Topics in Physics: Problem Set #6 (TA version)

Topics: electrostatics, magnetism

General TA instructions

- Give students the estimated time for each section, and plan to spend about 50% of the estimated time going through answers.
- Try not to go over time for the earlier sections, since many of the more interesting activities are at the end of the assignment.
- Ask for student volunteers to explain their answers and discuss discrepancies.
- Be stingy with hints, but if a student seems legitimately stuck or won't finish the section in time, try to walk them through how to solve the problem without giving them the answer. You could also have them discuss with any students who have finished the section.

Practice Problems (approx. 45 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. A cathode ray tube television works by shooting a beam of electrons at a phosphor screen, causing it to glow. An electric field is used to steer the beam to particular parts of the screen. If the electrons are traveling at 5×10^6m/s along the x axis through a 5cm steering device, what electric field should be applied to give the electrons a final velocity of 1×10^6m/s along the y axis? By what angle are the electrons deflected?

   (a) Electrons pass through the steering device in $\Delta t = \Delta x / v_x = 5\text{cm}/5 \times 10^6\text{m/s} = 10\text{ns}$. The necessary acceleration is a $a_y = \Delta v_y / t = 10^6\text{m/s}/10\text{ns} = 10^{14}\text{m/s}^2$. The electric field required to cause that acceleration is $F = ma = qE \rightarrow E = ma/q = 570N/C$.

2. It takes energy to push positive charges (e.g. protons) towards each other. If you arrange a bunch of protons into a ball of radius R, the energy stored in its electric field will be $U = \frac{Q^2}{8\pi\varepsilon_0}$. If all the electrons in your pinky finger were to suddenly vanish, with how much energy would the resulting ball of protons explode? (You can ignore the presence of neutrons - they won't change the energy of the explosion). Compare this to the largest atomic bomb ever tested, which had a yield of 50 Megatons of TNT.

   (a) $Q =$ electron charge $\times$ ~1/10 of a mole. So $U = (1.6 \times 10^{-19}C \cdot 6 \times 10^{22})^2/8\pi\varepsilon_0$ which is about 200 times stronger than 50 megatons of TNT.

3. Wire A carries current along the x axis in the positive direction, while wire B carries the same amount of current along the y axis in the positive direction. Describe/sketch the magnetic field in the xy plane. Specifically, which regions are the magnetic fields zero?

   (a) Magnetic field from a wire: $B = \mu_0 I/2\pi r$, and the direction comes from the right-hand rule. When x and y have the same sign, the fields from each wire point in opposite directions (along the z axis). Along the line $x = y$, the magnetic field is zero. When x and y have opposite signs, the fields from each wire point in the same direction.

4. In the earth's magnetosphere, dangerous solar wind particles are deflected away from the planet. Suppose the solar wind particles are traveling at 10^7 m/s and the magnetic field strength is 40 nT. What is the radius of the circular trajectory of an electron? What is the radius of the circular trajectory of a proton? If the deflections are happening 9 Earth radii away from the planet, will either the electrons or protons come close to hitting the earth?

   (a) Cyclotron radius as derived in class is $r = mv/qB$. Using $v = 10^7$ m/s, r for an electron is 1.4 km and r for a proton is 2600 km. Neither will hit Earth.

5. A velocity selector is a device which sorts charged particles according to their velocity. Only particles with a particular velocity v_0 will travel in a straight line through the selector. The device uses a magnetic field an an electric field which cause forces in opposite directions. The forces cancel perfectly for particles traveling at velocity v_0. In which direction should the magnetic field point relative to the electric field? How strong should the magnetic field be relative to the electric field?

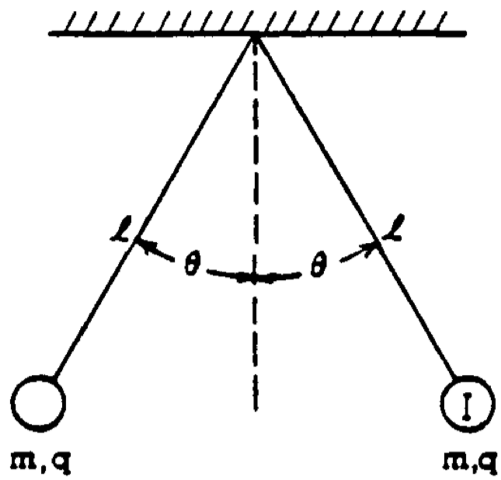
   (a) The electric and magnetic fields should be perpendicular to each other. $F = qE = qvB$, so $F = 0$ when $E = v_0 B$ so $E/B = v_0$.

