11. Multiple quantifiers & The Identity Predicate

1. Multiple quantifiers &
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a. Two quantifiers

Formulas expressing relations

- A formula $\mathcal{A}x$ with one free variable expresses a **property**
- A formula $\mathcal{B}xy$ with two free variables expresses a **relation**
- $(\forall x)(\forall y)\,\mathcal{B}xy$ is a sentence:
- It's true iff **every pair** of objects α, β stand in the relation expressed by $\mathcal{B}xy$.
- $(\exists x)(\exists y)\,\mathcal{B}xy$ is a sentence:
- It's true iff **at least one pair** of objects α, β stand in the relation expressed by $\mathcal{B}xy$.

Multiple uses of a single quantifier: $\forall$

- Axy: x admires y.
- $(\forall x)(\forall y)\, Axy$: for every pair $\langle \alpha, \beta \rangle$, α admires β.
- In other words: everyone admires everyone.
- Note: "every pair" includes pairs $\langle \alpha, \alpha \rangle$, i.e.,
- $(\forall x)(\forall y)\, Axy$ is true only if all pairs $\langle \alpha, \alpha \rangle$ satisfy Axy.
- That means, everyone admires themselves, in addition to everyone else.
- So: $(\forall x)(\forall y)\, Axy$ does **not** symbolize "everyone admires everyone **else**." (To handle that, we'll need identity!)

Multiple uses of single quantifier: $\exists$

- $(\exists x)(\exists y)\, Axy$: for at least one pair $\langle\alpha, \beta\rangle$, α admires β.
- In other words: at least one person admires at least one person.
- Note: includes pairs $\langle\alpha, \alpha\rangle$, i.e.,
- $(\exists x)(\exists y)\, Axy$ is already true if a single pair $\langle\alpha, \alpha\rangle$ satisfies Axy.
- That means, we could just have one person admiring themselves.
- So: $(\exists x)(\exists y)\, Axy$ does **not** symbolize "someone admires someone **else**." (again, for that, we'll need the identity predicate)

Alternating quantifiers

1. $(\forall x)(\exists y)\, Axy$
   Everyone admires someone
   (possibly themselves)
2. $(\forall y)(\exists x)\, Axy$
   Everyone is admired by someone
   (not necessarily the same person)
3. $(\exists x)(\forall y)\, Axy$
   Someone admires everyone
   (including themselves)
4. $(\exists y)(\forall x)\, Axy$
   Someone is admired by everyone
   (including themselves)

Convergence vs. uniform convergence

- A function f is **point-wise continuous** if

$$(\forall\epsilon)(\forall x)(\forall y)(\exists\delta)(|x-y|<\delta\supset|f(x)-f(y)|<\epsilon)$$

- A function f is **uniformly continuous** if

$$(\forall\epsilon)(\exists\delta)(\forall x)(\forall y)(|x-y|<\delta\supset|f(x)-f(y)|<\epsilon)$$

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b. Multiple Determiners

“Determiner phrases” say what?

- Determiners: quantifiers and indefinite or definite articles (also possessives and demonstratives)
- e.g. many, some, a, the, his, their, this, that
- Determiner phrases: combine a determiner with a (possibily modified) noun:
- ‘all heroes’; ‘a cape’
- ‘some woman’; ‘the donkey’

Symbolizing multiple determiners

- What if your sentence contains more than one determiner phrase?
- Deal with each determiner separately!
- Think of determiner phrase as replaced with name or variable—result has one less determiner.
- When you're down to one determiner, apply known methods for single quantifiers.
- This results in formulas that express properties or relations, but themselves contain quantifiers.

