Electrostatics + Magnetism !

Electrostatics (15 min)

First discovered as the triboelectric effect:

rubbing materials together can transfer charge

Triboelectric Series:

ranking of materials from most positive to least positive

Use acrylic + fur to move paper

+ lose electrons

- air
- skin
- fur
- acetate
- glass
- nylon
- paper
- wool
- silk
- cotton

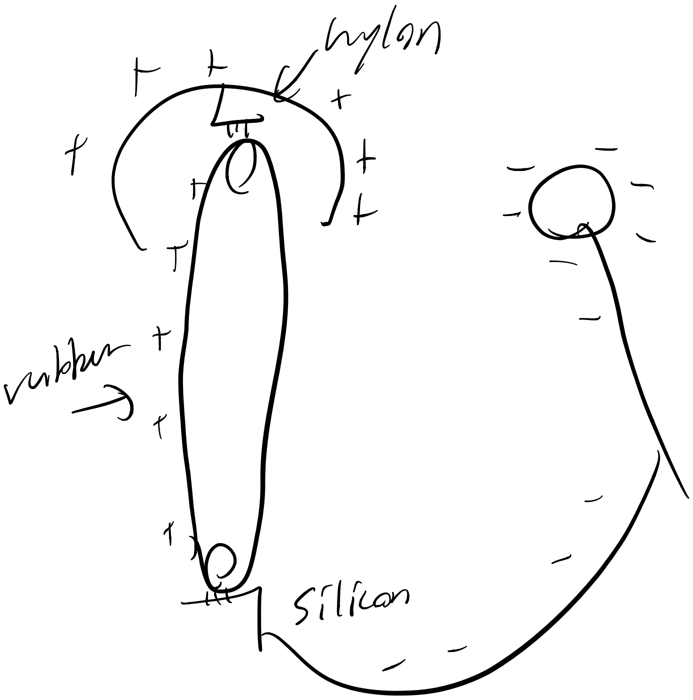
$\}$ (~0) — wool, silk, cotton

- steel
- wood
- amber
- wax
- rubber
- silver
- gold
- platinum
- acrylic
- silicon

− gain electrons

electron
from Latin electrum
from Greek ἤλεκτρον
(elektron)
= "amber"

Van de Graaf generator (15 min)



have them play with it for ~10 min

↳ why is hair standing on end?

Coulomb's law (10 min)

Coulomb's law: $\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$k = 8.99 \times 10^9 \frac{Nm^2}{C^2}$

ε_0 = vacuum permittivity
$8.85 \times 10^{-12}\ C^2/Nm^2$

Ex)

Two 1kg spheres 1m apart each have a charge of $q = +1C$. What is ratio of $\frac{F_E}{F_g}$?

$F_g : G \frac{1 \cdot 1}{1^2} = 6.67 \times 10^{-11}\ N$

$F_E = k \cdot \frac{1 \cdot 1}{1^2} = 8.99 \times 10^9\ N$

20 orders of magnitude!

Electric fields (10 min)

similar to gravitational field: a vector field that assigns an electrostatic acceleration vector to each point in space

↳ mathematica charge field lines demo

how to draw electric fields:

arrows go from ⊕ to ⊖

lines never cross!

$$\vec{F} = q\vec{E}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\downarrow$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

$$\downarrow$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\hat{r}}{r^2}\,dq$$

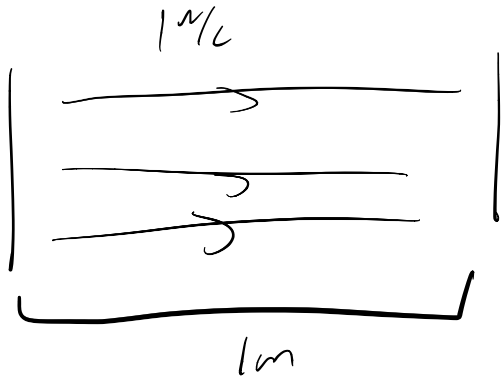
Electric potential (10 min)

Van de Graaff generator can produce 100s of kV

$\downarrow$

What is a Volt?

↳ potential difference between two plates 1m apart w/ a field of 1 N/C



units?

$$V = \frac{\text{energy}}{\text{charge}} = \frac{kg\frac{m^2}{s^2}}{A\cdot s} = \frac{kg\, m^2}{A\cdot s^3}$$

$$V = \int_{\infty}^{r_0} \vec{F}\cdot d\vec{r} = \int_{\infty}^{r_0} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0}$$

$$V = \int_{\infty}^{r_0} \vec{E}\cdot dr = \int_{\infty}^{r_0} \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}$$

$$\vec{E} = \frac{-\partial}{\partial r} V$$

$\downarrow$

$$\vec{E} = -\vec{\nabla} V$$

gradient!

