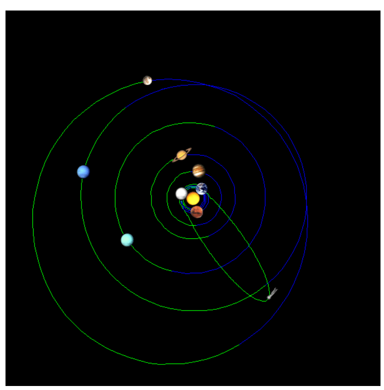
Topics in Physics: Problem Set #3

Topics: circular motion, angular momentum, gyroscopes

Practice Problems (approx. 45 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. At the equator, the centripetal acceleration from Earth's rotation partially cancels out gravity. How strong is the centripetal acceleration relative to gravity?
2. We are also subject to centripetal acceleration from the rotation of earth around the sun. Would this centripetal acceleration cancel or add to Earth's gravity? (Hint: the answer depends on the time of day.) How big is this acceleration relative to Earth's gravitational acceleration?
3. An astronaut uses a can of compressed air to spin himself. Starting with $\omega = 0$, he wants to accelerate and then decelerate so that he stops spinning just as he finishes 1 full revolution. If his maximum angular acceleration is 0.1rad/s^2, how quickly can he complete this maneuver?
4. Halley's comet follows a highly eccentric elliptical orbit around the sun, shown below, with a period of about 76 years. Its aphelion (farthest orbital radius) is about 5.3 billion km, where it orbits at a snail's pace[1] of 910m/s. The comet's perihelion (closest point to the sun) is 88 million km, where it whips past the sun at blazingly fast speeds. The comet has a mass of 2.2×10^{14}kg.

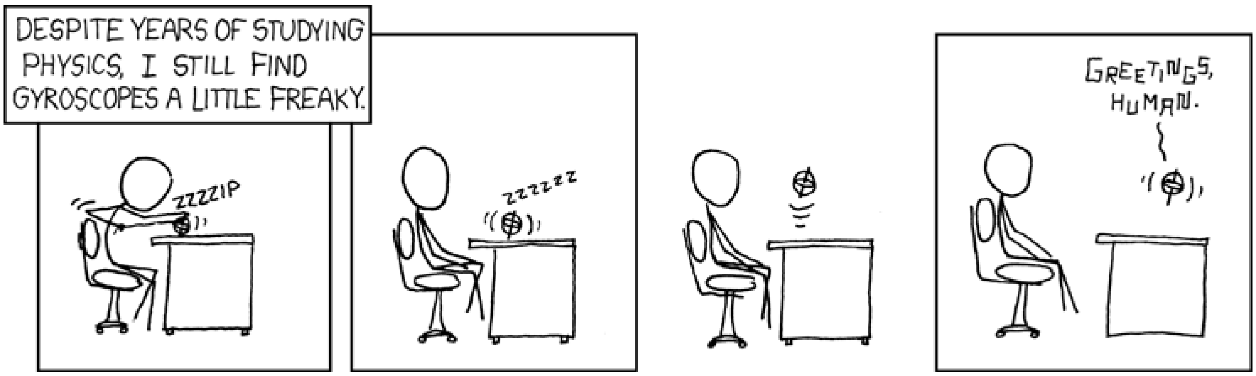


   (a) Calculate the comet's angular momentum at its aphelion.
   (b) How fast is the comet traveling when it is at its perihelion?

5. A soccer ball with radius 20cm is rolling at 5 rotations per second and stops after 20m. What is its angular acceleration? How long does it take to stop?

[1] We're of course talking about space snails, which still travel pretty fast by human standards.

Experiment: fun with gyroscopes (approx. 60-120 min)



https://xkcd.com/332/

In this exercise, we'll analyze the physics behind gyroscopes to try to get an estimate of how fast you can spin the flywheel. Gyroscopes are a hard subject to understand (this is probably college level material if you had to do this from scratch!) but we'll walk through the analysis step by step. This is probably the hardest derivation you will do in this course! Take your time with each question and try to think deeply about what is going on.

Note: this experiment follows some derivations which are easy to stumble across if you search for gyroscope precession, so please don't search for anything related to gyroscope precession until you have finished this lab. (General searches for angular kinematics, vectors, etc. are fine.) If you get stuck or are confused, ask a TA!

Consider your gyroscope in the configuration shown below. The gyroscope has m, the moment of inertia of the flywheel about its axle is I, and the center of mass of the flywheel is offset from the pylon by a vector $\vec{r}$, which makes an angle ϕ relative to vertical. Let S denote the contact point between the pylon and the pivot.

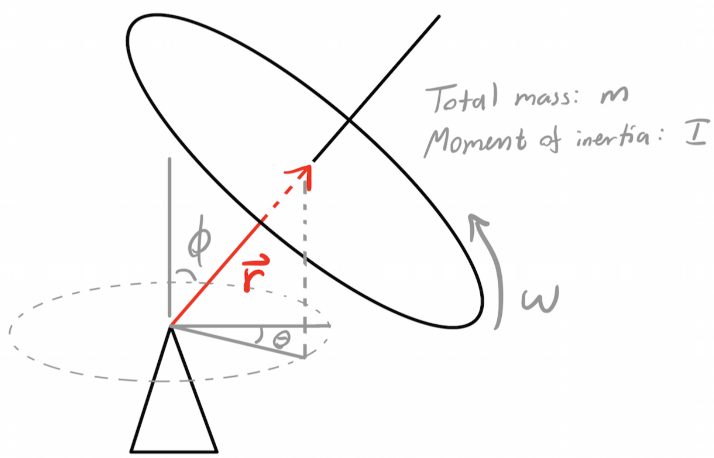


Figure 1: The configuration of your gyroscope as described above.

