

Lx: $\_\_\_\_\__x$ is an item of clothing.

Ex: $\_\_\_\_\__x$ is a cape

Rxy: $\_\_\_\_\__x$ wears $\_\_\_\_\__y$

Hx: $\_\_\_\_\__x$ is a hero

Ix: $\_\_\_\_\__x$ inspires

Yxy: $\_\_\_\_\__x$ is younger than $\_\_\_\_\__y$

Axy: $\_\_\_\_\__x$ admires $\_\_\_\_\__y$

Oxy: $\_\_\_\_\__x$ owns $\_\_\_\_\__y$

Expressing properties, revisited

- One-place predicates express properties, e.g.,

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- Using quantifiers, we can express even more complex properties, e.g.,

  $(\exists y)(Py \,\&\, Axy)$ expresses "x admires someone"

Finding, using properties expressed

- If you can say it for Greta, you can say it for x.

Ex: $\_\_\_\_\__x$ is a cape
Rxy: $\_\_\_\_\__x$ wears $\_\_\_\_\__y$

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  - Greta admires a hero.
    $(\exists y)(Hy \,\&\, Agy)$
  - x admires a hero.
    $(\exists y)(Hy \,\&\, Axy)$
- If you can say it for x, you can say it for Greta.
  - x wears a cape.
    $(\exists y)(Ey \,\&\, Rxy)$

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Examples

- x wears a cape.

Px	$\_\_\_\_\__x$ is a person	Lx	$\_\_\_\_\__x$ is an item of clothing
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11. Multiple quantifiers & The Identity Predicate

h. Multiple determiners: worked example

Mary Astell, 1666–1731

- British political philosopher
- *Some Reflections upon Marriage* (1700)
- In preface to 3rd ed. 1706 reacts to William Nicholls' claim (in *The Duty of Inferiors towards their Superiors, in Five Practical Discourses* (London 1701), Discourse IV: The Duty of Wives to their Husbands), that women are naturally inferior to men.

Astell TL;DR

- What can Nicholls possibly mean by "women are naturally inferior to men"?

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- The obviously intended meaning must be: **all** women are inferior to **all** men.
- But that can’t be right, for then “the greatest Queen ought not to command but to obey her Footman.”
- It can’t even be just: **all** women are inferior to **some** men.
- Since “had they been pleased to remember their Oaths of Allegiance and Supremacy, they might have known that *One* Woman is superior to *All* the Men in these Nations.”

Symbolizing Astell

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- Together:

$$(\exists x)(Wx \,\&\, (\forall y)(My \supset Sxy))$$

Formalizing Astell

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11. Multiple quantifiers & The Identity Predicate

i. Quantifier scope ambiguity

More scope ambiguity

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$$(Aai \vee Aal) \;\&$$
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- Two logically distinct, natural readings:

1) Autumn admires Isra or Luisa, **and** so does Greta.

$$(Aai \lor Aal)\ \&$$
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2) Autumn and Greta both admire Isra, or they both admire Luisa.

$$(Aai \,\&\, Agi) \lor$$
$$(Aal \,\&\, Agl)$$

Negation and the quantifiers

- "All heroes don't inspire"

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- “All heroes don’t inspire”
  - Denial of “all heroes inspire”. Ask: “Do all heroes inspire (Answer: No, *it’s not the case that* all heroes inspire ”)

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$$\sim(\forall x)(Hx \supset Ix)$$
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  - All heroes are not inspiring, i.e., No heroes inspire

$$(\forall x)(Hx \supset \sim Ix)$$
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Multiple quantifiers and ambiguity

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- A (probably bad) joke: "Every day, a tourist is mugged on the streets of New York. He's going through a lot of wallets."

11. Multiple quantifiers & The Identity Predicate

j. Donkey sentences

Happy farmers

"Every farmer who owns a donkey is happy"

- Step-by-step symbolization: "All As are Bs"

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$$Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)$$

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$$Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)$$

- Every farmer who owns a donkey is happy

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- Every farmer who owns a donkey is happy

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- Notice how 'a donkey' is bound by an existential here

Unhappy donkeys :(

"Every farmer who owns a donkey beats it"

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- PROBLEM: 'y' is unbound! So this is not a QL sentence. Gasp!

Save the donkeys: a failed attempt

- This was our problem: a donkey lay beaten and unbound:

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  But he still sounds like kind of a jack@$$!

Symbolizing donkey sentences

"Every farmer who owns a donkey beats it"

- When is it false that every farmer who owns a donkey beats it? If there's a farmer who owns a donkey but doesn't beat it. Deny that!

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- But what about the case where at least one farmer with a donkey beats only one of his donkeys? #Quitting