FIRST PUBLIC EXAMINATION

Honour Moderations in Classics

INTRODUCTION TO LOGIC

HILARY TERM 2016

Wednesday 9 March, 2.30 pm – 5.30 pm

*Candidates should answer **FOUR** questions.*

If you answer more than four questions,
your four best answers will be used to determine your mark.

The numbers in brackets beneath each part question indicate the weight
that the Examiners expect to assign to each part of the question.

You may not answer both question 1 and question 2.

Marking scheme: 25% of overall mark for each question.

Do not turn over until told that you may do so.

YOU MAY NOT ANSWER BOTH QUESTION 1 AND QUESTION 2.

1. (a) Which of the following English connectives are truth-functional? In each case write down a partial truth-table. Justify each row of the truth-table, using examples where appropriate.

   (i) It is obligatory that A. [3]

   (ii) It's not true that A or Andy has good reasons to assume that A. [3]

   (iii) It is and has always been the case that A. [3]

   (b) Formalize the following argument as a valid argument in the language $\mathscr{L}_1$ of propositional logic, rewording its premisses as necessary. Demonstrate the validity of your formalization using truth-tables or Natural Deduction. Specify your dictionary carefully and note any difficulties or points of interest. [10]

   If both premisses are true, the conclusion is true. However, as we have just shown, the conclusion implies Steven's claim, which is clearly false. The first premiss is necessarily true. It follows that the second premiss is false.

   (c) Provide formalizations of the following sentences in the language $\mathscr{L}_2$ of predicate logic. The formalizations should be as detailed as possible. Specify your dictionary carefully and note any difficulties or points of interest.

   (i) Every belief is justified, or there is a belief that is not justified. [3]

   (ii) Every person has a body. [3]

YOU MAY NOT ANSWER BOTH QUESTION 1 AND QUESTION 2.

2. (a) Answer the following questions on propositional consistency. [8]

   (i) What does it mean for a set of English sentences to be logically consistent?

   (ii) What does it mean for a set of English sentences to be *propositionally* consistent?

   (iii) If a set of English sentences is propositionally consistent, does it follow that it is logically consistent? Explain your answer using examples if appropriate.

   (iv) Conversely, if a set of English sentences is logically consistent, does it follow that it is propositionally consistent? Explain your answer using examples if appropriate.

   (b) How many symmetric relations are there whose ordered pairs contain only Paris and London as components? List all such relations. [5]

   (c) Determine for each of the following relations whether it is

   - asymmetric on the set of all persons,
   - antisymmetric on the set of all persons, and
   - transitive on the set of all persons.

   Explain your answers and comment on any difficulties or points of interest.

   (i) the relation containing all pairs $\langle d, e \rangle$ where e is not taller than d [3]

   (ii) the relation containing all pairs $\langle d, e \rangle$ where d and e have the same parents but e is neither a brother nor a sister of d [3]

   (iii) the relation containing all pairs $\langle d, e \rangle$ where e is older or taller than d [3]

   (iv) the relation containing all pairs $\langle d, e \rangle$ where e is older and taller than d [3]

3. (a) Specify for each of the following (abbreviated) sentences an $\mathscr{L}_2$-structure in which it is true. In each case explain why the sentence is true in the $\mathscr{L}_2$-structure you have specified.

(i) $\forall x \exists y\, Rxy \wedge \forall x \neg Rxx$ [3]

(ii) $\forall z \exists x \exists y\, (Rxz \wedge Ryz \wedge Px \wedge \neg Py) \wedge \forall z\, (Pz \to \exists x\, Qzx \wedge \exists x \neg Qzx)$ [5]

(iii) $\forall x \forall y \forall z\, (\exists x_1 \neg Rxx_1 \wedge (\neg Ryz \to (\neg Rxy \to \neg Rxz \wedge \neg Ryx)))$ [6]

(b) Consider an $\mathscr{L}_2$-structure $\mathscr{S}$ satisfying the following conditions:

$$\begin{aligned} D_{\mathscr{S}} &= \{\text{France, Germany, United Kingdom}\} \\ |P|_{\mathscr{S}} &= \{d : d \text{ is a monarchy}\} \\ |R|_{\mathscr{S}} &= \{\langle\text{France, Germany}\rangle, \langle\text{France, United Kingdom}\rangle, \\ &\qquad \langle\text{Germany, United Kingdom}\rangle\} \end{aligned}$$

Let α be a variable assignment with:

$$\begin{aligned} |x|^{\alpha}_{\mathscr{S}} &= \text{France} \\ |y|^{\alpha}_{\mathscr{S}} &= \text{Germany} \\ |z|^{\alpha}_{\mathscr{S}} &= \text{United Kindom} \end{aligned}$$

Does α satisfy any of the following formulae in $\mathscr{S}$? Justify your answer. (You do not have to give a complete proof, but you should say why you think each formula is or is not satisfied in $\mathscr{S}$ by the assignment.)

