FIRST PUBLIC EXAMINATION

Honour Moderations in Classics

INTRODUCTION TO LOGIC

HILARY TERM 2017

Wednesday 8 March, 2.30 pm – 5.30 pm

Candidates should answer ***FOUR*** *questions.*

If you answer more than four questions,
your four best answers will be used to determine your mark.

The numbers in brackets beneath each part question indicate the weight
that the Examiners expect to assign to each part of the question.

You may not answer both question 1 and question 2.

Marking scheme: 25% of overall mark for each question.

Do not turn over until told that you may do so.

YOU MAY NOT ANSWER BOTH QUESTION 1 AND QUESTION 2.

1. (a) Define what it is for an English argument to be:

   (i) *logically valid* [2]

   (ii) *propositionally valid* [1]

   (b) Are the following arguments propositionally valid? Are they logically valid? Substantiate your answers, using formalization and truth tables (or Natural Deduction) where appropriate.

   (i) No one leaves unless everyone is finished. Consequently, not everyone is finished only if no one leaves. [6]

   (ii) Some non-city-dweller disagrees with Owen Smith. After all, most Brexiteers are non-city-dwellers. And most Brexiteers disagree with Owen Smith. [6]

   (c) Formalize the following English sentences in the language $\mathscr{L}_2$ of predicate logic, using the dictionary below:

   P: . . . is a great logician

   Q: . . . invented . . .

   R: . . . admires . . .

   a: Natural Deduction

   b: the semantics of $\mathscr{L}_2$

   (i) All great logicians admire either the semantics of $\mathscr{L}_2$ or Natural Deduction. [2]

   (ii) No great logician invented nothing. [3]

   (d) Using the same dictionary, translate the following sentences of $\mathscr{L}_2$ into idiomatic English:

   (i) $\neg\exists x(Px \wedge (Qxa \wedge Qxb))$ [2]

   (ii) $\exists x(Px \wedge \forall y(Py \rightarrow (Rxy \leftrightarrow \neg Ryy)))$ [3]

YOU MAY NOT ANSWER BOTH QUESTION 1 AND QUESTION 2.

2. (a) Which of the following English connectives are truth-functional? In each case, write down a partial truth table. Justify each row of the truth table, using examples where appropriate.

   (i) A but Donald Trump knows otherwise. [2]

   (ii) A and B are always true at the same time. [2]

   (iii) The argument with premiss A and conclusion B is logically valid. [4]

   (b) Are the following claims about the language $\mathscr{L}_1$ of propositional logic true or false? Substantiate your answers, using truth tables (or Natural Deduction) where appropriate.

   (i) $((\neg(P_2 \wedge Q) \vee \neg(\neg P_1)) \wedge R)$ is an $\mathscr{L}_1$-sentence. [2]

   (ii) $P \wedge Q \wedge R \wedge P_1$ is an abbreviation of the $\mathscr{L}_1$-sentence $(P \wedge (Q \wedge (R \wedge P_1)))$ obtained in accordance with the bracketing conventions. [2]

   (iii) $\{(P \to Q), (P \to \neg Q), (Q \to P)\}$ is semantically inconsistent. [3]

   (iv) $(R \leftrightarrow (P \leftrightarrow Q))$ and $(\neg R \leftrightarrow (P \leftrightarrow \neg Q))$ are logically equivalent. [5]

   (v) $(\neg P \vee Q), (P \vee R), \neg(R \wedge \neg P_1) \vDash ((Q \to \neg P_2) \to (P_2 \to P_1))$ [5]

3. (a) Explain the difference between a relation with arity n and a predicate letter with arity n. [4]

   (b) Which of the following binary relations are equivalence relations on the set of $\mathscr{L}_1$-sentences? Explain your answers, using examples where appropriate.

