
Performance analysis of a PDEVS simulator supporting multiple synchronization protocols

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Introduction

DEVS [1] is a popular formalism for modelling complex dynamic systems using a discrete-event abstraction. In fact, it can serve as a simulation “assembly language” to which models in other formalisms can be mapped [2]. A number of tools have been constructed by academia and industry that allow the modelling and simulation of DEVS models.

But with the ever increasing complexity of simulation models, parallel simulation becomes necessary to perform the simulation within reasonable time bounds. And while Parallel DEVS [3] was introduced to increase parallelism, this is often insufficient. Several synchronization protocols from the discrete event simulation community [4] have been applied to DEVS simulation. While several parallel DEVS simulation kernels exist, they are often limited to a single synchronization protocol.

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The reason for different synchronization protocols, however, is that their distinct nature makes them applicable in different situations, each outperforming the other in specific models. The applicability of parallel simulation capabilities of current tools is therefore limited.

This paper introduces DEVS-Ex-Machina* (“dxex”), our simulation tool which offers multiple synchronization protocols: no synchronization (sequential execution), conservative synchronization, or optimistic synchronization. The selected synchronization protocol is transparent to the simulated model: users should merely determine, which protocol they wish to use. Users who simulate a wide variety of models, with different ideal synchronization protocols, can simply run the same model with different synchronization protocols. We investigate in this paper how model allocation and uncertainty determine the choice between synchronization protocols. The synchronization overhead is demonstrated by reducing the computational load of a model to near zero.

Our tool is based on PythonPDEVS, but implemented in C++11 for increased performance, using features from the new C++14 standard when possible. Unlike PythonPDEVS dxex only supports multicore parallelism.

We implemented a model that, depending on a single parameter, changes its ideal synchronization protocol. We demonstrate using several models the factors influencing the performance under a given synchronization protocol. Dxex, then, is used to compare simulation using exactly the same tool, but with a varying synchronization protocol. With dxex users can always opt to use the fastest protocol available. To verify that our flexibility does not counter performance, we compare to adevs, currently one of the fastest DEVS simulation tools available [5, 6].

Dxex offers visualization of the simulation and in depth statistics. A modeller can then make a more informed decision on which synchronization protocol to use or even intervene during simulation and request a switch between protocols.

The remainder of this paper is organized as follows: Section 2 introduces the necessary background on synchronization protocols. Section 3 elaborates on our design that enables this flexibility. In Section 4, we evaluate performance of our tool by comparing its different synchronization protocols, and by comparing to adevs. Related work is discussed in Section 5. Section 6 concludes the paper and gives future work.

Background

This section briefly introduces the synchronization protocols used by dxex: conservative and optimistic synchronization.

Conservative Synchronization

The first synchronization protocol we introduce is *conservative synchronization* [4]. In conservative synchronization, a node progresses independent of all other nodes,

*<https://bitbucket.org/bcardoen/devs-ex-machina>

up to the point in time where it can guarantee that no causality errors happen. When simulation reaches this point, the node blocks until it can guarantee a new time until which no causality errors occur. In practice, this means that all nodes are aware of the current simulation time of all other nodes, and the time it takes an event to propagate (called *lookahead*). Deadlocks can occur due to a dependency cycle of models. Multiple algorithms are defined in the literature to handle both the core protocol, as well as resolution schemes to handle or avoid the deadlocks [4].

The main advantage of conservative synchronization is its low overhead if the lookahead is high. Each node then simulates in parallel, and sporadically notifies other nodes about its local simulation time. The disadvantage, however, is that the amount of parallelism is explicitly limited by the lookahead. If a node can influence another (almost) instantaneously, no matter how rarely it occurs, the amount of parallelism is severely reduced. The user is required to define the lookahead, using knowledge about the model's behaviour. Defining lookahead is not always a trivial task if there is no detailed knowledge of the model. Even slight changes in the model can change the lookahead, and can therefore have a significant influence on simulation performance.

Optimistic Synchronization

A completely different synchronization protocol is *optimistic synchronization* [7]. Whereas conservative synchronization prevents causality errors at all costs, optimistic synchronization allows them to happen, but corrects them. Each node simulates as fast as possible, without taking note of any other node. It assumes that no events occur from other nodes, unless it has explicitly received one at that time. When this assumption is violated, the node rolls back its simulation time and state to right before the moment when the event has to be processed. As simulation is rolled back to a time prior to the event must be processed, the event can then be processed as if no causality error ever occurred.

Rolling back simulation time requires the node to store past model states, such that they can be restored later. All incoming and outgoing events need to be stored as well. Incoming events are injected again after a rollback, when their time has been reached again. Outgoing events are cancelled after a rollback, through the use of anti-messages, as potentially different output events have to be generated. Cancelling events, however, can cause further rollbacks, as the receiving node might also have to roll back its state. In practice, a single causality error can have significant repercussions.

Further changes are required for unrecoverable operations, such as I/O and memory management. These are only executed after the lower bound of all simulation times, called *Global Virtual Time* (GVT), has progressed beyond their execution time.

The main advantage is that performance is not limited by a small lookahead, caused by a very infrequent event. If an (almost) instantaneous event rarely occurs, performance is only impacted when it occurs, and not at every simulation step. The disadvantage is unpredictable performance due to the arbitrary cost of rollbacks and their propagation. If rollbacks occur frequently, state saving and rollback overhead can cause simulation to grind to a halt. Apart from overhead in CPU time, a significant memory overhead is present: all intermediate states are stored up to a point where it

can be considered *irreversible*. Note that, while optimistic synchronization does not explicitly depends on lookahead, performance still implicitly depends on lookahead.

Multiple Synchronization Protocols

Historically, dxex is based on PythonPDEVS [5]. Python is a good language for prototypes, but performance has proven insufficient to compete with other simulation kernels [8]. Dxex is a C++11-based implementation of PythonPDEVS, but implements only a subset of PythonPDEVS, while making some of its own additions. So while the feature set is not too comparable, the architectural design, core simulation algorithm, and optimizations, are highly similar.

We will not make a detailed comparison with PythonPDEVS here, but only mention some supported features. Dxex supports, similarly to PythonPDEVS, the following features: direct connection [9], Dynamic Structure DEVS [10], termination conditions, and a modular tracing and scheduling framework [5]. But whereas PythonPDEVS only supports optimistic synchronization, dxex support multiple synchronization protocols (though only in parallel). This is in line with the design principle of PythonPDEVS: allow users to pass performance hints to the simulation kernel. In our case, a user can pass the simulation kernel the synchronization protocol to use for this model. Our implementation in C++11 also allows for optimizations which were plainly impossible in an interpreted language. Dxex will use new optimizations from the C++14 standard when possible.

Since there is no universal DEVS model standard, dxex models are incompatible with PythonPDEVS and vice versa. This is due to dxex models being grafted on C++11, whereas PythonPDEVS models are grafted on Python.

In the remainder of this section, we will elaborate on our prominent new feature: support for multiple synchronization protocols within the same simulation tool, which are offered transparently to the model.

Synchronization protocols

We previously explained the existence of different synchronization protocols, each optimized for a specific kind of model. As no single synchronization protocol is ideal for all models, a general purpose simulation tool should support multiple protocols. Currently, most parallel simulation tools choose only a single synchronization protocol due to the inherent differences between protocols. An uninformed choice on which one to implement is insufficient, as performance will likely be bad. We argue that a real general purpose simulation tool should support sequential, conservative, and optimistic synchronization, as is the case for dxex.

These different protocols relate to three different model characteristics. Conservative synchronization for when high lookahead exists between different nodes, and barely any blocking is necessary. Optimistic synchronization for when lookahead is unpredictable, or there are rare (almost) instantaneous events. Finally, sequential simulation is still required for models where parallelism is bad, where all protocols actually slow down simulation.

