11. Identity

1. Identity

1.1 The identity predicate

1.2 Numerical quantification

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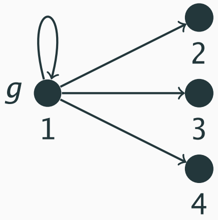
11. Identity

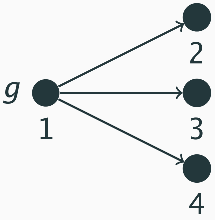
a. The identity predicate

Greta admires everyone (else)



Greta admires everyone.

$(\forall x)\, Agx$



Greta admires everyone **else**.

$(\forall x)(\text{“}x \text{ is not Greta”} \supset Agx)$

$(\forall x)(\sim x = g \supset Agx)$

The identity predicate

- A new, special two-place predicate: $=$
  - Written between arguments, **without parentheses**.
  - Needs no mention in symbolization key.
  - Always interpreted the same: extension of '$=$' is all pairs $\langle \alpha, \alpha \rangle$.
- '$a = b$' true iff 'a' and 'b' name one and the same object.
- $x = y$ satisfied by all and only the pairs $\langle \alpha, \alpha \rangle$.
- $\sim x = y$ is satisfied by a pair $\langle \alpha, \beta \rangle$ iff α and β are different objects.

MISTAKES! Ungrammatical expressions with identity

- $x = \sim y$ is not grammatical.

  $\sim$ can only go in front of a formula, and y is not one.

- $\sim(x = y)$ is also not grammatical.

  ‘$(x = y)$’ is also not a formula.

- *Carnap* will not tolerate this nonsense! Take heed!

'Something else' and 'everything else'

- Remember: different variables do NOT entail different objects.
- $(\exists x)(\exists y)\,Axy$ doesn't mean that someone admires someone else.
- It just means that someone admires someone (possibly themselves).
- To symbolize "someone else" add $\sim x{=}y$:

$$(\exists x)(\exists y)(\sim x{=}y \,\&\, Axy)$$

- $(\forall x)(\forall y)\,Axy$ says that everyone admires everyone (including themselves).
- To symbolize "everyone admires everyone else" add $\sim x{=}y$:

$$(\forall x)(\forall y)(\sim x{=}y \supset Axy)$$

‘Something else’ and ‘everything else’

- The closest quantifier (typically) determines whether you should use & or $\supset$:

$$(\forall x)(\exists y)(\sim x{=}y \,\&\, Axy) \quad \text{vs.} \quad (\exists x)(\forall y)(\sim x{=}y \supset Axy)$$

Everyone admires someone else vs. Someone admires everyone else

- If you have mixed domains, it works the same way:
- Recall predicate ‘Px’: “x is a person”
- Everyone admires someone **else**:

$$(\forall x)(Px \supset (\exists y)((Py \,\&\, \sim x{=}y) \,\&\, Axy))$$

- Someone admires everyone **else**:

$$(\exists x)(Px \,\&\, (\forall y)((Py \,\&\, \sim x{=}y) \supset Axy)$$

Other than, except

- "**Someone other than Greta** is a hero":

$$(\exists x)(\sim x = g \mathbin{\&} Hx)$$

- "**Everyone other than Greta** is a hero"; same as:
- "**Everyone except Greta** is a hero":

$$(\forall x)(\sim x = g \supset Hx)$$

‘No-one other than’ vs. Singular “only”

- “**No-one other than Greta** is a hero”:

  $\sim(\exists x)(Hx \,\&\, \sim x{=}g)$

  $(\forall x)(Hx \supset x{=}g)$

- “**Only Greta** is a hero”:

- Content: No-one other than Greta is a hero, **AND** Greta is a hero:

  $(\forall x)(Hx \supset x{=}g) \,\&\, Hg$

  $(\forall x)(Hx \equiv x{=}g)$

Uniqueness

- Non–unique: “There is at least one hero”:

$$(\exists x)\ Hx$$

- Unique: “There is exactly one hero”:
    - There’s at least one hero, AND
    - There are no others:

$$(\exists x)\,(Hx \,\&\, \sim(\exists y)\,(\sim y = x \,\&\, Hy))$$
$$(\exists x)\,(Hx \,\&\, (\forall y)(Hy \supset x{=}y))$$

    - Or more succinctly: $(\exists x)(\forall y)(Hy \equiv x{=}g)$

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b. Numerical quantification

Numerical Quantification: *n*–many as 'at least *n*'

- Cardinal numbers can be determiners:
  - **Three heroes** wear capes.
- Not always clear if "three heroes" means exactly vs. **at least** three hero
- We'll assume the latter.
  - Do you have two dollars? Yes, I have two dollars. (Uncontroversially true even if you have more than $2)
- Using QL, we can express the following kinds of sentences:
  - **At least** *n* people are ...
  - **Exactly** *n* people are ...
  - **At most** *n* people are ...
- i.e. we can count on QL!

