Questions on PS 4? (5 min)

Survey results (5 min)

- seems like people really liked the solar system lab!
- too many problems so I'll try to put some on the next sets
- about 3/4 of people think lecture pace is alright
- difficulty is about right
  - doing great, my are AP physics or harder!
- students have all been helpful in computing problems!
- seems like everyone is genuinely enjoying & learning a lot :)
- people wrote some nice stuff

Specific requests

show more steps

more analogies to explain math

more "big picture" stuff

I'll try to do both but feel free to ask if you don't understand or are confused

Big picture: where are we at? (10 min)

Last week: classical mechanics ($-\infty$ – ~1750)

- Describes the motion & evolution of macroscopic objects
- not most glitzy, but most applicable to everyday life
- given the full state of object, determine its future

vectors	"how do you describe spatial properties"
calculus	"how do you quantify change"
units	"how do you quantify physical things"
kinematics	"how do things move?"
forces/work/energy	"**why** do things move?"
angular dynamics	"how/why do things spin?"
gyroscopes	"GREETINGS, HUMAN!"
gravitation	"how do things attract each other"
computational physics	"how to simulate reality to study it"

~ mathematization of physics: Newton, Descartes, galileo

This week: "early modern physics" (~1750 – ~1900)

waves + oscillations

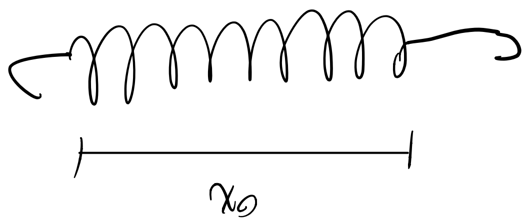
electrostatics + magnetism

Optics (electromagnetism)

Thermodynamics

Shortcomings of classical physics

Springs (cont'd) (toss jelly beans for questions)



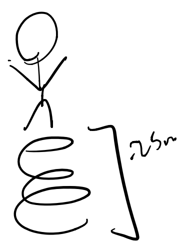
$F_s = -k(x - x_0)$

$= -k\Delta x$

k: spring constant (units? N/m)

intuition?

Ex a 75 kg person stands on a spring w/ $k = 5000$ N/m & $x_0 = 0.25$m. what is height of spring



$F = kx$

$\Delta x = \frac{F}{k} = \frac{mg}{k} = \frac{75 \cdot 10}{5000} = 0.15$m

$\therefore$ height $= .1$m

Energy Stored in a Spring:

$F = kx \quad E = \int F \cdot dx = \int kx\, dx = \frac{1}{2}kx^2$

Ex Pumpkin launchin: $k = 2000$ N/m, $x = 2$m, $m = 1$kg

$E = \frac{1}{2}kx^2 = \frac{1}{2} 2000 \cdot 4 = 4$ kJ $= \frac{1}{2}mv^2$

$v^2 = 8000$

$v = 89$ m/s $\rightarrow$ 200 mph

AK47: .008 kg bullet @ 715 m/s

$= 2$ kJ

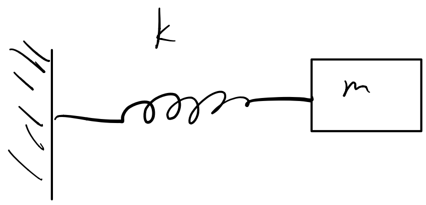
Simple harmonic oscillator (SHO)

(30 min)

Shows up everywhere in physics

- spring systems
- circuits
- pendulum
- electromagnetism
- everything in quantum physics

Show wikipedia list



$$F = ma$$

$$-kx = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x(t)$$

second-order differential equation

need $x(t)$ s.t. $\frac{d^3}{dx^2} x(t) \propto -x(t)$

What functions?

$x = \sin(\omega t)$ $\qquad + \cos(\omega t)$

$\omega \cos$ $\qquad -\omega \sin \omega t$

$-\omega^2 \sin(\omega t)$ $\qquad -\omega^2 \cos(\omega t)$

if $\frac{d^2}{dt} x(t) = -\omega^2 x(t) = -\frac{k}{m} x(t)$,

then $\omega = \sqrt{\frac{k}{m}}$

oscillation frequency of spring!

What about $x = A\sin(\omega t) + B\cos(\omega t)$?

$$\downarrow$$

$$\frac{d}{dt} = A\omega\cos(\ ) + -B\omega\sin(\ )$$

$$\downarrow$$

$$\checkmark \quad \frac{d^2}{dt^2} = -A\omega^2\sin(\ ) - B\omega^2\cos(\ )$$

How to determine A & B?

↳ initial conditions!

