# PHYS4007: Scientific Computing

## Week 4: Ordinary Differential Equations

After this laboratory session you should:

1. Solve ODEs using numerical methods,
2. Apply the methods to initial and boundary value problems,
3. Solve eigen value problems.

### Task 4.1 Initial Value Problem

A differential equation that occurs frequently in mathematical physics is Bessel's equation,

$$z^2 \frac{d^2 J_n}{dz^2} + z\frac{dJ_n}{dz} + (z^2 - n^2)J_n = 0,$$

where $\boldsymbol{J_n(z)}$ is a Bessel function of integer order n.

1. Solve this equation using the 2nd Order Runga-Kutta method for n=0 and boundary conditions

   $$J_0(0) = 1 \text{ and } \left.\frac{dJ_0}{dz}\right|_{z=0} = 0.$$

2. Find the solution using an inbuild ODE solver.
3. Compare your solutions with the inbuilt Bessel function generator.

### Task 4.2 Shooting Method.

The equation of motion of a cannon ball through air is

$$m\frac{d^2\vec{r}}{dt^2} = m\vec{g} - mk\,|v|\,\vec{v},$$

where $\vec{r}$ is the position vector, $\vec{v}$ is the velocity vector and $\vec{g}$ is the acceleration due to gravity. The motion may be assumed to be two-dimensional and $|v| = \left(v_x^2 + v_y^2\right)^{1/2}$ is the magnitude of the velocity. The drag constant $k$ is $10^{-4}$ $m^{-1}$. If the ball leaves the cannon with a speed of 200 $ms^{-1}$, what angle of elevation is needed to hit a target 3km away, assuming flat ground?

**Task 4.3 The 1D Schrödinger Equation**

The potential energy function for a particle confined by a surface can be written in the form

$$V(x) = \infty \qquad\qquad x \le 0$$

$$= -\frac{\hbar^2 V_0}{2ma^2} \qquad\qquad 0 < x \le a$$

$$= 0 \qquad\qquad x > a$$

where $V_0$ is a constant. The wavefunction satisfies the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + V(x)\varphi = E\varphi.$$

If $V_0 = 10$, find the ground state energy (in terms of $\hbar^2/2ma^2$) and normalised ground state wavefunction using the shooting method. As the potential is infinite for $x \le 0$ the wavefunction will be zero for all negative $x$.