(i) $\neg Px \vee \neg Pz$ [1]

(ii) $\exists x\, (\neg Px \wedge Rxy)$ [2]

(iii) $\forall x \exists y\, (Rxy \wedge Py)$ [3]

(iv) $(Pz \wedge \forall y\, (Ryz \to \neg Py)) \to \forall x\, Rxy$ [5]

4. (a) State the rule $\forall$Intro in words. Explain why the conditions on the occurrences of constants are required. [5]

(b) Is there a sentence ϕ of the language $\mathscr{L}_1$ of propositional logic that can be shown to be a tautology by the truth table method, but for which there is no proof in Natural Deduction without undischarged assumptions? Explain your answer. [3]

(c) Establish the following claims by means of proofs in the system of Natural Deduction.

(i) $\vdash Pa \lor \exists x\, Qx \to \exists y\, Qy \lor Pa$ [5]

(ii) $P \to (Q \to R_1) \vdash Q \to (P \to R_1 \lor R_2)$ [4]

(iii) $\forall x \forall y\, (Rxy \to Py) \vdash \exists x\, (\exists y\, Rxy \to Px)$ [8]

5. (a) Give an example of an English sentence that is a contradiction in predicate logic with identity but not in predicate logic (without identity). Substantiate your answer. [3]

(b) What is the scope of a quantifier? [2]

(c) In each of the following abbreviated $\mathscr{L}_=$-formulae determine the scope of the occurrence of the underlined connective or quantifier by writing down each formula with all brackets restored and underlining the scope of the marked occurrence.

(i) $\underline{\forall x_1}(Py \to Qy \land Ra)$ [2]

(ii) $\forall y\, \underline{\exists x}\, Rxy \leftrightarrow Rz_1 x \land x = y$ [2]

(iii) $\exists x\, Px \underline{\lor} \exists y Py \lor \exists x Qx \to \forall x\, (Px \to Qx)$ [2]

(d) Reveal and classify the ambiguities in the following sentences. Provide alternative formalizations in $\mathscr{L}_=$ of the sentences to clarify the ambiguities and specify dictionaries for the formalizations. Comment on any points of interest. [14]

(i) Glenn and Maggie read the same book.

(ii) Daryl and Carol are climbing up a tree.

(iii) It's not true that Rick is in the house or Carl put the file on the desk.

(iv) '¬t' and '?' and '+5z' are strings of symbols.

(v) Carol has two knives.

6. (a) Copy the following proof of $\vdash P \to (\neg P \to (P \wedge \neg P))$ and justify each step by writing next to each line which rules justifies the step, e.g., ¬Intro, ¬Elim, ∧Intro, ∧Elim1, ∀Intro, and so on. If there is more than one way of justifying the steps of a proof, mention all possible justifications. [5]

$$
\begin{prooftree}
\AxiomC{$[P]$}
\AxiomC{$[\neg P]$}
\BinaryInfC{$P \wedge \neg P$}
\UnaryInfC{$\neg P \to (P \wedge \neg P)$}
\UnaryInfC{$P \to (\neg P \to (P \wedge \neg P))$}
\end{prooftree}
$$

(b) Find any mistakes in the following attempted proofs. List all steps in the proof that are not licensed by a rule of Natural Deduction and explain why they are not permissible. If there is a repair, supply a correct proof. If not, show that the argument is not valid by means of a counterexample. (There is no need to prove that your structure is a counterexample; you only need to specify the structure.)

