Gravity + orbits ~45 min

Gravity mutual attraction between all things with energy

most accurately described by GR by curvature of space time

↳ things always travel locally in straight lines, but gravity warps what "straight" is

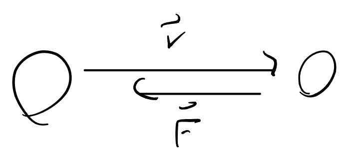
↳ geodesics

Newtonian gravity

$$F = G\frac{Mm}{r^2}$$

unit vector

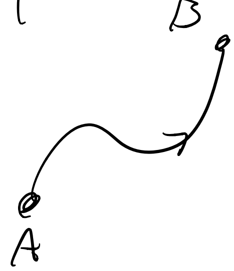
$$\vec{F} = -G\frac{Mm}{|\vec{r}|^2}\hat{r}$$



Gravitational field: vector field describing grav. force at all xyz

↳ conservative field: work is path independent

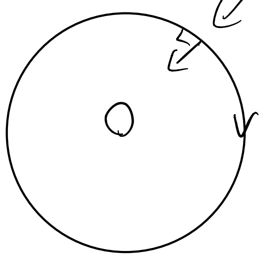
$$\int_P F_g(\vec{r}) \cdot d\vec{r} = \varphi(B) - \varphi(A)$$



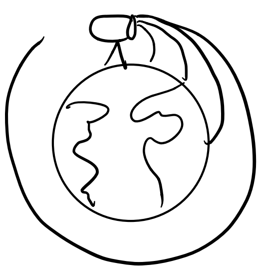
↳ $$\oint_C F_g(r) \cdot d\vec{r} = 0$$

90°; $\vec{F} \cdot d\vec{r} = 0$, no work

Orbit:

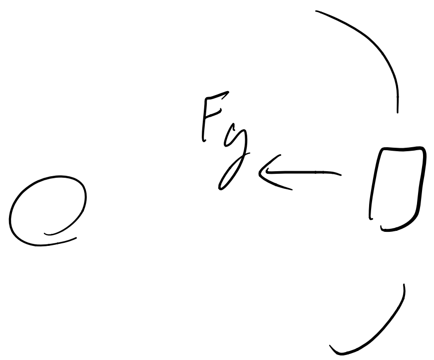


<u>Orbits</u> constantly falling, constantly missing



"Newton's cannonball"

Orbital speed



going in a circle

so:

$$F_g = F_c$$

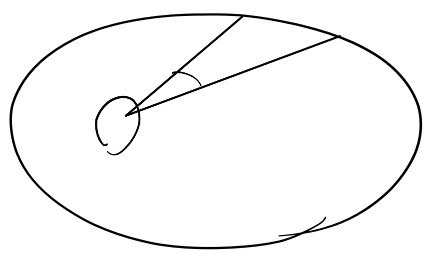
$$G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Gm}{r}}$$

Kepler's laws of planetary motion

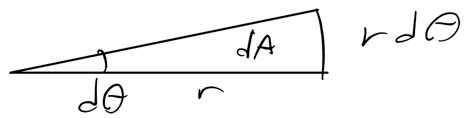
1. orbit of a planet is an ellipse
   w/ sun at one focus

2. line from planet to
   sun sweeps equal
   area in equal time



3. square of period $\propto$
   cube of semi-major axis
   $T^2 \propto r^3$

Law 2 is cons. of angular momentum:



$$dA = \frac{1}{2} bh$$
$$= \frac{1}{2} r\,(r\,d\theta)$$
$$\frac{dA}{dt} = \frac{1}{2} r \left(r \frac{d\theta}{dt}\right)$$
$$= \frac{1}{2} r\omega$$

$$L = I\omega$$
$$= mr\omega$$

$$\frac{dA}{dt} = \frac{1}{2} \frac{L}{m}$$

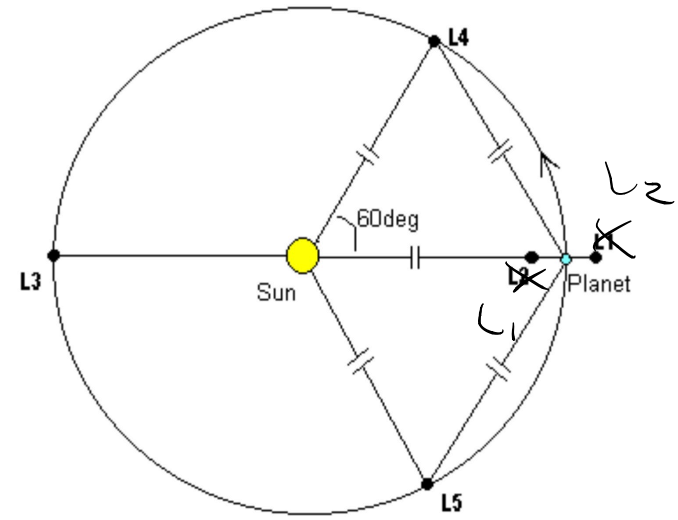
$= \underline{\text{const}}$ conserved

Law 3:

$$v = \sqrt{\frac{Gm}{r}} = \frac{2\pi r}{T}$$
$$\frac{Gm}{r} = \frac{4\pi^2 r^2}{T^2}$$
$$\frac{r^3}{T^2} = \frac{Gm}{4\pi^2} = \text{const}$$
$$\therefore r^3 \propto T^2$$

