# Topics in Physics: Problem Set #7 (TA version)

**Topics: electromagnetism, light, diffraction, geometric optics**

## General TA instructions

- Give students the estimated time for each section, and plan to spend about 50% of the estimated time going through answers.
- Try not to go over time for the earlier sections, since many of the more interesting activities are at the end of the assignment.
- Ask for student volunteers to explain their answers and discuss discrepancies.
- Be stingy with hints, but if a student seems legitimately stuck or won't finish the section in time, try to walk them through how to solve the problem without giving them the answer. You could also have them discuss with any students who have finished the section.

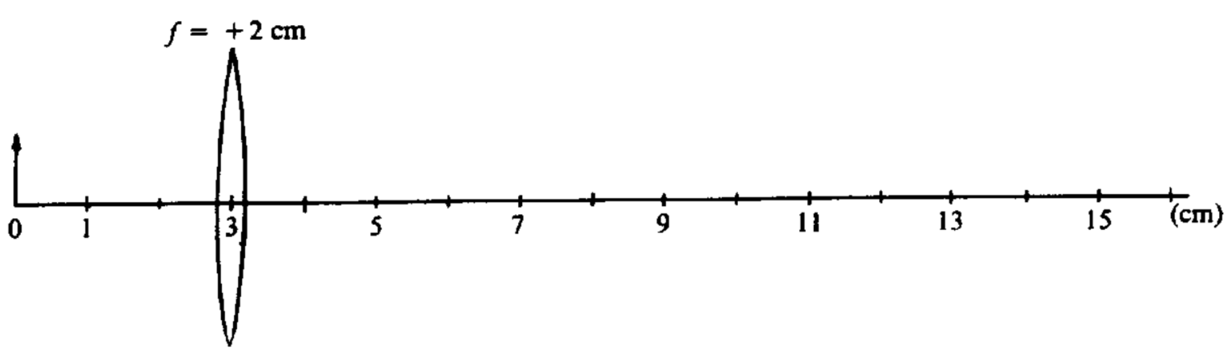
## Practice Problems (approx. 60 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. If you are underwater in a still pool, what range of angles are you able to see upwards out of?

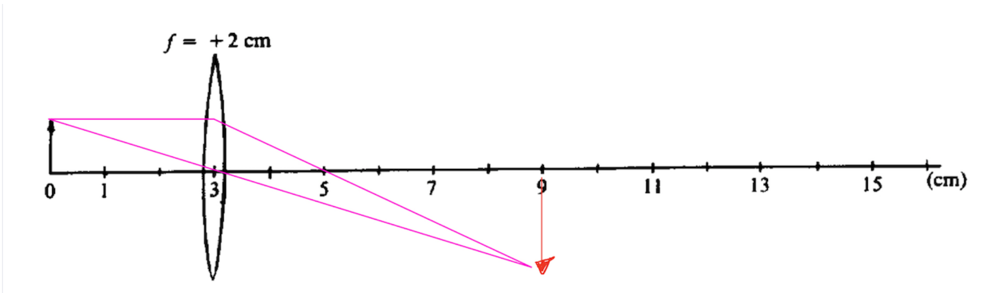
   (a) Total internal reflection will occur for $\theta$ such that $\frac{n_1}{n_2}\sin\theta_1 > 1$. Using $n_1 = 1.3$ (water) and $n_2 \approx 1$, then $\theta_1 > \arcsin\left(\frac{1}{1.3}\right) \approx 50°$. You can see upward out of any angle from vertical less than this critical angle.

2. An object is placed 3 centimeters to the left of a convex (converging) lens of focal length $f = 2$ centimeters, as shown below.



(a) Sketch a ray diagram on the scenario to construct the image.

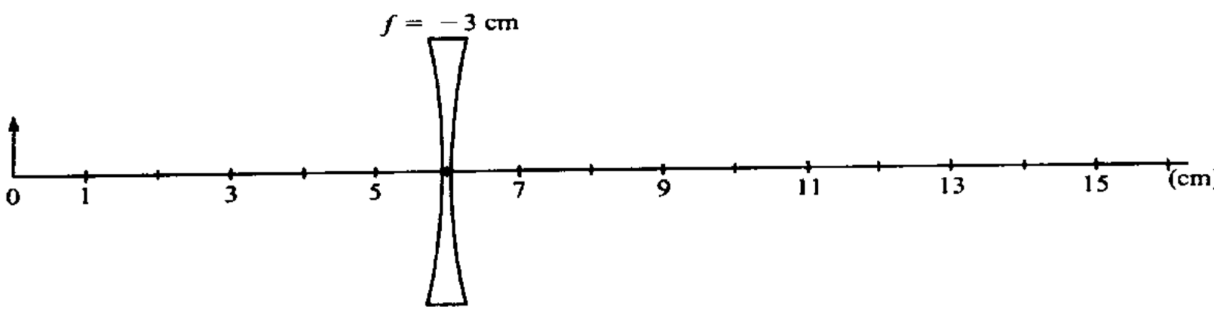
i. Looks like this:




(b) Determine the ratio of image size to object size.

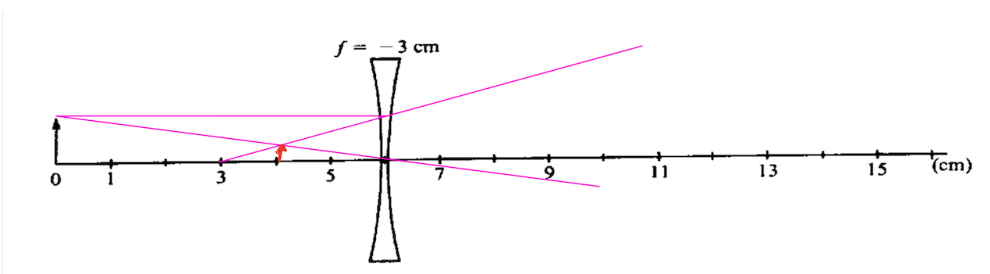
i. $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$, so $i=\left(\frac{1}{2}-\frac{1}{3}\right)^{-1}=6$. Then image size is $-\frac{i}{o}=-2$.

3. The converging lens is removed and a concave (diverging) lens of focal length $f=-3$ centimeters is placed as shown below.




(a) Sketch a ray diagram of the figure above to construct the image.

i. Looks like this:




(b) Calculate the distance of this image from the lens.

i. $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$, so $i=\left(\frac{1}{6}-\frac{1}{-3}\right)^{-1}=2$ and the image is at 4cm on the ruler.

4. Get a red laser pointer from the supplies, and use one of the $500\text{mm}^{-1}$ or $1000\text{mm}^{-1}$ diffraction gratings to determine its wavelength.

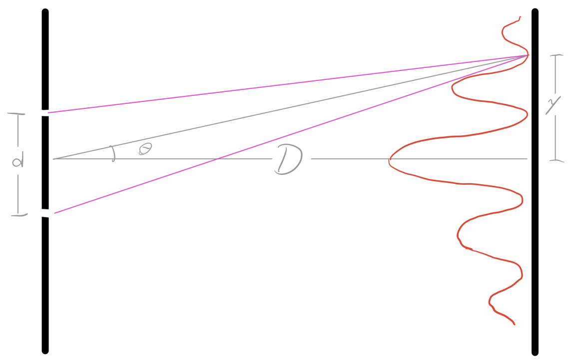
(a) Shine the laser through the grating at a wall. Measure the distance to the wall and the spacing of the fringes. Fringe maxima will occur at $d\sin\theta=n\lambda$, so $\lambda=\frac{d\sin\theta}{n}$. You can estimate $\theta$ as $\frac{\text{vertical fringe location}}{\text{horizontal distance to wall}}$.

# Challenge Problems (approx. 75 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

## Problem 1: deriving double-slit diffraction

The double-slit experiment is one of the most important experiments in modern physics. In 1801, Thomas Young first performed the experiment on a beam of light, but it would later be used to show in 1927 that electrons – and in fact all particles – have a wave-like nature. In this problem you'll derive an expression which determines where the fringes are located on a distant screen. You won't be given much guidance on this derivation, but it is similar to some of the ones we did in lecture.