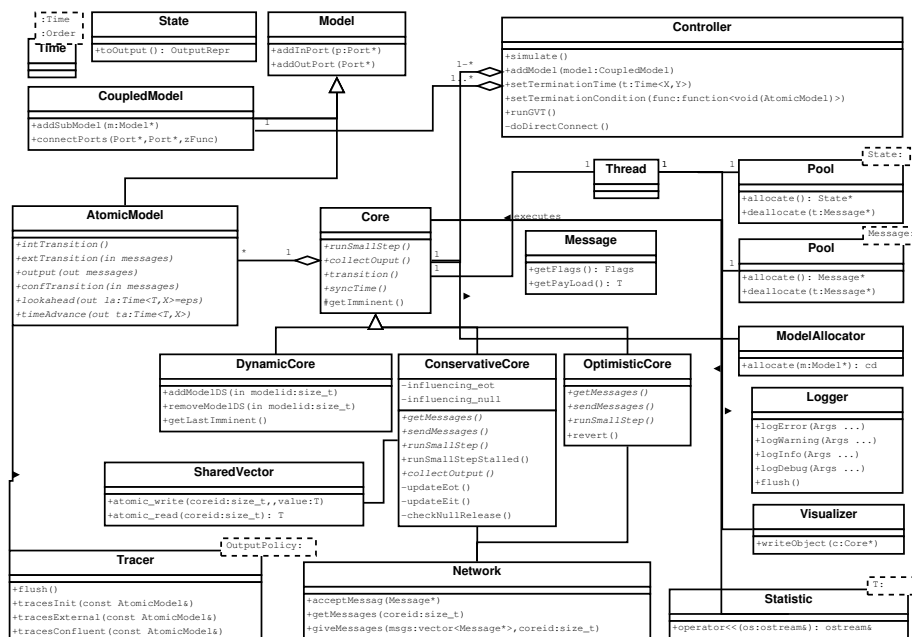


Figure 1. Drex design.

Sequential Our sequential simulation algorithm is very similar to the one found in PythonPDEVS, including many optimizations. Minor modifications were made, though, such that it can be overloaded by different synchronization protocol implementations. This way, the DEVS simulation algorithm is implemented once, but parts can be overridden as needed. In theory, more synchronization protocols (e.g., other algorithms for conservative synchronization) can be added without altering our design.

An overview of dxex’s design is given in Figure 1. It shows that there is a simulation Core, which simulates the AtomicModels connected to it. The superclass Core is merely the sequential simulation core, but can be used as-is. Subclasses define specific variants, such as ConservativeCore (conservative synchronization), OptimisticCore (optimistic synchronization), and DynamicCore (Dynamic Structure DEVS).

Conservative For conservative synchronization, each node must determine the nodes it is influenced by. Each model needs to provide a lookahead function, which determines the lookahead depending on the current simulation state. Within the returned time interval, the model promises not to raise an event. A node aggregates this information to compute its earliest output time (EOT). This value is written out in shared memory, where it can be read out by all other nodes.

Reading and writing to shared memory is done through the use of the new C++11 synchronization primitives. Whereas this was also possible in previous versions of the C++ standard, by falling back to non-portable C functions, it was not a part of the C++

language standard. C++11 further allows us to make the implementation portable, as well as more efficient: the compiler might know of optimizations specific to atomic variables or constant expressions which are heavily used in dxex.

Optimistic For optimistic synchronization, each node must be able to roll back to a previous point in time. This is often implemented through the use of state saving. This needs to be done carefully in order to avoid unnecessary copies, and minimize the overhead. We use the default: explicitly save each and every intermediate state. Mattern’s algorithm [11] is used to determine the GVT, as it runs asynchronously and uses only $2n$ synchronization messages. Once the GVT is found, all nodes are informed of the new value, after which fossil collection is performed, and irreversible actions are committed.

The main problem we encountered in our implementation is the aggressive use of memory. Frequent memory allocation and deallocation caused significant overheads, certainly when multiple threads do so concurrently. This made us switch to the use of thread-local (using `tcmlloc`) memory pools. Again, we made use of specific new features of C++11, that were not available in Python, or even previous versions of the C++ language standard.

Transparency

We define simulation kernel transparency as having a single model, which always can be executed on each supported synchronization kernel, without any modifications. User should thus only provide one model, implemented in C++11, which can be either using sequential execution, using conservative synchronization, or using optimistic synchronization. Switching between simulation kernels is as simple as altering the simulation termination time. The exception is conservative synchronization, where a lookahead function is required, which is not used in other synchronization kernels. Two options are possible: either a lookahead function must always be provided, even when it is not required and possibly not used, or we use a default lookahead function if none is defined.

Always defining a lookahead function might seem redundant, especially if users will never use conservative synchronization. Especially since defining the lookahead is often non-trivial and dependent on intricate model details. The more attractive option is for the simulation tool to provide a default lookahead function, such that simulation can run anyway, but likely not at peak performance. Depending on the model, simulation performance might still be faster than sequential simulation.

Defining a lookahead function is therefore recommended in combination with conservative synchronization, but is not a necessity, as a default *epsilon* (i.e., the smallest representable timestep) is used otherwise.

Features

We will enumerate some of the extra features dxex offers to modellers.

Tracing Like PythonPDEVS dxex has full tracing capabilities supporting a wide number of output formats : Json, XML, Cell, console, The user can implement his own outputpolicy and use it in our modular tracing design. The tracing functionality

itself is implemented asynchronous and thread-safe. By overriding the output functions in the Message or State classes the user can generate model specific output for use with an existing tracing output policy. Tracing can be completely disabled to maximize simulation performance should this be desired (e.g. benchmarks).

Memory Allocation As we will demonstrate in Memory management memory allocation is a performance bottleneck in simulation with high event frequency, especially in parallel. Dxex has a modular allocation interface that separates the object creation in kernels and models from the actual implementation. This allows us by using a single compilation parameter to change the allocation strategy per kernel type. This interface allows instrumentation of memory usage. Not only useful to detect memory leaks should this be a concern, it can also be of use to a model author to trace event creation in large models. A model author can rely on the default implementation as the fastest common configuration verified by the benchmarks.

Model Allocation A model author can specify which kernel a model should be allocated to, should such manual intervention be required. This is handled by the default model allocator. If no preference is given a simple striping scheme is followed. It should be obvious that this is not sufficient in most simulations. In Section Performance we analyze allocation schemes and their performance impact. By overriding the default allocator a model author can tune the allocation scheme to maximize parallel speedup. This interface can be in future work also be used to employ graph algorithms for an automatic allocation scheme, for example one that avoids cycles in the resulting kernel dependency graph.

Logging A high performance asynchronous buffered threaded logging module is used in dxex which offers a complete overview of the entire simulation. The logging functionality can be used in any model, it only requires linking to the dxex library. Logging statements are written to file with a granular level system. In case of a severe crash the logging system will ensure persistence of the log file itself up until the last statement. Logging can be completely disabled to remove the performance impact.

Visualization and Statistics Tracing offers a black box view of a DEVS model in a simulation, with transitions, events and states recorded at each state change. Dxex supplements this optionally with more in depth insights into the actual simulation. The allocation of models across kernels is visualized at the end of a simulation, with graphical representation of message frequency. This allows a user to fine tune future allocation schemes and the interface used by the visualization tool can also be used for more advanced purposes such as dynamic tracing of the simulation.

Invaluable during the development of dxex were runtime statistics collected by a simulation kernel. These record any measurable event in the simulation such as : events created/destroyed, inter kernel event frequency, reverts, GVT calculations, lookahead and eot values and insights into the fairness between the kernels themselves. Using this information it is possible to detect an imbalance between the simulation rounds that a single kernel executes compared to dependent or influencing kernels, or observe how balanced inter kernel message traffic actually is during a simulation. The same information can, as in the visualization tool, be used programmatically to extend dxex with visualization software that shows the simulation from a white box

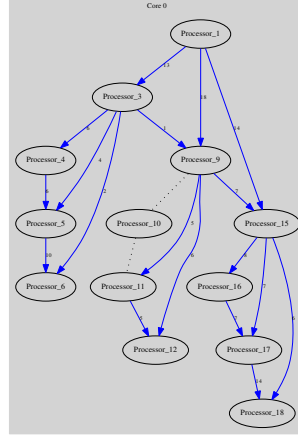


Figure 2. Visualization of PholdTree ($d=1, n=3, t=5000$) sequential simulation.

