Special Relativity

History + Motivation

Beginning of 20th century, there were some flaws in classical physics...

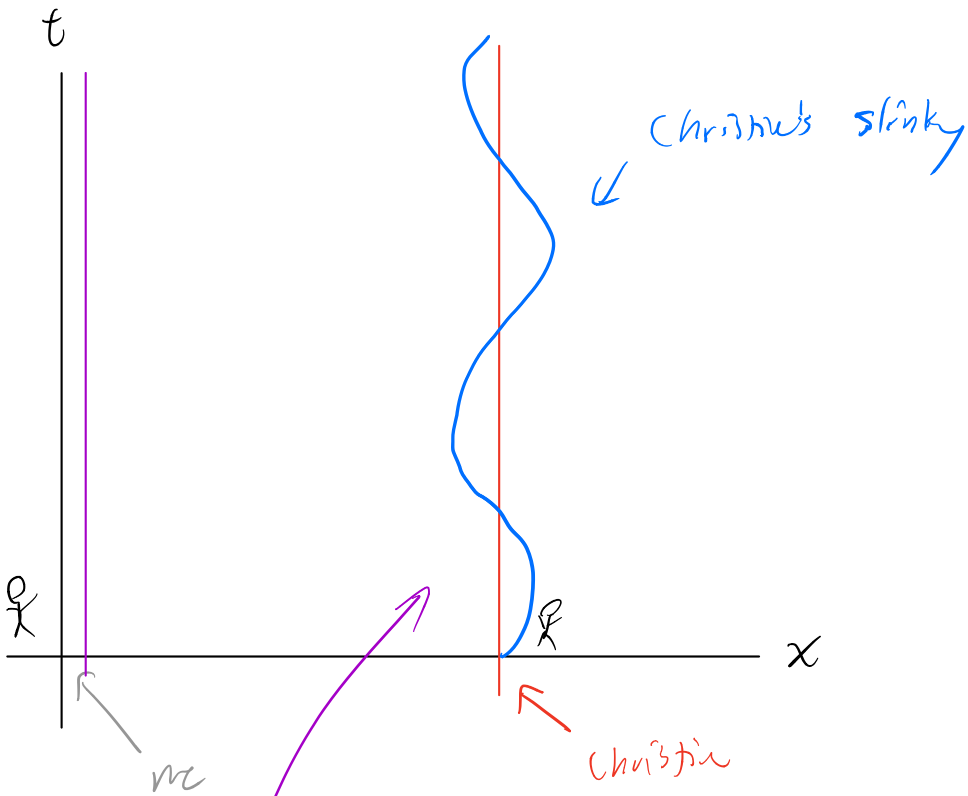
TO DO

Postulates of Relativity

1.
2.

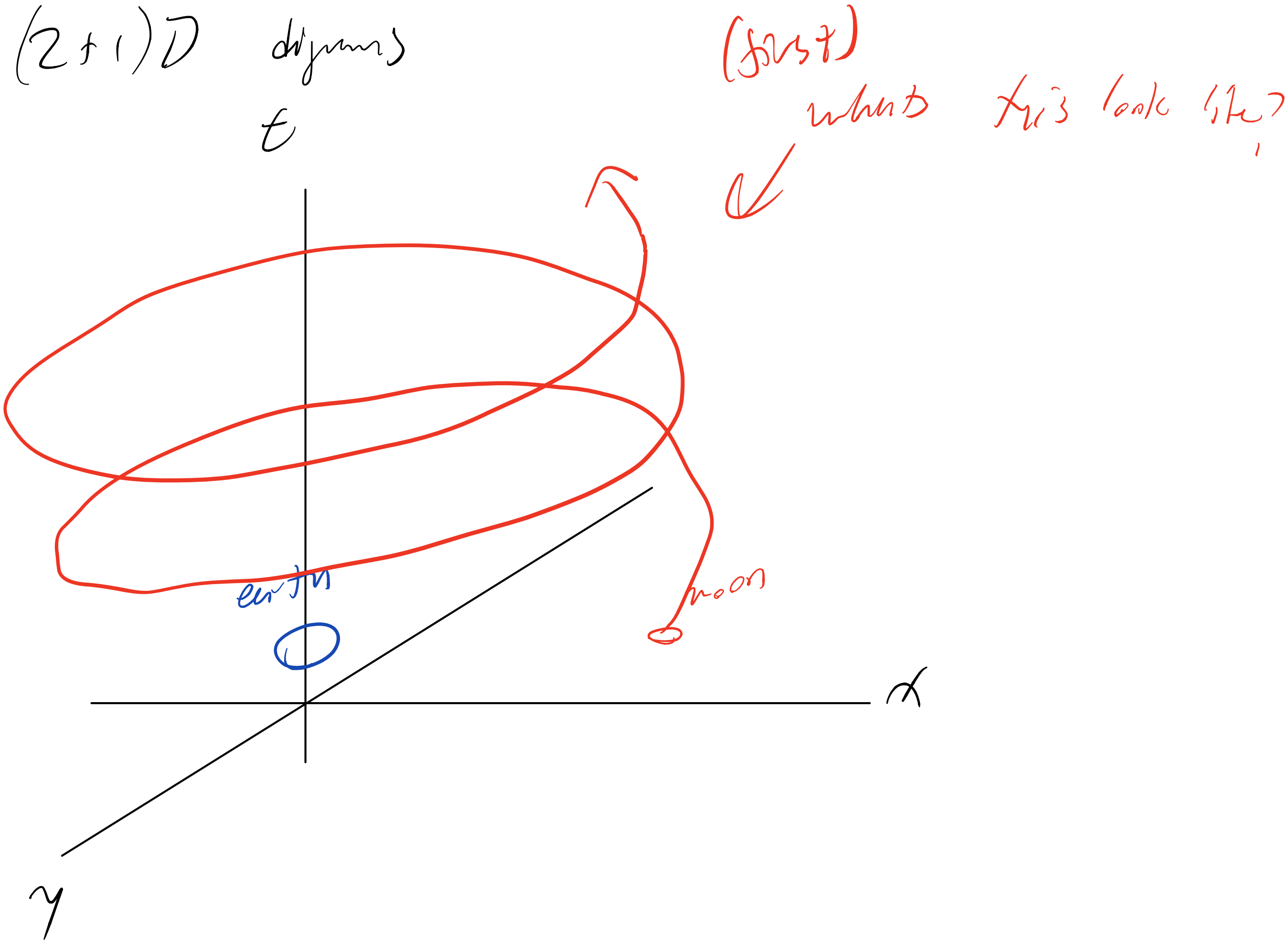
Space time diagrams!

