Midterm (Written Portion) for 24.241

100 ‘points’ (will scale to 50% of Midterm grade, i.e. 9 grade points)

To ease Symbolization on *Carnap*, below is the symbolization key for M1.1–M1.6:

- F: I have free will.
- S: I have a soul.
- B: I (will still) **B**elieve I am free.
- K: I **K**now that I am free.
- A: I fully and completely **A**ccept myself.
- E: I (will) recognize that I am **E**nough.

1. Test the following argument for validity *using the **tree** method* (STD).
   Label all relevant parts of your tree. Briefly justify your answer. [20 points]:

$$(\sim B \mathbin{\&} \sim D) \supset \sim A$$

$$- - - - - - - - - - - -$$

$$\therefore\ [A \supset (B \lor D)]$$

2. Consider a system STD* exactly like our system STD, except for the single indicated change to the rule for negated biconditional [15 + 15 = 30 points]:

   (a) Would the modified tree system be sound? If so, explain how to extend our inductive soundness proof to a system with this rule; if not, give a tree that is a counterexample to the soundness of STD*. [15 points]

   (b) Would the modified tree system be complete? If so, explain how to extend our inductive completeness proof to a system with this rule; if not, give a tree that is a counterexample to the completeness of STD*. [15 points]

Naughty Negated Biconditional (?∼≡)

$$\begin{array}{llcccl}
\text{m.} & & \multicolumn{2}{c}{\sim(\Phi \equiv \Psi)} & & \\
\vdots & & \multicolumn{2}{c}{\vdots} & & \\
 & & \diagup & \diagdown & & \\
\text{j.} & & \Phi \mathbin{\&} \sim\Psi & \sim\Phi \mathbin{\&} \Psi & & \text{m ?}{\sim}{\equiv} \\
\text{j+1.} & & N & \sim N & &
\end{array}$$

3. Call a string over $\{b, e\}$ a "4-beeb palindrome" if it is (i) a palindrome that has "*beeb*" as the middle four letters and (ii) has a string-length that is divisible by four (so a 4-beeb palindrome can have $4, 8, 12, 16, \ldots 4 \times n, \ldots$ many letters) [$15+15 = 30$ points]

   (i) Give a recursive definition of the set of "4-beeb palindromes", labeling each of the relevant clauses. [15 points]

   (ii) prove by induction that every 4-beeb palindrome has an even number of "b"'s. Please state your **induction hypothesis** clearly! [15 points]

4. *Using induction*, show that you cannot build a formula of propositional logic that is a contradiction using horseshoe ($\supset$) as your *only* connective (i.e. every contradiction in SL contains at least one other connective besides horseshoe). [20 points]

   – Hint: call a formula "**calm**" if: (a) it is not a contradiction or (b) it contains a "*stressed-connective*" tilde ($\sim$), a vel ($\vee$), an ampersand ($\&$), or a biconditional ($\equiv$). For shorthand, you may call these four non-horseshoe connectives the "stressed-connectives". Show by induction that every formula of propositional logic is "calm", and explain how this establishes what is to be proven.

   – You may use our "lazy induction schema" for SL. *Keep calm and logic on!*

REMINDERS for when (you think) you're done:

If time remains, check your work for silly mistakes!!!

DON'T LEAVE ANY QUESTION BLANK!!!! Plz WRITE SOMETHING, so that you can be awarded partial credit.

Make sure that you have answered EACH part of EACH question

Make sure you actually clicked 'Submit' on Carnap for EACH problem!

Make sure you've written **YOUR NAME** on any looseleaf
(and username if your first name is 'Daniel')

Make sure you actually finished the full proof above after proving that every SL sentence is calm! (or at least explain HOW you *would* finish the proof if you had successfully shown that every SL sentence is calm)