Problem Set 10

Partial Solutions: We Assume you will never share these!

Apologies for typos! Let me know if you catch any! (Gotta catch-em all!)

PS10.1

1	$Gb \supset Fb$	:PR
2	Gb	:PR
3	Fb	:1, 2 $\supset$E
4	$Fb \vee Hb$	:3 $\vee$I
5	$(\exists x)\,(Fx \vee Hx)$	:4 $\exists$I

PS10.2

1	$(\exists x)\,(Fx \,\&\, Gx)$	:PR
2	$\quad Fa \,\&\, Ga$	:AS for $\exists$E
3	$\quad Fa$	:2 &E
4	$\quad (\exists y)\,Fy$	:3 $\exists$I
5	$\quad Ga$	:2 &E
6	$\quad (\exists w)\,Gw$	:5 $\exists$I
7	$\quad (\exists w)\,Gw \,\&\, (\exists y)\,Fy$	:4, 6 &I
8	$(\exists w)\,Gw \,\&\, (\exists y)\,Fy$	:1, 2–7 $\exists$E

PS10.4

1	$(\forall x)\,(Fx \equiv Gx)$	:PR
2	$(\forall x)\,(Gx \equiv Hx)$	:PR
3	$Fa \equiv Ga$	:1 ∀E
4	$Ga \equiv Ha$	:2 ∀E
5	$\quad Fa$	:AS for ≡ I
6	$\quad Ga$	:3, 5 ≡E
7	$\quad Ha$	:4, 6 ≡E
	$--$	
9	$\quad Ha$	:AS for ≡ I
10	$\quad Ga$	:4, 9 ≡E
11	$\quad Fa$	:3, 10 ≡E
12	$Fa \equiv Ha$	:5–7, 9–11 ≡I
13	$(\forall x)\,(Fx \equiv Hx)$	:12 ∀I

Check that we have met the restrictions on Universal Introduction ∀I!

PS10.5

1	$(\forall x)\,Fx$	:PR
2	$(\exists x)\,Bzz$	:PR
3	$\quad Baa$	:AS for ∃E
4	$\quad Fa$	:1 ∀E
5	$\quad Baa\,\&\,Fa$	:3, 4 &I
6	$\quad (\exists y)\,(Byy\,\&\,Fy)$	:5 ∃I
7	$(\exists y)\,(Byy\,\&\,Fy)$	:2, 3–6 ∃E

Notice that we must apply Existential Introduction WITHIN the subproof, so that the final line of the subproof has no free occurances of our instantiating constant 'a'. We thereby obey the three restrictions on Existential Elimination.

PS10.7

1	$(\forall x)\,(Fx \equiv Lx)$	:PR
2	$(\exists x)\,Fy$	:PR
3	$\quad Fb$	:AS for $\exists$E
4	$\quad Fb \equiv Lb$	:1 $\forall$E
5	$\quad Lb$	:3, 4 $\equiv$E
6	$\quad (\exists x)\,Lx$	:5 $\exists$I
7	$(\exists x)\,Lx$	:2, 3–6 $\exists$E