Can represent motion of object relative to some reference point over time



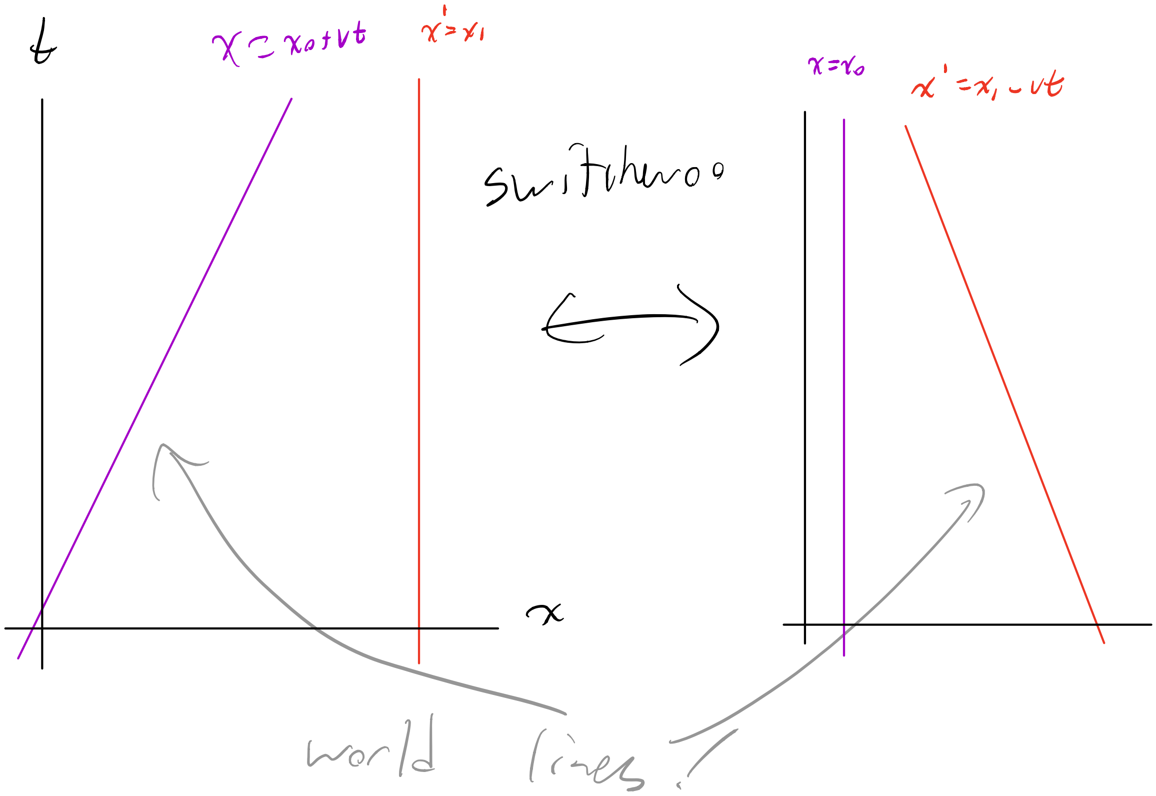
What would a car accelerating toward me look like?

(2+1)D diagrams



Back in 1D....

If I move to here, how does distance change?



Switcheroo changes relative things like position & velocity, but not absolute things like separation or whether we can cross

so you can shift reference frames & the same events behave the same way

This is called

Ye Olde Principle of Relativity

and is postulate #1

To understand Postulate #2, let's look at Michelson-Morley experiment

what does light travel through?

→ late 1800's: "Luminiferous Aether"

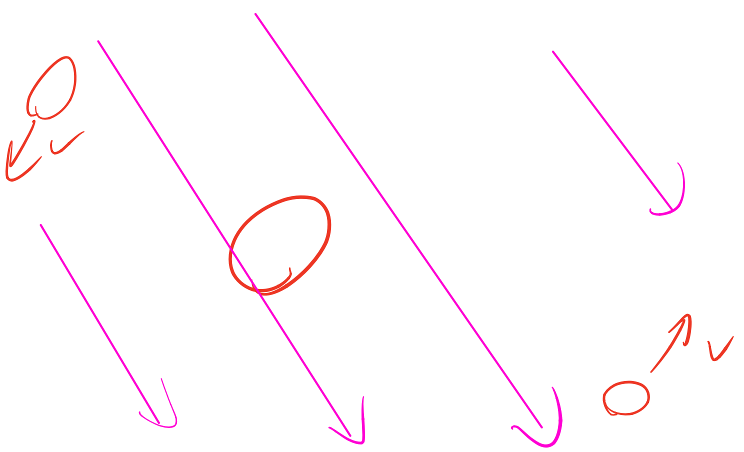
Probably has unusual properties:

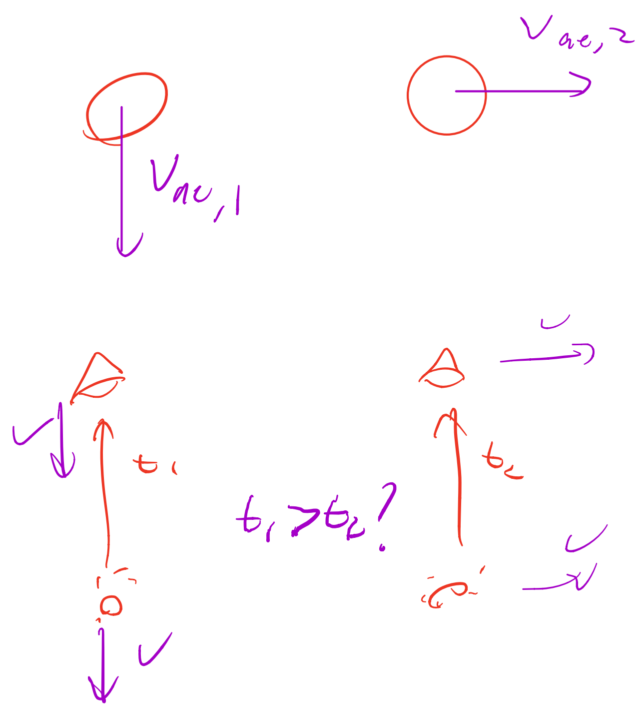
→ speed of light is huge!

→ no drag on massive bodies moving through it like Earth

The experiment: (1887)

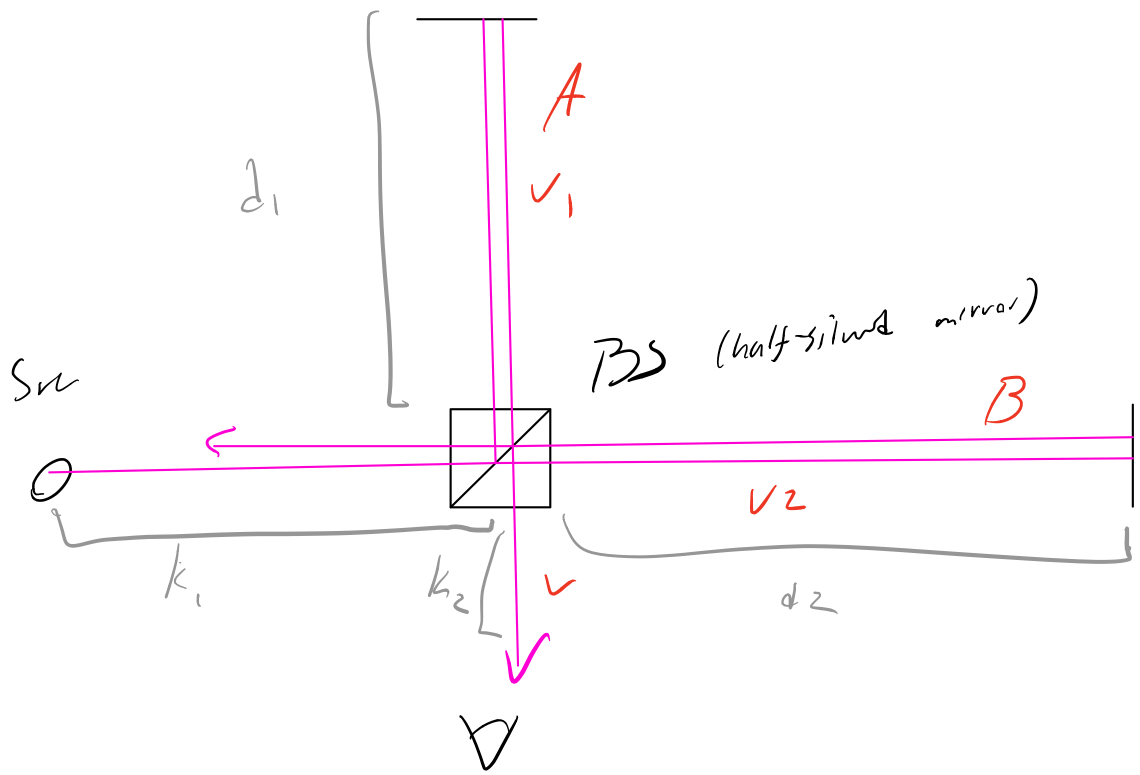
earth orbits sun at $\sim 3 \times 10^4$ m/s