Two separate determiner phrases

- All heroes wear a cape
- All heroes satisfy "x wears a cape"

$$(\forall x)(Hx \supset \text{“}x\text{ wears a cape”})$$

- x wears a cape

$$(\exists y)(Ey \,\&\, Rxy)$$

- Together:

$$(\forall x)(Hx \supset (\exists y)(Ey \,\&\, Rxy))$$

Determiner within determiner phrase

- All heroes who wear a cape admire Greta.
- All things that satisfy "x is a hero who wears a cape" admire Greta.

$$(\forall x)(\text{“x is a hero who wears a cape”} \supset Axg)$$

- x is a hero who wears a cape

$$Hx \mathbin{\&} (\exists y)(Ey \mathbin{\&} Rxy)$$

- Together:

$$(\forall x)((Hx \mathbin{\&} (\exists y)(Ey \mathbin{\&} Rxy)) \supset Axg)$$

“Any” is sometimes existential

- Any (every) cape is worn by a hero:

$$(\forall x)(Ex \supset (\exists y)(Hy \,\&\, Ryx))$$

- No hero wears any cape:

$$(\forall x)(Hx \supset \sim(\exists y)(Ey \,\&\, Rxy))$$

$$\sim(\exists x)(Hx \,\&\, (\exists y)(Ey \,\&\, Rxy))$$

- No hero wears every cape:

$$(\forall x)(Hx \supset \sim(\forall y)(Ey \supset Rxy))$$

$$\sim(\exists x)(Hx \,\&\, (\forall y)(Ey \supset Rxy))$$

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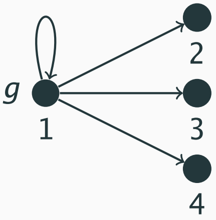
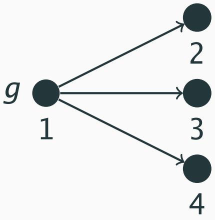
c. The identity predicate

Greta admires everyone (else)



Greta admires everyone.
$(\forall x)\ Agx$



Greta admires everyone **else**.
$(\forall x)$(“x is not Greta” $\supset Agx$)
$(\forall x)(\sim x = g \supset Agx)$

The identity predicate

- A new, special two-place predicate: $=$
  - Written between arguments, **without parentheses**.
  - Needs no mention in symbolization key.
  - Always interpreted the same: extension of '$=$' is all pairs $\langle \alpha, \alpha \rangle$.
- '$a = b$' true iff 'a' and 'b' name one and the same object.
- $x = y$ satisfied by all and only the pairs $\langle \alpha, \alpha \rangle$.
- $\sim x = y$ is satisfied by a pair $\langle \alpha, \beta \rangle$ iff α and β are different objects.

MISTAKES! Ungrammatical expressions with identity

- $x = \sim y$ is not grammatical.

  $\sim$ can only go in front of a formula, and y is not one.

- $\sim(x = y)$ is also not grammatical.

  ‘$(x = y)$’ is also not a formula.

- *Carnap* will not tolerate this nonsense! Take heed!

‘Something else’ and ‘everything else’

- Remember: different variables do NOT entail different objects.
- $(\exists x)(\exists y)\,Axy$ doesn’t mean that someone admires someone else.
- It just means that someone admires someone (possibly themselves).
- To symbolize “someone else” add $\sim x{=}y$:

$$(\exists x)(\exists y)(\sim x{=}y \;\&\; Axy)$$

- $(\forall x)(\forall y)\,Axy$ says that everyone admires everyone (including themselves).
- To symbolize “everyone admires everyone else” add $\sim x{=}y$:

$$(\forall x)(\forall y)(\sim x{=}y \supset Axy)$$

‘Something else’ and ‘everything else’

- The closest quantifier (typically) determines whether you should use $\&$ or $\supset$:

  $(\forall x)(\exists y)(\sim x{=}y \,\&\, Axy)$ vs. $(\exists x)(\forall y)(\sim x{=}y \supset Axy)$

  Everyone admires someone else vs. Someone admires everyone else

- If you have mixed domains, it works the same way:
- Recall predicate ‘Px’: “x is a person”
- Everyone admires someone **else**:

$$(\forall x)(Px \supset (\exists y)((Py \,\&\, \sim x{=}y) \,\&\, Axy))$$

- Someone admires everyone **else**:

$$(\exists x)(Px \,\&\, (\forall y)((Py \,\&\, \sim x{=}y) \supset Axy)$$

Other than, except

- "**Someone other than Greta** is a hero":