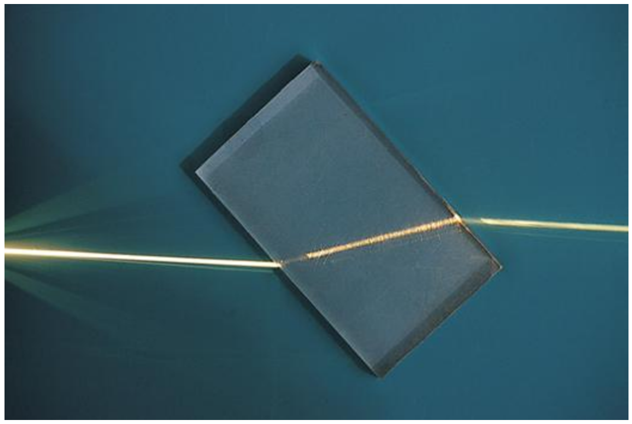



Refer to the figure above. A double-slit with slit separation $d$ is a distance $D \gg d$ away from a screen. Light with wavelength $\lambda$ is incident on the setup and creates a fringe pattern on the screen. Denote the vertical displacement from the center fringe as $y$. Determine expressions for the location of maximum intensity fringes $y_{\max}$ and for the location of minimum intensity fringes $y_{\min}$, assuming in both cases that $D \gg y$. Both of your expressions should involve only $\lambda$, $D$, $d$, and $n$, where $n \in \mathbb{Z}$.

ANSWER: $y_{\max}$ occurs at an angle $\theta$ satisfying $d \sin\theta = n\lambda$, just like the diffraction grating problem we derived in lecture. Then $\tan\theta = \frac{y}{D}$, and since $D \gg y$, $\theta$ is small, so $\tan\theta \approx \sin\theta \approx \theta \approx \frac{y}{D}$. Then $d \sin\theta = d\frac{y}{D} = n\lambda$, so $y_{\max} = n\frac{\lambda D}{d}$. For $y_{\min}$, we take the same approach, except using the condition that $d \sin\theta = \left(n + \frac{1}{2}\right)\lambda$, since the condition for minimum intensity is when the path difference is off by an odd multiple $\frac{\lambda}{2}$, resulting in a relative phase of $\pi$ and destructive interference. Then solving as above, $y_{\min} = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}$.

## Problem 2: refraction action

Prove that a beam entering a planar transparent block, as shown below, emerges parallel to its initial direction. Derive an expression for the lateral displacement of the beam.



ANSWER: Suppose you have a beam propagating through a material with index of refraction $n_1$ and a plate with refractive index $n_2$ and thickness $d$ oriented such that the normal vector of the front of the plate is at an angle $\theta_1$ with respect to the beam axis. Then Snell's law gives us that the beam propagates in the material at an angle of $\theta_2$ relative to the normal vector of the plate, given by $\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$. Upon exiting the plate, the beam propagates at an angle $\theta_3$ given by:

$$\begin{aligned}\theta_3 &= \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)\\ &= \sin^{-1}\left(\frac{n_2}{n_1}\frac{n_1}{n_2}\sin\theta_1\right)\\ &= \theta_1,\end{aligned}$$

so the beam always emerges parallel to the initial direction.

BONUS: The beam propagates at an angle $\theta_2$ relative to the normal vector of the plate, so it travels a total distance $d/\cos\theta_2$ through the block. The angle of the beam propagation in this block relative to the original beam is $\theta_2 - \theta_1$, so the lateral displacement $\delta y$ of the beam is given by

$$\delta y = d\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2}.$$

## Problem 3: electric generators

An electric generator works exactly as the inverse of an electric motor. Suppose you have a generator which consists of 100 turns of wire formed into a rectangular loop 50.0 cm by 30.0 cm, placed entirely in a uniform magnetic field with magnitude $\|\vec{B}\| = 3.5\text{T}$. What is the maximum value of the voltage produced when the loop is spun at 1000 rev/min about an axis perpendicular to $\vec{B}$?

ANSWER: The voltage is $V = -\frac{d\Phi_B}{dt}$, and $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\omega t$. Since $B$ and $A$ are unchanging, $-\frac{d\Phi_B}{dt} = \omega BA\sin\omega t$. The effective area of the loop is $100 \times 50\text{cm} \times 30\text{cm} = 1.5 \times 10^5\text{cm}^2$, and the angular frequency is $\omega = 2\pi f = \frac{100}{3}\pi\text{Hz} \approx 104.72\text{s}^{-1}$. The maximum voltage will occur when the loop is orthogonal to the field, at which point $\sin\omega t = 1$. Plugging everything in and letting WolframAlpha do the unit conversion, $V = \omega BA = 104.72\text{s}^{-1} \times 3.5\text{T} \times 2.5 \times 10^5\text{cm}^2 \approx 5500V$.

## Problem 4: mirror mirror on the horizontal wall...

Why does your reflection in a mirror look flipped horizontally but not vertically?