At least n

- At least 1 hero is inspiring:

$$(\exists x)(Hx \mathbin{\&} Ix)$$

- At least 2 heroes are inspiring:

$$(\exists x)(\exists y)(\sim x{=}y \mathbin{\&} ((Hx \mathbin{\&} Ix) \mathbin{\&} (Hy \mathbin{\&} Iy)))$$

- At least 3 heroes are inspiring:

$$(\exists x)(\exists y)(\exists z)\Big((\sim x{=}y \mathbin{\&} (\sim y = z \mathbin{\&} \sim x = z)) \mathbin{\&} \\ ((Hx \mathbin{\&} Ix) \mathbin{\&} ((Hy \mathbin{\&} Iy) \mathbin{\&} (Hz \mathbin{\&} Iz)))\Big)$$

At least n

- There are at least n As, i.e. "$(\exists^{\geq n} x)\, Ax$":

$$
\begin{aligned}
(\exists x_1) \ldots (\exists x_n)\Big((\sim x_1 = x_2 \,\&\, (\sim x_1 = x_3 \,\&\, \ldots \,\&\, (\sim x_1 = x_n \,\&\, \\
(\sim x_2 = x_3 \,\&\, \ldots \,\&\, (\sim x_2 = x_n \,\&\, \\
\ddots \\
\sim x_{n-1} = x_n) \ldots) \,\&\, \\
(Ax_1 \,\&\, (Ax_2 \,\&\, \ldots \,\&\, Ax_n) \ldots)\Big)
\end{aligned}
$$

At least n

- Note: must state that **every pair** of variables is different, e.g.,

$$(\exists x_1)(\exists x_2)(\exists x_3)((\sim x_1 = x_2 \mathbin{\&} \sim x_2 = x_3) \mathbin{\&} \\ (Hx_1 \mathbin{\&} (Hx_2 \mathbin{\&} Hx_3)))$$

  only says "There are at least two heroes"!
  - Take extension of Hx to be: 1, 2
  - Then 1 can play role of x_1 and x_3, 2 role of x_2.
  - Both "$\sim 1 = 2$" and "$\sim 2 = 3$" are true.
- At least n Bs are Cs: substitute '$Bx \mathbin{\&} Cx$' for 'Ax':

$$(\exists^{\geq n} x)(Bx \mathbin{\&} Cx)$$

Exactly one

- There is exactly one hero:

$$(\exists x)(Hx \,\&\, {\sim}(\exists y)(Hy \,\&\, {\sim}x{=}y))$$

- This is equivalent to:

$$(\exists x)(Hx \,\&\, (\forall y)(Hy \supset x{=}y))$$

- In general: "g has property A **uniquely**":

$$Ag \,\&\, (\forall y)(Ay \supset g{=}y)$$

or just: $$(\forall y)(Ay \equiv g{=}y)$$

Exactly n

- There are exactly n As, i.e. "$(\exists^{=n}x)\, Ax$":

$$
\begin{aligned}
(\exists x_1) \ldots (\exists x_n)\Big((\sim x_1 = x_2 \;\&\; (\sim x_1 = x_3 \;\&\; \ldots \;\&\; (\sim x_1 = x_n \;\&\; \\
(\sim x_2 = x_3 \;\&\; \ldots \;\&\; (\sim x_2 = x_n \;\&\; \\
\ddots \\
\sim x_{n-1} = x_n) \ldots) \;\&\; \\
(Ax_1 \;\&\; (Ax_2 \;\&\; \ldots \;\&\; Ax_n) \ldots)) \;\&\; \\
(\forall y)(Ay \supset (y = x_1 \vee \cdots \vee y = x_n))\Big)
\end{aligned}
$$

- Exactly n Bs are Cs:

$$(\exists^{=n}x)(Bx \;\&\; Cx)$$

Exactly n

- There are exactly n As, i.e. "$(\exists^{=n}x)\,Ax$":

$$
\begin{aligned}
(\exists x_1)\ldots(\exists x_n)\Big((\sim x_1 = x_2\ \&\ (\sim x_1 = x_3\ \&\ \ldots\ \&\ (\sim x_1 = x_n\ \&\\
(\sim x_2 = x_3\ \&\ \ldots\ \&\ (\sim x_2 = x_n\ \&\\
\ddots\\
\sim x_{n-1} = x_n)\ldots)\ \&\\
(\forall y)(Ay \equiv (y = x_1 \vee \cdots \vee y = x_n))\Big)
\end{aligned}
$$

