"The Quantum Theory of the Electron"
Presented by: Giovanni Chiappone

LaTeX Author: Benjamin Cammett

Meeting Held: April 24, 2016

This is part of a weekly journal club series held by the Society of Physics Students under the University at Buffalo chapter.

For the original paper discussed at this meeting, click: here

1 Introduction

We began the meeting by reviewing the energy and momentum operators, which are as follows:

$$E = i\hbar \frac{\partial}{\partial t} \tag{1}$$

$$p = \frac{\hbar}{i} \frac{\partial}{\partial x} \tag{2}$$

and then we include a recap of special relativity, which gives us the relationship:

$$E_0^2 = E^2 - c^2 p^2 = m_0^2 c^4 \tag{3}$$

which reduces to:

$$E_0 = \sqrt{E^2 - c^2 p^2} \tag{4}$$

We note that the Schrödinger equation results from the conservation of energy, so applying the idea of energy conservation to the result from special relativity, we aim to describe relativistic quantum mechanics. This is the aim of the paper discussed.

2 Discussion

It should be noted that we did not fully discuss the paper at hand. A short review of topics we discussed will be listed here, but to fully grasp the derivation of the Dirac Equation, it is strongly recommended that the student read through *Introduction to Elementary Particles* by David Griffiths, as well as *Modern Particle Physics* by Mark Thomson. Not only will the student understand the context and historical pressure to produce such an equation, but there is also much to learn about the thought process behind such a formulation as well as many useful procedures and techniques used in a variety of fields within the physics domain. That being said, do not expect the derivation to be trivial nor should one underestimate what it represents both physically and historically for the physics community and our ability to understand Nature and the phenomena she produces, such that we must observe with diligence and care.

Now, onto what topics we did cover during the meeting.

We started with the relationship:

$$m_0c^4 = -\hbar^2 \frac{\partial^2}{\partial t^2} + \hbar^2 c^2 \frac{\partial^2}{\partial x^2} \tag{5}$$

which is a result of applying energy conservation - relativistic form - as well as taking the variables in their operator form. This represents the idea behind the Klein-Gordon equation, but is inadequate, since we later discover it to produce a negative probability, which is not physically realizable, thus it is not satisfactory for what we are trying to explain. However, the problem can be later fixed with a form of the continuity equation, where we define charge density and current in terms of four-vectors.

Introducing arbitrary quantities as the following, in the form of the Klein-Gordon equation, gives us:

$$m_0c^4 = \Big(-\hbar A \frac{\partial}{\partial t} + \hbar c B \frac{\partial}{\partial x} + \hbar c C \frac{\partial}{\partial y} + \hbar c D \frac{\partial}{\partial z} \Big) \Big(-\hbar A \frac{\partial}{\partial t} + \hbar c B \frac{\partial}{\partial x} + ... \Big) \tag{6}$$

In order to have this reduce to the form we desire, i.e., no cross terms, we require that the arbitrary quantities we introduce anti-commute. For example:

$$AB + BA = 0 \tag{7}$$

$$CD + DC = 0 \tag{8}$$

$$...etc.. \tag{9}$$

As well as require that they be normalized as such:

$$A^2 = B^2 = C^2 = D^2 = 1 \tag{10}$$

After some discussion, we determine that these quantities must be matrices, for reasons we will not discuss here.

A reasonable guess is the Pauli Matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{12}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{13}$$

However, they are too low of a dimensions to represent the system we are trying to describe. To resolve this, gamma matrices are introduced. Again, this will not be covered here; one should go through the full derivation to see why and how the steps described here are taken.

3 Conclusion

We ended with discussing the principle ideas of this journal club series for future note. We invoked the following questions after more discussion of the system we are trying to describe:

· Why does the Dirac equation satisfy special relativity?

· The Dirac equation, as it is often stated, describes a free particle, i.e., there is no potential term. What are the consequences of this?

A hint to the first question: Experimentally, one would make measurements in different frames to test predictions and see if the Lorentz invariant quantity agrees between the two frames. So, what is the theoretical equivalent to this? How would one show this theoretically using the mechanisms provided in the Dirac equation?

I refrain from stating the Dirac equation here for stating it without proper derivation would be an injustice to the student. There are no shortcuts in understanding.