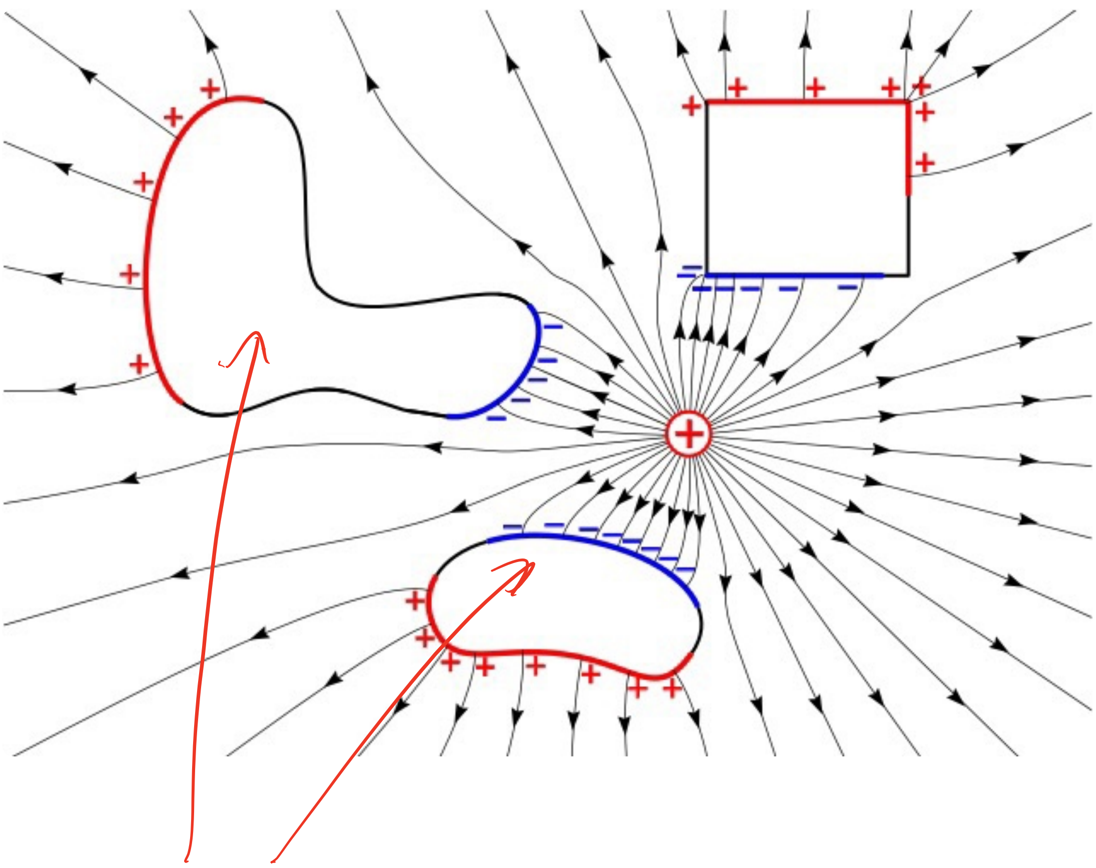
charge • voltage = energy!

1eV = energy to accelerate 1e through 1V

Electric fields on conductors (Smith)

conductors have totally free charges, so the charges move along the boundary to cancel electric fields since $\vec{F} \propto \vec{E}$

↳ No electric fields inside a conductor



Faraday cage! (why you don't have service in an elevator)

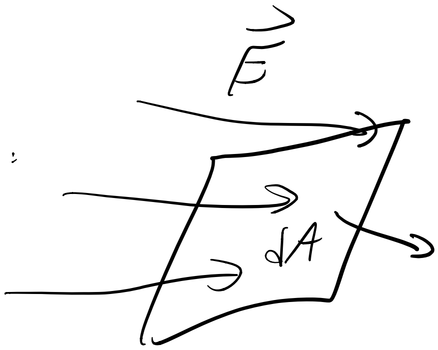
Q: does a conductor shield fields from charges it encloses?

A: no! Gauss's law!