- Exactly n Bs are Cs:

$$(\exists^{=n}x)(Bx\ \&\ Cx)$$

At most n

- There are **at most n** As $\Leftrightarrow$ There are **not at least $n+1$** As

$$(\exists^{\leq n} x)\, Ax \Leftrightarrow {\sim}(\exists^{\geq(n+1)} x)\, Ax$$

- For instance: There are at most two heroes:

$${\sim}(\exists x)(\exists y)(\exists z)((Hx \,\&\, (Hy \,\&\, Hz)) \,\&\, ({\sim}x = y \,\&\, ({\sim}x = z \,\&\, {\sim}y = z)))$$

$$(\forall x)(\forall y)(\forall z)((Hx \,\&\, (Hy \,\&\, Hz)) \supset (x = y \vee (x = z \vee y = z)))$$

- ${\sim}(\exists^{\geq(n+1)} x)\, Ax$ is equivalent to:

$$\begin{aligned}(\forall x_1) \ldots (\forall x_{n+1})((Ax_1 \,\&\, \ldots \,\&\, Ax_{n+1}) \supset \\ (x_1 = x_2 \vee (x_1 = x_3 \vee \cdots \vee (x_1 = x_{n+1} \vee \\ (x_2 = x_3 \vee \cdots \vee (x_2 = x_{n+1} \vee \\ \ddots \\ x_n = x_{n+1}) \ldots)))\end{aligned}$$

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c. Both ‘both’ and ‘neither’

Schematizing 'Both'

- "Both heroes inspire": this means that

  There are **exactly 2** heroes, and both inspire:

$$
\begin{aligned}
(\exists x)(\exists y)\Big(((\sim x{=}y \;\&\; (Hx \;\&\; Hy)) \;\&\\
(\forall z)(Hz \supset (z = x \vee z = y))) \;\&\\
(Ix \;\&\; Iy)\Big)
\end{aligned}
$$

- Note: "Both heroes inspire" implies "There are exactly two inspiring heroes", but not vice versa!

Schematizing ‘Neither’

- “Neither hero inspires”: this means that

  There are **exactly 2** heroes, and neither of them inspires:

$$
\begin{aligned}
(\exists x)(\exists y)\Big(((\sim x{=}y \;\&\; (Hx \;\&\; Hy)) \;\& \\
(\forall z)(Hz \supset (z = x \vee z = y))) \;\& \\
(\sim Ix \;\&\; \sim Iy)\Big)
\end{aligned}
$$

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d. ‘The’ Definite Description

Definite descriptions

- Definite description: **the so-and-so**
- Russell's analysis of definite description: to say

"The A is B"

  is to say:
- There is something, which:
  - is A,
  - is the only A,
  - is B.
- In QL:

$$(\exists x)(Ax \,\&\, (\forall y)(Ay \supset x{=}y) \,\&\, Bx)$$

- or more succinctly:

$$(\exists x)((\forall y)(Ay \equiv x{=}y) \,\&\, Bx)$$

“The” vs. “exactly one”

- Compare:
  1. The hero inspires:

$$(\exists x)(Hx \,\&\, (\forall y)(Hy \supset x{=}y) \,\&\, Ix)$$

  2. There is exactly one inspiring hero:

$$(\exists x)(Hx \,\&\, (\forall y)((Hy \,\&\, Iy) \supset x{=}y) \,\&\, Ix)$$

- (2) can be true without (1), but not vice versa.
- (Namely when there is exactly one inspiring hero, but also a non-inspiring hero.)
- So (1) entails (2), but not vice versa.

Strawson's analysis

- According to Russell, "The hero wears a cape" is **false** if there is no hero, or if there is more than one.
- P. F. Strawson disagrees with these truth conditions Nevertheless, we only succeed in making a statement if there is a unique hero.
- "There is a unique hero" is not part of what is **said** by a definite description, but is only presupposed.

Singular possessive

- Singular possessives make noun phrases, e.g., "Joe's cape"
- They work like definite descriptions: Joe's cape is the cape Joe owns. E.g.:
  - "Autumn wears Joe's cape" symbolizes the same as: "Autumn wears the cape Joe owns":

$$(\exists x)\Big(((Ex \,\&\, Ojx) \,\& \\ (\forall y)((Ey \,\&\, Ojy) \supset x{=}y)) \,\& \\ Wax\Big)$$

Singular vs. plural possessive

- Compare **plural** possessives: those are $\forall$'s:
  - "Autumn wears Joe's capes" symbolizes the same as:

    "Autumn wears every cape that Joe owns":

$$(\forall x)\big((Ex \,\&\, Ojx) \supset Wax\big)$$