↳ any n-th order diffeq requires n initial conditions

Ex |

A 1 kg mass on a spring with $k = 1\,\frac{N}{m}$

Starts with $x(0)$ 5 m & $v(0) = 0$.

Determine $x(t)$.

$$\omega = \sqrt{\frac{k}{m}} = 1$$

$$x = A\sin t + B\cos t$$

↳ $x(0) = \cancel{A\sin(0)} + B\underbrace{\cancel{\cos(0)}}_{1} = 5$

$$B = 5$$

$$\frac{dx}{dt}(0) = A\cos(0) + \cancel{-5\sin 0} = 0$$

$$A = 0$$

$$\therefore x(t) = 5\cos(t)$$

Ex 2)

Same, but $x(0) = 1\text{ m}$, $v(0) = 1\text{ m/s}$

$$x = A\sin t + B\cos t$$

$$x(0) = \cancel{A\sin 0} + B\cos 0 = 1$$

$$B = 1$$

$$\frac{dx}{dt}(0) = A\cos 0 + \cancel{-B\sin 0} = 1$$

$$A = 1$$

$$x(t) = \sin t + \cos t$$

Ex 3) Same problem. Determine $KE(t)$, $PE(t)$, $E(t)$

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2}(\cos t + -\sin t)^2$$

$$= \frac{1}{2}(\cancel{\cos^2 t + \sin^2 t} - 2\cos t\sin t)$$

$$= \frac{1}{2}(1 - 2\cos t\sin t)$$

$$PE = \frac{1}{2}kx^2$$

$$= \frac{1}{2}(\sin t + \cos t)^2$$

$$= \frac{1}{2}(\overline{\sin^2 t + \cos^2 t} + 2\cos\sin)$$

$$= \frac{1}{2}(1 + 2\cos t\sin t)$$

$$E = KE + PE$$

$$= \frac{1}{2}(1 - \cancel{2\cos t\sin t} + 1 + \cancel{2\cos t\sin t})$$

$$= 1.0\text{ J}$$

Can we write $x(t)$ any other way?

$$x = A \sin(\omega t + \phi)$$

amplitude

frequency

phase

$$\frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$$

One more way, but first...

Picard's existence theorem:

"existence & uniqueness of differential equations"

Consider initial value problem

$$\frac{d}{dt} y(t) = f(t, y(t)), \qquad y(t_0) = y_0$$

If f is continuous, then there exists a solution $y(t)$ and it is <u>unique</u>.

Any other ways to write $x(t)$?

$\to \frac{d}{dt} e^{t} = e^{t}$

$\to \frac{d}{dt} e^{ct} = ce^{t}$

Looking for $\frac{d^2}{dt^2} x(t) \propto -x(t)$

$$\frac{d^2}{dt^2} e^{ct} = c^2 e^{ct} = -e^{ct}$$

$$c^2 = -1$$

$$c = i$$

$$\therefore \boxed{x(t) = A e^{i(\omega t + \phi)}}$$

↳ phasor : represents sinusoidal function with constant A, ω, ϕ

consider $\frac{d^2}{d\theta^2}(x(\theta)) = -x(\theta)$, $x(0) = 1$

$\frac{d^2}{d\theta^2} e^{i\theta} = i^2 e^{i\theta} = -e^{i\theta}$ ✓

$\frac{d^2}{d\theta^2}(\cos\theta + i\sin\theta) = \frac{d}{d\theta}(-\sin\theta + i\cos\theta) = -\cos\theta - i\sin\theta$ ✓

$\therefore$ by Picard existence theorem,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

euler's formula

$e^{i\pi} = -1$

Waves

a wave is a propagating disturbance in a field

field is a physical quantity at each point in spacetime

rather than tell you, lets see for ourselves...

Examples of waves

...
...
...

Remember to allow for free ends!

Build a wave machine (15 min)

↳ ~ 1h15 - 1h30 in

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \rightarrow$$

↳ velocity of wave

Solutions:

$$u = Ae^{-i(kx+\omega t)} + Be^{i(kx-\omega t)}$$

forward & backward waves!

Properties of waves

Amplitude
frequency
phase

Reflection & Boundary conditions

hard boundary: $u_{end}(t) = 0$

soft boundary: $\frac{d^2}{dt^2} u_{end}(t) = 0$

no restoring force

refl. off hard boundary
refl. off soft boundary

standing waves → superposition

show wiki picture on the superposition → wave

→ harmonics

normal modes

impedance

discontinuity

$v = f\lambda$

Standing waves

Consider right & left waves

$$y(x,t) = \sin(kx-\omega t) + \sin(kx+\omega t)$$

Using identity that $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$,

$$y(t) = \sin(kx)\cos(\omega t) - \cancel{\cos(kx)\sin(\omega t)}$$
$$+\sin(kx)\cos(\omega t) - \cancel{\cos(kx)\sin(\omega t)}$$