(i) $P \wedge Q \vdash R \to (Q \wedge R)$. [4]

$$
\begin{prooftree}
\AxiomC{$P \wedge Q$}
\AxiomC{$[R]$}
\BinaryInfC{$P \wedge Q \wedge R$}
\UnaryInfC{$Q \wedge R$}
\UnaryInfC{$R \to (Q \wedge R)$}
\end{prooftree}
$$

(ii) $\forall x \forall z\, (Rxxz \to Pxz), \forall x \exists y\, Rxyc \vdash \exists x \exists y\, Pxy$ [6]

$$
\begin{prooftree}
\AxiomC{$\forall x \exists y\, Rxyc$}
\UnaryInfC{$\exists y\, Rayc$}
\AxiomC{$\forall x \forall z\, (Rxxz \to Pxz)$}
\UnaryInfC{$\forall z\, (Raaz \to Pac)$}
\UnaryInfC{$Raac \to Pac$}
\AxiomC{$[Raac]$}
\BinaryInfC{Pac}
\BinaryInfC{Pac}
\UnaryInfC{$\exists x\, Pxc$}
\UnaryInfC{$\exists x \exists y\, Pxy$}
\end{prooftree}
$$

(iii) $\forall x \forall y \forall z\, (Rxy \to (Ryz \to Qyy)), \forall x \exists y\, Rxy \vdash \exists z \neg\neg Qzz$ [10]

$$
\begin{prooftree}
\AxiomC{$\forall x \exists y\, Rxy$}
\UnaryInfC{$\exists y\, Ray$}
\AxiomC{$\forall x \exists y\, Rxy$}
\UnaryInfC{$\exists y\, Rby$}
\AxiomC{$[Rbc]$}
\AxiomC{$[Rab]$}
\AxiomC{$\forall x \forall y \forall z\, (Rxy \to (Ryz \to Qyy))$}
\UnaryInfC{$\forall x \forall z\, (Rxb \to (Rbz \to Qbb))$}
\UnaryInfC{$\forall z\, (Rab \to (Rbz \to Qbb))$}
\UnaryInfC{$Rab \to (Rbc \to Qbb)$}
\BinaryInfC{$Rbc \to Qbb$}
\BinaryInfC{Qbb}
\BinaryInfC{Qbb}
\UnaryInfC{$\neg\neg Qbb$}
\UnaryInfC{$\exists x \neg\neg Qxx$}
\AxiomC{$[\neg\neg Qbb]$}
\UnaryInfC{$\exists z \neg\neg Qzz$}
\BinaryInfC{$\exists z \neg\neg Qzz$}
\BinaryInfC{$\exists z \neg\neg Qzz$}
\end{prooftree}
$$

7. (a) Why is it not necessary to define $\mathscr{L}_=$-structures as structures different from $\mathscr{L}_2$-structures in order to provide semantics for the language $\mathscr{L}_=$ of predicate logic with identity? [2]

(b) Formalize each of the following sentences in the language $\mathscr{L}_=$ of predicate logic with identity in as much detail as possible. Specify your dictionary carefully and note any difficulties or points of interest.

(i) The thinking and the extended substance are distinct. [3]

(ii) Every man in room 3OB1 has an identical twin brother who stole a book from a library. [5]

(c) Formalize the following argument as a valid argument in the language $\mathscr{L}_2$ of predicate logic, rewording its premisses as necessary. Demonstrate the validity of your formalization using Natural Deduction. Specify your dictionary carefully and note any difficulties or points of interest. [15]

> Everything with a weight is physical. René weighs 79kg. René thinks that there are abstract objects. This refutes the claim that all thinking objects aren't physical.

8. (a) Give a precise definition of a free occurrence of a variable in a sentence of the language $\mathscr{L}_2$ of predicate logic. [4]

(b) What does it mean for sentences ϕ and ψ of $\mathscr{L}_2$ to be logically equivalent? [2]

(c) Provide for each of the following sentences a logically equivalent sentence of $\mathscr{L}_2$ not containing any occurrence of the negation symbol. You don't have to provide a proof of the logical equivalence.

(i) $\neg\forall x\,(Px \to \exists y\,(\neg Qy \wedge Rxy))$ [3]

(ii) $\exists x\,\neg((\neg Pxx \wedge \neg Qxa) \vee \forall z\,\neg(\neg Qzz \leftrightarrow \neg Rz))$ [5]

(d) For each of the following pairs of sentences show that they are *not* logically equivalent by providing suitable counterexamples. It is not necessary to prove that your counterexamples refute the logical equivalences.

(i) $P \to (Q \to R)$ and $(P \to Q) \to R$ [2]

(ii) $\forall x\,(\exists y\,Rxy \to Px)$ and $\forall x\,\exists y\,(Rxy \to Px)$ [4]

(iii) $\forall x\,\exists y\,Rxy \wedge \forall x\,\exists y\,(Rxy \to \neg Ryx)$ and
$\forall x\,\exists y\,Rxy \wedge \forall x\,\exists y\,(Rxy \to \neg Ryx) \wedge \forall x\forall y\forall z\,(Rxy \to (Ryz \to Rxz))$ [5]