  $(\exists x)(\sim x = g \,\&\, Hx)$

- "**Everyone other than Greta** is a hero"; same as:
- "**Everyone except Greta** is a hero":

  $(\forall x)(\sim x = g \supset Hx)$

‘No-one other than’ vs. Singular “only”

- “**No-one other than Greta** is a hero”:

  $\sim(\exists x)(Hx \,\&\, \sim x{=}g)$

  $(\forall x)(Hx \supset x{=}g)$

- “**Only Greta** is a hero”:
- Content: No-one other than Greta is a hero, **AND** Greta is a hero:

  $(\forall x)(Hx \supset x{=}g) \,\&\, Hg$

  $(\forall x)(Hx \equiv x{=}g)$

Uniqueness

- Non–unique: "There is at least one hero":

$$(\exists x)\ Hx$$

- Unique: "There is exactly one hero":
  - There's at least one hero, AND
  - There are no others:

$$(\exists x)\,(Hx \,\&\, {\sim}(\exists y)\,({\sim}y = x \,\&\, Hy))$$
$$(\exists x)\,(Hx \,\&\, (\forall y)(Hy \supset x{=}y))$$

  - Or more succinctly: $(\exists x)(\forall y)(Hy \equiv x{=}y)$

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d. Numerical quantification

Numerical Quantification: n–many as 'at least n'

- Cardinal numbers can be determiners:
  - **Three heroes** wear capes.
- Not always clear if "three heroes" means exactly vs. **at least** three hero
- We'll assume the latter.
  - Do you have two dollars? Yes, I have two dollars. (Uncontroversially true even if you have more than $2)
- Using QL, we can express the following kinds of sentences:
  - **At least** n people are . . .
  - **Exactly** n people are . . .
  - **At most** n people are . . .
- i.e. we can count on QL!

At least n

- At least 1 hero is inspiring:

$$(\exists x)(Hx \,\&\, Ix)$$

- At least 2 heroes are inspiring:

$$(\exists x)(\exists y)(\sim x{=}y \,\&\, ((Hx \,\&\, Ix) \,\&\, (Hy \,\&\, Iy)))$$

- At least 3 heroes are inspiring:

$$(\exists x)(\exists y)(\exists z)\Big((\sim x{=}y \,\&\, (\sim y = z \,\&\, \sim x = z)) \,\&\, \\ ((Hx \,\&\, Ix) \,\&\, ((Hy \,\&\, Iy) \,\&\, (Hz \,\&\, Iz)))\Big)$$

At least n

- There are at least n As, i.e. "$(\exists^{\geq n} x)\, Ax$":

$$
\begin{aligned}
(\exists x_1) \ldots (\exists x_n)\Big((\sim x_1 = x_2 \,\&\, (\sim x_1 = x_3 \,\&\, \ldots \,\&\, (\sim x_1 = x_n \,\&\, \\
(\sim x_2 = x_3 \,\&\, \ldots \,\&\, (\sim x_2 = x_n \,\&\, \\
\ddots \\
\sim x_{n-1} = x_n) \ldots) \,\&\, \\
(Ax_1 \,\&\, (Ax_2 \,\&\, \ldots \,\&\, Ax_n) \ldots)\Big)
\end{aligned}
$$

At least n

- Note: must state that **every pair** of variables is different, e.g.,

$$(\exists x_1)(\exists x_2)(\exists x_3)((\sim x_1 = x_2 \mathbin{\&} \sim x_2 = x_3) \mathbin{\&} (Hx_1 \mathbin{\&} (Hx_2 \mathbin{\&} Hx_3)))$$

  only says "There are at least two heroes"!
  - Take extension of Hx to be: 1, 2
  - Then 1 can play role of x_1 and x_3, 2 role of x_2.
  - Both "$\sim 1 = 2$" and "$\sim 2 = 3$" are true.
- At least n Bs are Cs: substitute '$Bx \mathbin{\&} Cx$' for 'Ax':

$$(\exists^{\geq n} x)(Bx \mathbin{\&} Cx)$$

Exactly one (i.e. Uniqueness; see above)