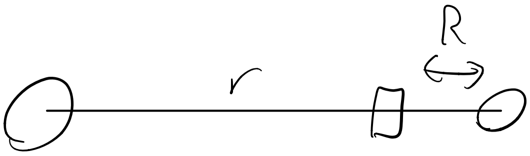
Lagrange points

points near 2 orbiting bodies where an object will maintain constant relative position



explain intuition behind 3, 1, 2

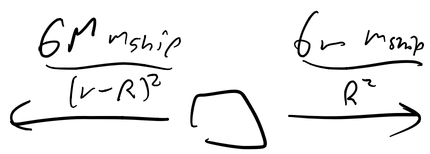
Derive L_1 distance



$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

$$G\frac{M}{r} = v^2 \rightarrow v = \frac{2\pi r}{T}$$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$$



$$a_{tot} = G\frac{M}{(r-R)^2} - G\frac{m}{R^2} = a_c = \frac{v_{ship}^2}{r-R}$$

what is v_{ship}? $\quad v_{ship} = \frac{2\pi(r-R)}{T}$

$$\frac{GM}{(r-R)^2} - \frac{Gm}{R^2} = \frac{(2\pi(r-R)/T)^2}{r-R}$$

$$\frac{GM}{(r-R)^2} - \frac{Gm}{R^2} = \frac{4\pi^2(r-R)}{T^2}$$

Solve with mathematica

Numerical integration

Differential eqn: relates some function to its derivatives. Initial conditions specify behavior

ex1 $\frac{dx}{dt} = 5$, $x(0) = 0$ $\rightarrow x = 5t$

$\frac{dx}{dt} = t$ $\rightarrow x = \frac{1}{2}t^2$

$x(0) = 0$

$\frac{dx}{dt} = x$ $\rightarrow x = e^t$

$x(0) = 1$

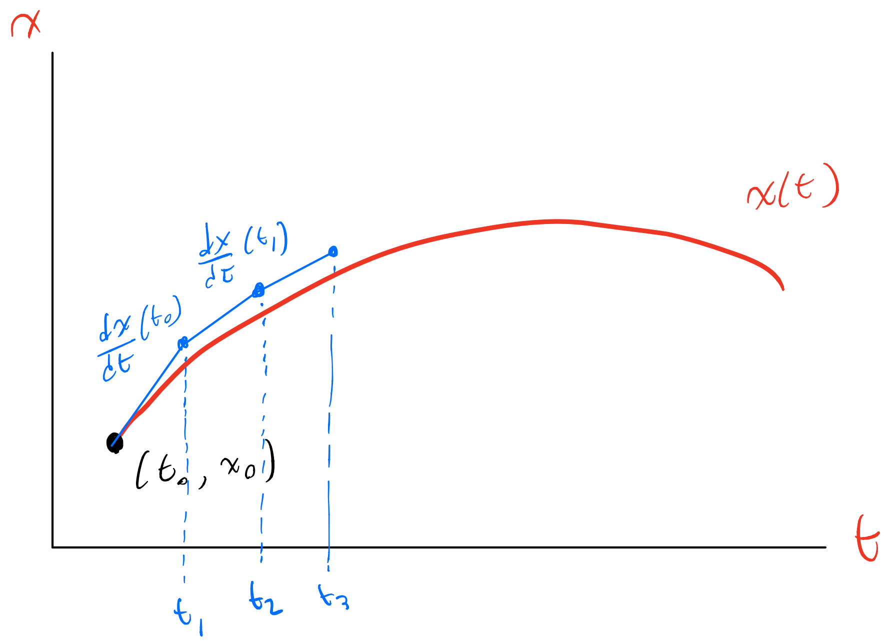
Suppose we have

$$\frac{dx}{dt} = f(x, t), \qquad x(t_0) = x_0$$

Solving diffeqs by hand is hard & often impossible!

can use numerical methods...

Want to approximate solution $x(t)$



$$x \approx x_0 + \frac{dx}{dt}\Delta t = x_0 + f(x_0, t_0)\Delta t$$

$$x_1 = x_0 + f(x_0, t_0)\Delta t$$
$$t_1 = t_0 + \Delta t$$

$$x_2 = x_1 + f(x_1, t_1)\Delta t$$

$$t_2 = t_1 + \Delta t$$

$$\vdots$$

$$x_{n+1} = x_n + f(x_n, t_n)\Delta t$$

$$t_{n+1} = t_n + \Delta t$$