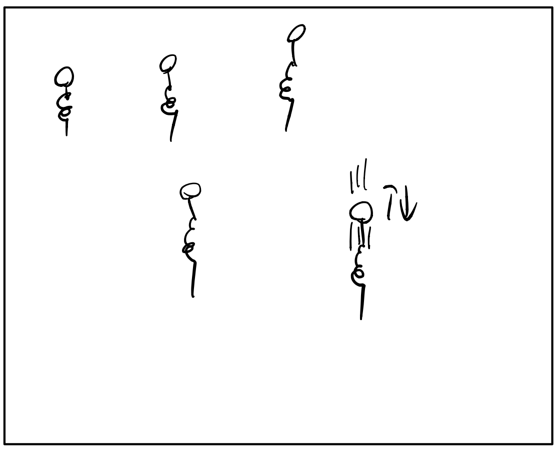
Quantum mechanics

Origins of QM:

Blackbody radiation

accelerated charged particles radiate light

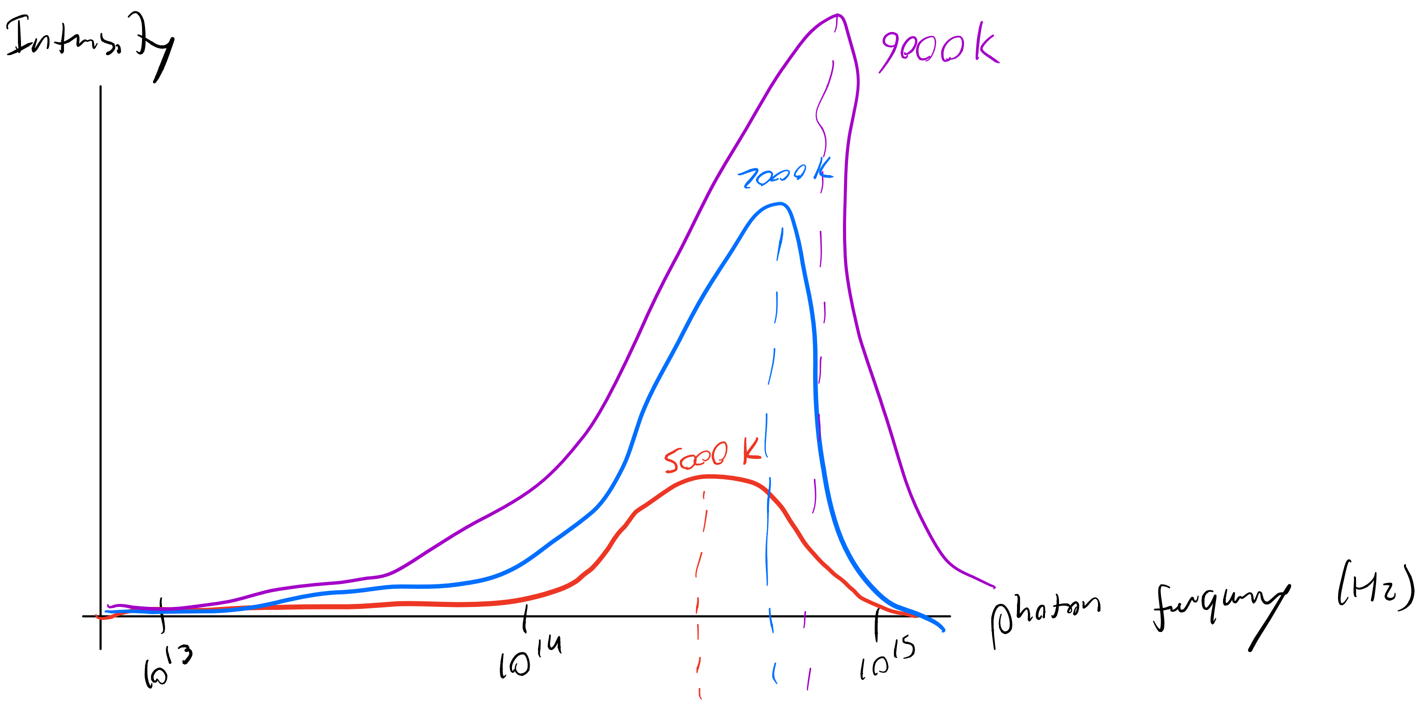


faster jiggling = higher frequency and more emitted light

Sun emits visible spectrum because $T \approx 6000K$

you glow in IR @ $T \approx 300K$

Blackbody spectrum carefully measured by end of 1800s



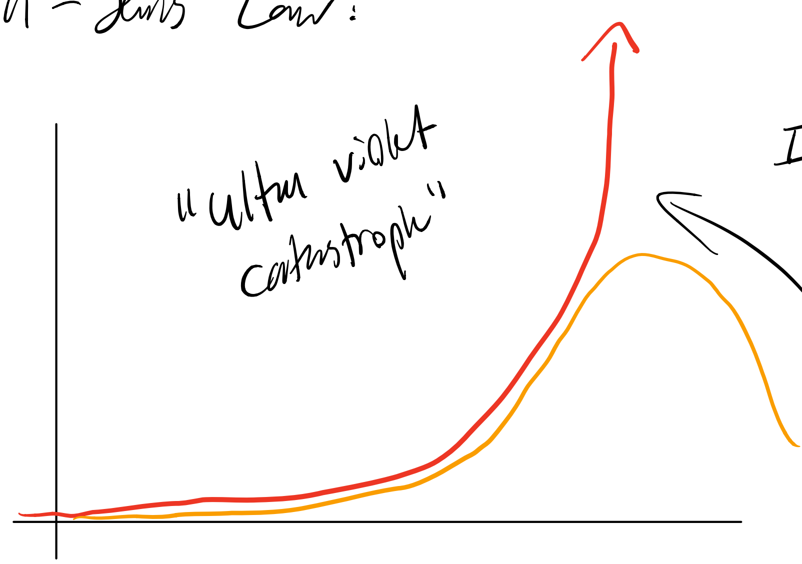
The cause of shape of curve was still unknown

Rayleigh + Jeans: equipartition theorem

→ heat jiggles particles in all ways they can be jiggled

→ energy is equally partitioned over all possible energy states

Rayleigh-Jeans Law:



$$I = \frac{\frac{\omega^2}{2\pi} k_B T}{c^2}$$

brightness way too high!

$I \to \infty$ as $\omega \to \infty$

The problem:

assumes vibrations can be infinitely subdivided

can subdivide into infinitesimally small high-ν states

Max Planck: (out of desperation) particles can only vibrate with $E = hf \cdot n$

quantized energy states!

— h had yet to be measured, just a small number
— would become Planck's constant!

— quantizing energies allowed Planck to equipartition energies in a way that solved the ultraviolet catastrophe

— why?

→ limited how much energy the high-freq vibrations could hold!

Planck's law

$$B = \frac{2hf^3}{c^2} \frac{1}{e^{hf/k_BT} - 1}$$

described the shape of a BB spectrum perfectly!

Planck didn't think quantization was real

↳ expected $h = 0$ → no interval between energy states,
no quantization,
no min energy

Einstein: photoelectric effect (1921 nobel prize)

→ realized that actually light is quantized!

