Black hole entropy

~~Black holes seem like they should have low entropy, but actually contain most of the entropy in universe~~

Black holes predicted by GR

- observed by:
    - gravitational lensing
    - gravitational waves
    - M87 black hole picture
        - Katie Bouman!

Pose a conflict w/ entropy + 2nd law

~~Jakob Bekenstein (60s–70s)~~

~~↳ black holes + information theory~~

No-hair theorem

Black holes fully characterized by

- mass
- ang. mom.
- charge

Most of info about stuff in BH is lost to outside universe

why?

mass: Outside universe can't see the mass inside event horizon but it is remembered by gravitational field

Gauss's law: $$\oint\!\!\oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss's law for mass: $$\oint\!\!\oint_{\partial V} \vec{g} \cdot d\vec{A} = -4\pi G M$$

for a non-rotating BH, event horizon is closed sphere w/ mass at singularity (point mass)

distribution of mass doesn't matter!

Charge: Gauss's law for electricity!

mass + charge are special:

1. infinite in range
2. arise from conserved quantities

What about angular momentum?

↳ encoded in gravitational field! frame-dragging

↳ Gravity probe B measured frame dragging due to Earth's rotation

infalling material adds or subtracts $\vec{J}$ from the frame dragging

What information is lost?

→ baryon #

→ lepton #

→ other conserved quantities

} mediated by strong/weak force which is not ∞-range

Quantum mechanics: information cannot be created or destroyed

↳ message on paper, burn paper, can reconstruct message in theory

conservation of information arises from conservation of probability, which arises from unitary evolutions

↓

two independent states can't evolve to the same state

↓

reverse: No-cloning theorem!

What about measurement? Depends on interpretation...
everett conserves, pilot waves conserves, copenhagen does not...

Black holes evaporate!
↳ Hawking radiation

Where does information go?
Where does entropy go?

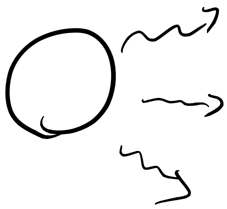
Black hole information paradox

Black hole information paradox

black holes evaporate over unthinkably long timescales

↳ Hawking radiation, which looks like blackbody radiation ~ thermal "noise"

↳ fields outside BH look like moving particles... energy must come from BH



destruction of information is

BH info paradox

quantum information cannot be created or destroyed

TO DO

No-hair theorem seems contradictory: information is still there! Conservation of info doesn't require information to be in an accessible part of universe, just that it exists somewhere

Hawking rad. causes BH to evaporate into a random buzz of radiation that contains none of original information about the black hole.

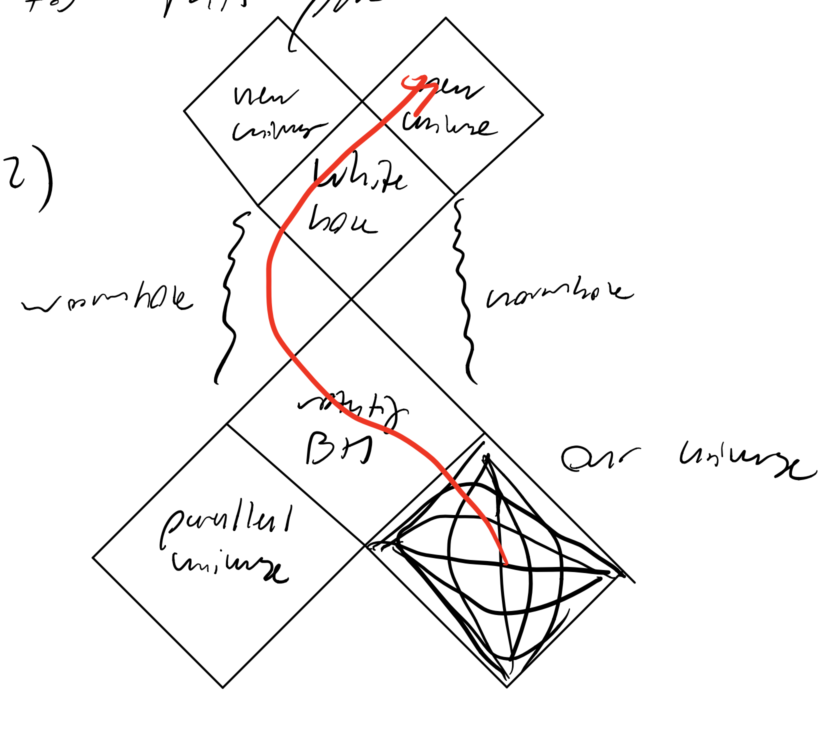
The key is that Hawking radiation doesn't depend on what BH is made of:

$$T_H = \frac{\hbar c^3}{8\pi k_B G M} \propto \frac{1}{M}$$

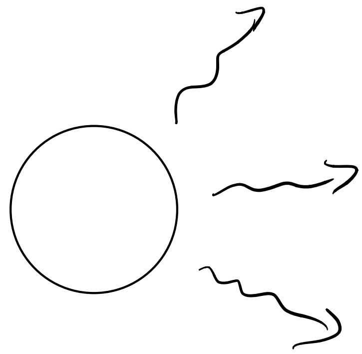
only depends on mass

Early Solutions to this paradox:

Einstein-Cartan theory (1922)
(something like this)



Info imprinted on hawking radiation itself



no new universes required!

Motivation: from outside observer, nothing actually falls into a black hole

↓

everything crossing horizon remains frozen in time, smeared across the surface of the horizon

1997: Preskill / Hawking – Thorne bet

P: info leaks to universe

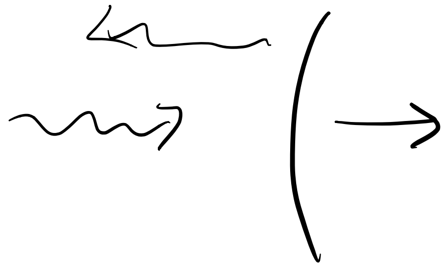
HT: info is lost from universe

reward: encyclopedia

2 problems:

1. Mechanism?
2. still breaks QM...

Problem 2: transfer of quantum info to hawking radiation still violates conservation of information



infalling info is copied!
violates no-cloning theorem

is this a problem? interior of BH is inaccessible ~ Leonard Susskind

→ Black hole complementarity:
interior & exterior of BH are not simultaneously knowable
(like $\Delta x \Delta p \geq \frac{\hbar}{2}$)
← complementary variables

Problem 1: mechanism:

infalling material doesn't freeze at horizon, but distorts surface of horizon, creating a "lump" at point of crossing which contains info about infalling material

↳ influences Hawking radiation?
→ how? open question

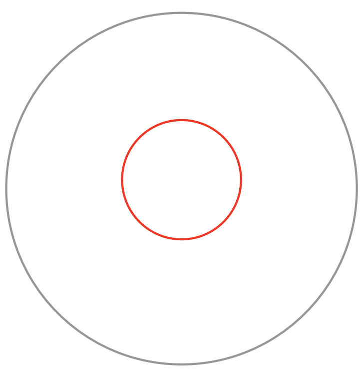
hawking proposed quantum hair, gave preskill encyclopedia

Gerard 'T Hooft

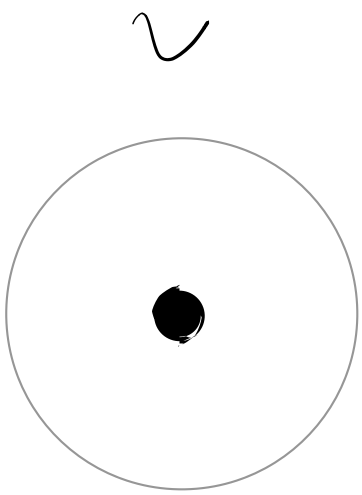
described mechanism by which
info on infalling particles
could be preserved &
encoded on surface of
event horizon
↳ imprinted on Hawking radiation
and transformed back into
the universe

1999 Nobel prize for
electroweak interactions

Let's make a black hole!



lots of entropy of collapsing
star
↳ we have almost no
information about it
but info still exists
in the universe...



go from knowing almost nothing about system to knowing everything we can know about it!

↓

can easily measure M, J, q

Region of space containing the BH seems to have gone from high $S \longrightarrow S = 0$ in an instant

↳ but whaddabout muh 2^{nd} law!

If you can encode <u>information</u> on surface of BH, you can store <u>entropy</u> on surface & radiate it back into the universe

Resolution to information paradox also saves 2nd law!

This got Jakob Bekenstein thinking about information + BH's

Surface area of BH event horizon can never decrease (according to GR)

If nothing can escape BH, they can only grow, never shrink in mass/radius

↳ Not quite true!