To measure the speed of aether,
Michelson built an interferometer

Interferometer uses interference of waves to measure varying properties



Path A: $k_1 + 2d_1 + k_2$

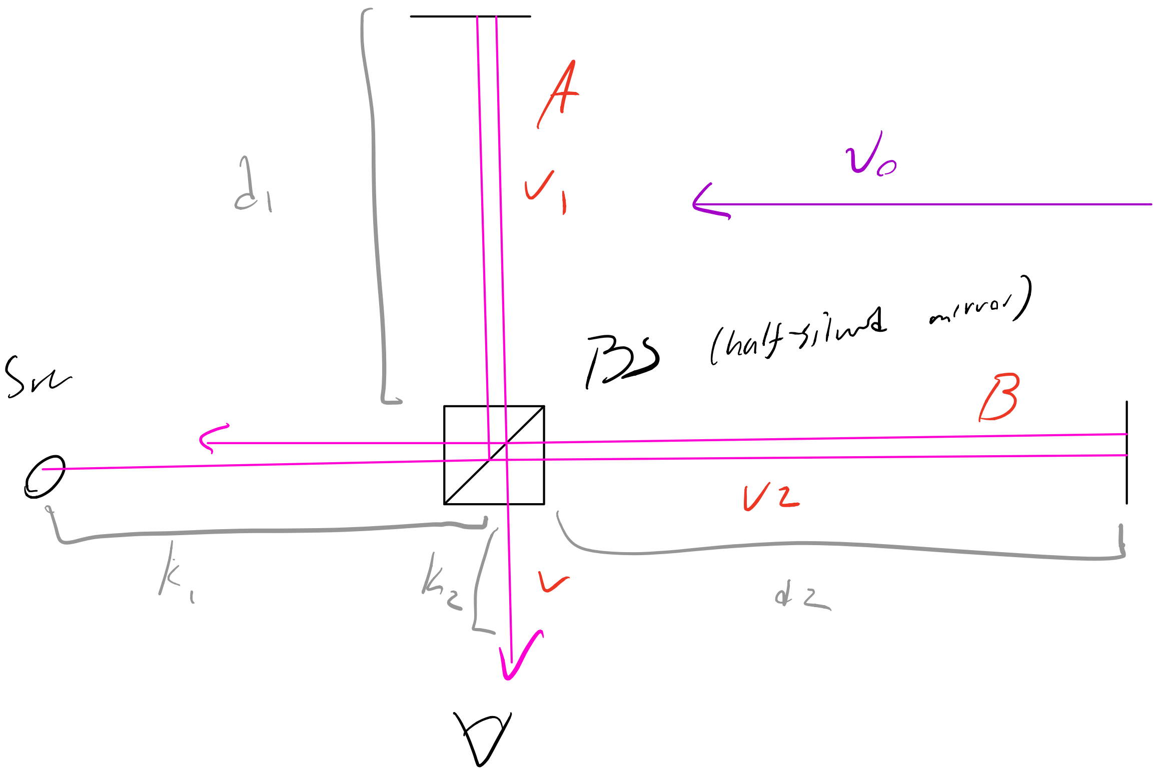
Path B: $k_1 + 2d_2 + k_2$

$\Delta x = 2(d_1 - d_2)$

constructive interference @
$\Delta x = n\lambda$
destructive @
$\Delta x = (n + \frac{1}{2})\lambda$

$\Delta t = 2\left(\frac{d_1}{v_1} - \frac{d_2}{v_2}\right)$

$\Delta \phi = \left(\frac{v}{\lambda} \Delta t\right) 2\pi$



$$\Delta t = \frac{d_1}{v} + \frac{d_1}{v} + \frac{d_2}{v - v_0} + \frac{d_2}{v + v_0}$$

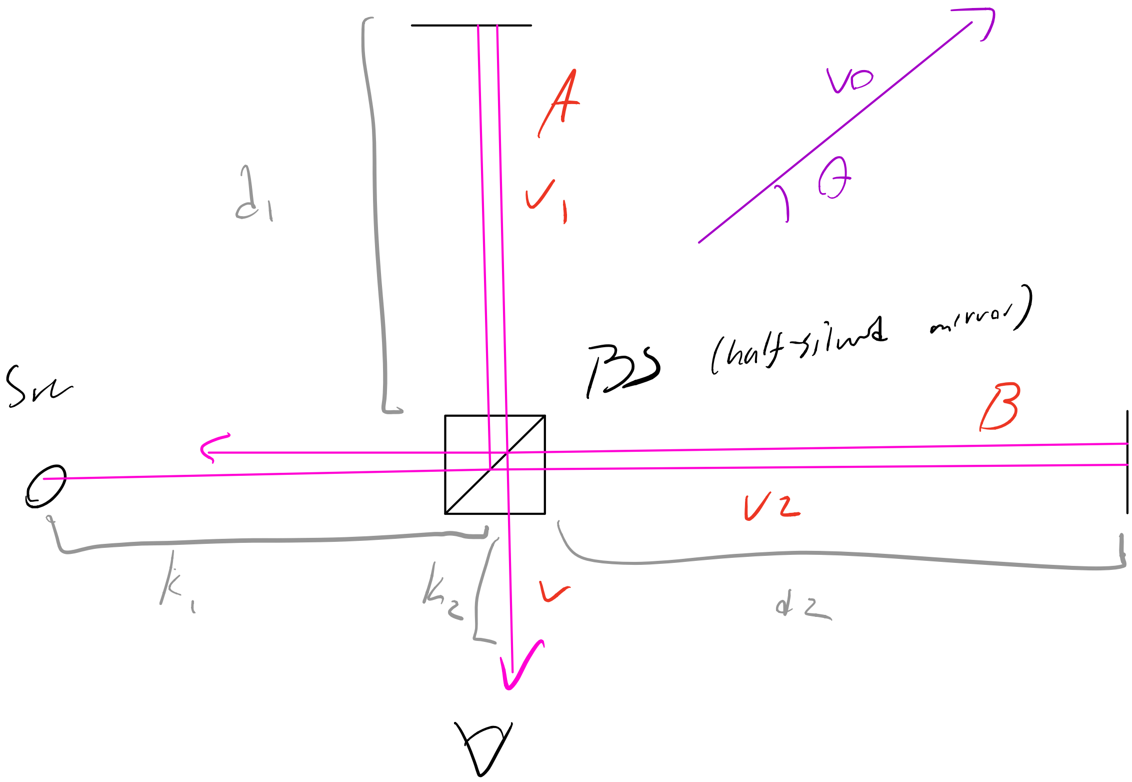
$$= \frac{2d_1}{v} + \frac{d_2(v + \cancel{v_0}) + d_2(v - \cancel{v_0})}{v^2 - v_0^2}$$

