“On the Quantum Mechanics of Neutrino Oscillation”
Presented by: Lisong Chen

LaTeX Author: Yuichi Okugawa

Meeting Held: April 10, 2016

This is part of a weekly journal club series held by the Society of Physics Students under the University at Buffalo chapter.

For the original paper discussed at this meeting, click: here

1 Introduction

Neutrino oscillation is a phenomenon when a flavor of neutrino (electron, muon, tau) transition to different flavor. Its expectation value will change through time with certain period. This is due to the mass neutrino carries which is not predicted in standard model (SM). In this paper, they discussed about neutrino oscillation using some knowledge from quantum mechanics. At the club, we discussed about the characteristics of neutrino as well as its importance in SM, then we discovered mathematical derivation regards to the mechanisms of the oscillation.

2 Discussion

2.1 Neutrino

Neutrino is lepton under family of fermion particles. In SM, leptons carries 0 color charge, particle and anti-particle of them carries -1 and +1 electric charge respectively. There are particles with three flavors, electron (e), muon (μ) and tau (τ). Each of them has their own neutrinos and they do not carry neither color nor electric charges. It is said that neutrino almost has no mass in SM yet the neutrino oscillation predicts it is non-zero.

Since neutrino does not have any charges, it does not interact with maters in the universe. It was first predicted by Wolfgang E. Pauli in order to explain conservation of energy and momentum regards to weak interaction such as β decay. Nevertheless the experimental observation is extremely hard due to its characteristic. In 1987, Japanese neutrino observatory Super-Kamiokande first

Generations	1	2	3
Leptons	e	μ	τ
	ν_e	ν_μ	ν_τ

Figure 1: Leptons

observed neutrino originated from supernova in Large Magellanic Cloud, using water tank installed under the ground of Kamioka.

2.2 Formalism

The neutrino flavor transition can be explained by neutrino oscillation. The mass eigenstates is essentially the Hamiltonian in free particle system that satisfy,

$$\hat{H}\psi = i\frac{\partial \psi}{\partial t} = E\psi \tag{1}$$

By using separation of variable,

$$\psi(\mathbf{x}, t) = \phi(\mathbf{x})e^{-iEt} \tag{2}$$

We define eigenstate of neutrinos for each flavor to be $\nu_f = (\nu_e, \nu_\mu, \nu_\tau)$ and we can express this state in terms of mass eigenstates i.e.

$$\nu_f = \sum_m U_{fm}\nu_m \tag{3}$$

More implicitly,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{4}$$

For neutrino produced with flavor f, wave function will become

$$\psi(x, t) = \sum_m U_{fm}\nu_m e^{ip_\nu x} e^{-iE_m t} \tag{5}$$

Now using energy momentum conservation law, we can define energy to be

$$\begin{aligned} E_m(p_\nu) &= (p_\nu^2 + M_m^2)^{1/2} \\ &\approx p_\nu + \left(\frac{M_m^2}{2p_\nu}\right) \end{aligned} \tag{6}$$

Since neutrino moves almost with speed of light, we can approximate $x \approx t$. Therefore

$$\psi(x,t) \approx \sum_m U_{fm}\nu_m e^{-i\left(\frac{M_m^2}{2p_\nu}\right)x} \tag{7}$$

From this, we can say the probability $|\alpha_{ff'}(p_\nu, x)|^2$ of finding neutrino having flavor f' at distance x from the source to be

$$|\alpha_{ff'}(p_\nu, x)|^2 = \sum_m U_{f'm}^2 U_{fm}^2 + \sum_{m' \neq m} U_{f'm}U_{f'm'}U_{fm}U_{fm'} \cos\left(\frac{2\pi x}{\Delta\phi_{mm'}}\right) \tag{8}$$

Phase difference of neutrino oscillation can be defined as

$$\Delta\phi_{mm'} \approx 2\pi \frac{M_m^2 - M_m'^2}{2p_\nu} \tag{9}$$

Thus we conclude if neutrino have phase difference in its flavor(i.e. $\Delta\phi \neq 0$), they have mass.

3 Conclusion

This time, we discovered that neutrino indeed has mass by deriving the expectation value of eigenstate for flavor transformed neutrino. At the end of the day, neutrino can have mass due to its flavor phase transition and this is the reason why we can immediately tell if neutrino has mass just by observing phase transition in practical experiment.