“A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae”
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This is part of a weekly journal club series held by the Society of Physics Students under the University at Buffalo chapter.

For the original paper discussed at this meeting, click: here

1 Introduction

The discussion this week was on Edwin Hubble's original paper “A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae” in which he first notices a linear relationship between the velocities and distances to distant objects located a few megaparsecs from our solar system, and proposes the possibility that such motion can be expressed by a constant, which later becomes the well-known Hubble Constant that represents the expansion of the universe.

Even though most of what we known about Hubble's Constant today came after this original work, I am going to note a few items so that the student can put some of this work in context.

First, the constant “K” that Hubble discusses is what later becomes the Hubble Constant that we know as H_0, however Hubble is quite off in the term he defines (possibly due to his equipment at the time, or possible statistical error), and as such, they are not equal, but are describing the same idea. H_0 has the units of km/s/Mpc - kilometers per second per megaparsec - which reduces to, effectively, inverse seconds, s^{-1}, and is defined (again, in later works) as:

$$H \equiv \frac{\dot{a}(t)}{a(t)} \tag{1}$$

where a(t) is a scale factor and $\dot{a}$(t) is the time derivative of the scale factor. The scale factor is used in other aspects of large-structure formation, and has a somewhat circular definition, but is outside the scope of this discussion. I encourage the student to read a bit about its context in the Friedmann Equation.

Second, a Cepheid (here) is a radially pulsating star, and as such, has its brightness changing with a well-defined stable period and amplitude. We are concerned with these types of objects because they act as our "standard candle" - of which carries importance in the fields of astronomy, astrophysics, and cosmology (not cosmetology...don't ever, ever make this mistake in front of an actual cosmologist...you won't have a fun day). A standard candle is somewhat like our rock to hang on to - we know its behavior very well, it's consistent, etc. - so we take it as our reference when measuring other objects nearby the given standard candle.

Third - which is too lengthy to put here, nor would this be an ideal medium to explain such - is Olber's Paradox. I strongly encourage the curious student to look up information about this, if not for the thought process, then at least for the joke associated with it...but I'm obviously not going to tell you the joke, that would ruin the fun.

2 Discussion

We began the discussion by noting the definition of luminosity as the energy emitted per unit time, and SI units of J/s, i.e., watts, and is given by a perfect blackbody as the following:

$$L = \sigma A T^4 \tag{2}$$

Hubble takes how luminosity can be used to measure the distance to objects, how Cepheids fit into this framework, as well as practical methods for data collection of these results. He continues on with discussing possible preliminary statistical errors that would result from these measurements. We took this opportunity to discuss how changes in luminosity, i.e., a redshift in the observed frequency, can be used to determine such results. We also discussed the possibilities of interstellar dust and gas clouds blocking the emitted light, but the resolution came about that they are treated as a blackbody, in that any incident radiation on them would would be the same frequency and energy of radiation emitted from them, so we can essentially ignore them. This discussion of comoving and accelerating bodies invoked a discussion about special relativity, as there reaches a point where the bodies might appear to satisfy v $\gg$ c, a clear violation of the special theory of relativity, but the resolution is that (as we learn in the years after this experiment) that it is not the bodies themselves that are moving, but the space between them that is expanding, which as far as we know spacetime can expand however fast it pleases. So special relativity still works. Yay Einstein! (...actually, funny enough, he'll come back into this story at the end of this, stay tuned.) As far as the concern of comoving bodies, we are satisfied with simply saying that the Robertson–Walker metric takes care of the invariant quantities.

After reviewing his data, Hubble continues on with discussing similar types of measurements made by another research team, however the results from this other team were made quite a bit before him, and seemed to be off a bit from his results. The data and methods implemented by this other team were of a reasonable standard and quality, as well were Hubble's (given the limitations of his time), so Hubble began to make note of a linear relationship between the distances and velocities for these objects, over this time period between the other measurements and his measurements. As a result, Hubble noted that there seemed to be an increase in the velocities of these objects over time, i.e., over time, these objects were moving away from us faster than they were prior to Hubble's measurements. From this point on, Hubble refers to this as "peculiar motion." This is where he starts to attach numerical bonds on the values of K, based on the calculated values for the velocities.

However, this all said, Hubble was still to cautious of these results, due to his limitations in experimental methods. He proceeds to have other check his calculations and data collection methods (he puts all of this in, I believe, just to show the extend to which statistical errors were reduced, in order to be sure of his conclusions). We see that K has units of H_0, as expected from what we currently know.

