Absolute Generality

PARADOX AND INFINITY
Benjamin Brast-McKie
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Absolutism

Gloss: It is possible to quantify over absolutely everything.

- By 'possible' we mean 'there is an interpretation $I, \hat{a}$ of the language'.

Minimal Language: Consider a language $\mathcal{L}$ with just '$\forall$', 'x', and 'F' as primitive symbols.

- The extension $I(F) \subseteq D_I$ interprets the predicate 'F'.
- $\hat{a}$ is a variable assignment for $\mathcal{L}$ *iff* $\hat{a}(x) \in D_I$.
- 'Fx' is true in $I, \hat{a}$ *iff* $\hat{a}(x) \in I(F)$.
- '$\forall x Fx$' is true in $I, \hat{a}$ *iff* 'Fx' is true in $I, \hat{b}$ for every x-variant $\hat{b}$ of $\hat{a}$.

Restatement: For anything y there is an interpretation I of $\mathcal{L}$ where $y \in D_I$.

- Everything belongs to the domain of quantification D_I.
- $\forall y \exists D(y \in D)$.
- But this is just the claim that there is a universal set.

Direct Method: We must learn to understand absolute quantification directly.

- Model theory does not provide an adequate account.
- At most we can say: $\forall x \exists y(x = y)$.
- But this is trivial, failing to communicate the substance of absolutism.

No Domain Theory

Absolute Plurality: There is a plurality of everything, i.e., $\exists xx \forall y(y \prec xx)$.

- "They lifted the piano," "They won the championship," etc.
- We read '$y \prec xx$' as 'y is *one of* the xxs'.

Quinian Doubts: Some have thought that $\exists xx\varphi$ is just shorthand for $\exists x(S(x) \wedge \varphi)$.

- Similarly, '$y \prec xx$' is just shorthand for '$y \in x$'.
- But this flattens the absolutists current attempt to state their thesis.
- Instead the absolutist must take plural quantifiers to be primitive.
- We do seem to have plural quantifiers in English.

Plural Separation: For any*things*, there is a set of those that are such that φ.

- Formally: $\forall xx \exists y \forall x(x \in y \leftrightarrow x \prec xx \wedge \varphi)$ where 'y' doesn't occur in φ.
- Naive comprehension follows: $\exists y \forall x(x \in y \leftrightarrow \varphi)$.

Plural Comprehension: $\exists yy \forall x(x \prec yy \leftrightarrow \varphi)$ where 'yy' does not occur in φ.

Indefinitely Extensible?

Extensions: Predicates are interpreted by assigning them to sets.

- But how are we to interpret '`set`'?
- Suppose $I(\texttt{set}) \subseteq D$.
- But since $I(\texttt{set}) \notin I(\texttt{set})$, there is a set not in the extension of '`set`'.

Relativism: Claims that we can always extend any extension of the predicate '`set`'.

- $I \subseteq J$ *iff* $I(\kappa) \subseteq J(\kappa)$ for any predicate κ.
- $I \subset J$ *iff* $I \subseteq J$ and $J \nsubseteq I$.
- For any I of $\mathcal{L}$, there is some J of $\mathcal{L}$ where $I \subset J$.
- Every extension of '`set`' has a broader extension.

Absolutism: Instead of sets, suppose extensions are taken to be pluralities.

- The intended extension of '`set`' is the plurality of all sets.
- $\exists yy \forall x(x \prec yy \leftrightarrow \texttt{set}(x))$.
- The relativist may claim that this fails to capture indefinite extensibility.
- The absolutist is happy to avoid indefinite extensibility.