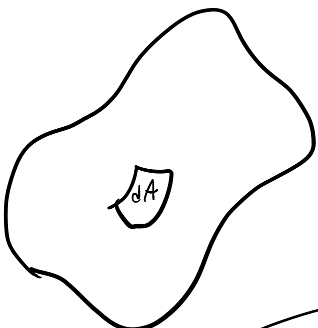
Gauss's law (10 min)

net electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the enclosed charge

Flux:



$$d\Phi = \vec{E} \cdot d\vec{A}$$



$$\Phi = \iint_A \vec{E} \cdot d\vec{A}$$

Gauss's law: $$\oiint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

If no charge is enclosed, any field lines coming in also exit, so $\Phi = 0$

~75 minutes in

Ex) Earth has surface field of
$E \simeq 100 \frac{V}{m}$ pointed outwards.
What is charge of earth?

$$\oiint_{earth} E \cdot dA = 100 \tfrac{V}{m} \cdot 4\pi R_\oplus^2 = \frac{Q}{\varepsilon_0}$$

$$Q = 452.6 \text{ kC}$$

What is # electrons / area?

$$\sigma = \frac{Q}{4\pi r^2} = \frac{452.6 \text{ kC} \cdot 6.24E18 \, e/c}{4\pi R_\oplus^2} = 5.53 \times 10^9 \frac{e}{m^2}$$

Why?

Thunderstorms

(10 min)

Break + AMA

Current and resistance (10 min)

Current: $I = \frac{dQ}{dt}$ ← rate of charge flowing through wire

Resistance: opposition to flow of current

Ohm's law: $\frac{V}{I} = R$

analogy: water flow

$\frac{\text{pressure of water}}{\text{water flow rate}} \propto$ cross section of orifice

Power dissipated by a circuit:

$$P = \frac{\Delta E}{t} = \frac{Q\Delta V}{t} = IV$$

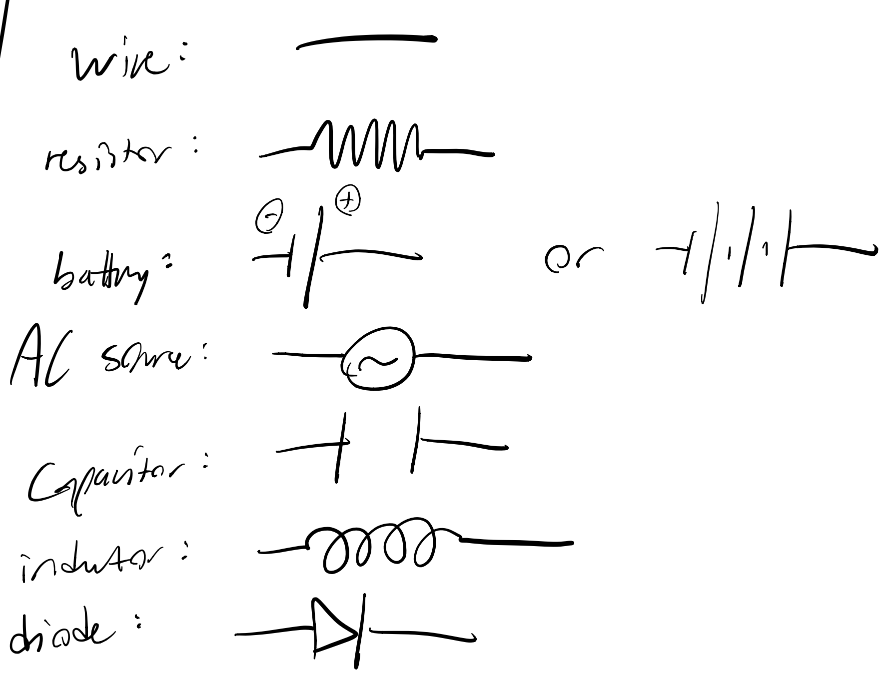
$V = IR$ $\quad$ $I = \frac{V}{R}$

$$P = I^2 R \qquad P = \frac{V^2}{R}$$

Circuits: (5 min)

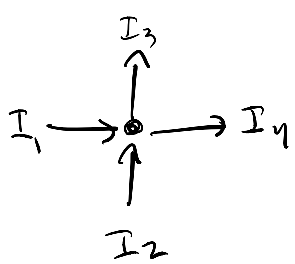
a closed loop of electrical components

common symbols:



Kirchhoff's circuit laws: (5 min)

Current law: current in = current out



$$I_1 + I_2 = I_3 + I_4$$

$$\sum_k I_k = 0$$

Voltage law: total potential difference around any closed loop is zero

$$\sum_k V_k = 0$$

Magnetism (15 min)

How do magnets work?

