Topics in Physics: Problem Set #9

Topics: black hole thermodynamics

Reduced-length problem set

This problem set is designed to take only a portion of the afternoon session to complete. Plan to spend the remaining time in the session working on your final projects.

Challenge Problems (approx. 60 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

1. Approximately what is the information density (in bits per page) of typical single-sided printed text on paper?

2. Bekenstein bound states that the maximum entropy contained in any region of space $S \leq \frac{2\pi k_B RE}{\hbar c}$. In the case of a black hole, this inequality is saturated, so $S_{BH} = \frac{2\pi k_B RE}{\hbar c}$, where R is the Schwarzschild radius ($R = \frac{2GM}{c^2}$) and E is the total mass-energy of the black hole. The information contained in a black hole is $I_{BH} = \frac{1}{k_B \log 2} S_{BH}$, which represents the number of bits contained in the quantum states contained inside the event horizon. Show that this can be written as:

$$I_{BH} = \frac{4\pi GM^2}{\hbar c \log 2}.$$

3. How many pages of paper would it take to print out all of the information contained inside a black hole with the mass of the sun?

4. What would the mass of that many pages be? How does this compare to the mass of the known universe?

5. If you took that much paper and made a black hole out of it, how many sheets of paper would you need to print out to contain all of the information in the new black hole?