- There is exactly one hero:

$$(\exists x)(Hx \,\&\, {\sim}(\exists y)(Hy \,\&\, {\sim}x{=}y))$$

- This is equivalent to:

$$(\exists x)(Hx \,\&\, (\forall y)(Hy \supset x{=}y))$$

- In general: "g has property A **uniquely**":

$$Ag \,\&\, (\forall y)(Ay \supset g{=}y)$$

or just: $$(\forall y)(Ay \equiv g{=}y)$$

Exactly n

- There are exactly n As, i.e. "$(\exists^{=n}x)\,Ax$":

$$
\begin{aligned}
(\exists x_1)\ldots(\exists x_n)\Big((\sim x_1 = x_2 \;\&\; (\sim x_1 = x_3 \;\&\; \ldots \;\&\; (\sim x_1 = x_n \;\&\;\\
(\sim x_2 = x_3 \;\&\; \ldots \;\&\; (\sim x_2 = x_n \;\&\;\\
\ddots\\
\sim x_{n-1} = x_n)\ldots) \;\&\;\\
(Ax_1 \;\&\; (Ax_2 \;\&\; \ldots \;\&\; Ax_n)\ldots)) \;\&\;\\
(\forall y)(Ay \supset (y = x_1 \vee \cdots \vee y = x_n))\Big)
\end{aligned}
$$

- Exactly n Bs are Cs:

$$(\exists^{=n}x)(Bx \;\&\; Cx)$$

Exactly n

- There are exactly n As, i.e. "$(\exists^{=n}x)\,Ax$":

$$
\begin{aligned}
(\exists x_1)\ldots(\exists x_n)\Big(&(\sim x_1 = x_2 \,\&\, (\sim x_1 = x_3 \,\&\, \ldots \,\&\, (\sim x_1 = x_n \,\&\\
&(\sim x_2 = x_3 \,\&\, \ldots \,\&\, (\sim x_2 = x_n \,\&\\
&\ddots\\
&\sim x_{n-1} = x_n)\ldots) \,\&\\
&(\forall y)(Ay \equiv (y = x_1 \vee \cdots \vee y = x_n))\Big)
\end{aligned}
$$

- Exactly n Bs are Cs:

$$(\exists^{=n}x)(Bx \,\&\, Cx)$$

At most n

- There are **at most n** As $\Leftrightarrow$ There are **not at least $n+1$** As

$$(\exists^{\leq n} x)\, Ax \Leftrightarrow {\sim}(\exists^{\geq (n+1)} x)\, Ax$$

- For instance: There are at most two heroes:

$${\sim}(\exists x)(\exists y)(\exists z)((Hx \,\&\, (Hy \,\&\, Hz)) \,\&\, ({\sim}x = y \,\&\, ({\sim}x = z \,\&\, {\sim}y = z)))$$

$$(\forall x)(\forall y)(\forall z)((Hx \,\&\, (Hy \,\&\, Hz)) \supset (x = y \lor (x = z \lor y = z)))$$

- ${\sim}(\exists^{\geq (n+1)} x)\, Ax$ is equivalent to:

$$\begin{aligned}(\forall x_1) \ldots (\forall x_{n+1})((Ax_1 \,\&\, \ldots \,\&\, Ax_{n+1}) \supset\\ (x_1 = x_2 \lor (x_1 = x_3 \lor \cdots \lor (x_1 = x_{n+1} \lor\\ (x_2 = x_3 \lor \cdots \lor (x_2 = x_{n+1} \lor\\ \ddots\\ x_n = x_{n+1}) \ldots)))\end{aligned}$$

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e. Both ‘both’ and ‘neither’

Schematizing ‘Both’

- “Both heroes inspire”: this means that
  There are **exactly 2** heroes, and both inspire:

$$(\exists x)(\exists y)\Big(((\sim x{=}y \,\&\, (Hx \,\&\, Hy)) \,\&\\ (\forall z)(Hz \supset (z = x \vee z = y))) \,\&\\ (Ix \,\&\, Iy)\Big)$$