A: little pieces of gravity inside rocks

Magnetic fields! Units: Tesla, ~~Gauss~~ Symbol: $\vec{B}$

$1\,T = 10^4\,G$

Just like electric fields, but two key differences:

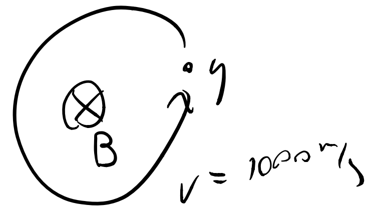
1. no magnetic monopoles (probably, open question)
2. Force cannot add energy to charged particles

Play w/ magnet toys (~5 min)

Force created by magnetic field: <u>Lorentz force</u>

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Ex| electron travels at $v = 1000\,m/s$ in a $B = 1\,mT$ field. What is its radius?



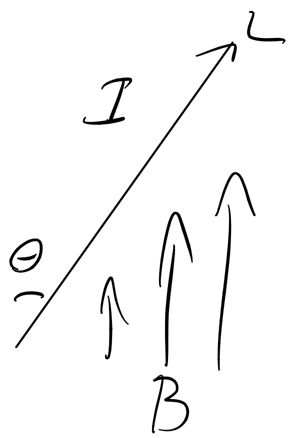
$$F_B = F_c$$

$$qvB \cancel{\sin\theta}_{1} = \frac{mv^{\cancel{2}}}{r}$$

$$r = \frac{mv}{qB} = 5.68\,\mu m$$

cyclotron radius

Force on wires:



$$qv = \frac{qL}{t} = IL$$

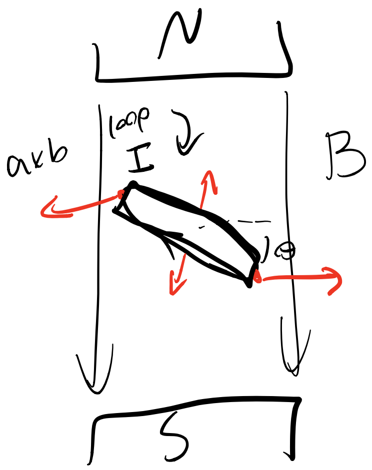
$$F = q\vec{v} \times \vec{B}$$

$$= I\vec{L} \times \vec{B}$$

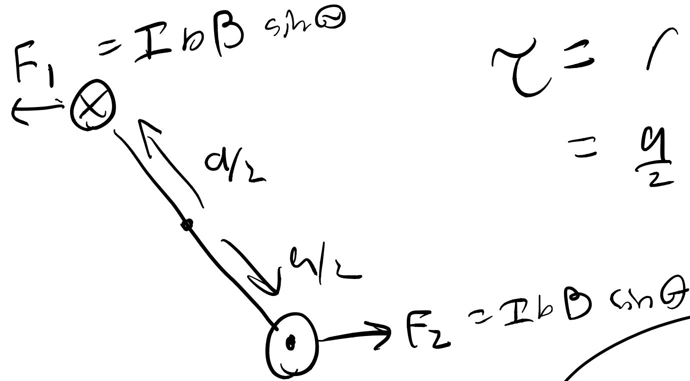
$$= ILB \sin\theta$$

Torque on a current carrying loop (5min)

→ This is how motors work!