ANSWER: Imagine a blue dot and a red dot. They are in front of you, and the blue dot is on the right. Behind them is a mirror, and you can see their image in the mirror. The image of the blue dot is still on the right in the mirror. What's different is that in the mirror, there's also a reflection of you. From that reflection's point of view, the blue dot is on the left.

The answer is that a mirror doesn't flip an image horizontally (or vertically). What the mirror really does is flip the order of things in the direction perpendicular to its surface. Going on a line from behind you to in front of you, the order in real space is:

- Your back
- Your front
- Dots
- Mirror

The order in the image space is:

- Mirror
- Dots
- Your front
- Your back

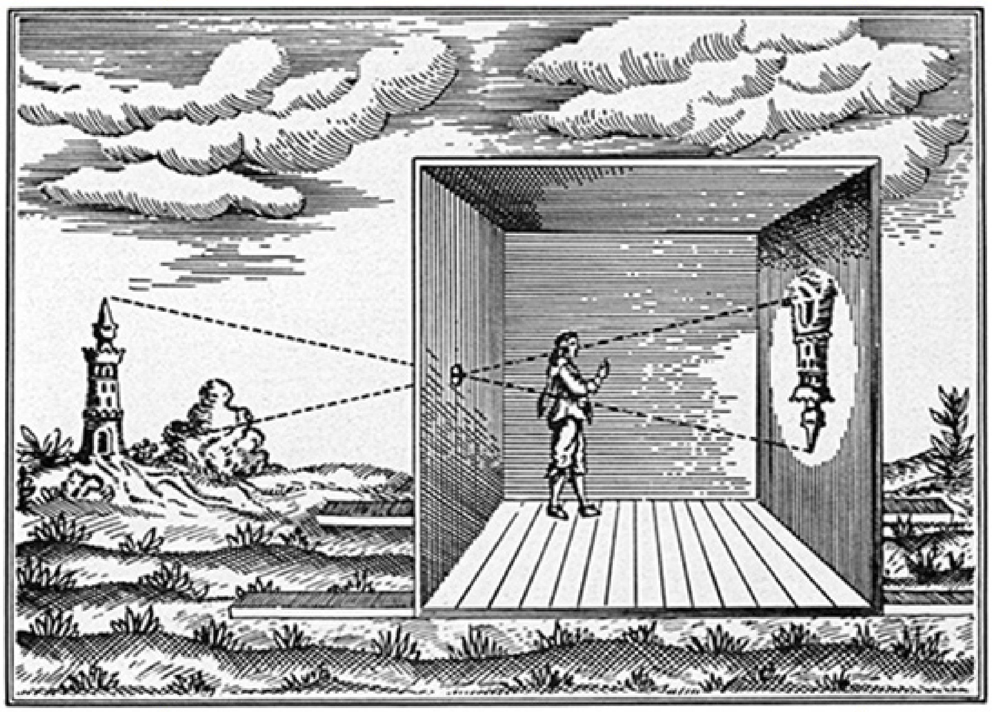
Although left and right are not reversed, the blue dot, which in reality is lined up with your right eye, is lined up with your left eye in the image.

It is easy to anthropomorphize your reflection and think of it as being flipped horizontally. (It is much harder to imagine a human being inverted along the $\hat{z}$ axis!) Thinking of the problem as ray tracing along colored objects removes some of this confusion.

# Activity: room-scale camera obscura (approx. 45 min)

A camera obscura, also referred to as pinhole image, is the natural optical phenomenon that occurs when an image of a scene at the other side of a screen is projected through a small hole in that screen as a reversed and inverted image. The surroundings of the projected image have to be relatively dark for the image to be clear, so many historical camera obscura experiments were performed in dark rooms.

The camera obscura was used as a means to study eclipses, without the risk of damaging the eyes by looking into the sun directly. As a drawing aid, the camera obscura allowed tracing the projected image to produce a highly accurate representation, especially appreciated as an easy way to achieve a proper graphical perspective.



In this activity, you'll make room-scale camera obscura! The procedure is very simple. Find a room with a window that has a large blank wall opposite of it. Using masking tape and the large roll of black paper, black out all of the windows in the room. Finally, use a knife or pair of scissors to cut a small circular hole (about 1 inch diameter, but this isn't critical) in the paper. Ideally, the room (1) has a large flat surface to project on, (2) has an interesting view, and (3) is easily made dark (e.g. connected to any other rooms by closable doors). Take some pictures when you're done!