↳ vibration quantized because can only emit/absorb
1 photon at a time

Photoelectric effect

Cs + Zn emit electrons when exposed to light

If light is just a wave, what happens if we turn up brightness?

↳ electrons have more energy

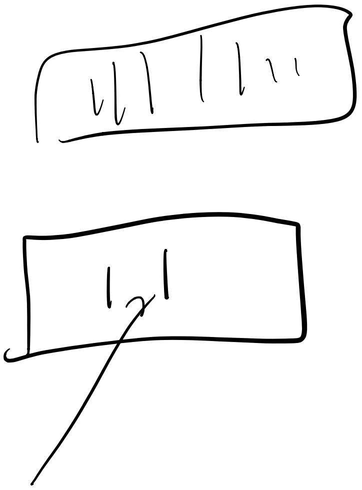
Not the case!

Energy of emitted electrons directly proportional to frequency of photons, independent of brightness

1905: einstein proposes particles of light called photons with $E \propto f$

Double slit experiment

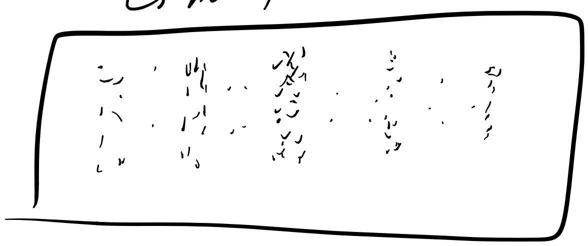
light → waves ⇒ interference



works with single electrons too!

↳ molecules!

↳ buckyballs!



electron chooses "landing" spots randomly, but distribution follows interference pattern!

↳ each electron travels through both slits as wave! wave interacts with itself

↳ somehow, wave resolves itself to a single position when you measure it

interference peaks → more probable regions to find particle

"probability wave"

other properties (momentum, spin) follow similar wave behaviors

distribution of properties: wave function ψ

Describing wavefunctions is the heart of QM!

What is wave made of? Why does it collapse to a point when you measure position?

↳ Open question, open to interpretation

Bohr + Heisenberg: Copenhagen interpretation

wavefunction doesn't have physical location

wave tracks all possible configurations of particles

measurement → wavefunction collapse

universe simulates all possible realities at once!

↳ realities can interfere with each other!

outcome is fundamentally random

- QM + Copenhagen produces stunningly accurate predictions of reality!