$$F = I\vec{L} \times \vec{B}$$

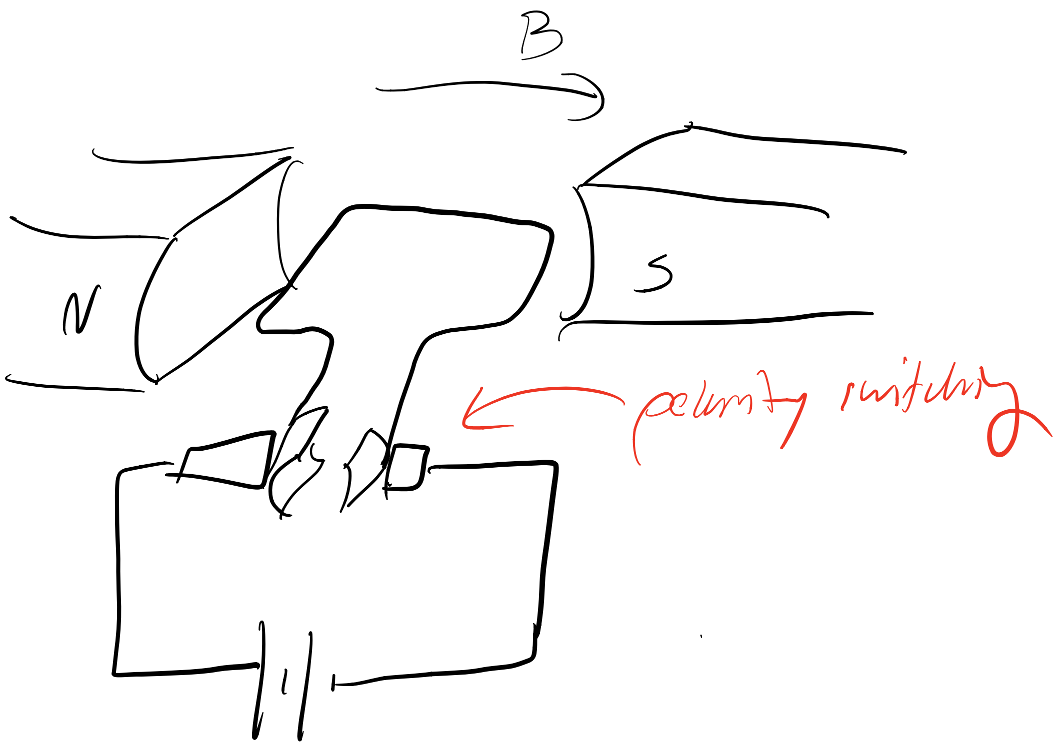


$$\tau = r \times F$$

$$= \frac{a}{2} I b B \sin\theta \times 2$$

$$\tau = Iab\,B\sin\theta$$

Motors (5min)



Currents produce magnetic fields! (10 min)

Biot-Savart law

$$B(r_0) = \frac{\mu_0}{4\pi} \int_C \frac{\vec{I} \times \vec{r}}{|r|^3} \, dl$$

Vacuum permeability

For a loop of wire at center:

$$B(r_0) = \frac{\mu_0}{4\pi} \oint_C \frac{\vec{I} \times \vec{r}}{\vec{r}^3} \, dl$$

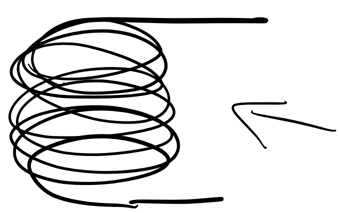
$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \oint_C dl$$

$$= \frac{\mu_0}{2\cancel{4\pi}} \frac{I}{r^{\cancel{2}}} \cdot \cancel{2\pi r}$$

$$= \frac{\mu_0 I}{2r}$$

If you have a coil:

$$B = \frac{\mu_0 I}{2r} \cdot N$$



solenoid

Capacitors (10 min)

a two-terminal device which stores energy in an electric field

capacitance: $\dfrac{\text{charge separated}}{\text{voltage applied}}$ $\qquad C = \dfrac{Q}{V}$

Units: Farads $= \dfrac{1 \text{ coulomb}}{1 \text{ volt}}$

1F is a lot of capacitance!
most capacitors around in pF

Energy stored in a capacitor:

$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

since $Q = CV$,

$$U = \frac{1}{2}\frac{Q^3}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Parallel plate capacitor: (10 min)

charge density (2D)

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{\varepsilon_0}{\cancel{\sigma}} \frac{\cancel{\sigma} A}{d} = \frac{\varepsilon_0 A}{d}$$

E for a plane of charge is $E = \frac{\sigma}{\varepsilon_0}$

$\boxed{\text{Ex 1}}$ A PP capacitor has $C = 1$ Farad & $d = 1mm$.
what is area?

$$\frac{\varepsilon_0 A}{d} = 1F$$

$$A = \frac{1F \cdot 1mm}{\varepsilon_0} = 1.12 \times 10^8 m^2 \rightarrow 43 mi^2 !$$

yoooge!

Ex) PP cap. w/ $A = 1m^2$ & $d = 1mm$
what is energy stored if voltage diff. is 100V?

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\varepsilon_0 \, 1m^2}{1E\text{-}3m} \cdot (100V)^2 = 44.3 \mu J$$

Other major circuit component: Inductors!

To do this, we need to understand magnetism!

Currently @ 160 min

more PI to challenge