$$= \frac{2d_1}{v} + \frac{2d_2 v}{v^2 - v_0^2}$$

$$= \frac{2d_1}{v} + \frac{2d_2}{v} \frac{v^2}{v^2 - v_0^2}$$

$$= \frac{2d_1}{v} + \frac{2d_2}{v} \frac{1}{\frac{v^2}{v^2} - \frac{v_0^2}{v^2}}$$

$$= \frac{2d_1}{v} + \frac{2d_2}{v} \left(\frac{1}{1 - \frac{v_0^2}{v^2}} \right)$$



Performed experiment @ any θ,
could never find any interference!

either: 1. aether being dragged along with earth
or 2. doesn't exist.

this + other experiments all indicated
speed of light in a vacuum is
constant regardless of motion of
src / obs.!

Postulates of special relativity

1. laws of physics are invariant over all inertial reference frames

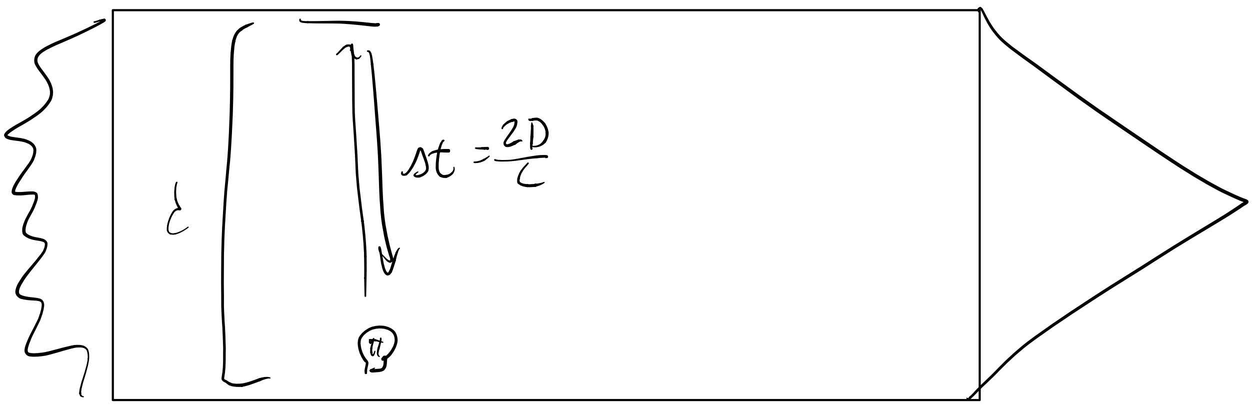
2. speed of light in a vacuum is the same for all observers & sources

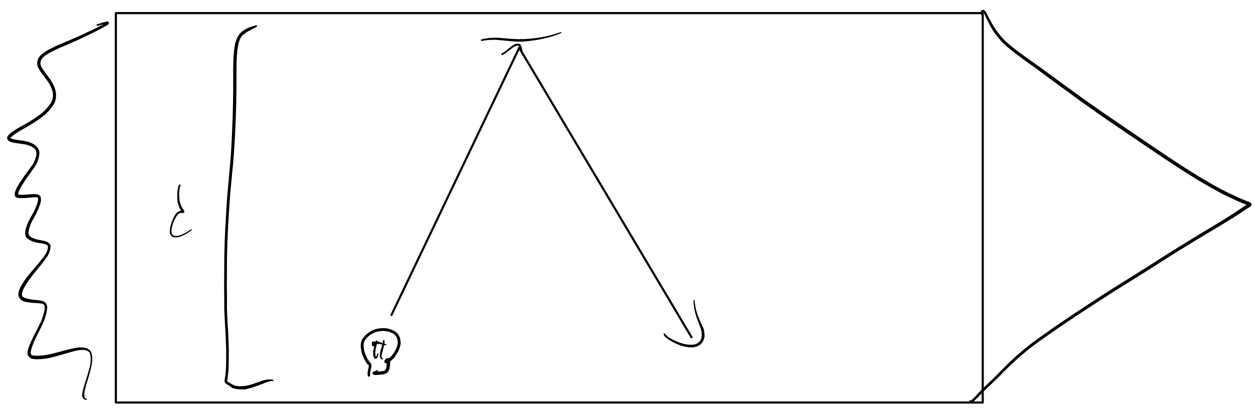
#1 was previously stated by Galileo + Newton

#2 revealed experimentally to be true by many tests + MM experiment

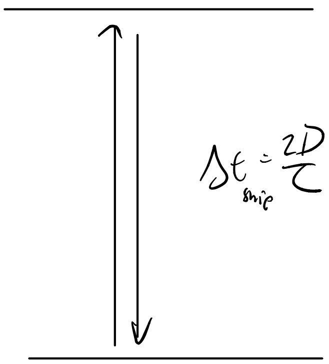
Time dilation

Consider a "light clock"

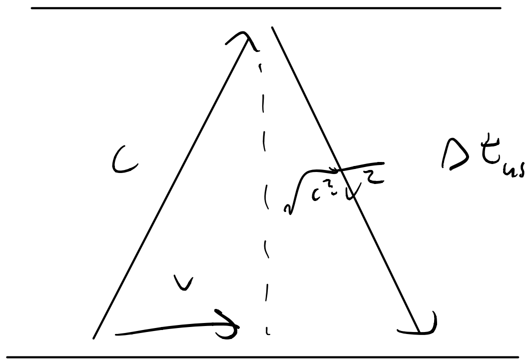




Ship's perspective



$\Delta t_{ship} = \frac{2D}{c}$



$$\Delta t_{us} = \frac{2D}{\sqrt{c^2-v^2}} = \frac{2D}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Ship's time runs slow to us by a factor of $\frac{1}{\sqrt{1-v^2/c^2}}$!

Evidence

no direct evidence when SR was proposed

→ Cosmic rays: protons striking nuclei creating muons.
$\tau \approx 2.2\mu s$, atm $L_0 = 10^4 m$.
so ~16 lifetimes to reach ground, only
1 in ~20 000 hit earth

↳ comparing high alt & low alt flux

→ supernova light intensity changes
↳ (Type Ia) 10^{44} J

→ gps satellites

Length contraction

Transverse lengths don't contract

↳ Symmetric knife-runner thought experiment

Clock traveling over ruler stick

Stop clock @ end

$$\Delta t = \frac{D}{v}\frac{1}{\sqrt{1-v^2/c^2}}$$ → should be $\frac{D}{v}$ ...

how?