Challenge Problem (approx. 30 min)

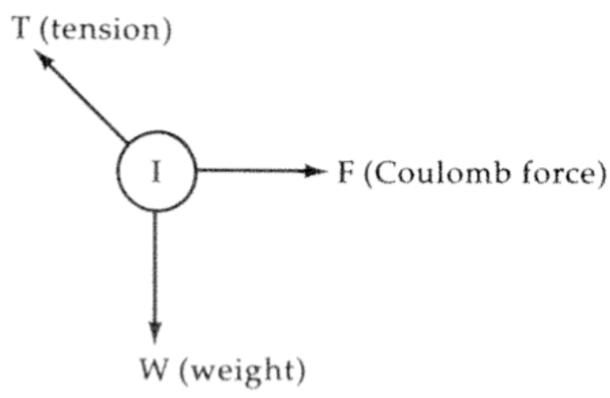
You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

Problem 1: charged spheres

Two small spheres, each of mass m and positive charge q, hang from threads of length l, with each thread making an angle θ from vertical, as shown below.



1. Draw a free-body diagram and label all the forces on sphere I.



2. Develop an expression for the charge q in terms of m, l, θ, g, and Coulomb's constant $k = \frac{1}{4\pi\varepsilon_0}$.

   (a) Resolving the tension into components we have $T\cos\theta = W$ and $T\sin\theta = F$, where $W = mg$ and $F = kq^2/r^2$ and $r = 2l\sin\theta$, giving $F = kq^2/\left(4l^2\sin^2\theta\right)$. Dividing the two expressions we get $\tan\theta = F/mg = kq^2/\left(4l^2\sin^2\theta mg\right)$. Solving yields $q^2 = 4mgl^2\sin^2\theta\tan\theta/k$.

Activity: simulating a cyclotron (approx. 30-45 min)

This activity is very similar to the solar system simulator activity. In fact, most of the code was taken straight from that project! Take a moment to look over the code for the simulator in `cyclotron_sim.py`; it should look pretty familiar. For the first part of this activity, you need to fill in the `compute_electric_force()` and `compute_magnetic_force()` functions to compute the forces on the particle due to the electrostatic and magnetic fields present in the cyclotron. You'll also need to use Euler integration to update the velocities and positions of the particle.

Once you're done with this and your simulation is working, try to think about how the cyclotron is working. Look at the value of $|v|$ for the electron. When does $|v|$ change? When does it increase? I've set the magnitudes of $|E|$ and $|B|$ to values which make the simulation work, but there are sliders at the top to change them and you should feel free to play around with them!

ANSWER: The complete code can be found in `cyclotron_sim_complete.py`. The program looks like this when run with default parameters when everything is implemented correctly.

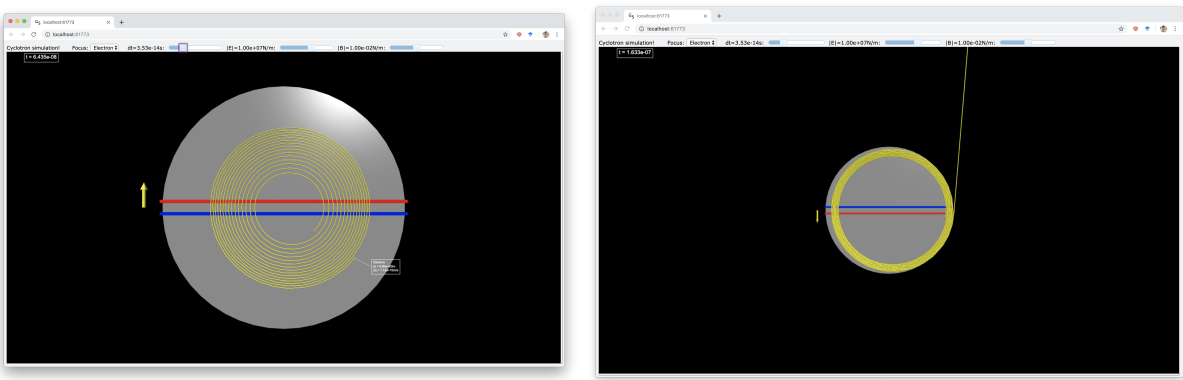


Figure 1: Screenshots of the cyclotron simulator

Activity: building an electric motor (approx. 45 min)

Go have fun and build a motor! Think about the principles we learned in class for torque acting on a current-carrying loop. How does contact (or lack thereof) of the spinners with the metal brushes allow the polarity to switch to the motor keeps spinning in the same direction?

Remaining time: research final project topics

Your final project topic proposals are due by this Thursday. Please email me a ~1 paragraph description (`benbartlett@stanford.edu`) of what you would like to do as the topic project some time before Thursday morning (along with who your teammates are, if you are working in a group). If you are unsure about your topic idea, feel free to approach me in/after lecture to run it by me.

Spend your remaining time in this session doing a bit of research into what you would like to do for your final project. If you're stuck trying to come up with a topic, ask a TA or another student for ideas! You'll have about 90 minutes during the afternoon session per day for five days (starting Thursday 7/04) to work on your final project, so make sure that it represents an appropriate amount of time. (You're welcome to work extra on it in your free time but this of course is not required or expected.)