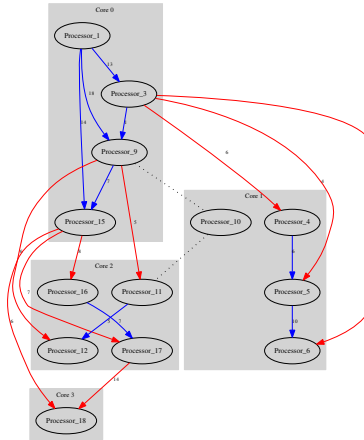


Figure 3. Visualization of PholdTree ($d=1, n=3, t=5000$) parallel simulation with breadth first allocation and 4 kernels.

perspective or allow activity tracking. Figures 2,4,3 give an example of the modular visualization output generated by dxex after simulation the PholdTree model. These figures demonstrate the effect of allocation visually, the high inter kernels event count will degrade performance, as we will demonstrate in Allocation . Dotted lines represent connections between models that did not see events exchanged, directed edges represents events with frequency as weight and color to distinguish inter versus intra kernel events.

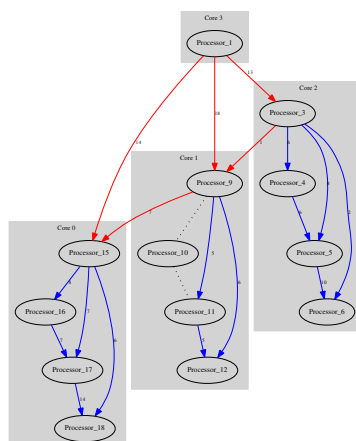


Figure 4. Visualization of PholdTree ($d=1, n=3, t=5000$) parallel simulation with depth first allocation and 4 kernels.

Performance

In this section, we evaluate the performance of different synchronization protocols in dxex. We also compare to adevs, currently one of the most efficient simulation kernels [6], to show that our modularity does not impede performance. CPU time and memory usage is compared for both sequential and parallel simulation.

We start off with a comparison of sequential simulation, to show how adevs and dxex relate in this simple case. For the parallel simulation benchmarks, results are presented for both conservative and optimistic synchronization.

For all benchmarks, results are well within a 5% deviation of the average, such that only the average is used in the remainder of this section. The same compilation flags were used for both adevs and dxex benchmarks (“-O3 -fno”). To guarantee comparable results, no I/O was performed during benchmarks. Before benchmarking, simulation traces were used to verify that adevs and dxex return exactly the same simulation results. Benchmarks were performed using Linux, but our simulation tool works equally well on Windows and Mac. The exact parameters for each benchmark can be found in the repository, as well as the data used in this paper.

Benchmarks

We use four different benchmarks, which cover different aspects of the simulation kernel:

1. The *Queue* model, based on the *HI* model of DEVStone [12], creates a chain of hierarchically nested atomic DEVS models. A single generator pushes events into the queue, which are processed by the processors after a fixed or random delay. It takes two parameters: width and depth, which determine the width and depth of the hierarchy. This benchmark shows how the complexity of the simulation kernel behaves for an increasing amount of atomic models, and an

increasingly deep hierarchy. An example for width and depth 2 is shown in Figure 5.

2. The *PHOLD* model, presented by [13], creates n atomic models, where each model has exactly $n - 1$ output ports. Each atomic model is directly connected to every other atomic model. After a random delay, an atomic model sends out an event to a randomly selected output port. Output port selection happens in two phases: first it is decided whether the event should be sent to an atomic model at the same node. Afterwards, a uniform selection is made between the remaining ports. The model takes two parameters: the percentage of remote events, which determines the fraction of messages routed to other nodes, and the percentage of priority events. Priority events are events generated in a very short time after the previous event. This benchmark shows how the simulation kernel behaves in the presence of many local or remote events and when the average time advance is larger than the lookahead in conservative simulation. An example for four models, split over two nodes, is shown in Figure 7.
3. The *Interconnect* model, a merge of PHOLD [13] and the *HI* model of DEVStone [12], creates n atomic models, where each model has exactly one output port. Similar to PHOLD, all models are connected to one another, but all through the same port: every model receives each generated event. The model takes one parameter: the number of models. This benchmark investigates the complexity of event routing, and how the simulation kernel handles many simultaneous events. An example for four models is shown in Figure 6.
4. The *PHoldTree* model, a hierarchical model based on the semantics of PHOLD [13]. The model consists of recursively defined trees connected to each other, similar to the structure of an AST. All links can be configured to be bidirectional. Unlike the *Queue* model the hierarchical nature is still present in the topology of the atomic models after the directconnect stage. This model allows us to investigate parallel speedup in several key dimensions in combination or isolation: model allocation, message density, uncertainty and hierarchy. The model has a depth parameter, fanout (childcount), probability of priority message and linkage direction. We demonstrate the importance of atomic to kernel allocation, in this model this can be configured as a breadth-first versus a depth-first scheme. *PHoldTree*, while a synthetic benchmark, can be used to model gossiping in social networks, hierarchical resource sharing or load balancing. Unlike *Queue* this model does not lend itself easily to parallelism and thus highlights the difficulties model authors face in the real world when employing PDEVS solutions. The number of atomic models in this benchmark is given by

$$\frac{n^{d+2} - 1}{n - 1} \quad (1)$$

Whereas the number of links between the models is given by

$$(2n - 1) \sum_{i=0}^d n^i \quad (2)$$

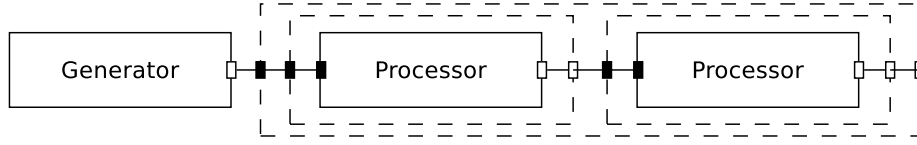


Figure 5. Queue model for depth and width 2.

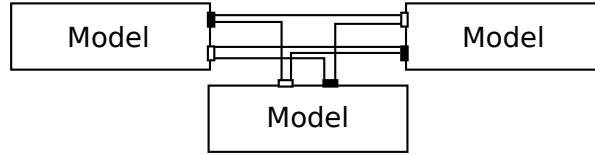


Figure 6. Interconnect model for three models.

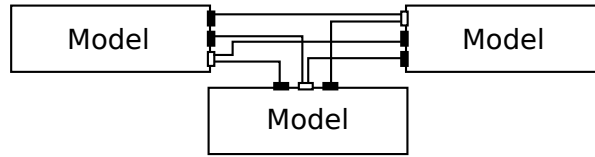


Figure 7. PHOLD model for three models.

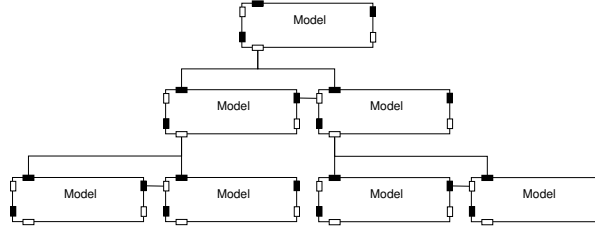


Figure 8. PHOLDTree model for depth 1, width 2.

Sequential Simulation

Queue For the first benchmark, we tested the effect of hierarchical complexity of the model in the performance of the simulator. A set of three tests was performed, where each test has the same number of models but an increasing depth. The results can be seen in Figure 9. Since dxex symbolically flattens the model, there is no performance hit when the depth is increased. The overhead of running the directconnect algorithm is one time only and negligible when the end time of the simulation is sufficiently large. Adevs on the other hand does suffer from the increased depth. With every new hierarchical layer, routing an event from one atomic model to the next becomes more expensive, resulting in an increase in runtime.

Interconnect In the Interconnect model, we increase the number of atomic models, thus quadratically increasing the number of couplings and the number of external

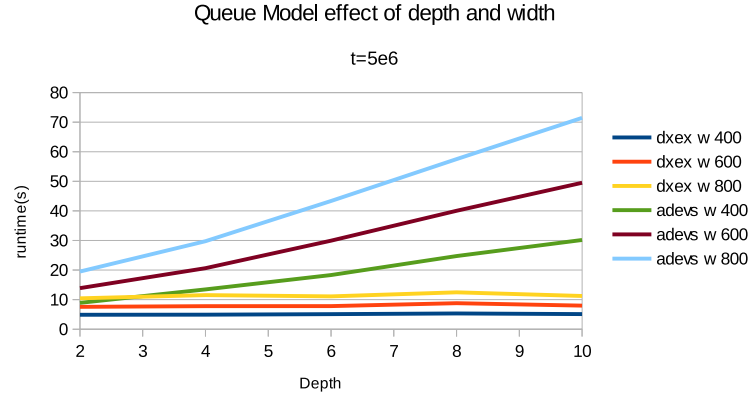


Figure 9. Queue benchmark results for sequential simulation.

