Problem Set 5 (24.241 Symbolic Logic)

Due Fri. **October 14th** by **5pm** Eastern

Please scan and upload to Canvas as a pdf; feel free to *also* turn in a paper copy to Philosophy Dept on 8th floor Stata Center, Dreyfoos-wing

Answer FOUR questions total: 1 and 2, 3 Xor 4, 5 Xor 6 (exclusive or's!)

Question 0: if you worked with up to two classmates, please list their names!

1. Consider the argument with premise set $\{\Phi, \Psi\}$ and conclusion Θ. Suppose that you are able to make a tree for this argument in which all branches close, even though you make no use of Φ and Ψ: i.e. you do not resolve these sentences, and when you close a branch, it is never because it contains Φ or because it contains Ψ. What is the most informative semantic property you can ascribe to Θ (you can take for granted that our system is sound and complete)?
   (feel free to use symbols 'P', 'Q', and conclusion 'C' if you don't feel like writing Greek letters or you worry about your Greek handwriting!)

2. Prove that in a sound and complete tree system, no argument G is both tree-valid and tree-invalid. Let 'G' be an arbitrary argument with premise set Γ and conclusion Θ.

   (*Hints*: Show that if an argument is tree-invalid, then it is not tree-valid. To do this, apply **a key fact** coming from our proof of completeness, and then apply the soundness result. Alternatively, you could do a proof by contradiction, using the same results.)

Pick **one** of questions 3 or 4 to answer (then PROCEED TO 2nd PAGE!):

3. Briefly explain why we could augment our nine tree rules with their 'syntactic equivalents' and not get into trouble with our Soundness and Completeness results (e.g. swapping the order of branches in the 'splitting rules', or the order of sentences in the 'stacking rules'). Your brief argument should note both (i) why our system would remain Sound ('reasoning from the top-down') and (ii) why our system would remain Complete ('reasoning from the bottom-up').
   NB: I'm *not* asking you to formally extend the Soundness and Completeness proofs to include these additional $6 + 7 \times 2 = 20$ rules. But if you prefer to make it really concrete, feel free to just focus on a disjunction rule where the right disjunct is placed on the left branch, and the left disjunct is placed on the right branch of our new node.

4. Our textbook's Chapter 5 does not set-up the inductive proof for completeness properly. In lecture, we showed how to properly set-up the proof for soundness.
   Do the same for completeness noting (i) what you are doing induction over;
   (ii) the base case(s); (iii) the induction hypothesis;
   (iv) what you would need to show in the induction step.
   NB: I'm not asking you to actually re-do the proof, just to set it up!
   Warning: this problem seems considerably trickier than #3 to answer completely correctly.

Pick **one** of questions 5 or 6 to answer; do parts (a) AND (b):

What follows are two modifications to our SL tree system. For each, imagine a system STD* exactly like our system STD, except for the single indicated change.

(a) Would the modified tree system be sound? If so, explain how to extend our inductive soundness proof to a system with this rule; if not, give a tree that is a counterexample to the soundness of STD*.

(b) Would the modified tree system be complete? If so, explain how to extend our inductive completeness proof to a system with this rule; if not, give a tree that is a counterexample to the completeness of STD*.

5. *Crunk Conditional* (C$\supset$)

$$\begin{array}{llll}
\text{m.} & \multicolumn{2}{c}{\mathit{\Phi} \supset \mathit{\Psi}} & \\
\vdots & \multicolumn{2}{c}{\vdots} & \\
\text{j.} & {\sim}\mathit{\Phi} \vee \mathit{\Psi} & \mathit{\Psi} & \text{m C}\supset \\
\text{j+1.} & & \Theta &
\end{array}$$

Note that Θ is an arbitrary wff of SL

6. *Negligent Negated Conditional* (N$\sim\supset$)

$$\begin{array}{lllll}
\text{m.} & \multicolumn{3}{c}{{\sim}(\mathit{\Phi} \supset \mathit{\Psi})} & \\
\vdots & \multicolumn{3}{c}{\vdots} & \\
\text{j.} & \mathit{\Phi} & {\sim}\mathit{\Psi} & (\mathit{\Phi} \,\&\, {\sim}\mathit{\Psi}) \,\&\, (P \vee {\sim}P) & \text{m N}{\sim}\supset
\end{array}$$