- But, not only interpretation:
  - Many worlds
  - De Broglie - Bohm → particles have definite positions guided by pilot wave eqn
  - Objective collapse (Ghirardi - Rimini - Weber)
  - Consciousness causes collapse (von Neumann - Wigner)

Dynamics of the wavefunction

continuous!

$\psi(x,t)$ — wavefunction

$$\int_a^b |\psi(x,t)|^2 dx = \text{prob. of finding particle between } a \text{ \& } b \text{ at time } t$$

$|\psi|^2 = \psi^* \psi$

corollary:

normalization

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$$

wavefunction evolution determined by Schrödinger eqn:

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -i\hbar\frac{\partial\psi}{\partial t} + V\psi(x,t)$$ time dependent

wave equation!

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi(x) - V(x)\psi(x)$$ time independent

Expectation of an operator

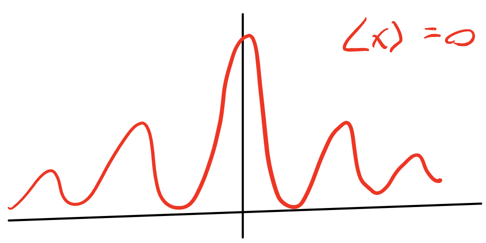
properties get promoted to "operators"

$\langle \hat{Q} \rangle$ = expected value of $\hat{Q}$

$$\langle \hat{Q} \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\ \hat{Q}\ \psi(x,t)\, dx$$

$\hat{x} = x$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, x\, \psi(x,t)\, dx = \int_{-\infty}^{\infty} x\, |\psi(x,t)|^2\, dx$$

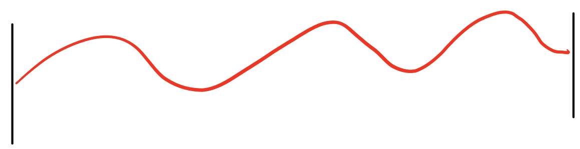


$\hat{p} = -i\hbar \frac{\partial}{\partial x}$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, \frac{\partial}{\partial x}\, \psi(x,t)\, dx$$

Particle in a box

New motion:



Potential "Box"

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

At all points:

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = (E-V)\psi(x)$$

Outside well: $\psi(x) = 0$ is only sensible solution

Inside well: $\psi(x) = A \sin kx + B \cos kx$

$\psi(0) = 0$: $B = 0$

$\psi(a) = 0$: $ka = \pm n\pi \rightarrow k = \frac{n\pi}{a}$

$($

$$\psi(x) = A \sin\left(\frac{n\pi}{a} x\right)$$

find A: $\int_{-\infty}^{\infty} \psi^2(x)\, dx = \int_0^a A^2 \sin^2\left(\frac{n\pi}{a} x\right) = A^2 \frac{a}{2} = 1$

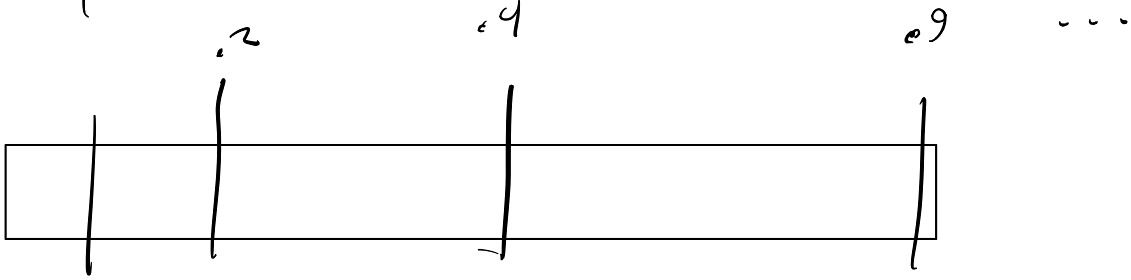
$$A = \sqrt{\frac{2}{a}}$$

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$$

possible energies: $E_n = \frac{\hbar^2}{2m} k^2$

$$= \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2$$

Spectrum:



$E = \frac{\hbar^2 \pi^2}{2ma^2} \cdot 1$

Eigen states: ↙ ignoring normalization

$\psi_1(x) = \sin\left(\frac{\pi x}{a}\right)$ is a valid wavefunction... so is

$\psi_2(x) = \sin\left(2\frac{\pi x}{a}\right)$.

Any linear combination $A\psi_1(x) + B\psi_2(x)$ is also a valid solution!