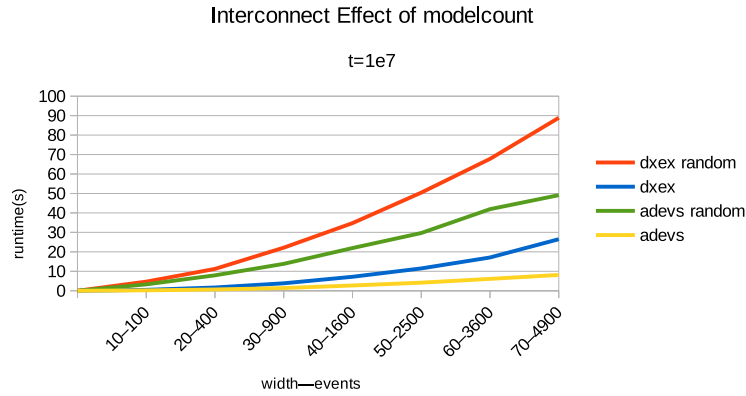


Figure 10. Interconnect benchmark results for sequential simulation.

transitions. As can be seen in Figure 10, adevs outperforms dxex by a fair margin. Analysis showed that this is caused by the high amount of exchanged events: event creation is much slower in dxex than it is in adevs, despite dxex's use of memory pools. Dxex reuses the same event type for all synchronization algorithms. To shield the user from threading and deallocation concerns dxex provides an event superclass from which the user can derive to create a specialized event type. Copying and deallocation semantics are solved by the kernels at a runtime cost in simulations where event frequency is very high. Profiling the diverging benchmarks clearly shows the increasing cost of output generation (event creation) and deallocation as the determining factor in the gap in performance. We refer the interested reader to the dxex repository for the profiling call graphs for the different benchmarks.

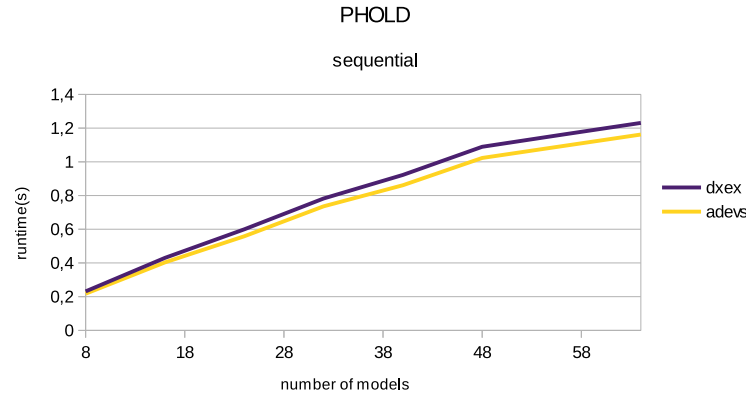


Figure 11. PHold benchmark results for sequential simulation.

PHold The PHold model is very similar to the Interconnect model. The biggest difference is that the amount of messages sent is much lower. The number of events scales linear with the number of models, not quadratic. Figure 11 shows that the performance of dxex and adevs are very close to each-other, with adevs slightly outperforming dxex.

PHoldTree The PHoldtree, like Queue, is a highly hierarchical model but one where the flattened structure cannot be partitioned into a chain as in Queue. This topology is interesting since it highlights the effects of allocation which we have shown already in PHold and Interconnect to be of vital importance. This model allows us to investigate in depth the effects of non-cyclic allocation strategies and measure parallel speedup. Since adevs does not use the directconnect algorithm, we expect some performance penalty between dxex and adevs. Conversely the messaging intensity in PHoldTree can be configured to highlight dxex's performance disadvantage in benchmarks with a high event frequency as shown in the Interconnect benchmark.

Event frequency Similarly when the probability on a priority message (p) increases the amount of messages generated will increase and thus the runtime. In Figure 12 we observe that the messaging probability has a near linear impact on performance, with the shape of the curve due to rounding errors. The advantage adevs has in simulations where event frequency is high has been demonstrated by the interconnect benchmark. Dxex chooses to provide the user with an event type where the user need not consider deallocation or copy semantics, at a cost in performance where event frequency is high. Counterintuitively the probability of a high priority event in this model does not affect the performance difference between conservative and optimistic. The key parameter quickly becomes the load of a kernel in models. Our conservative implementation in an uncertain (trivial lookahead) simulation is very sensitive to a high load in models, whereas optimistic has no such limitation.

Hierarchy To establish a performance baseline for the parallel simulation benchmarks we instrument how the PHoldTree benchmark performs sequential when

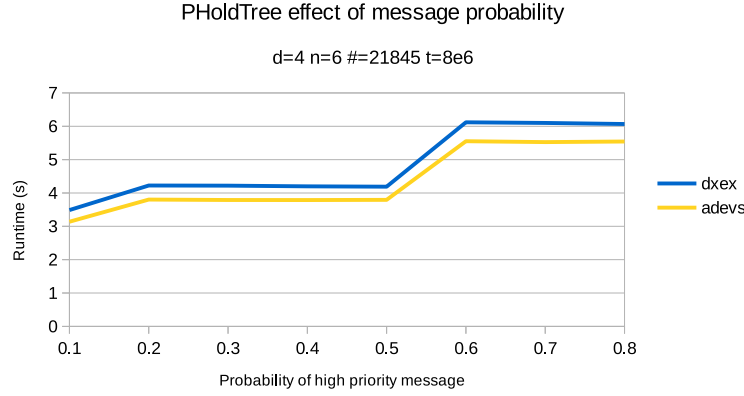


Figure 12. PHoldTree for increasing probability of priority message.

we vary each of the model's parameters. When we increase the depth and fanout of the model, we expect to see an increase in runtime. In Figure 13 the n parameter (fanout) determines the performance penalty adevs suffers w.r.t dxex but we need to look at the profiling call graphs to determine what causes adevs' performance graph to behave non linear.

If we compare the instances $d=2, n=2$ with $d=2, n=4$ which show a clear deviation in performance between adevs and dxex for the latter, we conclude that the increase in width per subtree (n) leads to higher overhead for adevs. Dxex in contrast due to the directconnect algorithm has no such overhead. The intuition behind this effect, and especially why the d parameter does not have such an impact, is that an increase in n will rapidly increase the number of connections 2 (and thus the routing problem adevs faces), whereas d will increase modelcount 1 faster but have a lesser impact on connections. Both kernels scale linear in increasing number of atomic models.

Parallel Simulation

Queue

Allocation The Queue model is one single chain of models. Each kernel gets one connected part of this chain. The result is that the kernels themselves also form a chain where events only travel in one direction.

Strong and Weak Scaling Figure 15 shows the speedup compared to dxex sequential for a fixed problem size. As the amount of kernels increases, the optimistic kernel quickly becomes the worst choice. The difference between dxex conservative and adevs becoming smaller. The same effect can be seen for weak scaling in Figure 14. In Figure 16 the allocation of the Queue model is visualized. This trace allows us to demonstrate the benchmark results and explain why optimistic is not ideal for such a chain topology as present in Queue. Kernel 2 is dependent on events from kernels 0 and 1 but by definition of the optimistic synchronization protocol it will simulate ahead without waiting for events it depends on. Any simulation it performs is immediately

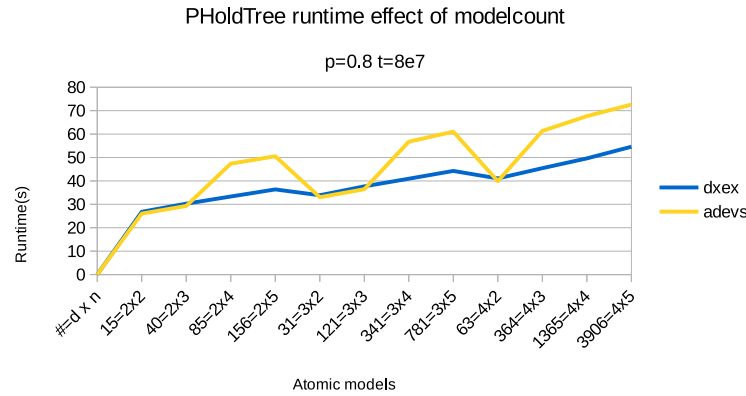


Figure 13. PHoldTree : Effect of hierarchy.

void and will have to be reverted as soon as events from kernels 0 and 1 arrive. When kernel 2 reverts, kernel 3 will inevitably have to revert as well since any events it received from kernel 2 are invalidated and marked as such by antimessages. This leads to severe performance degradation, with an increasing probability of cascading reverts as the number of kernels and models per kernels increases. The speedup of adevs is always in comparison with the runtime of the corresponding dxex sequential benchmark.