   (i) $\{\langle \phi, \psi \rangle : \phi \vDash \psi\}$ [3]

   (ii) $\{\langle \phi, \psi \rangle : \phi \vDash \psi$ or $\psi \vDash \phi\}$ [4]

   (iii) $\{\langle \phi, \psi \rangle : \phi \vDash \psi$ and $\psi \vDash \phi\}$ [4]

   (c) Given a binary relation R, and an item d, define the sets Rd and dR as follows:

$$Rd = \{e : \langle e, d \rangle \in R\}$$
$$dR = \{e : \langle d, e \rangle \in R\}$$

   (i) Show that if R is symmetric, then $Rd = dR$ for every item d. [5]

   (ii) Is it also true that if $Rd = dR$ for every item d, then R is symmetric? Explain your answer. [5]

4. Let a PN-string be any finite string of symbols each of which is either a sentence letter or one of the letters 'N', 'K', 'A', 'C', and 'E'.

For example, here are three PN-strings:

$$P \qquad\qquad NNNQ_{26} \qquad\qquad KAPQR$$

Given an $\mathscr{L}_1$-sentence ϕ, its PN-translation $[\phi]^{\mathrm{PN}}$ is the PN-string defined as follows:

$$\begin{aligned}
[\phi]^{\mathrm{PN}} &= \phi \qquad \text{if } \phi \text{ is a sentence letter} \\
[\neg\phi]^{\mathrm{PN}} &= N[\phi]^{\mathrm{PN}} \\
[(\phi \wedge \psi)]^{\mathrm{PN}} &= K[\phi]^{\mathrm{PN}}[\psi]^{\mathrm{PN}} \\
[(\phi \vee \psi)]^{\mathrm{PN}} &= A[\phi]^{\mathrm{PN}}[\psi]^{\mathrm{PN}} \\
[(\phi \to \psi)]^{\mathrm{PN}} &= C[\phi]^{\mathrm{PN}}[\psi]^{\mathrm{PN}} \\
[(\phi \leftrightarrow \psi)]^{\mathrm{PN}} &= E[\phi]^{\mathrm{PN}}[\psi]^{\mathrm{PN}}
\end{aligned}$$

Note that $K[\phi]^{\mathrm{PN}}[\psi]^{\mathrm{PN}}$ is the PN-string that results from writing 'K', then the PN-string $[\phi]^{\mathrm{PN}}$, then the PN-string $[\psi]^{\mathrm{PN}}$. The analogous convention applies in the other cases. For example, the PN-translation of $(P \vee \neg P)$ is $APNP$.

(a) Compute the PN-translations of the following $\mathscr{L}_1$-sentences (eliminating all $\mathscr{L}_1$-connectives and parentheses):

(i) $\neg(P \wedge Q)$ [2]

(ii) $(P \wedge (Q \vee R))$ [3]

(iii) $(\neg(P_1 \to (P \leftrightarrow (Q \vee \neg R))) \leftrightarrow R_2)$ [3]

(b) Find $\mathscr{L}_1$-sentences which have the following PN-translations:

(i) $KPAPCPP$ [3]

(ii) $KAPPCPP$ [3]

(iii) $KACPPPP$ [4]

(c) Are the following claims true or false? Explain your answers.

(i) Every $\mathscr{L}_1$-sentence has exactly one PN-string as its PN-translation. [4]

(ii) Every PN-string is the PN-translation of exactly one $\mathscr{L}_1$-sentence. [3]

5. (a) Define what it is to be a *variable assignment over an $\mathscr{L}_2$-structure*? Under what conditions are two variable assignments said to *differ in the variable v at most*? [4]

(b) Consider an $\mathscr{L}_2$-structure $\mathscr{C}$ satisfying the following conditions:

$$D_{\mathscr{C}} = \{0, 1, 2\}$$
$$|P|_{\mathscr{C}} = \{0,1\} \qquad |b|_{\mathscr{C}} = 0$$
$$|R|_{\mathscr{C}} = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,0\rangle\} \qquad |c|_{\mathscr{C}} = 1$$

Are the following (abbreviated) $\mathscr{L}_2$-sentences true in $\mathscr{C}$? Justify your answers.

(i) $\neg(Rbb \vee Rbc) \wedge Pc$ [3]

(ii) $\neg\forall x \forall y \neg Rxy$ [4]

(iii) $\forall x(Px \rightarrow \exists y Ryx)$ [4]

(c) Are there $\mathscr{L}_2$-sentences ϕ_1 and ϕ_2 with the following properties? In each case, give an example of a sentence with the property or explain why no sentence has the property.