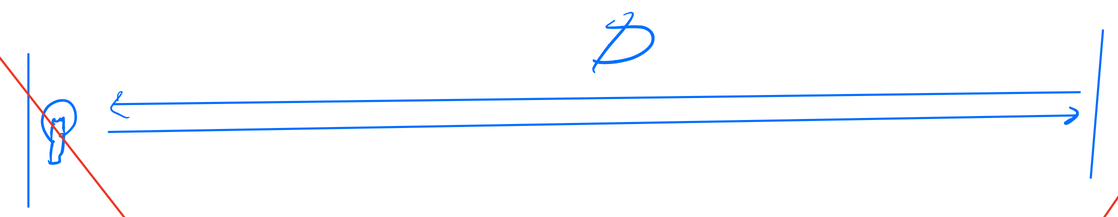
Length contraction!

$$D \Rightarrow D\sqrt{1-\frac{v^2}{c^2}}$$

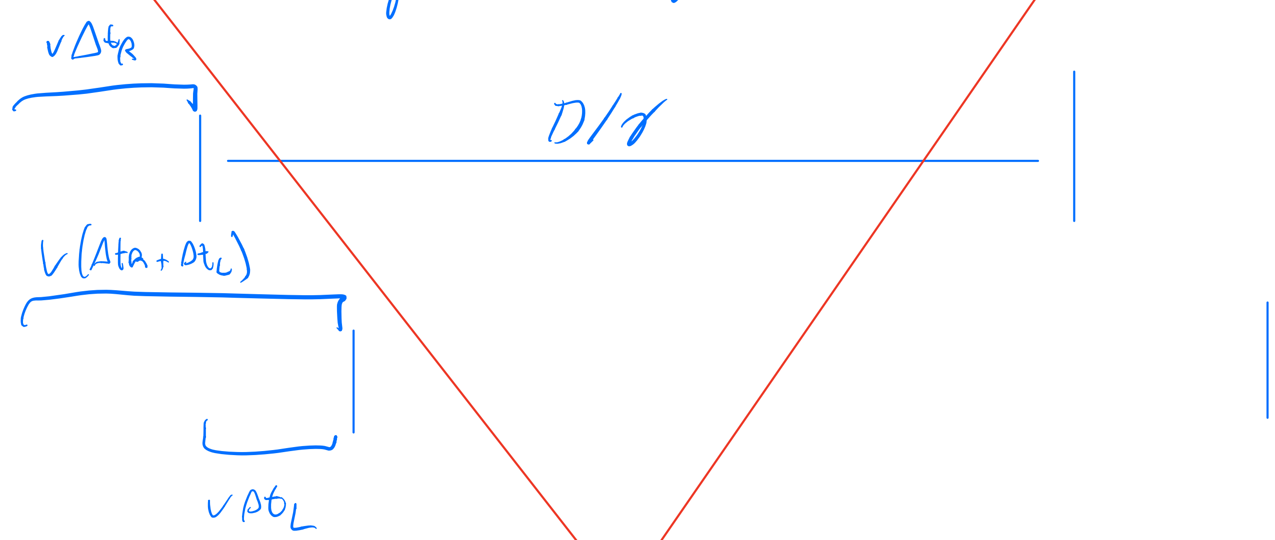
$$= \frac{1}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$: Lorentz factor

Longitudinal light clock



moving to right:



$$c\Delta t_R = \frac{D}{\gamma} + v\Delta t_R$$

$$c\Delta t_L = \frac{D}{\gamma} - v\Delta t_L$$

$$\therefore \quad \Delta t = \Delta t_R + \Delta t_L$$

$$= \frac{D/\gamma}{c-v} + \frac{D/\gamma}{c+v}$$

$$= \frac{D/\gamma\,(c+v) + D/\gamma\,(c-v)}{c^2-v^2}$$

$$= \frac{2D}{\gamma}\,\frac{c}{c^2-v^2}$$

$$= \frac{2D}{\gamma}\,\frac{1}{c}\,\frac{c^2}{c^2-v^2} = \frac{2D}{\gamma}\,\frac{1}{c}\,\frac{1}{1-\frac{v^2}{c^2}}$$

$$= \frac{2D}{c}\,\frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} = \boxed{\frac{2D}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2D}{c}\gamma}$$

looks a lot like MM experiment!

Ex electrons @ SLAC (2 mi long) travel at

$$v = 0.999\ 999\ 999\ 7\ c$$

how long does accelerator appear to electrons?

$$D = \frac{2\,\text{mi}}{\gamma} \quad \rightarrow \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \left(\sqrt{1 - .9999999997^2}\right)^{-1}$$

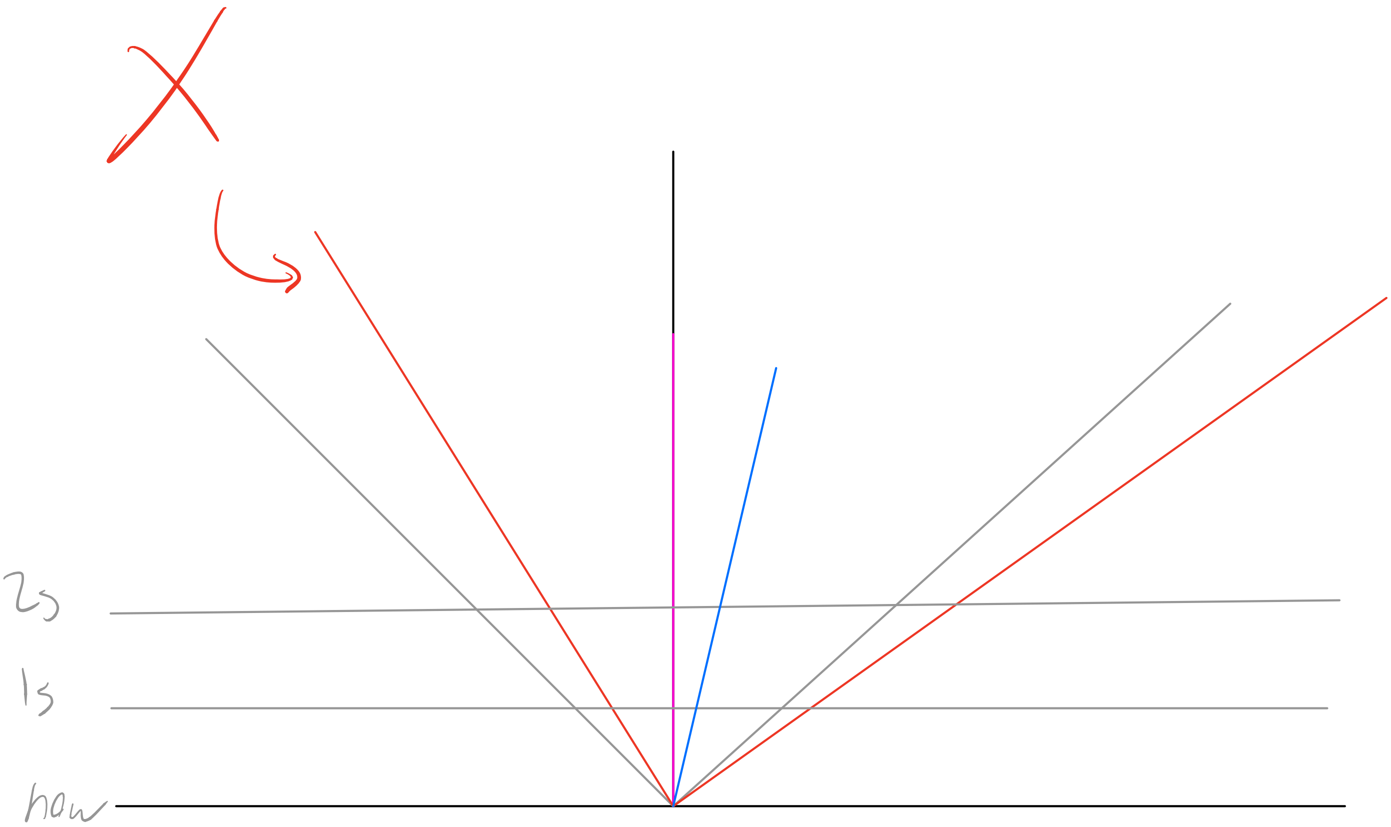
$$\approx 40\ 000$$

$$= 5\text{E-}5\ \text{mi}$$

$\approx$ 3 inch (credit card length)

So... how does anything move?

