The Modern History of Modal Logic

1   Cartesian Semantics

Countermodels: Evaluate the following, providing a proof or $\mathcal{L}^{\mathrm{B}}_{\times}$ countermodel.

1. *If* $\Gamma \vDash \varphi$, *then* $\Box\Gamma \vDash \Box\varphi$.

2. $\vDash \Box\varphi \rightarrow \varphi$.

3. $\vDash \Box\varphi \rightarrow \Box\Box\varphi$.

4. $\vDash \varphi \rightarrow \Box\Diamond\varphi$.

5. $\vDash \Box\varphi \rightarrow \triangledown\varphi$.

6. $\vDash \boxtimes\varphi \leftrightarrow \Box\triangle\varphi$.

7. $\vDash \boxtimes\varphi \rightarrow \varphi$.

8. $\vDash \boxtimes\varphi \rightarrow \boxtimes\boxtimes\varphi$.

9. $\vDash \varphi \rightarrow \boxtimes\diamondtimes\varphi$.

10. $\vDash \boxtimes\varphi \rightarrow \triangledown\varphi$.

2   Task Semantics

Validity: Evaluate the following, providing a proof or $\mathcal{L}^{\mathrm{B}}$ countermodel.

1. $\vDash \Box\varphi \rightarrow \Box\boxed{\mathrm{F}}\varphi$.

2. $\vDash \Box\varphi \rightarrow \triangle\varphi$.

3. $\vDash \Box\varphi \rightarrow \boxed{\mathrm{F}}\Box\varphi$.

4. $\vDash \triangledown\varphi \rightarrow \Diamond\varphi$.

3   Extensions

Regimentation: Regiment the following in $\mathcal{L}^{\mathrm{B}}$ and $\mathcal{L}^{\mathrm{B}}_{+}$ disambiguating as needed.

1. If it could rain, it will eventually stop.

2. Either it will always eventually rain or it could eventually always rain.

3. If it has always been raining, then it will eventually stop.

4. Either it could always rain, or it has been raining and will rain again.

5. If it could eventually always rain, then it eventually it could always rain.

6. Either it has been raining or it will eventually rain.

7. If it could eventually stop raining, then it has been raining.

8. If it will be raining, then it could have always been raining.

9. If it has always been raining, then it could eventually stop.

10. If it could eventually always rain, then it will always rain.

Mixed Modals: Evaluate the following, providing a proof or $\mathcal{L}^{\mathrm{B}}_{+}$ countermodel.

11. $\boxdot\varphi \vDash \boxvlines\varphi$.

12. $\boxvlines\varphi \wedge \boxvrines\varphi \vDash \varphi$.

13. $\boxed{!}\varphi \vDash \boxed{\text{P}}\boxed{\text{!}}\varphi$

14. $\boxdot\varphi \vDash \boxvrines\varphi$.

15. $\varphi \vDash \boxvlines\varphi \wedge \boxvrines\varphi$.

16. $\Diamondblack\varphi \vDash \boxed{\text{P}}\Diamondblack\varphi$

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