(i) ϕ_1 is true in a structure with the domain $\{0,1,2,3\}$ but not true in any structure with the domain $\{0,1,2\}$. [4]

(ii) ϕ_2 is true in a structure with the domain $\{0,1,2\}$ but not true in any structure with the domain $\{2,3,4\}$. [6]

6. (a) Carefully state the Natural Deduction introduction and elimination rules for the existential quantifier, $\exists$Intro and $\exists$Elim. [5]

(b) Establish the following claims by means of proofs in the system of Natural Deduction:

(i) $\vdash P \rightarrow (Q \rightarrow (P \rightarrow P))$ [2]

(ii) $\vdash P \rightarrow ((Q \rightarrow P) \rightarrow P)$ [2]

(iii) $\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$ [5]

(c) Are the following claims true or false? Substantiate each of your answers by giving a proof in the system of Natural Deduction, or by specifying a counterexample, as appropriate.

(i) $\vdash \forall x(Px \rightarrow Qx) \rightarrow (\exists x Px \rightarrow \exists x Qx)$ [3]

(ii) $\vdash (\forall x Px \rightarrow \forall x Qx) \rightarrow \forall x(Px \rightarrow Qx)$ [3]

(iii) $\vdash (\neg\forall x Px \rightarrow \forall x \neg Qx) \rightarrow \forall x(Qx \rightarrow Px)$ [5]

7. (a) The first part of this question concerns the syntax of the language $\mathscr{L}_1$ of propositional logic.

(i) Carefully state the definition of an $\mathscr{L}_1$-*sentence*. [4]

(ii) Explain the following terms:

- the negation of an $\mathscr{L}_1$-sentence ϕ
- the conjunction of $\mathscr{L}_1$-sentences ϕ and ψ [3]

(b) Consider the following English argument:

> For any $\mathscr{L}_1$-sentence ϕ, its negation is true if and only if ϕ is not true. Similarly, a conjunction $(\phi \wedge \psi)$ is true if and only if both ϕ is true and ψ is true. Take the $\mathscr{L}_1$-sentence $\neg(P_1 \wedge P_2)$, then: this can only ever be true if not both of the sentence letters P_1 and P_2 are true.

- Formalize the argument as a valid argument in the language $\mathscr{L}_2$ of predicate logic, using the dictionary below. You may reword its premisses as necessary, and add any further assumptions upon which the speaker might naturally be expected to be relying.

Q_1: ...is the negation of $\mathscr{L}_1$-sentence ...

Q_2: ...is the conjunction of $\mathscr{L}_1$-sentences ...and ...

R: ...is a true $\mathscr{L}_1$-sentence

a_1: the sentence letter P_1

a_2: the sentence letter P_2

b: the $\mathscr{L}_1$-sentence $(P_1 \wedge P_2)$

c: the $\mathscr{L}_1$-sentence $\neg(P_1 \wedge P_2)$

- Demonstrate the validity of your formalization using Natural Deduction.
- Comment on any difficulties or points of interest. [18]

8. (a) Explain how the syntax and semantics of the language $\mathscr{L}_=$ of predicate logic with identity differ from the syntax and semantics of the language $\mathscr{L}_2$ of predicate logic. [5]

(b) Reveal and classify the ambiguities in the following sentences. Provide alternative formalizations in $\mathscr{L}_=$ of the sentences to clarify the ambiguities and specify dictionaries for the formalizations. Comment on any points of interest.

(i) Only rebellious Labour MPs voted against article 50. [2]

(ii) The *proxime accessit* only comes behind the winner. [3]

(iii) Sally will bet aggressively and win the pot if she has anything better than a pair. [3]

(iv) Two wines are always served at Common table. [4]

(v) '!!!' and '???' and '!?!' are strings of symbols. [4]

(c) Consider the relations R_1 and R_2:

$$R_1 = \{\langle d, e\rangle : d \text{ and } e \text{ are numerically identical material objects}\}$$
$$R_2 = \{\langle d, e\rangle : d \text{ and } e \text{ are qualitatively identical material objects}\}$$

Briefly discuss whether the following claims are true or false:

(i) Every pair in R_1 is also in R_2. [2]

(ii) Every pair in R_2 is also in R_1. [2]