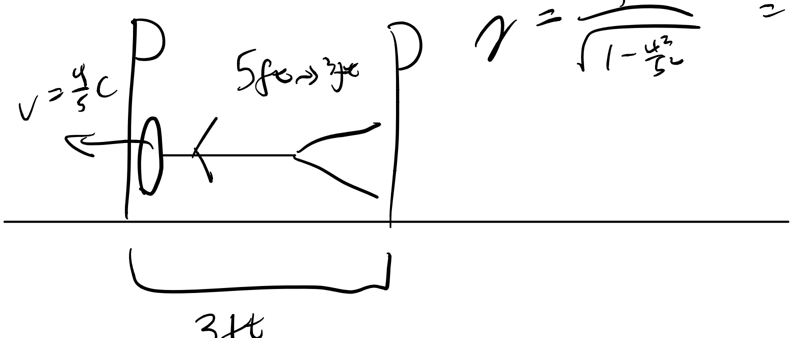
constant speed of light:



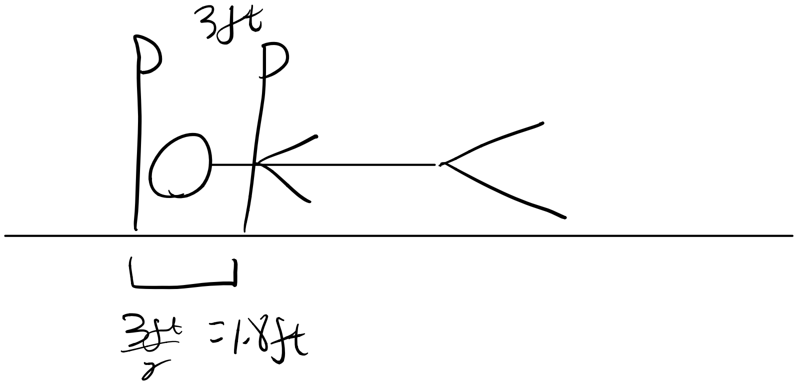
Thought experiment: Magician's assistant

$$\gamma = \frac{1}{\sqrt{1-\frac{4^2}{5^2}}} = \frac{5}{3}$$

our frame:



her frame:

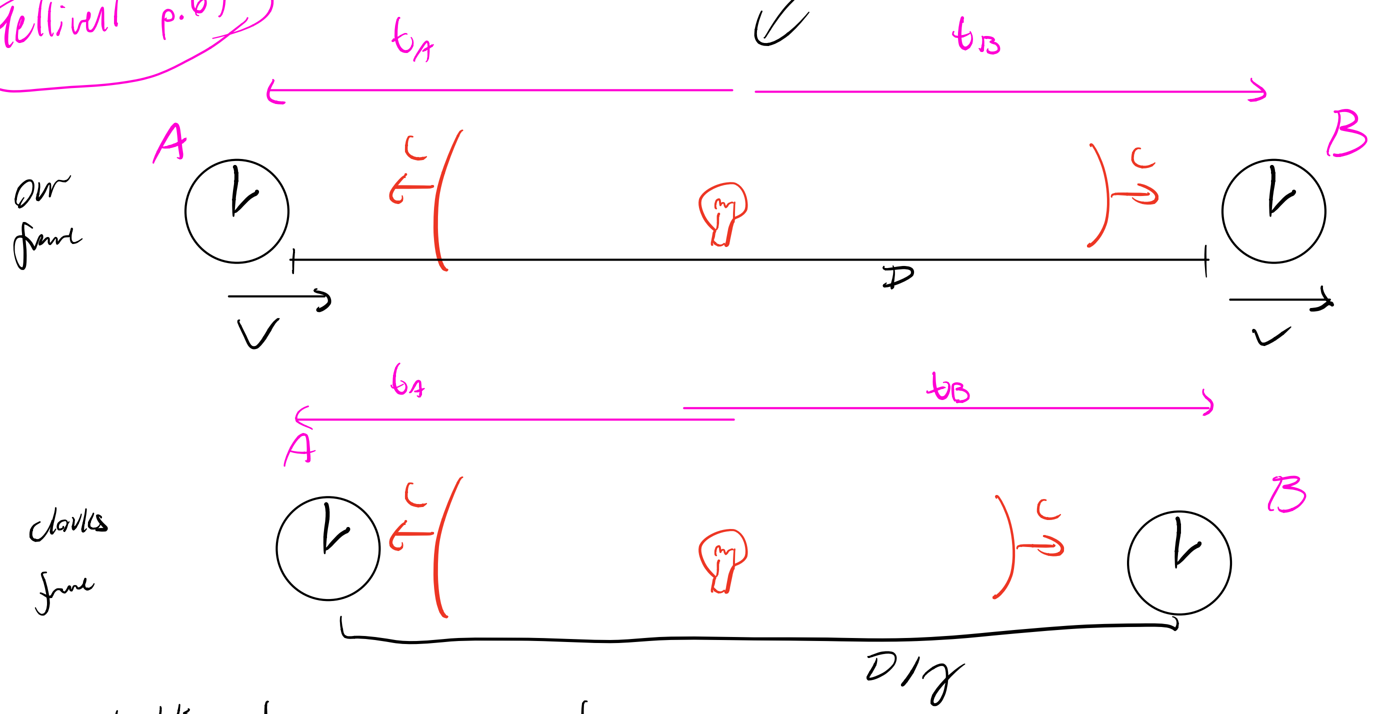


what fits?

