Final (Written Portion) for 24.241

100 'points' (will scale to 50% of Final grade, i.e. 13.5 grade points)

To ease Symbolization on *Carnap*, below is the symbolization key for F1.1–F1.6:

- Domain of Discourse: people (your peeps, my peeps, all the peeps)
- **u**: A name for yourself, i.e. '**u**rself'
- **j**: Little **j**ohnny, our fictional hero
- Sx: x is a **S**cholar
- Axy: x **A**ccepts y
- Pxy: x has the **P**ower to heal y

1. Consider a system *SND** just like *SND* except that we add the rule "negated disjunction intro." ($\sim\vee$I): $\{\sim\mathcal{P}, \sim\mathcal{Q}\} \vdash \sim(\mathcal{P} \vee \mathcal{Q})$ (*see back of page*). Does this rule preserve soundness? If so, extend our inductive proof by adding a case (showing that the new line is righteous); If not, provide a concrete counterexample to *SND** soundness. [14pts]

   For problems 2 and 3, let Γ and Δ be possibly infinite sets of QL-sentences:

2. If $\Gamma \cup \{\sim\mathcal{P}\}$ is unsatisfiable and $\Delta \cup \{\sim\mathcal{Q}\}$ is unsatisfiable, prove or provide a counterexample to the following: $\{\Gamma \cup \Delta\} \vdash_{QND} (\mathcal{P} \,\&\, \mathcal{Q})$. [10pts]

3. If $\Gamma \vdash_{QND} \mathcal{P}$ and $\Delta \cup \{\mathcal{H}\}$ is unsatisfiable, prove or provide a counterexample to the following: $\{\Gamma \cup \Delta\} \vDash \big((\mathcal{P} \vee \mathcal{R}) \,\&\, \sim\mathcal{H}\big)$ [10pts]

4. Assume that Γ^* is a maximally-SND-consistent set. Prove that the following membership condition holds: $\sim\mathcal{P} \supset \sim\mathcal{Q} \in \Gamma^*$ if and only if either $\mathcal{P} \in \Gamma^*$ or $\mathcal{Q} \notin \Gamma^*$.

   In your proof, you may use case (a) of our membership lemma (book's 6.4.11a) but no other cases! Note that you will need to provide two non-trivial schematic natural deductions (probably written on paper). [24pts]

5. Using the Soundness and Completeness theorems (or the Consistency Lemma) for system *QND*, prove that Quantifer Logic (without identity) is compact. [14pts]

6. Prove **ONE** of the following two claims [14 points]:

   (a) Let Γ be a set of QL-sentences. Prove that if Γ is satisfiable, then Γ is QND-consistent (i.e. syntactically consistent in derivation system QND).

   (b) If c does not occur in a QND-Consistent set $\Gamma_k \cup \{(\exists x)\,\mathcal{Q}\}$, then $\Gamma_k \cup \{(\exists x)\,\mathcal{Q}, \mathcal{Q}[c/x]\}$ is QND-consistent.
   (NB: *The Logic Book* mentions a lemma to prove this claim that is IRRELEVANT (Lemma 11.1.10). Instead, you must proceed purely syntactically.)

7. Briefly describe the significance of both (i) complete open branches and (ii) maximally-SND-consistent sets in the completeness proofs of STD and SND, respectively.
   How are these ideas connected or similar? (i.e. what functional role(s) do they play in the completeness proofs of these derivation systems?) [14pts]

REMINDERS for when (you think) you're done:

If time remains, check your work for silly mistakes!!!

DON'T LEAVE ANY QUESTION BLANK!!!! Plz WRITE SOMETHING, so that you can be awarded partial credit.

Make sure that you have answered EACH part of EACH question

Make sure you actually **clicked 'Submit'** on Carnap for EACH problem!

Make sure you've written **YOUR NAME** on the first page of looseleaf
(and username if your first name is 'Daniel')

Make sure you gave the requisite natural deductions in SND for Question #4 (membership lemma). (for partial credit: describe the kind of deductions you *would* need.)

For #1: Syntactic Definition of new rule "**Negated Disjunction Intro**." ($\sim\vee$I):

$$
\begin{array}{l|ll}
h & \sim\mathcal{P} & \\
 & \vdots & \\
j & \sim\mathcal{Q} & \\
k+1 & \sim(\mathcal{P} \vee \mathcal{Q}) & :h,\ j \sim\vee\mathrm{I}
\end{array}
$$