Surprise Exam Paradox

PARADOX AND INFINITY
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Disbelief

Knowledge: Sometimes the argument is developed in terms of knowledge.

- Were going to stick with belief.

Belief: Can the students believe the instructor?

- Yes, easily so long as they don't do too much reasoning (bad answer).
- Let $S = (E_m \wedge \neg\mathcal{B}_m(E_m)) \vee (E_w \wedge \neg\mathcal{B}_w(E_w)) \vee (E_f \wedge \neg\mathcal{B}_f(E_f))$.
- The interesting question is whether $\mathcal{B}_m(S)$ given *Closure*, etc.

Logic: Can logically omniscient students believe S on Monday?

- It might seem that the arguments show that the answer is 'No'.
- But it seems like there can be surprises, and so S could be true.
- So are the logically omniscient students missing out on a true belief?

Repost: Perhaps the good reasons for belief are overturned by the argument.

- Even the most expert testimonies can be overturned, why not this?
- Remains to accommodate the possibility of a surprise exam.
- But we also want to maintain reasonably strong epistemic principles.

Doubts

Setup: Why can't the students believe S?

- One explanation claims that S happens to be false.
- But surely S is possible, and if so, assume such a case.
- Another strategy looks to spot the mistake in our reasoning before.

One Day: Can there be an announced surprise exam on just one day?

- Announcement: "There is a surprise exam on Monday."
- Seems that the announcement ensures that it is false.

Two Days: Can there be an announced surprise exam on one of two days?

- Suppose the exam is held on Monday (as opposed to Wednesday).
- Would it come as a surprise to the students?
- On Monday, how could they be sure that it wasn't on Wednesday?
- Because if it was on Wednesday, it wouldn't be a surprise *then*.
- But we might be surprised *today* to find out that it is on Wednesday.

Surprise

Timing: It would come as a surprise on Monday that it is/isn't on Monday.

- It wouldn't be a surprise on Wednesday if it didn't happen Monday.
- Do we need to maintain that it is a surprise on the day of?
- Why not take something to be a surprise by referencing the day before?

Analysis: Let E_i be a *surprise iff* $E_i \wedge \neg\mathcal{B}_{i-1}(E_i)$.

- Assume $m - 1 = f'$ (on the week before), $w - 1 = m$, and $f - 1 = w$.
- If E_f, then since $\neg\mathcal{B}_{f-1}(E_f) = \neg\mathcal{B}_w(E_f)$, so it is a surprise.
- Really the surprise takes place on Wednesday.
- On Wednesday, the surprise is about whether E_w or E_f.

Surprise: Does this new analysis capture a natural notion of surprise?

- No less reasonable than the first analysis, and blocks the argument.
- So there can be surprise exams, just not of the first kind of surprise.
- Is this adequate?

Learning: Compare learning something new: you go from $\neg\mathcal{B}(X)$ to $\mathcal{B}(X)$.

- Suppose that you learn something now about something in the future.
- Suppose Ali will go on a walk tomorrow.
- Learning this today, must we be surprised?
- You might say, "I'm not surprised," since Ali often goes on walks.
- But this has the same form as before: $\texttt{Walk}_i \wedge \neg\mathcal{B}_{i-1}(\texttt{Walk}_i)$.

Belief: Could weaken our analysis to a mere necessary condition.

- Partial analysis risks being fairly weak, though still true.
- Consider the exclamations: "I don't believe it!", "I am in disbelief!".
- We say these things when we believe something that surprises us.
- It's not just that we learn something new, it has to be anticipated.

Credences: But couldn't we anticipate Ali's walk without being surprised?

- Merely contemplating a future event is not enough to anticipate it.
- Instead of changing our beliefs, consider updating our credences.
- The bigger the jump in credences, the more surprising.
- When no test is given Wednesday, we go from $\frac{1}{2}$ to 1 that it is on Friday.
- Couldn't our expectation that Ali goes on a walk be similar?
- What makes the exam a surprise and Ali's walk anything but?

Stakes: One thought is that the *stakes* play a role.

- The higher the stakes, the more surprising something can be.