Another thought experiment: synchronizing clocks

Set $t=0$ at flash

Helliwell p.69



Clock's frame:

$t_A = t_B$

Our frame:

$$t_A = \frac{D/2\gamma}{c+v}$$

$$t_B = \frac{D/2\gamma}{c-v}$$

$$\Delta t = t_B - t_A = \frac{D\sqrt{1-v^2/c^2}}{2(c+v)} - \frac{D\sqrt{1-v^2/c^2}}{2(c-v)}$$

$$= \frac{Dv\sqrt{1-v^2/c^2}}{c^2-v^2}$$

Δt runs slow by γ in our frame, so divide by γ to get into A's frame

$$\Delta t_A = \Delta t/\gamma = \frac{Dv\left(1-\frac{v^2}{c^2}\right)}{c^2-v^2}$$

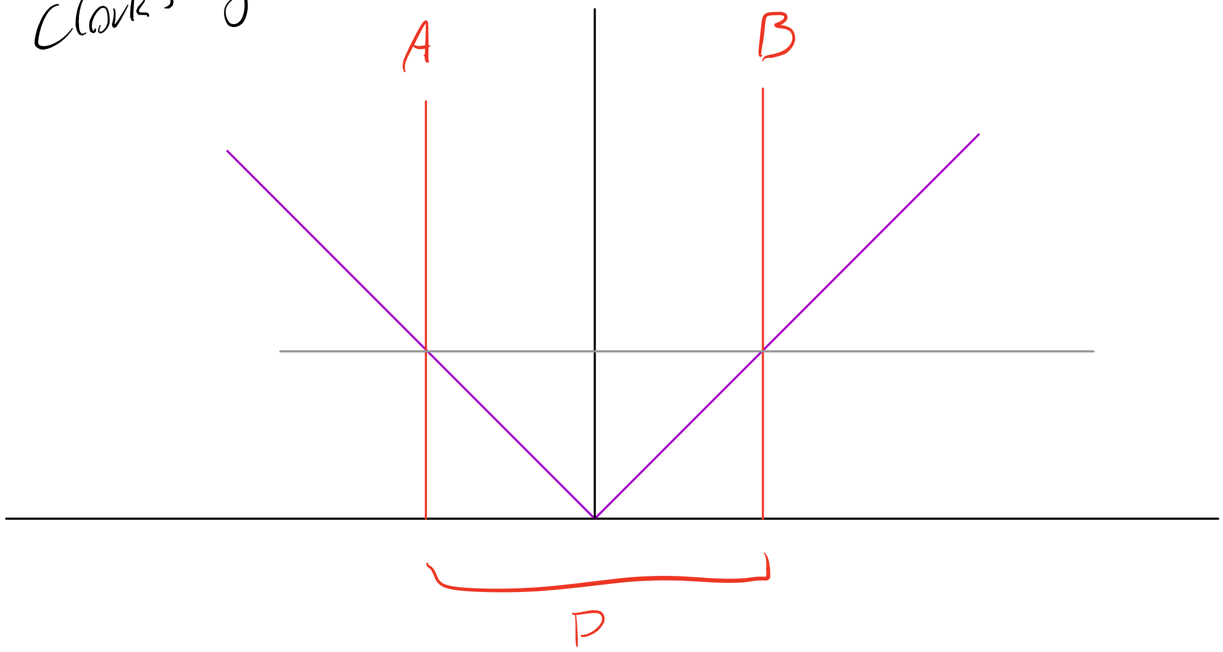
$$= \frac{Dv}{c^2}\frac{c^2-v^2}{c^2-v^2}$$

$$= \frac{Dv}{c^2}$$

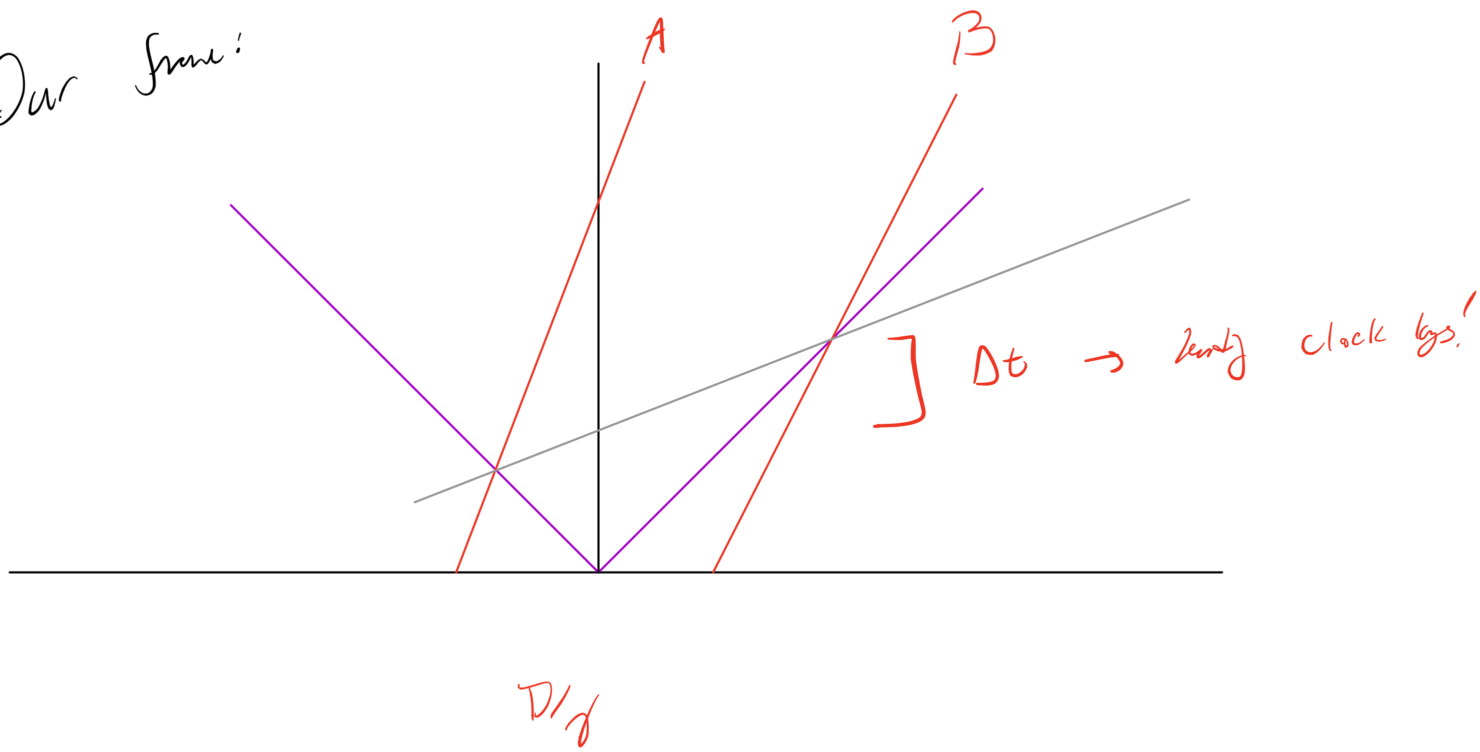
Clock A runs behind clock B by $\Delta t = \frac{2D}{c^2}$!

