

Problem Set: Task Semantics

Well-Formedness: Determine which of the following are well-formed sentences of \mathcal{L}^B :

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|------------------------------------|---|
| 1. $\Box p_1 \rightarrow \Box p_2$ | 5. $p_1 \Box p_2$ |
| 2. $\Box \Box p_1$ | 6. $\Box \rightarrow p_1$ |
| 3. $\Delta \nabla p_1$ | 7. $\Delta (\Box p_1 \leftrightarrow \Box p_2)$ |
| 4. $\Box \Box \Diamond p_1$ | 8. $\Diamond \Diamond \Box p_1$ |

Abbreviations: Expand the following abbreviated formulas into primitive \mathcal{L}^B notation (using only $\perp, \rightarrow, \Box, \Box, \Box$):

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|--------------------------|--|
| 1. Δp_1 | 3. $\Diamond \varphi \vee \Diamond \psi$ |
| 2. $\Diamond \nabla p_1$ | 4. $\Box \varphi \wedge \Delta \psi$ |

Scope Ambiguity: Provide fully parenthesized versions of the following to clarify intended scope:

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|---|--|
| 1. $\Box \varphi \rightarrow \Box \psi \vee \chi$ | 2. $\Delta \varphi \wedge \Box \psi \rightarrow \Diamond \chi$ |
|---|--|

Countermodels: Evaluate the following, providing a proof or countermodel:

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|---|---|
| 1. $\models \Box \varphi \rightarrow \Delta \varphi$ | 4. $\models \Diamond \nabla \varphi \rightarrow \Diamond \varphi$ |
| 2. $\models \nabla \varphi \rightarrow \Diamond \varphi$ | 5. $\models \Box \varphi \rightarrow \Box \Box \varphi$ |
| 3. $\models \Box \varphi \rightarrow \Box \Delta \varphi$ | 6. $\models \Box \varphi \rightarrow \Box \Box \varphi$ |

Duality: Show that the following pairs are duals by providing semantic argument or counterexample:

1. $\Box \varphi$ and $\Diamond \varphi$
2. $\Box \varphi$ and $\Diamond \varphi$
3. $\Delta \varphi$ and $\nabla \varphi$