Interconnect In the Interconnect model, we determine how broadcast communication is supported across multiple nodes. The number of models is now kept constant at eight. Results are shown in Figure 17. When the number of nodes increases, performance decreases due to increasing contention in conservative simulation and an increasing number of rollbacks in optimistic simulation. All models depend on each other and have no computational load whatsoever, negating any possible performance gain by executing the simulation in parallel. In interconnect there is no allocation scheme possible that avoids cyclic dependencies between simulation kernels, as shown in the trace 18 of a simulation with 4 models. Such a cycle forces sequential operation of the kernels with no speedup possible.

Phold In the Phold model, we first investigate the influence of the percentage of remote events on the speedup. A remote event in this context is an event that is sent from a model on one kernel to a model on another simulation kernel. When remote events are rare, optimistic synchronization rarely has to roll back, thus increasing performance. With more common remote events, however, optimistic synchronization quickly slows down due to frequent rollbacks. Conservative synchronization, on the other hand, is mostly unconcerned with the number of remote events: the mere fact that a remote event can happen, causes it to block and wait. Even though a single synchronization protocol is always ideal in this case, it already shows that different synchronization protocols respond differently to a changing model. Adevs is significantly slower during conservative synchronization. Analysis of profiling

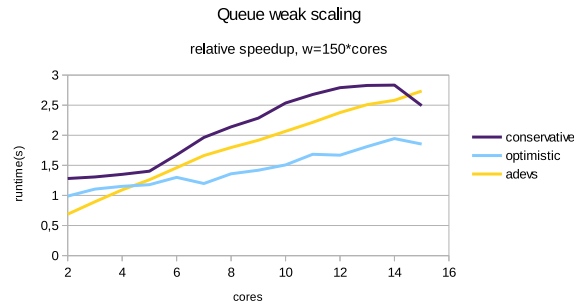


Figure 14. Queue model weak scaling speedup compared to dxex sequential.

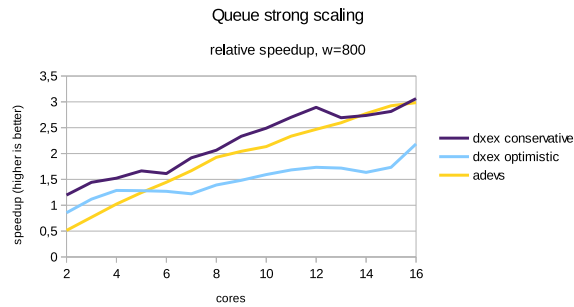


Figure 15. Queue model strong scaling speedup compared to dxex sequential.

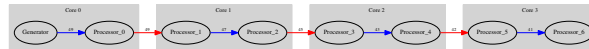


Figure 16. Queue model ($d=2$, $w=7$, $t=5000$, random timeadvance) allocation and simulation trace across 4 kernels.

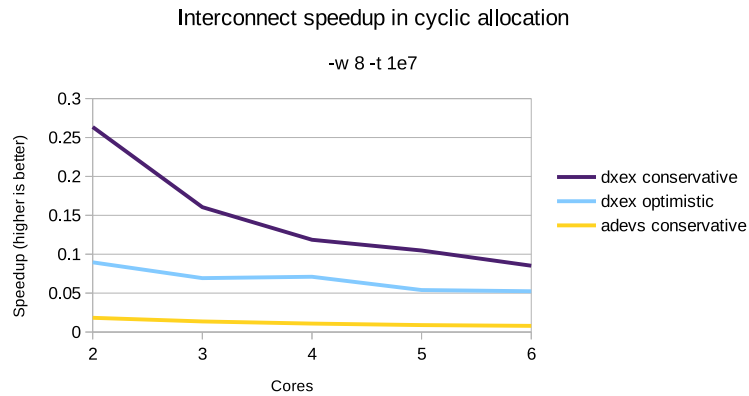


Figure 17. Interconnect benchmark results for parallel simulation.

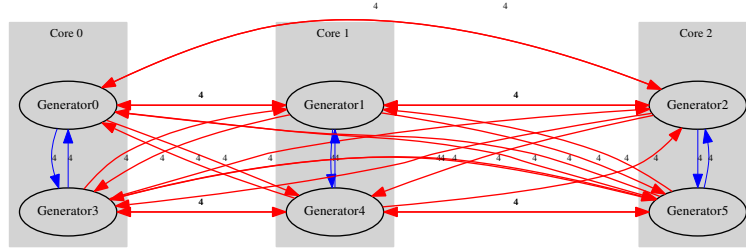


Figure 18. Interconnect parallel simulation trace for 6 models on 3 kernels.

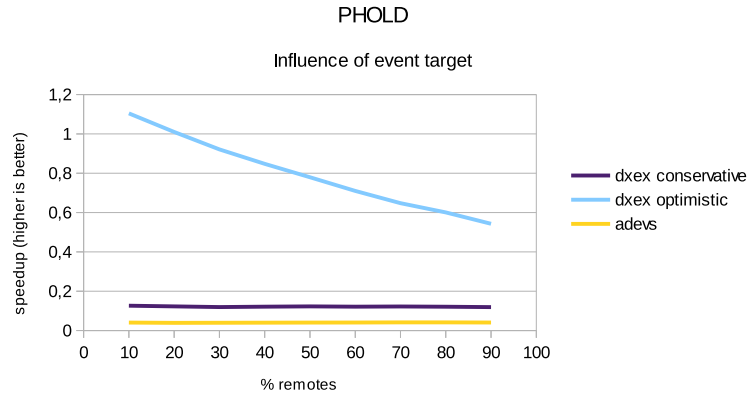


Figure 19. Phold benchmark results for parallel simulation using four kernels, four atomics per node, with varying percentage of remote events.

results shows that exception handling in adevs is the main cause. To keep the models equivalent, the adevs version does not provide the `{begin,end}Lookahead` methods, which accounts for the exception handling. These functions require the user to implement a state saving mechanism in contrast to PythonPDEVS and dxex's optimistic kernels which handle this inside the kernel. We feel this would lead to an unfair comparison as we would like to keep the models synchronization-agnostic across all benchmarks.

We slightly modified the Phold benchmark, to include high-priority events. Contrary to normal events, high-priority events happen almost instantaneously, restricting lookahead to a very small value. Even when normal events occur most often, conservative synchronization always blocks until it can make guarantees. Optimistic synchronization, however, simply goes forward in simulation time and rolls back when these high-priority events happen. This situation closely mimics the case made in the comparison between both synchronization algorithms by [4]. In Figure 21 it is clear that

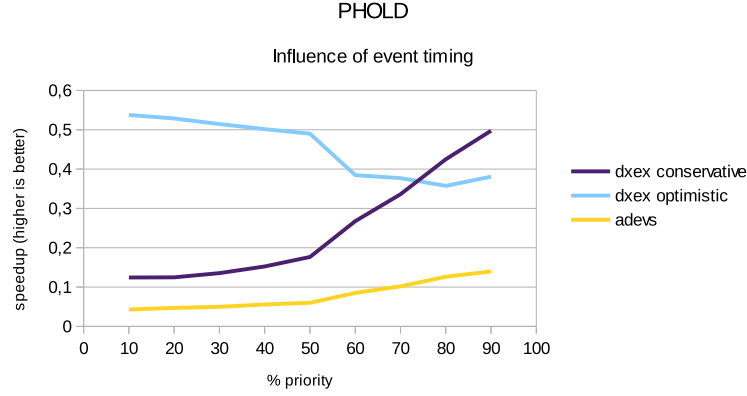


Figure 20. Phold benchmark results for parallel simulation using four kernels, with varying amount of high-priority events.

