Writing Informal Proofs

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Proofs

A proof is an argument which consists of assumptions often called *premises*, inferences from those premises, and a *conclusion*. Although all arguments aim to provide reasons to believe the conclusion on the basis of believing the premises, not all arguments are successful, and some arguments are stronger than others. Unlike arguments in the law or elsewhere in day-to-day dealings, *proofs* aim to draw an especially strong connections between the premises and their conclusion.

Common Locutions

ASSUME: Use 'Assume X' when you need to proof something of the form 'If X, then Y'. You should then try to make use of your assumption to establish Y. You may then conclude that if X, then Y as needed. (Note that this is not the only case in which you might use 'Assume'.)

SO: Use 'So' or 'It follows that', to indicate to your reader that what you are now saying follows from the line above. Of course, if the proof is going to work, your claim must actually follow from the line above.

SINCE: Use 'Since', 'Having already shown that', 'Given that', or 'Because' to indicate that you are about to restate something that you have already established on a line prior to the previous line to go on to derive some further claim. For instance, we might write 'Having already shown that X, it follows that Y' to indicate that we are drawing on X (perhaps together with the previous line) to conclude that Y is the case.

CASES: Use 'Consider the following cases' or 'Either of the following must then hold' before labeling the different possibilities you want to consider in order. You may then proceed case by case, citing the labels that you have provided. (This is easier to read and write than using columns to consider cases in parallel.)

RAA: Use 'Assume for RAA' to indicate that you are assuming the opposite of what you want to show for a *reductio ad absurdum* style proof. Alternatively, you could use 'Assume for contradiction' for a similar effect. Then you have to show that the assumption leads to a contradiction, drawing the negation of your assumption as a conclusion

$\perp$: After deriving both P, and then at some latter point deriving $\neg P$ (or *vice versa*, you may use 'But this contradicts P' followed by '$\perp$' as a sentence of its own, or just '$\perp$' when the contradiction is clear.

BUT: Use 'But' or 'However' to recall some past conclusion which contradicts the claim just above (perhaps after some further reasoning). This is a common devise used in writing RAA proofs.

LET: Use 'Let α be an arbitrary F', or 'Let $\alpha \in X$', or etc., to establish a general claim of the form 'All α which are F are also G'. After choosing such an arbitrary α which is F (or a member of X, etc.), you should aim to show that α is G before concluding that every α which is F is also G.

THUS: Use 'Thus', 'Therefore', or 'We may then conclude that' to mark that your proof, or a substantial part of it, is coming to a close.

Try to divide longer proofs into natural sections in the same way that you might divide an essay into paragraphs. It can help a lot to write a quick outline or "sketch" of your proof before you try to write it in more careful detail. Most hard proofs require more than one version before they make sense and clearly communicate what you aim to establish. Much like other writing, proofs require editing.