(*A NOTE TO THE WISE*: if you are not an expert already on induction, I highly recommend coming to class this week, where we will work through analogue problems whose solutions will probably not appear in lecture slides. It'll be fun! You'll love it!)

Problem Set 3 (24.241 Symbolic Logic)

Due Fri. **Sept 30** by **5pm** Eastern

Please scan and upload to Canvas as a pdf; feel free to *also* turn in a paper copy to Philosophy Dept on 8th floor Stata Center, Dreyfoos-wing

For EACH problem, *label the base case*(s) AND *label the induction step*/hypothesis.

1. Call a string over $\{a, b\}$ an "a-palindrome" if it is a palindrome that has "a" as a middle letter. (An a-palindrome therefore must have an odd number of letters.)

   (i) Give a recursive definition of the set of "a-palindromes", and
   (ii) prove by induction that every a-palindrome has an even number of "b"'s.

2. Here is the recursive definition of n! (read "n factorial"):
   $1! = 1$
   $(n+1)! = (n+1) \times n!$

   That is, $n! = \underbrace{(n \times (n-1) \times \ldots 3 \times 2 \times 1)}_{n\ times}$

   Prove by induction: For every n greater than or equal to 5, $3^{n-1} < n!$

   Hint: A simple bit of algebra will be useful here: if a, b, c, d are all natural numbers, with $a < b$ and $c < d$, then $a \cdot c < b \cdot d$.

3. Prove by induction that if you just have 4 and 11 cent stamps, you can get a combination of stamps for 30 cents, and *any* amount greater than 30.
   (Hint: The base case(s) you need in this one needs to be crafted carefully. You will need to prove more than one case.)

4. Prove by induction that the product of n odd numbers (with $2 \leq n$) is odd.

   (You may find this fact useful: Any odd natural number m can be written as $2k+1$, for some other natural number k.)

5. Prove that no well-formed formula of sentential logic ever contains consecutive atomic formulas [e.g. nothing like '$(PP\&Q)$'.]