1. The gyroscope is affected by gravity (believe it or not), and it is held up by the pylon, so there will be a torque resulting from the gravitational force.

   (a) What is the magnitude of the torque about S resulting from the force of gravity?

(b) What direction does the torque point when the gyroscope is in the configuration shown in Figure 1?

(c) As we learned in class, $\vec{\tau} = \frac{d\vec{L}}{dt}$. Write an expression for the amount $d\vec{L}$ that the angular momentum will change over a time dt.

2. Assume the gyroscope is spinning with a constant angular velocity with magnitude ω and angular momentum $\vec{L}$.

   (a) What is the magnitude of $\vec{L}$ in terms of the parameters listed above?

   (b) What direction does $\vec{L}$ point in?

   (c) What are the magnitudes of the vertical and horizontal components of $\vec{L}$? (Hint: think about limiting behavior of ϕ!)

3. The diagram shown in Figure 1 describes the gyroscope angle in spherical coordinates: ϕ denotes the *polar angle*, which is analogous to latitude (except measured from the poles, not the equator), and θ denotes the *azimuthal angle*, which is analogous to longitude.[2] In the first problem, you should have gotten an answer for $d\vec{L}$ which is orthogonal to $\vec{L}$, as shown in Figure 2.

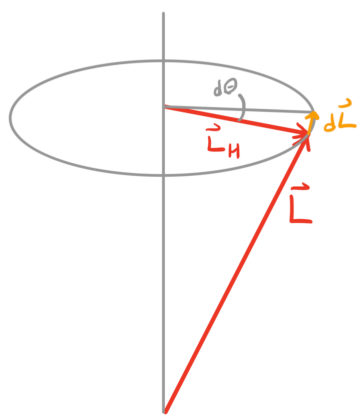


Figure 2: Illustration depicting the relation between $d\theta$ and dL.

   (a) Referring to Figure 2, if the angular momentum $\vec{L}$ changes by some amount $d\vec{L}$ over a time dt, what is the resulting change in the angle $d\theta$? (Hint 1: think inverse trig functions!) (Hint 2: remember that θ is defined relative to the horizontal, so you may need to use the decomposed vectors you found in the previous problem in your expression.)

   (b) For small values of x, $\arctan(x) \approx x$. Since the change in angular momentum $d\vec{L}$ can be made arbitrarily small by taking $dt \to 0$,[3] we can replace our expression for $d\theta$ with $\arctan(x) \to x$. Using this, write an expression that determines the angle $d\theta$ (from the previous part) that the gyroscope precesses through in a time dt in terms of r, m, g, ϕ, I, ω and dt. (Hint: don't think too hard on this: at this point, aside from replacing $\arctan(x) \to x$, all you are doing is plugging in expressions you have found from the previous problems.)

4. You now have an expression which relates $d\theta$ to dt! As we learned in class, the angular frequency (the magnitude of angular velocity) is $\omega' = \frac{d\theta}{dt}$. (I'm using ω' because we're referring to a separate quantity here than ω: ω' measures the frequency of precession, while ω measures the angular frequency of the flywheel.) To find period of the precession, we need to use $T = \frac{2\pi}{\omega'}$.

   (a) What is the period of the gyroscope precession?

[2] Annoyingly, it is just as common to use θ as the polar angle and ϕ for the azimuthal angle. This conflict has raged between scientists for years and covered the lands in mathematical bloodshed.

[3] Remember, this is what the definition of a derivative is!

5. You're almost there! Finally, since we don't know the mass of the flywheel itself, we would like to get an expression for T which is independent of m. We're going to make the (maybe slightly bad[4]) approximation that the flywheel comprises all of the mass of the gyroscope and that it is distributed in a ring with a radius r_{ring}.
   (a) What is the moment of inertia I of the gyroscope?
   (b) Substitute this into your expression for the precession period T and solve for the flywheel rotation rate ω. Check that your units work out.

Congratulations! You've made your way through some very sophisticated physics and you now have an expression for ω that is solely in terms of things that you already know or that you can easily measure. Now let's see how well experiment agrees with theory.

1. Using a ruler, measure the values of r (the offset from the pivot) and r_{ring} (the radius of the flywheel).
2. Spin your gyroscope and let it precess. Use a stopwatch to get an estimate of its precession period.
3. How many rpm are you able to spin your flywheel at?

[4] When I wrote this lab last night, I hadn't actually held the model of gyroscope I'll have given to you tomorrow, only seen pictures of it. The flywheel looks to comprise most of the mass of the gyroscope, but it's hard to tell.

Setting up for next class (10 minutes + download time)

Tomorrow's lecture will deal with computational physics, which is an exciting topic! Please bring your laptop and its charger to class tomorrow. So we don't waste time in the morning session, please set up a Python installation before then.

- Go to `https://www.anaconda.com/distribution` and download the Python 3.7 version for your OS (which should be automatically selected).
- Run the graphical installer and follow all the instructions.
- If prompted to check a box which includes something along the lines of "Add Python to PATH variable", check the box.
- When you are done, restart your computer, then open a terminal (Mac/Linux) or command prompt (Windows) and run the following commands to make sure things work:
  - `python`: will open a python interpreter. Press control+D to exit it.
  - `jupyter lab`: will open the Jupyterlab environment. Press control+C to exit.
- If you have any issues, get a TA to help you.