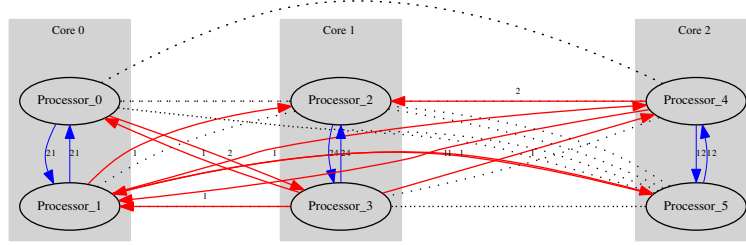


Figure 21. Phold benchmark trace for parallel simulation using three kernels.

in Phold it is possible for dependency cycles to form between kernels which as we have shown for Interconnect degrades performance for both optimistic and conservative. This is also the cause of the sublinear speedup observed in our Phold benchmark.

Figure 20 shows how simulation performance is influenced by the fraction of these high-priority events. If barely any high-priority events occur, conservative synchronization is penalized due to its excessive blocking, which often turned out to be unnecessary. When many high-priority events occur, optimistic synchronization is penalized due to its mindless progression of simulation, which frequently needs to be rolled back. Results show that there is no single perfect synchronization algorithm for this model: depending on configuration, either synchronization protocol might be better.

PholdTree We further verify that our contribution fulfills our projected use case: a single model that can be tweaked to favor either conservative or optimistic synchronization. We will demonstrate that allocation is critical to achieve a parallel speedup and that the parameters of the model will give either conservative or optimistic an advantage.

Allocation The PholdTree benchmark can be configured to use 2 allocation schemes: breadth first and depth first. The breadth first allocation scheme will result in kernels that will form a dependency chain with multiple branches, much like in the Queue model. Such a linear dependency chain can result in a parallel speedup as we demonstrated with the Queue model, but this is not always true as we will demonstrate in this section. A single kernel that has an unbalanced number of atomic models or models with unequal computation load in transition functions will slow down the remainder of the chain. This effect is also apparent if the OS unfairly schedules the thread a kernel runs on. In conservative this will lead to excessive polling of the other kernels' eot values, whereas in optimistic this will lead to a cascade of reverts since dependent kernels will simulate ahead of the slower kernel. In Figures 3 and 4 the simulation trace is visualized for both allocation schemes highlighting the remarks made in this paragraph.

Strong Scaling In Figure 23 we see that the difference in performance for all 4 kernel configurations compared to that shown in Figure 22 is a constant factor. The probability of the priority message has no extra impact on performance other than that shown in sequential performance. The difference between depth first and breadth first allocated kernels is striking, the first results in sublinear speedup for both synchronization variants. The breadth first allocation scheme will lead to a very high number of inter-kernel connections, which is detrimental for any parallel synchronization algorithm. An event exchanged between kernels cannot be securely deallocated without a GVT algorithm and all the complexity this entails. Even an asynchronous GVT algorithm will involve in a shared memory architecture some form of synchronization, and this will slow down the kernels.

Similar to the Queue benchmark the breadth first allocation scheme leads to a chain topology for the kernels, but PholdTree differs in the number of links between the kernels in the chain. In Queue there is only a single model on the edge of a kernel exchanging messages to a single other model in a neighbouring kernel, this is not the case in the PholdTree under breadth first allocation. This explains the difference in speedup between Queue and PholdTree despite the similarity in kernel topology.

Depth first allocation still has a higher inter kernel connection count, but not on the same order as breadth first allocated PholdTree. Depth first allocation will converge to a star topology which in this benchmark leads to a significant speedup.

The performance drop observed for 2 kernels for all allocation schemes and synchronization algorithms is in part due to the higher link count between kernels. With more kernels this effect will be spread across more kernels and result in a relative lower performance penalty.

Conservative synchronization suffers a further penalty in this benchmark when there are only a few kernels. The PholdTree model has a lookahead of ϵ leading the kernel to query each allocated atomic model for a lookahead value on each simulation round. As the atomic models are spread over more kernels this effect is reduced leading to an increase in performance. With 4 kernels conservative surpasses optimistic in speedup. This is clear evidence that conservative synchronization is a good candidate for parallel simulation even in simulations with uncertainty.

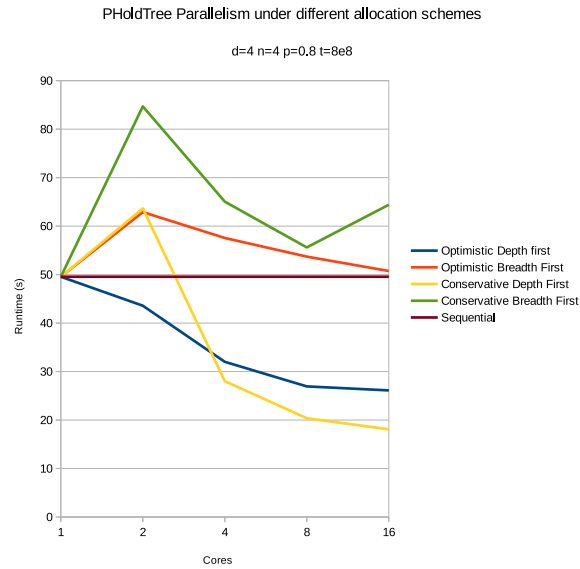


Figure 22. PholdTree model performance under different allocation schemes with low message probability

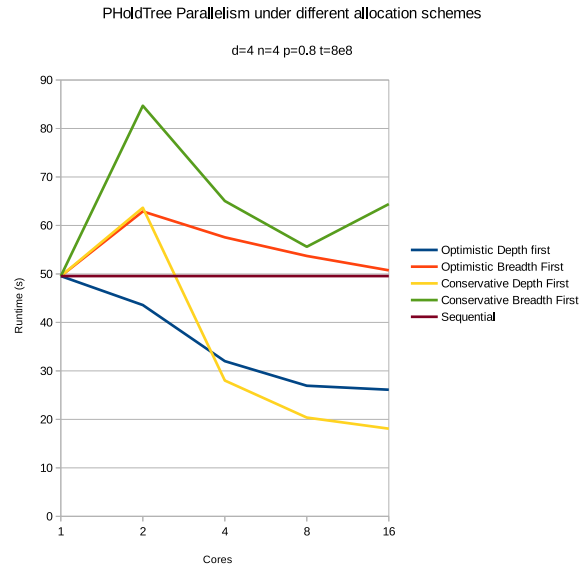


Figure 23. PholdTree model performance under different allocation schemes with high message probability

Weak Scaling In this paragraph we investigate dxex’s parallel performance when the number of atomic models distributed over a fixed number of kernels varies. From 1 we know that increasing d will lead to a very rapid increase in models, we want to observe what happens in a closer to linear increase in model count per kernel, so we vary n .

Adevs’ conservative implementation cannot handle the uncertainty (lack of non ϵ lookahead) here and is omitted from the comparison. Its parallel performance is two orders of magnitude slower than the sequential implementation. Upon investigation we conclude that this slowdown is caused by not implementing the `{begin/end}lookahead()` functions, which when not overridden in a model trigger an exception on each invocation. This exception handling completely stalls the kernel. We do not implement this function in our PholdTree variant for adevs since this would force us to implement state saving in the model code and not use the provided state saving functionality in the kernel, as is done in dxex or PythonPDEVs’s optimistic. In dxex state saving is never required for a conservative simulation, this is handled transparently for the user who need not implement this behaviour. By using the state saving technique inside the model code we feel we would no longer compare identical models across different synchronization algorithms, although we do not doubt adevs’ increased performance when these functions are overridden.

In Figure 24 we observe that dxex’s optimistic kernels are slightly sensitive to the number of atomic models allocated to them. In the worst case with 2 kernels there is no real speedup observable, as soon as the number of kernels increases we see a converging speedup trend, that except for 32 kernels is almost constant despite the increase in n . Note that only depth first allocated kernels are measured here, breadth first as we can see in 23 has no speedup advantage. The probability parameter is kept at 0.1 for the same reason, it only induces a linear increase in load.

Conservative has a more nuanced speedup behaviour in this benchmark. We have detailed how a conservative kernel in dxex scales linear in the number of atomic models (if lookahead is ϵ), this effect is more clearly visible here.