ψ_1 has energy $E_1 = \frac{\hbar^2}{2m} \cdot 1$

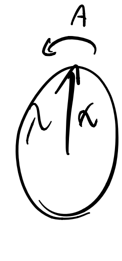
ψ_2 has energy $E_2 = \frac{\hbar^2}{2m} \cdot 4$

→ If we have $\psi = \psi_1 + \psi_2$ and we measure energy of particle, we get 50/50 chance of E_1 or E_2

→ If we have $\psi = 1\psi_1 + 2\psi_2 + 3\psi_3$, we get $\frac{1}{1^2+2^2+3^2} = \frac{1}{14}$ chance E_1, $\frac{2^2}{14}$ chance E_2, $\frac{3^2}{14}$ chance E_3

→ States with definite property values are called <u>eigenstates</u> of the observable operator. The value of the property is the <u>eigenvalue</u>.

Linear algebra: eigenvectors: $A\vec{x} = \lambda\vec{x}$

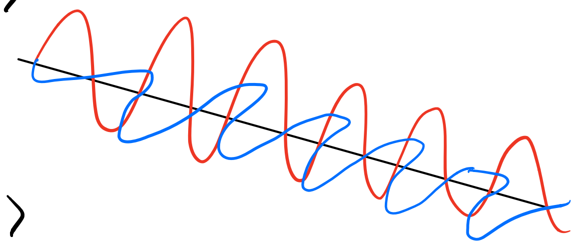


Measurement examples: linear polarizers

Light has polarization, which describes geometric orientation of E-field

$|V\rangle$

Linear, vertical:

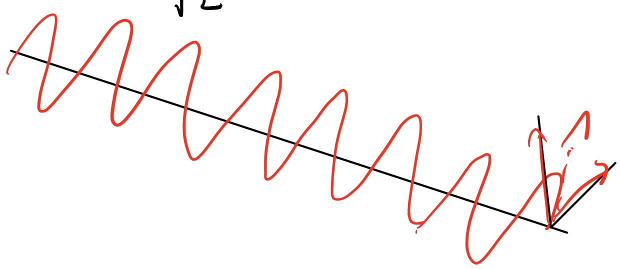


$|H\rangle$

Linear, horizontal:

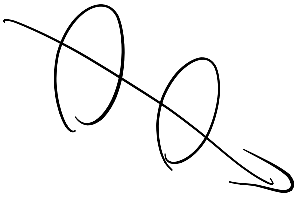
$|D\rangle = \frac{|V\rangle + |H\rangle}{\sqrt{2}}$

Linear, diagonal:

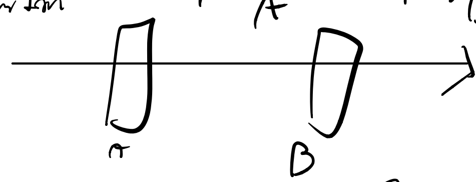


Polarizer measures polarization & only allows one polarization to pass

2 polarizers:



random $|V\rangle_A$ $|V\rangle_B$



rotate by θ: amt. transmitted is $\cos^2\theta$

Bras & Kets:

$\psi(x)$ is position of <u>state vector</u> $|\psi\rangle$, which encodes everything about the particle's state

other properties: polarization

$$|\psi\rangle = A|V\rangle + B|H\rangle$$

Probability of observing particle in $|\psi\rangle$ to be in eigenstate $|\phi\rangle$ is $\langle\phi|\psi\rangle = \int_{-\infty}^{\infty} \phi^*(x)\,\psi(x)\,dx$

inner product, just like vectors but complex

$\langle A|B\rangle = 0$ if A, B orthogonal

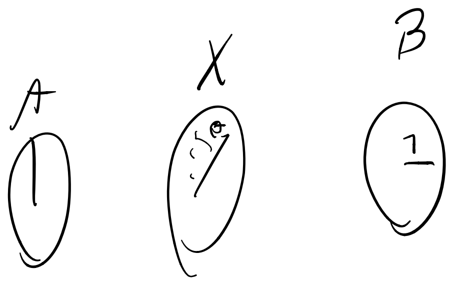
<u>2 polarizers @ 90°:</u>

Pre-A: random polarization

A: $|V\rangle$, 50% transmitted

B: $|H\rangle$, 0% transmitted because $\langle H|V\rangle = 0$

Add a 3^{rd} polarizer:



A: $|V\rangle$, 50% transmitted

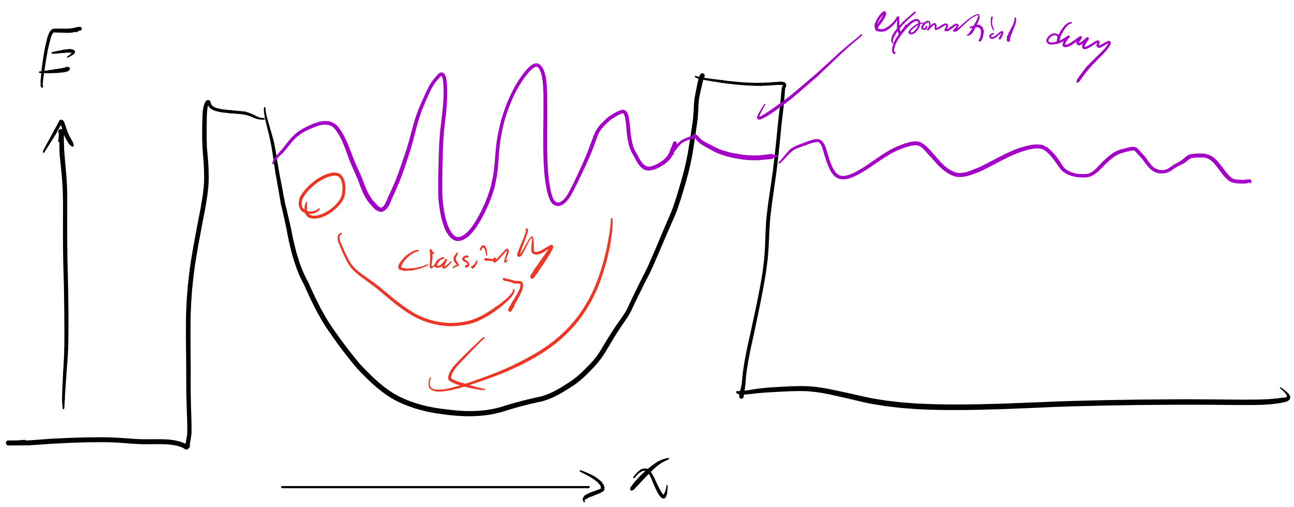
X: $\langle X|V\rangle$, $\cos^2\theta$ transmitted

$\hookrightarrow = \frac{1}{2}$ if $\theta = 45°$

B: $\langle B|X\rangle$, $\cos^2(90-\theta)$ transmitted

$= \frac{1}{2}$ if $\theta = 45°$

Quantum tunneling

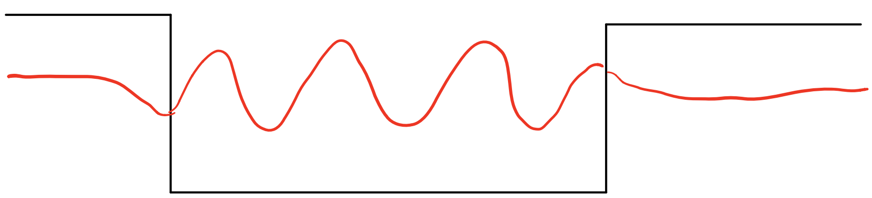


protons, neutrons, α particles $\rightarrow$ fusion!

no sunlight without tunneling!

Example: finite square well

$$V(x) = \begin{cases} -V_0 & -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$



particle has some energy $E < 0$

if $-a \le x \le a$:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = (E + V_0)\psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -l^2 \psi, \quad l \equiv \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

if $|x| > a$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial x^2} = E\psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\kappa^2 \psi$$

$$\kappa = \frac{\sqrt{2mE}}{\hbar}$$

$\psi \propto e^{ikx}$

$l : \dfrac{\sqrt{2m(E+V_0)}}{\hbar} > 0$

$k : \dfrac{\sqrt{2mE}}{\hbar}$

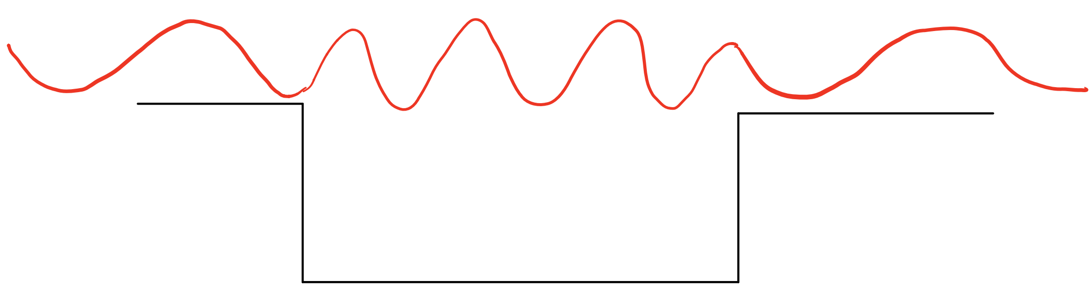
$\psi \propto e^{ilx}$

$A\cos(lx) + B\sin(lx)$

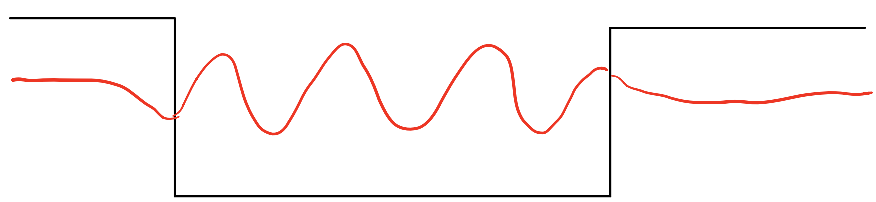
so $\psi \sim e^{ikx}$

if $E > 0$: k is real

$\psi = A\cos(kx) + B\sin(kx)$



if $E < 0$: k is imaginary so $e^{ikx} \to e^{-\frac{\sqrt{-2mE}}{\hbar}x}$



The Heisenberg uncertainty principle

uncertainty principle: fundamental limit to the precision with which certain complementary variables, (e.g. x & p) can be known

