Problem Set 4 Solutions

Keep it secret! Keep it safe!

Note that I ought to have line justifications next to the '×' symbol for closed branches. But alas, I was lazy! I appreciate that most of you included these justifications in your own work!

For questions 1 and 2:
(i) Schematize the following argument into the language of sentential logic.
(ii) Then, investigate its validity using the tree method (STD):

1. "If the lawyer did it, then the doctor did not. Therefore, if the doctor did it, then the lawyer did not."

    - Symbolization Key: B = the lawyer did it; G = the doctor did it

    Solution:

    Schematization: $B \supset {\sim}G$. Therefore $G \supset {\sim}B$

    $B \supset {\sim}G \vdash_{STD} G \supset {\sim}B$

    1. $B \supset {\sim}G$ ✓ Assumption
    2. ${\sim}(G \supset {\sim}B)$ ✓ ∼Conclusion
    3. G 2 ∼⊃
    4. ${\sim}{\sim}B$
    5. ${\sim}B$ ${\sim}G$ 1 ⊃

    × ×

    4, 5 3, 5

    Upshot: since each branch closes, the argument is valid.

2. "If naïve realism is true, then naïve realism is false. Therefore, naïve realism is false."

    Solution:

    Schematization: $N \supset {\sim}N$. Therefore: ${\sim}N$

    $N \supset {\sim}N \vdash_{STD} {\sim}N$

    1. $N \supset {\sim}N$ ✓ Assumption
    2. ${\sim}{\sim}N$ ∼Conclusion
    3. ${\sim}N$ ${\sim}N$ 1 ⊃

    × ×

    2, 3 2, 3

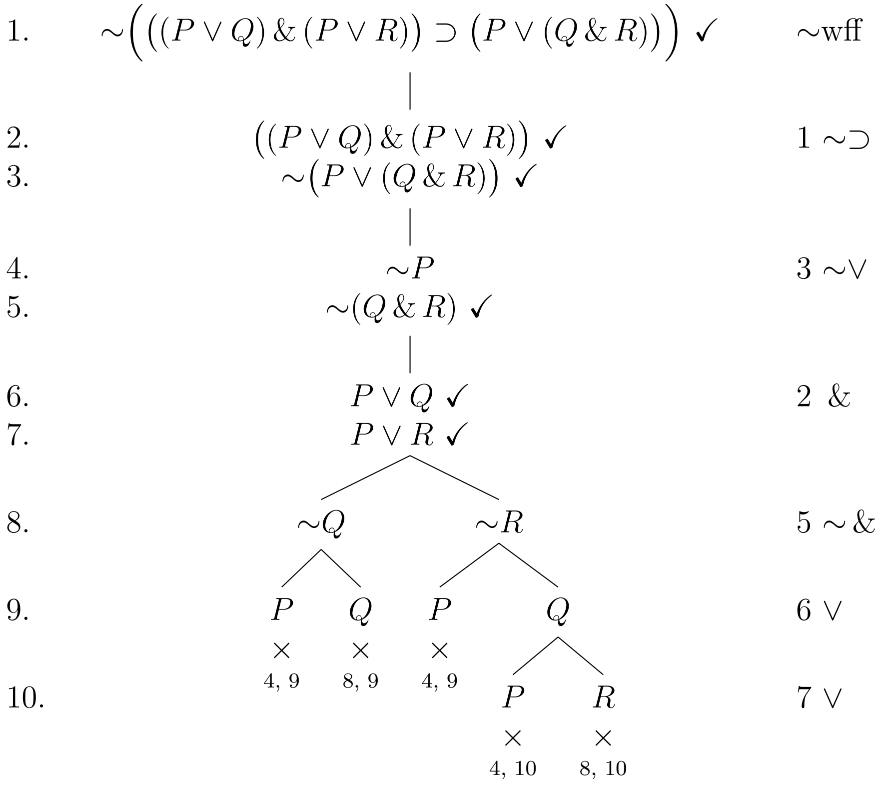
    Upshot: since each branch closes, the argument is valid.

3. Show via the tree method that the following is a tautology:

$$\big((P \vee Q) \,\&\, (P \vee R)\big) \supset \big(P \vee (Q \,\&\, R)\big)$$

Solution (albeit not the most efficient tree):

$\vdash_{STD} \big((P \vee Q) \,\&\, (P \vee R)\big) \supset \big(P \vee (Q \,\&\, R)\big)$:



