Absolute Generality

PARADOX AND INFINITY
Benjamin Brast-McKie
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Restricted Quantifiers

Coffee: There is no more coffee.

- Of course there is some coffee somewhere.

Bags: I have packed everything.

- But not everything in existence.

Restriction: The domain of quantification is, somehow, restricted by context.

- Context does not change the meaning of 'there is' or 'everything'.
- The quantifiers have the same semantics, just the domain shifts.

Unrestricted Quantifiers

Biology: All humans are mortal.

Identity: Everything is self-identical.

Sets: No set is a member of itself.

- It appears that we can quantify over *everything*.
- If not, quantified claims are not as informative as they could be.

Question: Are we able to quantify over everything, at least sometimes?

Russellian Doubts

Reductio: Consider the following argument against unrestricted quantification:

1. Assume that we can quantify over all sets.
2. The domain of quantification must include all sets.
3. Quantifier domains are sets.
4. So there is a universal set of all sets.
5. We can derive naive comprehension from separation.
6. A contradiction follows by Russell's paradox.
7. Hence we cannot quantify over all sets.
8. Sets are things.
9. Thus we cannot quantify over everything.

Domain Free Quantification

All-in-One: Cartwright rejects (3) above.

- Can we quantify over everything though there is no set of everything?
- "There is no set that has as members all and only those things that are not members of themselves. But the things that are not members of themselves can simultaneously be the values of the variables of a first-order language; so at any rate I claim." —Cartwright (1994, p. 3)

Absolute Generality: Why can't x be instantiated by w in $\forall x(x \in w \leftrightarrow x \notin x)$?

- Because there is no set w according to ZFC.
- Rather, by *Separation*, we get $\forall x(x \in w' \leftrightarrow (x \in z \wedge x \notin x))$ for some z.
- But there is no universal set, and so at most $w' = z$.
- But if we can quantify over everything, what justifies *Separation*?

Relatively Unrestricted Quantification

Indefinite Extensibility: In quantifying unrestrictedly, new entities can always be defined.

- In particular, w falls outside the domain of the quantifier.
- Naive comprehension may be preserved, but w cannot instantiate x.

Self-Defeating: It is not possible to quantify over everything.

- Hence I am not quantifying over everything.
- So there is something that I am not quantifying over.
- But this is self-defeating, and so false.

Context Domains: Not everything is quantified over in context c.

- For the reasons above, this theses is false in c.

Context Principle: For any c, there is some c' where something quantified over in c' is not also quantified over in c.

- So not everything is quantified over in c, which is false in c.
- But c was an arbitrary context, so the *Context Principle* is false in any c.

Show Don't Tell: The relativist might claim only to be able to always shift the context.

- Start in c, Russell's paradox moves us to c' with broader quantifiers.
- Should we trust a theory that we can't state?

Absolutism: Claiming that we can quantify over everything is not self-defeating.

- We may quantify over everything but are still beholden to *Separation*.
- Not as many sets exist as we might think we are able to naively define.