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

NOT a statement about:

- limited measuring device precision
- observation altering state of system

Consider sound waves:

sound of perfect tone: sine wave

sound of orchestra: complicated

BUT

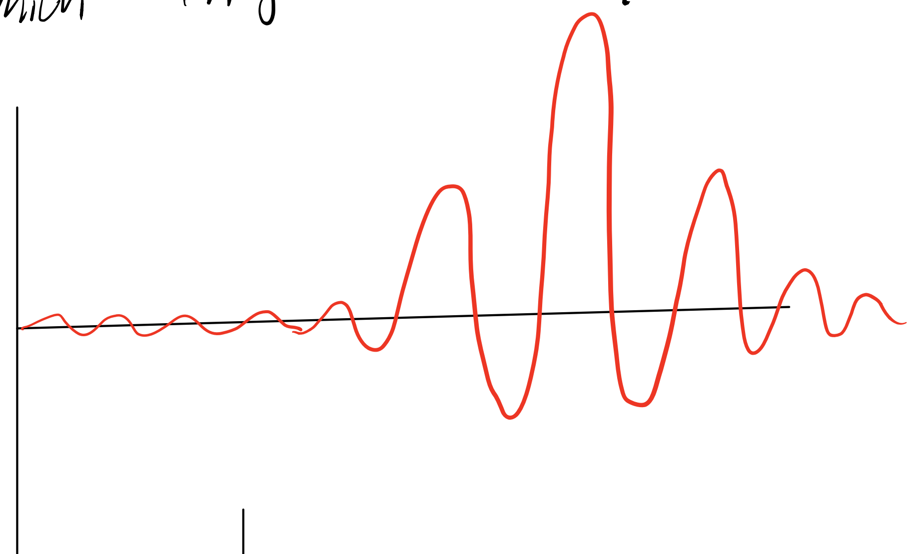
Any wave can be expressed as sum of pure sine waves

Fourier Series: $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$

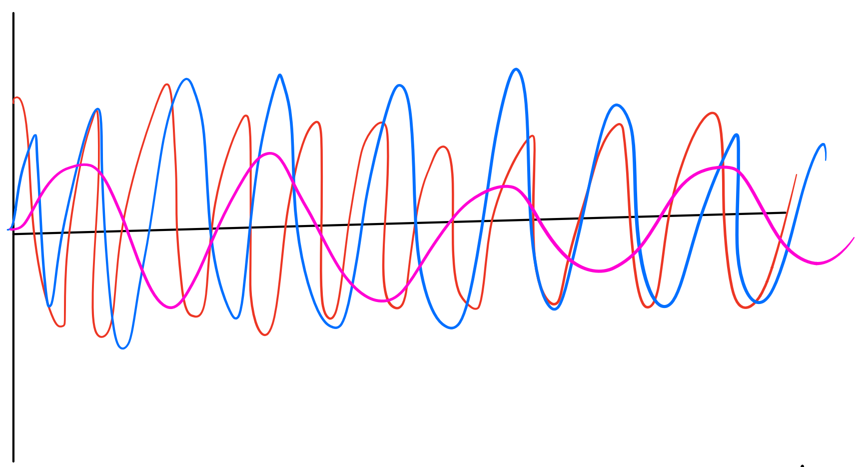
Sound wave can be represented in amplitudes or frequency basis

amplitude, frequency are Fourier pairs,
conjugate variables

Can make a wave packet with many frequencies which interfere destructively except @ locations

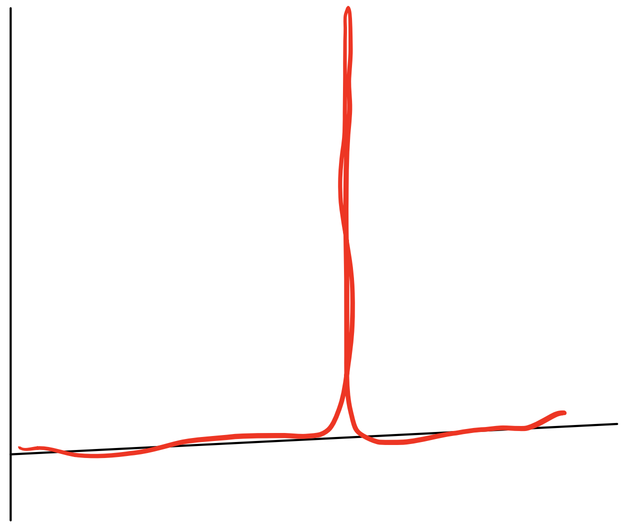


$\hookrightarrow$ =



To get steep edges around peak, need higher & higher frequencies because high freq. changes give rapid changes in intensity.