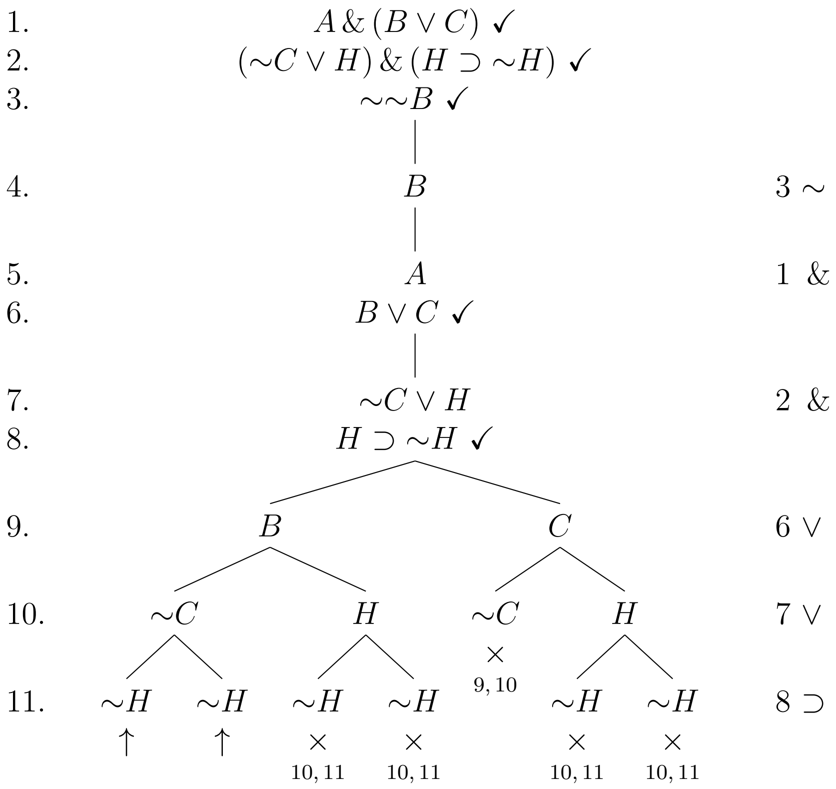
Upshot: since each branch closes, the sentence is a tautology. Our tree shows that the negation of this sentence is unsatisfiable, i.e. a contradiction.

4. Test the following argument for validity using the tree method (STD):

$$\begin{array}{r} A \,\&\, (B \vee C) \\ (\sim C \vee H) \,\&\, (H \supset \sim H) \\ -\,-\,-\,-\,-\,-\,-\,-\,-\,-\,- \\ \therefore \sim B \end{array}$$

Solution (albeit again, a more elegant tree is to be had:

$\{A \,\&\, (B \vee C), (\sim C \vee H) \,\&\, (H \supset \sim H)\}?? \vdash_{STD} ??\sim B$:



Upshot: the argument is invalid since there is a complete open branch. Hence, we can make the premises true and the conclusion false by assigning True to A and B and False to C and H.

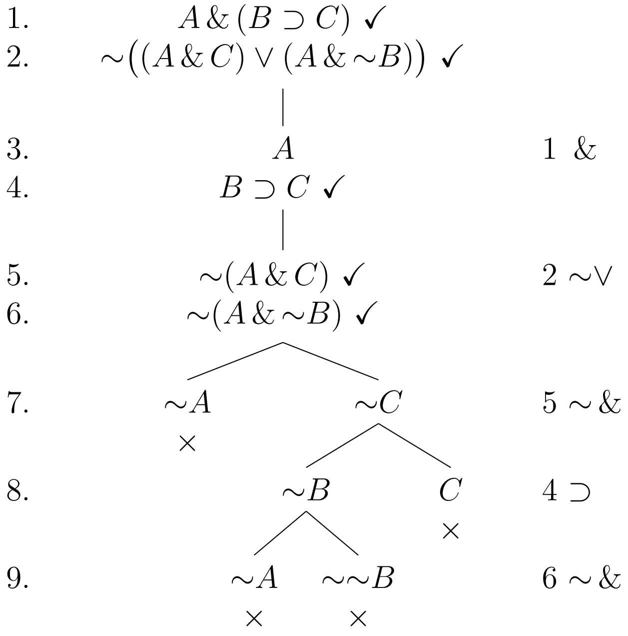
N.B.: we could have stopped making the tree as soon as we reached a complete open branch (e.g. the branch with the leftmost ∼H). So in completing the tree, I have shown more work than necessary.

5. Test the following argument for validity using the tree method (STD):

$$\begin{array}{r} A \,\&\, (B \supset C) \\ \text{— — — — — — — — — —} \\ \therefore\ (A \,\&\, C) \vee (A \,\&\, {\sim}B) \end{array}$$

Solution

$A \,\&\, (B \supset C) \vdash_{STD} (A \,\&\, C) \vee (A \,\&\, {\sim}B)$:



Upshot: since each branch closes, the argument is tree-valid. Given the soundness of our system STD, we can conclude that there is no truth-value assignment that makes the premises true but the conclusion false.

6. Use a tree to check whether the following formula is a tautology. State your conclusion. If the formula is *not* a tautology, then use the tree to find a truth value assignment that makes the formula false:

$$\big(P \supset (Q \supset R)\big) \supset \big((P \supset Q) \supset (P \supset R)\big)$$

Solution:

The following tree shows that this sentence is a tautology, since all branches close of a tree with its negation in the root.

$\vdash_{STD} \big(P \supset (Q \supset R)\big) \supset \big((P \supset Q) \supset (P \supset R)\big)$:

1. $\sim\Big(\big(P \supset (Q \supset R)\big) \supset \big((P \supset Q) \supset (P \supset R)\big)\Big)$ ✓
2. $\big(P \supset (Q \supset R)\big)$ ✓ — 1 $\sim\supset$
3. $\sim\big((P \supset Q) \supset (P \supset R)\big)$ ✓
4. $P \supset Q$ ✓ — 3 $\sim\supset$
5. $\sim(P \supset R)$ ✓
6. P — 5 $\sim\supset$
7. $\sim R$
8. $\sim P$ $\times$ | Q — 4 $\supset$
9. $\sim P$ $\times$ | $Q \supset R$ ✓ — 2 $\supset$
10. $\sim Q$ $\times$ | R $\times$ — 9 $\supset$