- Note: “Both heroes inspire” implies “There are exactly two inspiring heroes”, but not vice versa!
- e.g. if there are exactly two inspiring heros and one (or more) not–inspiring hero(s)

Schematizing ‘Neither’

- “Neither hero inspires”: this means that

  There are **exactly 2** heroes, and neither of them inspires:

$$(\exists x)(\exists y)\Big(\big((\sim x{=}y \,\&\, (Hx \,\&\, Hy)) \,\&\, (\forall z)(Hz \supset (z = x \vee z = y))\big) \,\&\, (\sim Ix \,\&\, \sim Iy)\Big)$$

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f. ‘The’ Definite Description

Definite descriptions

- Definite description: **the so-and-so**
- Russell's analysis of definite description: to say

  "The *A* is B"

  is to say:
- There is something, which:
  - is A,
  - is the **only** A (i.e. the unique thing that is A),
  - is B.
- In QL:

$$(\exists x)(Ax \mathbin{\&} (\forall y)(Ay \supset x = y) \mathbin{\&} Bx)$$

- or more succinctly:

$$(\exists x)(\forall y)((Ay \equiv x = y) \mathbin{\&} Bx)$$

Example: ‘The author of Waverley is blah’

- Schematize “The author of *Waverley* is Scottish”:
- Use the following symbolization key:
- Ax: x is an author; Wxz: x wrote z; Sx: x is Scottish; ℓ: *Waverley*

$$(\exists x)(Ax \,\&\, Wx\ell \,\&\, (\forall y)((Ay \,\&\, Wy\ell) \supset x{=}y) \,\&\, Sx)$$

"The" vs. "exactly one"

- Compare:
  1. The hero inspires:

  $$(\exists x)(Hx \,\&\, (\forall y)(Hy \supset x{=}y) \,\&\, Ix)$$

  2. There is exactly one inspiring hero:

  $$(\exists x)(Hx \,\&\, Ix \,\&\, (\forall y)((Hy \,\&\, Iy) \supset x{=}y))$$

- (2) can be true without (1), but not vice versa.
- (Namely when there is exactly one inspiring hero, but also a non-inspiring hero.)
- So (1) entails (2), but not vice versa.

Strawson's analysis (presuppositional theories)

- According to Russell, "The hero wears a cape" is **false** if there is no hero, or if there is more than one.
- Consider: "the present King of France is bald."
- P. F. Strawson disagrees with these truth conditions. Rather, we only succeed in making a statement if there is a unique hero (or a unique king of France).
- "There is a unique hero" is not part of what is **said** by a definite description, but is only presupposed.

Singular possessive (a definite description)

- Singular possessives form noun phrases, e.g., "Joe's cape"
- They work like definite descriptions:
  Joe's cape is the cape Joe owns. E.g.:
  - "Autumn wears Joe's cape" symbolizes the same as:
    "Autumn wears the cape that Joe owns":

$$(\exists x)\Big(\big((Ex \,\&\, Ojx) \,\&\, (\forall y)((Ey \,\&\, Ojy) \supset x{=}y)\big) \,\&\, Wax\Big)$$

Singular vs. plural possessive

- Compare **plural** possessives: those are '$\forall$'s':
  - "Autumn wears Joe's cape**s**" symbolizes the same as:

    "Autumn wears every cape that Joe owns":

$$(\forall x)((Ex \,\&\, Ojx) \supset Wax)$$

- So plural possessives are NOT definite descriptions.

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g. Using quantifiers to express properties

Our symbolization key

Domain: people alive in 2022 and items of clothing

a: Autumn

g: Greta

Px: $\_\_\_\_\__x$ is a person

Lx: $\_\_\_\_\__x$ is an item of clothing.