To compress wave to an arbitrarily fresh spike, need ∞-ly many sine waves



How's this relate to quantum stuff?

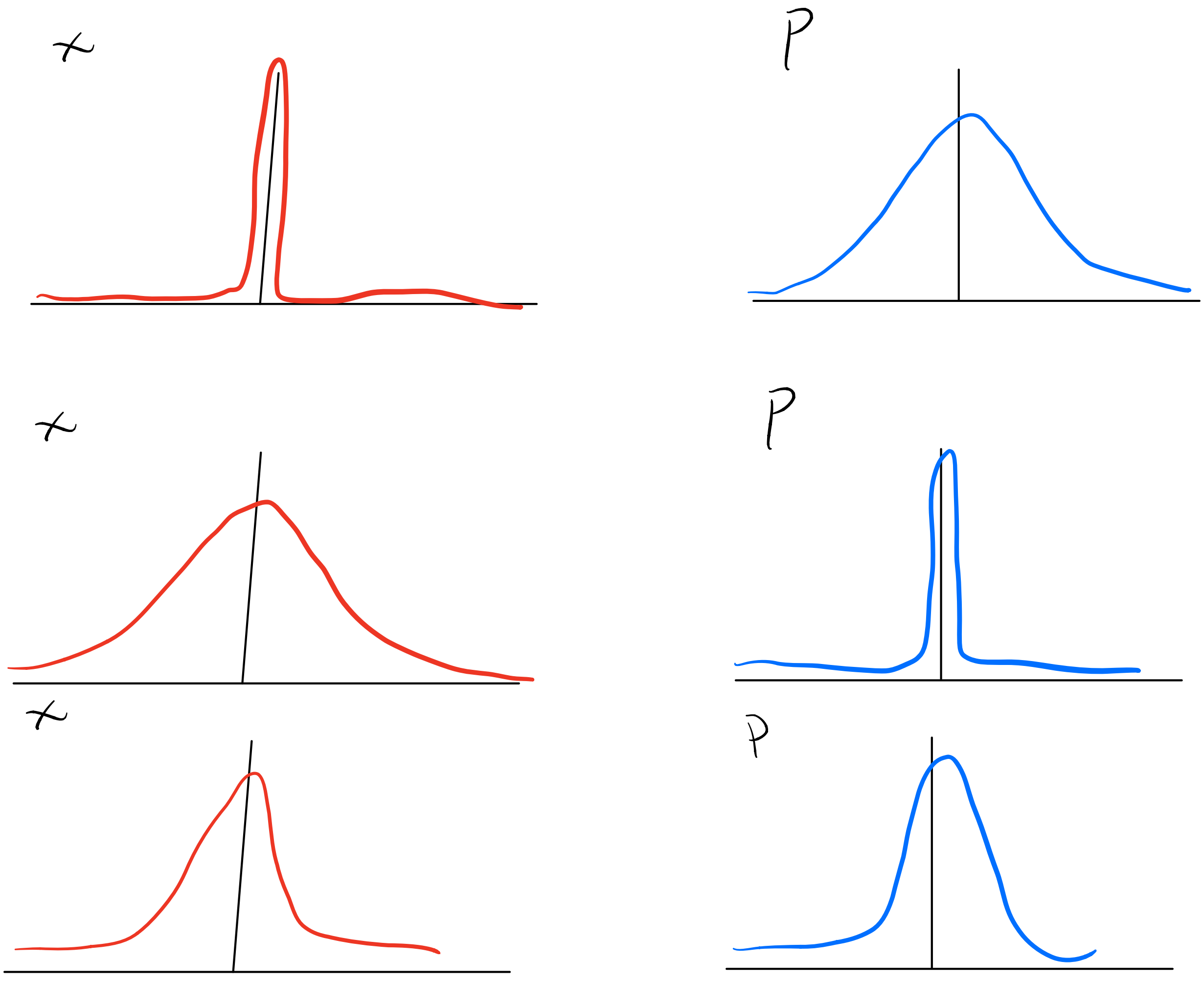
position: amplitude of wave at position x