↪ "Leading clock lags"

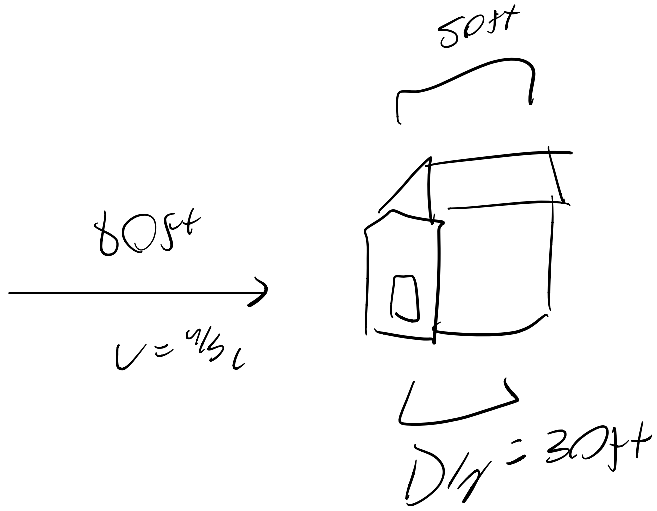
Clock's frame

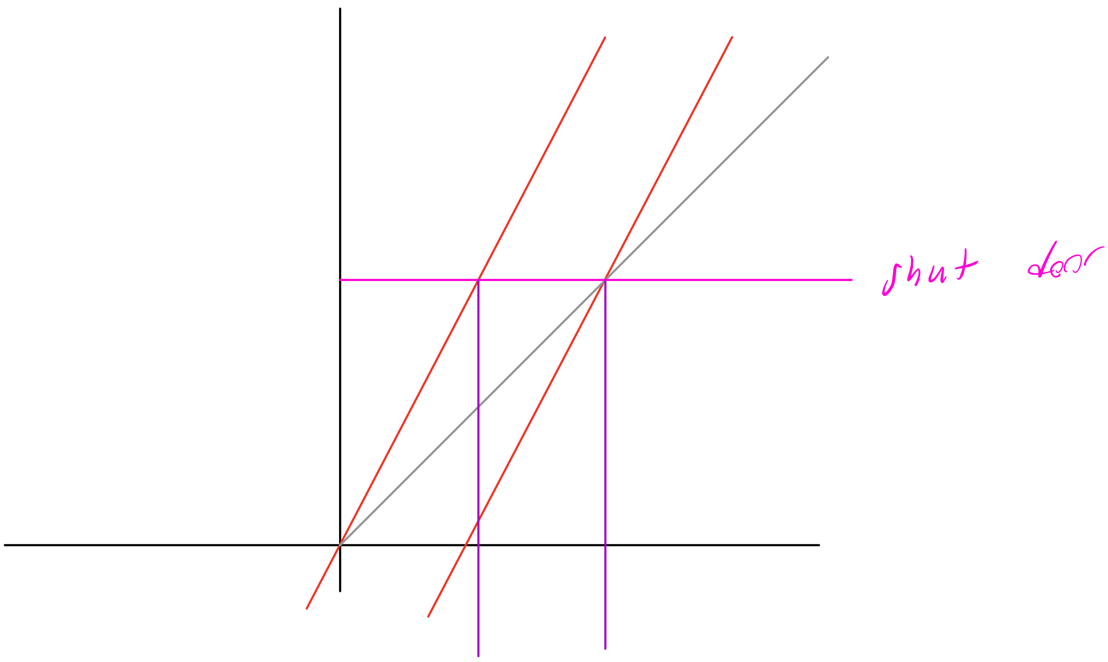


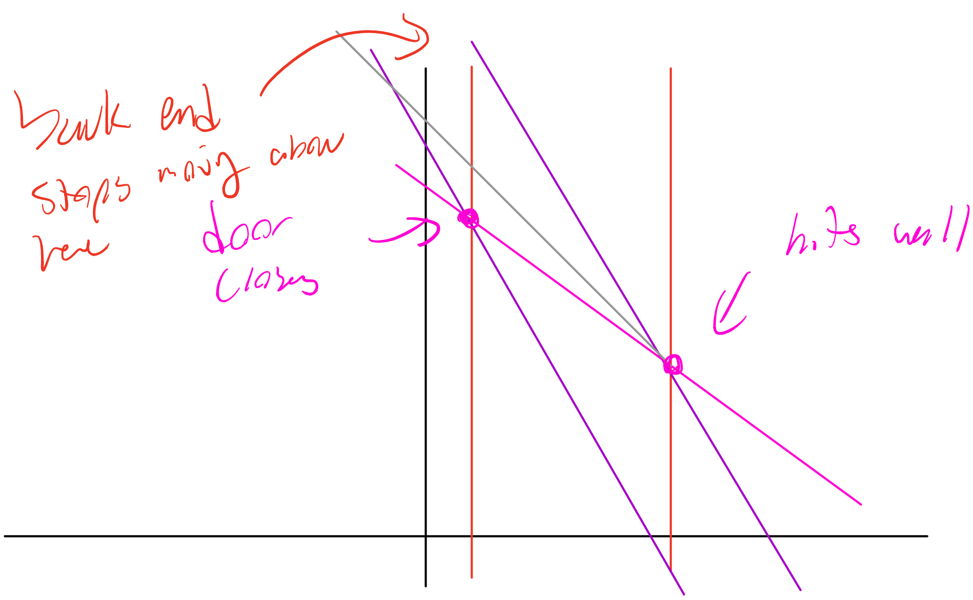
Our frame:



Pole vaulter in barn paradox







Lorentz Transformations

Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Einstein's relativity:

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^2)$$

$\updownarrow$

$$x = \gamma(x' + vt')$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t' + vx'/c^2)$$

can be found easier by $v \to -v$ or by switching prime coordinates

Spacetime intervals

→ light cone - everything inside c - lines

→ causally connected / disconnected

interval: normal physics.

$$\|(\Delta x, \Delta y, \Delta z)\| \text{ is invariant}$$

in relativity, interval is

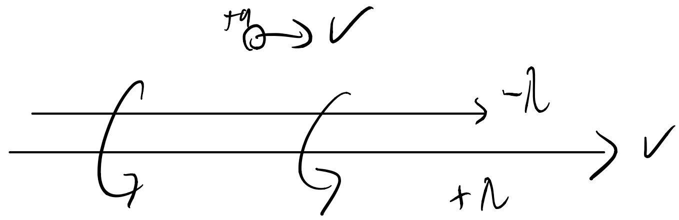
$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2$$

$\Delta s > 0$: spacelike

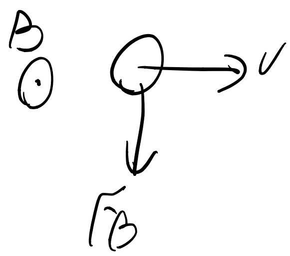
$\Delta s < 0$: timelike

$\Delta s = 0$: null or light like

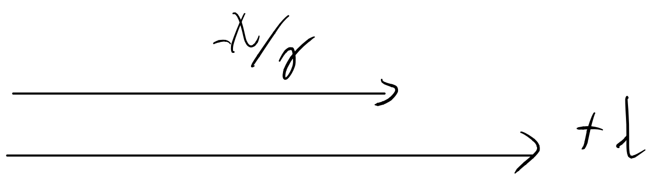
EM fields:



Our frame



e^- frame



Here, $\boldsymbol{\beta} = (\beta, 0, 0)$ is used. These results can be summarized by

$$\begin{aligned}
\mathbf{E}_{\|'} &= \mathbf{E}_{\|} \\
\mathbf{B}_{\|'} &= \mathbf{B}_{\|} \\
\mathbf{E}_{\perp'} &= \gamma\left(\mathbf{E}_{\perp} + \boldsymbol{\beta} \times \mathbf{B}_{\perp}\right) = \gamma(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})_{\perp}, \\
\mathbf{B}_{\perp'} &= \gamma\left(\mathbf{B}_{\perp} - \boldsymbol{\beta} \times \mathbf{E}_{\perp}\right) = \gamma(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E})_{\perp},
\end{aligned}$$