Ex: $\_\_\_\_\__x$ is a cape

Rxy: $\_\_\_\_\__x$ wears $\_\_\_\_\__y$

Hx: $\_\_\_\_\__x$ is a hero

Ix: $\_\_\_\_\__x$ inspires

Yxy: $\_\_\_\_\__x$ is younger than $\_\_\_\_\__y$

Axy: $\_\_\_\_\__x$ admires $\_\_\_\_\__y$

Oxy: $\_\_\_\_\__x$ owns $\_\_\_\_\__y$

Expressing properties, revisited

- One-place predicates express properties, e.g.,

  Hx expresses property “being a hero”

- Combinations of predicates (with connectives, names) can express derived properties, e.g.,

  Axg expresses “x admires Greta”

  $Hx \,\&\, Cx$ expresses “x is a hero who wears a cape”

- Using quantifiers, we can express even more complex properties, e.g.,

  $(\exists y)(Py \,\&\, Axy)$ expresses “x admires someone”

Finding, using properties expressed

- If you can say it for Greta, you can say it for x.
  - Greta admires a hero.
    $(\exists y)(Hy \,\&\, Agy)$
  - x admires a hero.
    $(\exists y)(Hy \,\&\, Axy)$
- If you can say it for x, you can say it for Greta.
  - x wears a cape.
    $(\exists y)(Ey \,\&\, Rxy)$
  - Greta wears a cape.
    $(\exists y)(Ey \,\&\, Rgy)$

Ex: $\_\_\_\__x$ is a cape
Rxy: $\_\_\_\__x$ wears $\_\_\_\__y$

Examples

- x wears a cape.
  $(\exists y)(Ey \,\&\, Rxy)$
- x is admired by everyone.
  $(\forall y)(Py \supset Ayx)$
- x admires a hero.
  $(\exists y)(Hy \,\&\, Axy)$
- x admires only heroes.
  $(\forall y)(Axy \supset Hy)$
- x is unclothed (i.e. naked).
  $\sim(\exists y)(Ly \,\&\, Rxy)$
  $(\forall y)(Ly \supset \sim Rxy)$

Px	$\_\_\_\__x$ is a person	Lx	$\_\_\_\__x$ is an item of clothing
Ex	$\_\_\_\__x$ is a cape	Rxy	$\_\_\_\__x$ wears $\_\_\_\__y$

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h. Multiple determiners: worked example

Mary Astell, 1666–1731

- British political philosopher
- *Some Reflections upon Marriage* (1700)
- In preface to 3rd ed. 1706 reacts to William Nicholls' claim (in *The Duty of Inferiors towards their Superiors, in Five Practical Discourses* (London 1701), Discourse IV: The Duty of Wives to their Husbands), that women are naturally inferior to men.

Astell TL;DR

- What can Nicholls possibly mean by "women are naturally inferior to men"?
- It can't be that some woman is inferior to some man, since that's "no great discovery."
- After all, surely some men are inferior to some women.
- The obviously intended meaning must be: **all** women are inferior to **all** men.
- But that can't be right, for then "the greatest Queen ought not to command but to obey her Footman."
- It can't even be just: **all** women are inferior to **some** men.
- Since "had they been pleased to remember their Oaths of Allegiance and Supremacy, they might have known that *One* Woman is superior to *All* the Men in these Nations."

Symbolizing Astell

- Some woman is superior to every man
- Some woman satisfies "x is superior to every man"

$$(\exists x)(Wx \,\&\, \text{“}x \text{ is superior to every man”})$$

- x is superior to every man

$$(\forall y)(My \supset Sxy)$$

- Together:

$$(\exists x)(Wx \,\&\, (\forall y)(My \supset Sxy))$$

Formalizing Astell

- Some woman is superior to some man.

  $(\exists x)(Wx \,\&\, (\exists y)(My \,\&\, Sxy))$

- Every woman is superior to every man.

  $(\forall x)(Wx \supset (\forall y)(My \supset Sxy))$

- Every woman is superior to some man.

  $(\forall x)(Wx \supset (\exists y)(My \,\&\, Sxy))$

- Some woman is superior to every man.