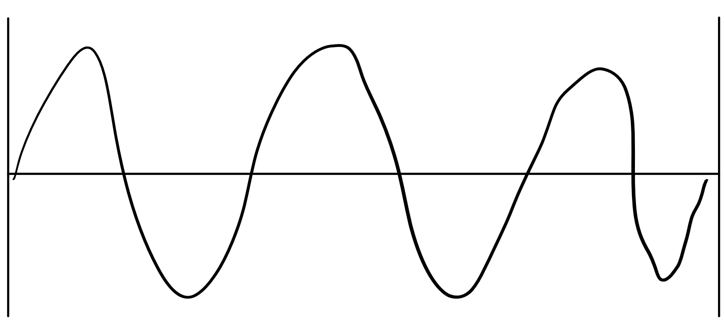
$$= 2\sin(kx)\cos(\omega t)$$

Standing wave! Independent x, t terms

$\sin(kx) = 0 \leftarrow$ nodes

$\sin(kx) = \pm 1 \leftarrow$ antinodes

Harmonics: boundary conditions enforce $\mathbb{Z}$ number of wavelengths (will talk more later w/ particle in a box)



Superposition & interference

Superposition: for linear systems, the net response caused by two stimuli is the sum of the individual responses

examples:

- rolling = rotation + translation
- waves on a pond
- incident & reflected waves on wave machine
- chords

interference: two waves superpose to form a resulting wave of greater or lesser amplitude

ex) $\sin(x) + \sin(x+\pi) = 0$

$\sin(x) + \sin(x) = 2\sin(x)$

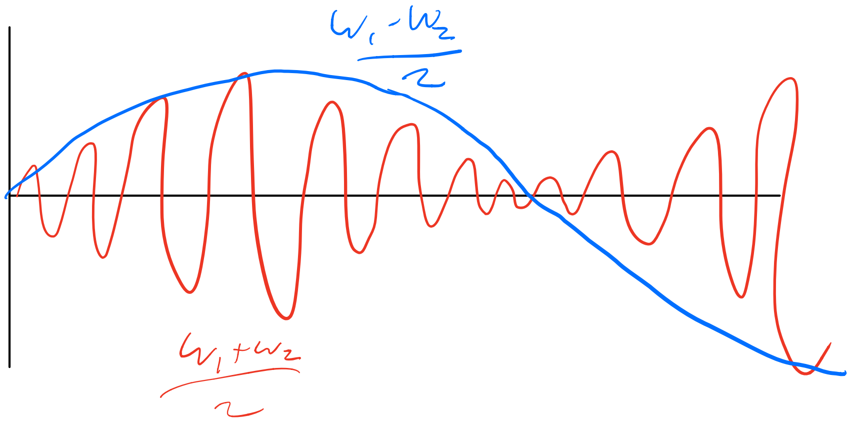
depends on relative phase of waves

Beats

$$\cos(\omega_1 t) + \cos(\omega_2 t) = 2\cos\left(\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$$

if $\omega_1 \simeq \omega_2$, $\frac{\omega_1+\omega_2}{2} \approx \omega_1$

$\frac{\omega_1-\omega_2}{2}$ = low frequency

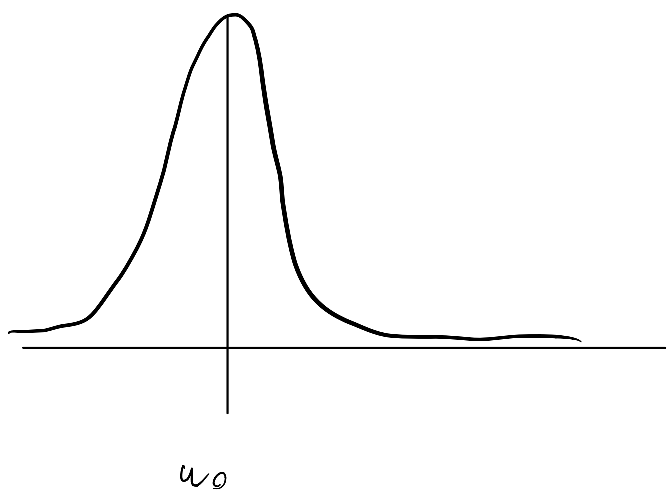


Tuning fork demo

Resonance

Increase in oscillation amplitude when a driving force is applied near a natural (resonant) frequency of a system.

ex) kid on a swing
Tacoma narrows bridge
electrical response in circuits
absorption / emission of photons
laser cavities
etc...



Q - factor → "how good is the resonator?"

$$Q = 2\pi \times \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

Normal modes (from parity)