The equation of motion Hubble defines is as follows:

$$rK + X\cos(\alpha)\cos(\delta) + Y\sin(\alpha)\cos(\delta) + Z\sin(\delta) = v \tag{3}$$

where spherical motion has been accounted for, but the main point Hubble proceeds to show is that the motion is concentrated in the rK term, where r is the distance to the objects being studied, and thus we see a preliminary form of Hubble's Law:

$$rK = v \tag{4}$$

which becomes what we know as:

$$v = H_0 r \tag{5}$$

Hubble then proceeds to plot this radial velocity against the distance, as shown in Figure 1. The ironic part in this is that, while everything is consistent, the data has been combed through thoroughly, the calculations checked, and the laws correct, Hubble still somehow messed up the units on the y-axis of his plot. It is indeed that data for the radial velocities, but for some reason he just made the units km instead of km/s...we have no clue why this is. Maybe he put a graduate student in charge of this task. Regardless, we see this does indeed follow a linear relationship, and the best-approximated line has a slope of K, with units of km/s/Mpc.

Hubble also mentions that a similar conclusion was determined through studying the photographic luminosity changes, as well as frequency of the light, but does not go into too much depth on the experimental end of this. He does

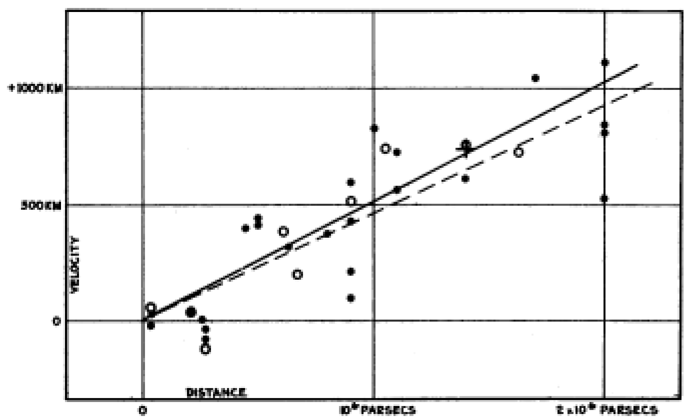

FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

Figure 1: Original Plot by Edwin Hubble - Note: Incorrect y-axis Units

state that having reached a similar verdict through different means, compared to the velocity vs. distance data, that this reinforces his conclusions, but is still hesitant to shout “Eureka!” as this is quit the discovery, should it to hold true in the general case, because of just how big of a deal it would be. We wouldn’t live in a static universe, we would live in one that is expanding at a continuously increasing rate. I can see why one would be hesitant to jump to such conclusions, based on the limited data.

Another group is put together in order to study this idea; they look at a similar data pool, collect independent results, as well as formulate their own conclusions (take note, this is science in action!) - the result being in agreement with Hubble’s. Scienced. Boom. Mary’s your aunt.

However, this is just the beginning of this idea - Hubble still wants larger teams to be put together in order to study more regions of the sky, just to be sure that this holds in the general case, and not just in the (possibly) specific case. While the universal result is still in question, Hubble publishes this paper to show their findings and make the rest of the community aware of what might lie ahead.

We ended the discussion with our own questions. Does Hubble’s Law have higher-order expansions as, by analogy, Hooke’s Law does under a Taylor expansion? If so, what does this mean? We discussed properties related to the energy density of the universe - that might case such acceleration observed in Hubble's Law - as well as large-scale structures and their formation in terms of the Friedmann equation, in its most general case. The question also arose as to whether or not this result (Hubble's Law) is characteristic to our universe or if it is a geometric property of the spacetime, i.e., is it only in our universe that such expansion happens - due to our geometry and curvature - or does it hold in all possible universes, and as such, wouldn't be a geometric property, but rather a property of spacetime itself?

3 Conclusion

In conclusion, Hubble showed the recessional velocities of extra-galactic nebulae was increasing - at least, for a reasonably sized data collection - which later turned out to be the general case. He did this by means of matching the radial velocity to the distance of such objects, and noticing a linear relationship with positive slope, after having removed a decent amount of possible statistical errors that could have interfered with such results. He confirmed these results by means of another group studying the same regions and achieving independent results that were in agreement with his.

This is an excellent example of the scientific process, even when such large discoveries are the topic of concern. Most of what we know about the expansion of the universe started with this discovery, and we owe it to such pivotal moments in the history of scientific progress and research to remember the work that went into them and what they meant for our understanding of the cosmos.

As an interesting historical note: Einstein had a term, know as the cosmological constant, in his equations for general relativity, that predicted an expanding universe. But Einstein discarded this idea, for he didn't think it was possible for us to be in a non-static universe. He called this discarded term his "Biggest Mistake." However, after Hubble's discovery, Einstein put this term back into his equations in order to predict other phenomena that we know today. In 1931, Einstein took a trip to Mount Wilson to thank Hubble for "providing the observational basis for modern cosmology." We should take care to note the intellectual courage displayed by Hubble in publishing these results.