  $(\exists x)(Wx \,\&\, (\forall y)(My \supset Sxy))$

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i. Quantifier scope ambiguity

More scope ambiguity

- "Autumn and Greta admire Isra or Luisa."
- Two logically distinct, natural readings:

1) Autumn admires Isra or Luisa, **and** so does Greta.

$$(Aai \lor Aal)\ \&$$
$$(Agi \lor Agl)$$

2) Autumn and Greta both admire Isra, or they both admire Luisa.

$$(Aai\ \&\ Agi) \lor$$
$$(Aal\ \&\ Agl)$$

Negation and the quantifiers

- "All heroes don't inspire"
  - Denial of "all heroes inspire". Ask: "Do all heroes inspire (Answer: No, *it's not the case that* all heroes inspire ")

$$\sim(\forall x)(Hx \supset Ix)$$
$$(\exists x)(Hx \,\&\, \sim Ix)$$

  - All heroes are not inspiring, i.e., No heroes inspire

$$(\forall x)(Hx \supset \sim Ix)$$
$$\sim(\exists x)(Hx \,\&\, Ix)$$

Multiple quantifiers and ambiguity

- "All heroes wear a cape"
  - "A cape" in the scope of "all heroes", i.e., "For every hero, there is a cape they wear"

$$(\forall x)(Hx \supset (\exists y)(Ey \,\&\, Rxy))$$

$$(\forall x)(\exists y)(Hx \supset (Ey \,\&\, Rxy))$$

  - "All heroes" in scope of "a cape", i.e., "There is a cape which every hero wears"

$$(\exists y)(Ey \,\&\, (\forall x)(Hx \supset Rxy))$$

$$(\exists y)(\forall x)(Ey \,\&\, (Hx \supset Rxy))$$

- A (probably bad) joke: "Every day, a tourist is mugged on the streets of New York. He's going through a lot of wallets."

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j. Donkey sentences

Happy farmers

"Every farmer who owns a donkey is happy"

- Step-by-step symbolization: "All *A*s are *B*s"
- x is a farmer who owns a donkey ...

$$Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)$$

- Every farmer who owns a donkey is happy

$$(\forall x)((Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)) \supset Hx)$$

- Notice how 'a donkey' is bound by an existential here

Unhappy donkeys :(

"Every farmer who owns a donkey beats it"

- Step-by-step symbolization: "All As are Bs"
- x is a farmer who owns a donkey . . .

$$Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)$$

- Every farmer who owns a donkey beats it:

$$(\forall x)((Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)) \supset Bxy)$$

- PROBLEM: 'y' is unbound! So this is not a QL sentence. Gasp!

Save the donkeys: a failed attempt

- This was our problem: a donkey lay beaten and unbound:

$$(\forall x)((Fx \,\&\, (\exists y)(Dy \,\&\, Oxy)) \supset Bxy)$$

- Can we simply extend the scope of the existential?

$$(\forall x)(\exists y)((Fx \,\&\, (Dy \,\&\, Oxy)) \supset Bxy)$$

- ‘y’ is now bound, but alas, this sentence is trivially true:
- Provided at least one thing in our UD is not a donkey, that thing makes the antecedent of the conditional false, making the conditional trivially true, for any *x*.
  In particular, our farmer is not a donkey.
  But he still sounds like kind of a jack@$$!

Symbolizing donkey sentences

"Every farmer who owns a donkey beats it"

- When is it false that every farmer who owns a donkey beats it? If there's a farmer who owns a donkey but doesn't beat it. Deny that!

$$\sim(\exists x)(Fx \,\&\, (\exists y)(Dy \,\&\, Oxy \,\&\, \sim Bxy))$$

- For every farmer and every donkey they own: the farmer beats the donkey.

$$(\forall x)(\forall y)((Fx \,\&\, (Dy \,\&\, Oxy)) \supset Bxy)$$

- Every farmer beats every donkey they own.

$$(\forall x)(Fx \supset (\forall y)((Dy \,\&\, Oxy) \supset Bxy))$$

- But what about the case where at least one farmer with a donkey beats only one of his donkeys? #Quitting