It is clear that as the fanout (n) of the model increases the kernel performance degrades rapidly. The intersection of the performance graph of each configuration is shifted to the right, the actual 1-speedup point increases as the kernelcount increases. For a $d=4$, $n=5$ configuration with 4 kernels each kernel has a load of 1300 atomic models when it crosses the 1-speedup line.

An 8 kernel configuration with $d=4$ $n=6$ can sustain a load of 2700 atomic models before it reaches that point. From the results we see that the 32-kernel configuration is not yet slowing down with the current parameters.

The modelcount is only a part of the explanation of this effect, as the kernelcount increases the amount of inter kernel messages will be split into more distinct sets which can be handled with more concurrency in dxex’s architecture. That this effect is not always true can be observed from the $d=4$ $n=3$ 32-kernel datapoint. Note that in this configuration 341 atomic models are distributed over 32 kernels, a relative low model load can more easily highlight synchronization overhead, even if allocation is optimal.

As with optimistic we do not include breadth first allocated benchmarks in this speedup plot since none of those achieve a speedup higher than 1.

If we combine both to determine which synchronization protocol is more optimal we see in Figure 26 that for these configurations conservative with 8 kernels is an ideal configuration up until $n=5$, where the modelcount starts to degrade performance. Optimistic with the same kernelcount is almost insensitive to the increase in modelcount and is therefore a more robust choice. Note how optimistic and conservative with 32 kernels at this point still have a reasonable speedup which is surprising given the synchronization overhead in such a large set of kernels.

Measuring Synchronization Overhead

The simulation kernels have a non trivial overhead compared to a sequential kernel. In this section we will try to isolate this overhead. Profiling indicates that there are in the most trivial of models only 2 causes for computational load outside of the actual simulation : memory management and random number generation. By reducing the cost of both we will isolate the synchronization overhead. We use the Queue benchmark since it is highly parallel and its computational load in transitioning is only determined by random number generation and event/state allocation.

Memory management In dxex a model author is given access to automatic memory management for events and states. A model written for a sequential simulation will run correctly in a conservative or optimistic simulation without altering (from the point of view of the model author) the (de)allocation semantics of events or states.

In sequential and conservative simulation the kernel gives the guarantee that no state will be copied. Dxex's simulation control hides the implementation details of this management, a model author can focus on the actual semantics of the model. The kernels use memory allocators backed by a thread-aware pooling library to reduce the performance impact of memory management as much as possible. A sequential simulation kernel no allocated event will persist beyond a single time advance, this enables the use of an arena-style allocator. Conservative and optimistic simulation need to use generic pool allocators since events are shared across kernels and thus have a different lifetime.

An important observation to make is that intra-kernel events can be pooled more aggressively, whereas inter-kernel events need a GVT algorithm to determine when safe deallocation can occur, even in conservative synchronization. A simulation with high inter kernel events will suffer a performance hit, whereas the impact of high intra kernel events can be optimized using arena allocators.

The allocation interfaces are instrumented in dxex to trace memory usage in a simulation and can serve as a basic memory leak detector. Shielding the user from (de)allocation complexity comes at a cost in performance, as the comparison with adevs in simulations with a high event frequency 10 of events shows.

Figure 27 shows an experiment where we measure peak memory usage (maximum resident set size (kB)) for each synchronization protocol and determine how the memory usage differs with respect to sequential simulation. Both conservative kernels require only a small constant increase in memory usage.

Dxex uses Boost Pool[14] allocators in parallel simulation kernels, and can use arena-style allocators for sequential simulation. The latter can be faster, but at the cost of extra configuration. The allocators are supplemented by the library tcmalloc [15]. In

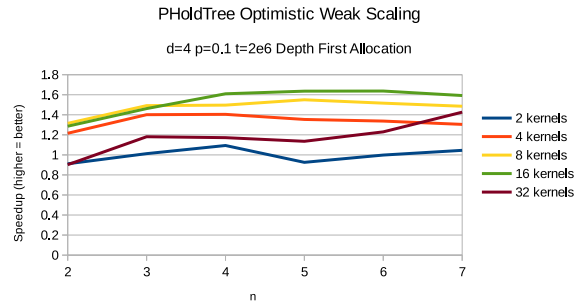


Figure 24. PholdTree model weak scaling under varying fanout and optimistic synchronization

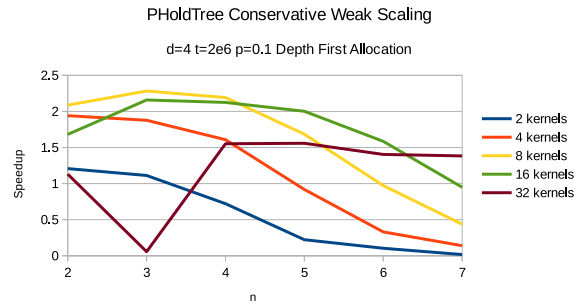


Figure 25. PholdTree model weak scaling under varying fanout and conservative synchronization

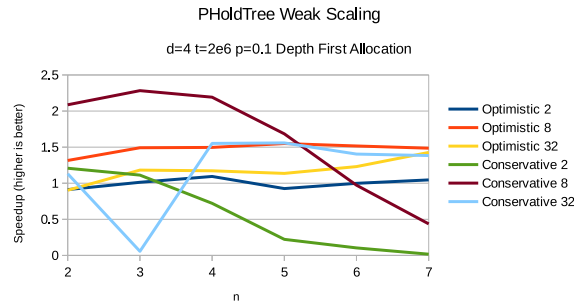


Figure 26. PholdTree model weak scaling under varying fanout and different synchronization algorithms

Figure 28 the Interconnect model is benchmarked under the different combinations of allocation strategies.

Interconnect is a perfect test case since it generates each round a set of events quadratic in size to the number of atomic models in the simulation. With each event dynamically

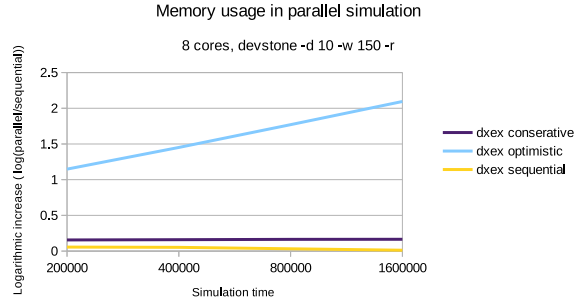


Figure 27. Memory usage in parallel synchronization

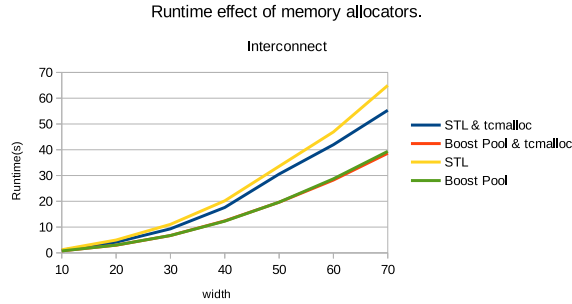


Figure 28. Effect of memory allocators on sequential runtime.

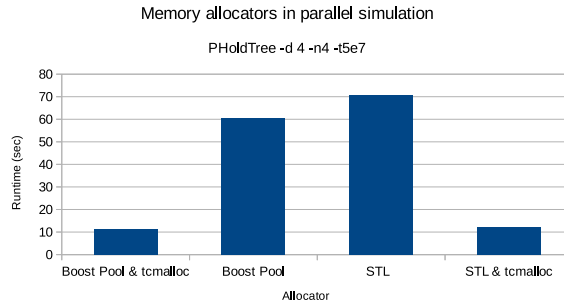


Figure 29. Effect of memory allocators on parallel runtime.

allocated this makes it the perfect stress test for any allocation strategy. Both tcmalloc and Boost pools result in an advantage, but not necessarily combined.

We next investigate the effect of both strategies in an optimistic simulation. We have already demonstrated that conservative simulation differs little in memory usage from sequential, so we only benchmark the optimistic kernel. From the results it is clear that optimistic simulation benefits greatly from the use of tcmalloc, regardless of the allocator. Nonetheless the pool allocator still reduce the allocation overhead.

