24.118: Paradox and Infinity, Spring 2024
Response Set 2: Type Theory

Please include your name and list all of your collaborators on the last page of your response set, preferably a separate page so that no one sees it till the end. *Failing to list collaborators constitutes a violation of academic integrity.*

How your answers will be graded:

- There is no quiz this week so all answers are to be submitted by PDF.
- Each response should be a maximum of 250 words.
- Responses will be evaluated by the clarity, concision, and the degree to which they reflect careful reading and comprehension of the text. Begin by working out your thoughts separately as you might with a problem set, writing up your response only once you have a good sense of what you want to say and how you want to say it.
- You are encouraged to read critically rather than accepting every word of a text as true. At the same time, it is important to be charitable to what the author intends. When something in the text is less than clear, it is important to do your best to get clarity about what the author intends to express, assuming the best interpretation consistent with the text before delivering any criticisms. For instance, saying that something in the text is not clear is not a good criticism even if it is true.
- You may consult published literature and the web, but you must credit all sources where failure to do so constitutes plagiarism and can have serious consequences. For advice about how and when to credit sources see `https://integrity.mit.edu/`. Note that merely citing a source does *not* count as a good justification.
- Reading old texts is hard! So don't worry if you struggle to understand every point. Instead, do your best to learn what the author is trying to express, using this as a chance to sharpen your close reading skills.

All submissions must be in PDF format. Type-written submissions may be strongly preferred by your TA; handwritten submissions are acceptable only if:

1. Your handwriting is easily legible (as judged by your TA);
2. You produce a clean version of the document (as opposed to the sheet of paper you used to work out the problems); and
3. Your manuscript has been scanned to high enough standards (as judged by your TA). Consider using, e.g. *Scannable* or *Adobe Scan*.

Ramsey (1925, p. 29) writes, "The principal mathematical methods which appear to require the Axiom of Reducibility are mathematical induction and Dedekindian section, the essential foundations of arithmetic and analysis respectively. Mr Russell has succeeded in dispensing with the axiom in the first case, but holds out no hope of a similar success in the second. Dedekindian section is thus left as an essentially unsound method, as has often been emphasized by Weyl, and ordinary analysis crumbles into dust. That these are its consequences is the second defect in the theory of *Principia Mathematica*, and, to my mind, an absolutely conclusive proof that there is something wrong. For as I can neither accept the Axiom of Reducibility nor reject ordinary analysis, I cannot believe in a theory which presents me with no third possibility."

In "Mathematics and Logic" (1946, p. 6), Weyl accuses (in English as opposed to his earlier German publications) Russell of having, "cured the disease but, as shown by the Dedekind example, also imperiled the very life of the patient. Classical analysis, the mathematics of real variables as we know it and as it is applied in geometry and physics, has simply no use for a continuum of real numbers of different levels."

1. Briefly describe the disease that Russel is trying to cure. What part of Russell's cure does Weyl take to imperil the patient? Explain what the problem is.

2. Use Dedekind's construction of the real numbers to explain what Weyl means by 'different levels'. Why is this an unacceptable consequence for mathematics.

3. How does the Axiom of Reducibility avoid creating numbers of different levels? Why does Weyl reject the Axiom of Reducibility if it avoids dividing numbers into different levels? Do you find Weyl's rejection of the Axiom compelling?

In "The Foundations of Mathematics" (1925, p. 20), Ramsey divides the paradoxes with which Russell is concerned into two groups.

4. Explain why Ramsey divides the paradoxes into the groups that he does. How does his division relate to Russell's original solution? Do you feel that Ramsey is right to distinguish between paradoxes, or do you agree with Russell that they should be handled together?