"A Hamiltonian Approach to Thermodynamics"
Presented by: Elijah Ramsey

LaTeX Author: Benjamin Cammett

Meeting Held: April 17, 2016

This is part of a weekly journal club series held by the Society of Physics Students under the University at Buffalo chapter.

For the original paper discussed at this meeting, click: here

1 Introduction

The purpose of this paper was to determine if thermodynamic quantities expressed in the respective equations of state could be likewise expressed in terms of manifolds, i.e., apply a geometric approach to thermodynamic quantities. The idea is to do so in a phase space, so that the Hamiltonian mechanics may be utilized, and thus have the respective coordinates, similar to such in analytical mechanics. This is so that we may express the respective quantities in terms of canonical transformations.

Since not all present had studied thermodynamics, there was some review of terminology, as well as a brief review of the 4 laws related to thermodynamics, i.e., the zeroth, first, second, and third law.

We also reviewed Poisson Brackets as well as the idea behind a manifold.

2 Discussion

As a disclaimer: I, the writer, did not fully understand all aspects of this presentation. Thus, it is also the responsibility of the reader to read the paper themselves to fully grasp the ideas expressed, as well as have insights about the material I may have personally missed, or that we as a group missed during our discussion.

The idea is to reconstruct the following relationship:

$$Tds = dV + PdV \tag{1}$$

using a manifold, but specifically a 2-form, which is a nondegenerate surface.

We start with a function $\mathrm{H}\big(\mathrm{S,V}\big)$ - where if $\Delta\mathrm{S} = 0$, then $\mathrm{H}\big(\mathrm{S,V}\big) = 0$, and there is no progression in the system, i.e., where there is no change in the entropy of the system, the function describe these states does not change either, however, ΔS = 0 also implies that this is a reversible system.

We see the author of the paper takes the First Law (ΔE = 0, for a closed system, energy is conserved) to hold true, as an assumption.

From here, we move onto the manifold, where we note that the neighborhood within the vicinity of u must be non-overlapping with the neighborhood within the vicinity of v, as seen in Figure 1. For such points on a manifold, we define that the intersection of the neighborhood of u with the neighborhood of v must be a null set, i.e., we require the condition for the respective neighborhoods as such: $u \cap v = \varnothing$. From this condition, we are able to define a notion of distance, and thus a metric, and then from that the length of curves on such a manifold. It should be noted that there is another name for Figure 1, but I'll leave that to the reader to figure out; should the reader find it ill-taste, I'll forewarn them that is the mathematicians that dubbed it as such.

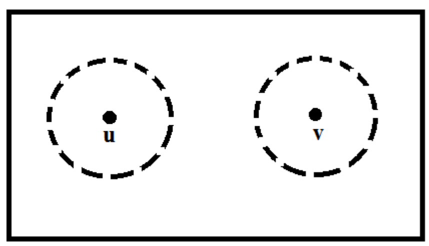

Figure 1: Neighborhoods of u and v

We defined $\left(q^i, p_i\right)$ where $i = 1, 2, 3, \ldots$ for an open set M (an open set for manifold, M). We find the wedge product for such a manifold to be defined as:

$$\omega = dp_i \wedge dq^i \tag{2}$$

We had a brief discussion about the terms defined and their relationship to the physical system we are trying to describe, and after clearing up some confusion, we continue on with the paper to find the the following relationship:

$$dU = TdS - PdV \tag{3}$$

as well as our earlier variables to hold for: $q^1 = S$, $p_1 = T$, $q^2 = V$, and $p_2 = -P$. From these, we find the Maxwell relation as follows:

$$\frac{\partial V}{\partial S}\bigg|_P = \frac{\partial T}{\partial P}\bigg|_S \tag{4}$$

We continued on with a discussion of more variables and spaces for this system, and from these, we find the energy terms for the surface, thus confirming that the thermodynamic quantities can be expressed in terms of the surface quantities for a manifold - which is what the paper originally set out to do.

3 Conclusion

In conclusion, we ended our discussion with discussing general concepts about the Hamiltonian approach for describing such a system, as well as marveling at the fact that it can be expressed in such a way. We can, in fact, define thermodynamic quantities in terms of manifolds, while utilizing phase space and the appropriate basis. So, we confirm that there is a complete duality between such analytical mechanics and our conventional description of thermodynamics.