- BH merger radiating mass via GWs

- Penrose process → can extract energy from rotating black holes

Penrose process

Penrose Process

TO DO

Mergers + Penrose process decrease mass + radiating but (other than Hawking radiation), one property that no process can reduce:

→ Surface area of Event horizon ←

anything you do to black holes results in

$$\Delta SA \geq 0$$

what's this look like?

Bekenstein realized:

Clear correspondence $\Delta S \geq 0 \leftrightarrow \Delta SA \geq 0$

$$\Delta S \geq 0 \qquad \qquad \Delta SA \geq 0$$

$$\downarrow \qquad \qquad \Downarrow$$

$$\Delta S = \frac{\Delta U}{T} \qquad \qquad \Delta SA = \frac{\Delta M}{\Theta}$$

thermodynamic entropy

looks the same as laws of (regular) thermodynamics

Bekenstein just discovered BH thermodynamics!

Laws of BH thermodynamics

Law 0: event horizon of a stationary black hole has constant surface gravity κ

Law 1: for a stationary black hole,

$\Delta U = Q - W$

$dU = T\,dS - P\,dV$

$$\Delta U = \frac{\kappa}{8\pi}\Delta A + \omega \Delta J + V \Delta q$$

energy

surface gravity

ΔEH area

angular vel

Δ ang. mom

charge

electrostatic potential

Law 2: $\Delta A \geq 0$ → superseded by Hawking radiation

Law 3: cannot have a BH w/ vanishing surface gravity ($\kappa = 0$ cannot be achieved)

Black hole entropy

Boltzmann eqn: $S = k_B \log \Omega$

→ To define BH entropy, need to calculate amt. of information that would be lost into a BH as it grows

To do this, build a BH out of idealized elementary particles containing a single bit of information

↳ TO DO

information in BH is proportional to its surface area

$l_p^2 = \frac{G\hbar}{c^3} = 2.6 \times 10^{-70}\ m^2$

↳ info ≈ $\log\left(\frac{SA}{\#\text{ planck areas}}\right)$

one "pixel" contains a single bit of information

multiply info × k_B ⟶ BH entropy!

connection between SA ⟷ S could have been a coincidence, but this...

1974: Stephen Hawking → Hawking radiation

↓

looks exactly like Blackbody rad!

↓

if BH have temperature, they also have entropy!

Thermodynamic entropy

$$\Delta S = \frac{\Delta U}{T}$$

Hawking temperature

$$T_H = \frac{\hbar c^2}{8\pi G M k_B}$$

Substitute $T \leftarrow T_H$

$U \leftarrow M_{BH} c^2$

Bekenstein - Hawking entropy:

$$S_{BH} = \frac{k_B A_{Hor}}{4 l_p^2}$$

BH = Black hole

= Bekenstein - Hawking! lol

Same amount of entropy if you calculate Δ information by building a black hole as if you calculate Δ info radiated via Hawking radiation!

2 consistent independent results... we're probably on to something!

↳ $\propto A_{hor}$!

2nd law is saved hence BHs do have entropy...

BH's actually contain maximal possible information/entropy density!

contain most of entropy in the universe!

Bekenstein bound

entropy / info S_{BH} derived for black holes, but S_{BH} also describes max amount of information you can fit into any volume of space!

Information bound:

non-rotating

$$I \leq \frac{A_{Hor}}{4\, l_p^2 \ln 2} = \frac{2\pi R E}{\hbar c}$$

$R \to \frac{2GM}{c^2}$

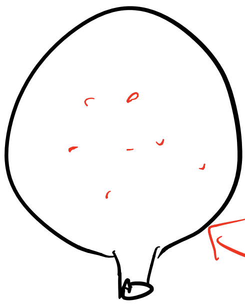
E/c^2

$$\frac{A}{4\, l_p^2 \ln 2} \to \frac{4\pi R^2}{4 \cdot \frac{G\hbar}{c^3} \ln 2} = \frac{4\pi R \cancel{\frac{2GM}{c^2}}}{4 \frac{G\hbar}{c^3} \ln 2} = \frac{2\pi r E}{\hbar c \ln 2}$$

But this is weird...

max amount of info you can fit inside a space should depend on volume, right?

bits balloon



$V \sim$ # particles $\sim$ # bits = information

1 bit per tiny volume element

Bekenstein bound: NOPE

1 bit per area element on a surface enclosing that space

corollary:

information needed to describe any volume of space, no matter the contents, is $\propto$ to area bounding the space

This simple idea led to...

The holographic principle

entire 3D volume of universe is just a projection of information encoded on a 2D surface surrounding the universe