momentum: ~wavelength of wavefunction $\quad p = \frac{h}{\lambda}$

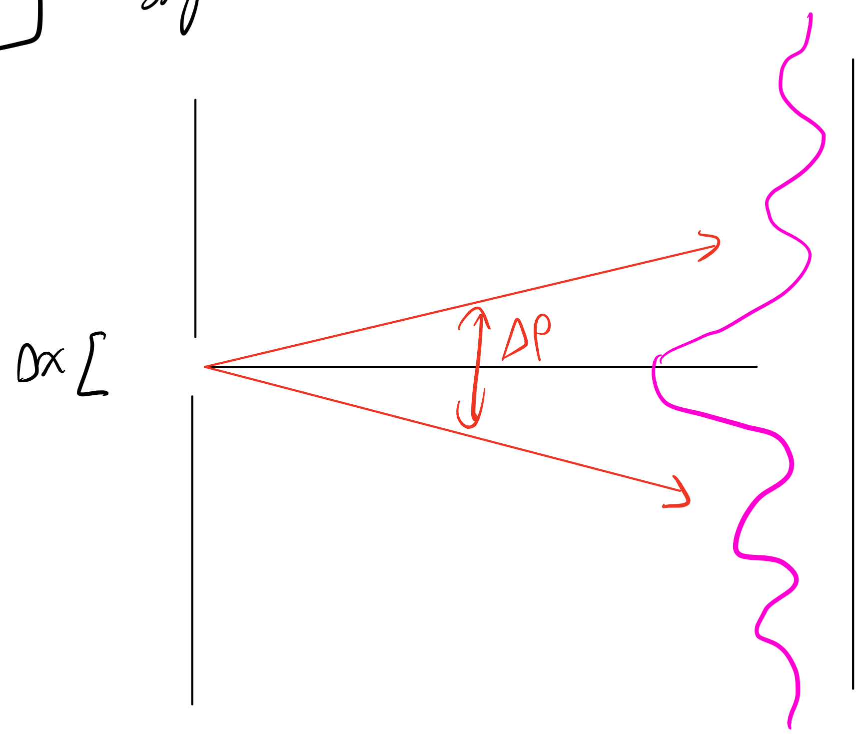
$|\psi(x)|^2 \sim$ probability of particle @ position x

$|\psi(p)|^2 \sim$ probability of particle @ momentum p

Measure position $\longrightarrow$ spike @ x $\longrightarrow$ wide range of momenta ex)



Ex] Single slit diffraction



Quantum entanglement

Einstein hated Copenhagen interpretation

↳ wavefunction is incomplete!

↳ universe should have "hidden variables"

To demonstrate silliness of wf + Copenhagen,

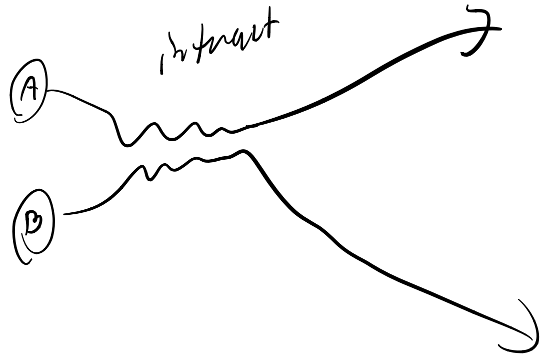
Einstein + Podolsky + Rosen showed that to

abandon assumption of realism, must abandon assumption

of <u>locality</u>

↳ each bit of universe only acts on immediate surroundings

EPR Paradox:



measure A's momentum

NONLOCAL

B's momentum is known by conservation of momentum

ψ can only be described by a product wavefunction

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

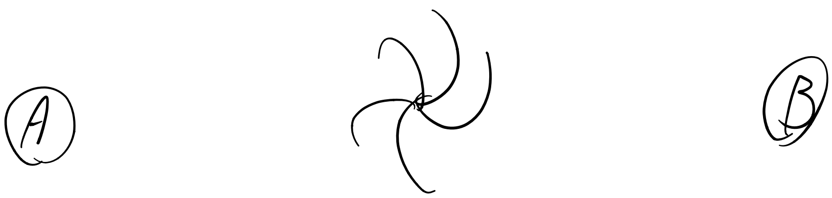
entanglement can violate locality & possibly causality!

↳ rick + morty problem

Bell's theorem:

create $e^- + e^+ \rightarrow$ spin is conserved so

$$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$$



no physical theory of local hidden variables can ever reproduce all predictions of QM

non-locality allows particles to affect each other instantaneously

BUT

entanglement doesn't allow you to send real information FTL

↳ correlations require (local) comparison of results

ONE big exception

Superdeterminism:

There is a way to escape the inference of superluminal speeds and spooky action at a distance. But it involves absolute determinism in the universe, the complete absence of free will. Suppose the world is super-deterministic, with not just inanimate nature running on behind-the-scenes clockwork, but with our behavior, including our belief that we are free to choose to do one experiment rather than another, absolutely predetermined, including the 'decision' by the experimenter to carry out one set of measurements rather than another, the difficulty disappears. There is no need for a faster-than-light signal to tell particle *A* what measurement has been carried out on particle *B*, because the universe, including particle *A*, already 'knows' what that measurement, and its outcome, will be. [5]

Do we have free will?