Regimentation and Relations

LOGIC I
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Restricting Quantifiers

Universals Quantifiers: Regiment the following sentences:

- All dogs go to heaven.
- Jim took every chance he got.
- All the monkeys that Amar loves love him back.
- Everyone who trained hard or got lucky made it to the top or else didn't compete.

Hidden Quantifiers: Regiment the following sentences:

- At least the guests that remained were pleased with the party.
- I haven't met a cat that likes Merra.
- Kiko's only friends are animals.

Existential Quantifiers: Regiment the following sentences:

- Something great is around the corner.
- One of Ken's statues is very old.
- Kate found a job that she loved.

Mixed Quantifiers

1. Nothing is without imperfections.
2. Every dog has its day.
3. Everyone loves someone.
4. Nobody knows everybody.
5. Everybody everybody loves loves somebody.
6. No set is a member of itself.
7. There is a set with no members.

Arguments

Love: Regiment the following argument:

- Cam doesn't love anyone who loves him back.
- May loves everyone who loves themselves.

$\therefore$ If Cam loves himself, he doesn't love May.

Bigger: Regiment the following argument:

- Whenever something is bigger than another, the latter is not bigger than the former.

$\therefore$ Nothing is bigger than itself.

Relations

Domain: Let the *domain* D be any set.

Relation: A *relation* R on D is any subset of D^2.

Reflexive: A relation R is *reflexive* on D *iff* $\langle x, x\rangle \in R$ for all $x \in D$.

Non-Reflexive: A relation R is *non-reflexive* on D *iff* R is not reflexive on D.

Question 1: What is it to be *irreflexive*?

Irreflexive: A relation R is *irreflexive* on D *iff* $\langle x, x\rangle \notin R$ for all $x \in D$.

Symmetric: A relation R is *symmetric iff* $\langle y, x\rangle \in R$ whenever $x, y \in R$.

Question 2: Why don't we need to specify a domain?

Question 3: Why is a relation reflexive or irreflexive with respect to a domain?

Asymmetric: A relation R is *asymmetric iff* $\langle y, x\rangle \notin R$ whenever $\langle x, y\rangle \in R$.

Question 4: What is it to be non-symmetric? How about non-asymmetric?

Task 1: Show that every asymmetric relation is irreflexive.

Transitive: A relation R is *transitive iff* $\langle x, z\rangle \in R$ whenever $\langle x, y\rangle, \langle y, z\rangle \in R$.

Intransitive: A relation R is *intransitive iff* $\langle x, z\rangle \notin R$ whenever $\langle x, y\rangle, \langle y, z\rangle \in R$.

Question 5: Is every symmetric transitive relation reflexive? (No: $R = \varnothing$)

Task 2: Show that every transitive irreflexive relation asymmetric?

Euclidean: A relation R is *euclidean iff* $\langle y, z\rangle \in R$ whenever $\langle x, y\rangle, \langle x, z\rangle \in R$.

Task 3: Show that every transitive symmetric relation is euclidean.