Both the pools and `tcmalloc` will try to keep memory allocated to them by the OS, this gives a somewhat misleading view in statistics collected from the OS. The `dxex` kernels themselves and the pool interfaces allow the user to instrument event allocation and deallocation exactly, useful for example in observing how efficient GVT-triggered deallocation is in practice.

In conclusion both techniques are required to reduce the overhead of memory allocations in `dxex`, and on by default.

Random Number Generators A non trivial amount of time in a simulation spent waiting for a random number generator can mask synchronization overhead. Several of our benchmarks use random number generators to test the effect of uncertainty in a simulation, either by random time advances or random destinations for events.

In a parallel simulation we also need to guarantee isolation between the random calls to avoid excessive synchronization on the `rng` object itself. `Dxex` uses the Tina random number generator collection (`trng`) [16] as an alternative random number generator as it is designed with performance and multithreading usage in mind. We also wrote a new version of our models that stores the `rng` object in the state instead of calling a shared thread local object to isolate the synchronization overhead.

From Figure 30 we see that storing the `rng` in the state is a very expensive operation for the default STL random number generator. The size of the `rng` object in the state triggers memory overhead but it is interesting to see that `adevs`' conservative is also sensitive to this effect even though in theory in conservative synchronization no state copying/saving is required. `Dxex`'s conservative kernel is insensitive to storing the `rng` object in the atomic model state, since no copying/state saving occurs in `dxex` conservative simulation. The overhead of initially doing so is therefore constant and not a factor in long simulations.

Replacing the STL `rng` with the threading optimized `trng` we are more accurately able to isolate synchronization overhead. Figure 30 shows that both `dxex` and `adevs` in sequential simulation gain a ≈ 3 speedup by using the faster random number generator. The parallel kernels have smaller speedup with `adevs`' conservative and `dxex` optimistic achieving an almost identical factor. `Dxex` conservative is almost insensitive to the changing of the `rng`. `Dxex` optimistic and `adevs` conservative are in the default configuration spend relatively more time waiting for random numbers than `dxex` conservative, demonstrated by the speedup gained when random number generation is accelerated. The actual synchronization overhead in all parallel kernels still dominates the cost of random number generation which is seen from the speedup difference between parallel and sequential.

Conclusion

We have shown that our contribution is invaluable for high performance simulation: depending on the expected behaviour, modellers can choose the most appropriate synchronization protocol. We will summarize the key aspects in achieving good parallel performance here.

Allocation Allocation of atomic models over kernels is critical to obtain a speedup. In the case of the Interconnect model it is not possible to allocate models without

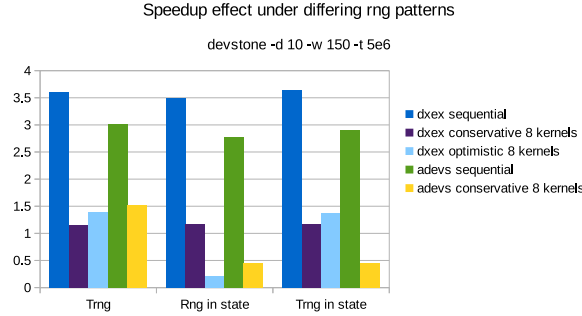


Figure 30. Speedup with different rng usage patterns in Queue model

introducing a cyclic dependency between kernels with severe performance degradation as a result. Even when no cycles exist in the topology between kernels a good allocation scheme will minimize the number of connections between kernels and aim for a star topology as in PHoldTree with depth first allocation or a chain topology as in the Queue model. Dxex can optionally offer the user a visualization of the behaviour of the kernels under a user supplied allocation scheme to allow for more insight. The same instrumentation could in future be used to optimize an existing allocation scheme.

Uncertainty A simulation where future behaviour cannot be predicted is typically not suited for conservative simulation, but we have demonstrated that dxex’s conservative kernel can still offer a non trivial speedup in this context. Optimistic is not constrained by this uncertainty and can offer a good speedup regardless of lookahead. Phold and PholdTree demonstrate that a single model can benefit from different synchronization protocols depending on a single parameter.

Limits Dxex’s conservative kernel is constrained by the amount of models it controls, as shown in the PHoldTree benchmark. This is especially the case when the minimal lookahead is ϵ requiring near constant polling of models for lookahead values. Optimistic is not constrained by the number of models it manages, but can quickly consume all available memory in a simulation. While this effect can be reduced with a thread aware allocation scheme, the underlying problem remains.

Related Work

Several similar DEVS simulation tools have already been implemented, though they differ in key aspects. We discuss several dimensions of related work, as we try to compromise between different tools.

In terms of code design and philosophy, dxex is most related to PythonPDEVS [5]. Performance of PythonPDEVS was still decent, through the use of simulation and activity hints from the modeler. This allowed the kernel to optimize its internal data structures and algorithms for the specific model being executed. All changes were completely transparent to the model, and were completely optional. In this spirit, we offer users the possibility to choose between different synchronization protocols.

This allows users to choose the most appropriate synchronization protocol, depending on the model. Contrary to PythonPDEVs, however, dxex doesn't support distributed simulation, model migrations [17], or activity hints [18].

While PythonPDEVs offers very fast turnaround cycles, due to the use of an interpreted language, simulation performance was easily outdone by compiled simulation kernels. In terms of performance, adevs [19] offered much faster simulation, at the cost of a significant compilation time. The turnaround cycle in adevs is much slower though, specifically because the complete simulation kernel is implemented using templates in header files. As a result, the complete simulation kernel has to be compiled again every time. Dxex compromises, as vle [20] or PowerDEVs [21], by separating the simulation kernel into a shared library. After the initial compilation of the simulation tool, only the model has to be compiled and linked to the library. This significantly shortens the turnaround cycle, while still offering good performance. In terms of performance, dxex is shown to be competitive with adevs. Despite its high performance, adevs does not support optimistic synchronization, which we have shown to be highly relevant.

Previous DEVs simulation tools have already implemented multiple synchronization protocols, though none have done it in a strictly modular way that allows straightforward protocol switching for a single given model. For example CD++ [22] has both a conservative (CCD++ [23]) and optimistic (PCD++) [24]) variant. Despite the implementation of both protocols, they are different projects entirely, and are incompatible with modern compilers. Dxex, on the other hand, is a single project, where switching between different synchronization protocols is as simple as switching any other configuration parameter. CD++, however, implements both conservative and optimistic synchronization for distributed simulation, whereas we limit ourselves to parallel simulation. By limiting our approach to parallel simulation, we are able to achieve higher speedups through the use of shared memory communication.

In summary, dxex tries to find the middle ground between the concepts of PythonPDEVs, the performance of adevs, and the multiple synchronization protocols of CD++.

Conclusions and future work

In this paper, we introduced DEVs-Ex-Machina ("dxex"), a new C++14-based Parallel DEVs simulation tool. Our main contribution is the implementation of multiple synchronization protocols for parallel multicore simulation. We have shown that there are indeed models which can be simulated significantly faster using either synchronization protocol. Dxex allows the user to choose between either conservative or optimistic synchronization as simple as any other configuration option. Notwithstanding this modularity, dxex achieves performance competitive to adevs, another very efficient DEVs simulation tool. Performance is measured both in elapsed time, and memory usage. Our empirical analysis shows that allocation of models over kernels is critical to enable a parallel speedup. Furthermore we have shown when and why optimistic synchronization can outperform conservative and vice versa.

Future work is possible in several directions. Firstly, our implementation of optimistic synchronization should be more tolerant to low-memory situations. In

its current state, simulation will simply halt with an out-of-memory error. Having simulation control, which can throttle the speed of nodes that use up too much memory, has been shown to work in these situations [4]. Faster GVT implementations, such as those presented by [25] and [26], might further help to minimize this problem. Secondly, the idea of activity can be implemented for our simulation kernels, making it possible to dynamically switch between conservative and optimistic synchronization when behavioural changes are detected. Thirdly, activity algorithms, as already implemented by PythonPDEVS, can also be implemented in dxex, to determine how they influence simulation performance. Finally automatic allocation is possible by analysis of the connections between models. This information is already used in dxex to determine the dependency graph for conservative synchronization and the directconnect algorithm. A graph algorithm that distributes models while avoiding cycles in the resulting kernels topology could be used to maximize parallel speedup in either optimistic or conservative synchronization.

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