Topics in Physics: Problem Set #10 (TA version)

Topics: special relativity

Reduced-length problem set

This problem set is designed to take only a portion of the afternoon session to complete. Plan to spend the remaining time in the session working on your final projects.

General TA instructions

- Give students the estimated time for each section, and plan to spend about 50% of the estimated time going through answers.
- Try not to go over time for the earlier sections, since many of the more interesting activities are at the end of the assignment.
- Ask for student volunteers to explain their answers and discuss discrepancies.
- Be stingy with hints, but if a student seems legitimately stuck or won't finish the section in time, try to walk them through how to solve the problem without giving them the answer. You could also have them discuss with any students who have finished the section.

Practice Problems (approx. 30 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. How fast do I have to throw a meterstick so that it appears to be 1 cm long?
   (a) $\gamma = 100$ so $D/\gamma = 1$cm. If $\gamma = 100$ then $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 100$, so $v = 0.99995c$.
2. If I want to live to see the year 3000, how fast must I travel relative to the Earth until then?
   (a) I am 24 and the average life expectancy in the US is 79, so I maybe have 55 years left to live. There are 981 years until the year 3000, so I need my time dilated by a factor of $\gamma = 981/79$. Solving for v, I obtain $v = 0.99675c$.
3. The Large Hadron Collider (LHC) is the world's largest and highest-energy particle accelerator. It is a sequence of connected synchrotrons, with the largest ring having a circumference 27km long. The ring is a pair of evacuated tubes in which particles rotate in opposite directions. At the four detector points, the beams cross and particle collisions can be measured. The particles are collimated into small "bunches" of 1.15×10^{11} protons, each of which has a size of around $1\mu m \times 1\mu m \times 30$cm. The LHC can inject 2556 bunches in the ring and accelerate them to the point that each proton has a total energy of 6.5TeV. The total energy of a fast-moving particle is $E = \gamma mc^2$, and the rest mass-energy of a proton is $m = 938\text{MeV}/c^2 \approx 1.76 \times 10^{-27}$kg.

(a) How fast are the protons traveling?

  i. $\gamma = 6.5 \times 10^{12}\text{eV}/938 \times 10^{6}\text{eV} = 6929.6$, so $v \approx 0.999999989587645c$.

(b) What is the rest mass of all particles in the ring (in kg)?

  i. Total mass is $2556 \times 1.15 \times 10^{11} \times 1.76 \times 10^{-27}\text{kg} = 5.17 \times 10^{-13}\text{kg}$, about $5\times$ the mass of a human red blood cell.

(c) What is the total energy of all particles in the ring? How does this compare to the kinetic energy of moving freight train?

  i. Total energy is $\gamma mc^2 = 6929.6 \times 2556 \times 1.15 \times 10^{11} \times 938\text{MeV} = 300\text{MJ}$. A freight train maybe weighs 10000 tons and travels at 60mph, so $\frac{1}{2}mv^2 = 54\text{MJ}$. The total energy of the particles, which together weigh less than a typical cell, is about 6x greater than a moving freight train!

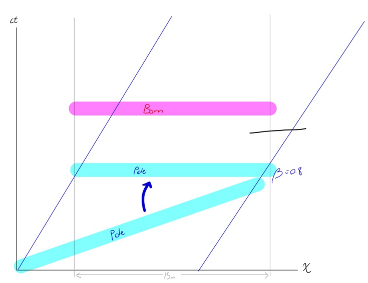
Challenge Problems (approx. 60 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

Problem 1: the pole in the barn

There is a barn 15 m long in the earth's frame. A pole, which is 20 m long in its own rest frame, is being carried toward the barn (by a very fast runner) at speed $\frac{dx}{dt} = 0.8c$ in the earth's frame. Draw a spacetime diagram in the earth's frame and use it to explain and justify your answers.

ANSWER: spacetime diagram looks like this:



(a) How long is the pole in the earth's frame?

ANSWER: If the pole is 20m long in its rest frame and is moving at $\beta = 0.8$, then its contracted length is $\sqrt{1-\beta^2} \cdot 20\text{m} = 12\text{m}$.

The barn door is initially open and immediately after the trailing end of the pole enters the barn, the door is rapidly shut.

(b) How long (in the earth's frame) after the door is shut does the front of the pole hit the other end of the barn? Compute the spacetime interval between the events of the door shutting and the pole hitting the wall. Is it timelike, spacelike, or null?

ANSWER: In the barn's frame, if the back of the pole entering the barn and the door shutting occurs at $(t, x) = (0, 0)$, then the front of the pole is at $(0, 12\text{m})$. Since it is moving at $\beta = 0.8$, the front of the pole hits the barn at $\left(\frac{15\text{m}-12\text{m}}{0.8c}, 15\text{m}\right)$, or $(1\mu\text{s}, 15\text{m})$. The two events are spacelike separated.

(c) In the runner's frame, what is the length of the barn and the pole?

ANSWER: In the runner's frame, the pole is 20m long and the barn is $\sqrt{1-0.8^2} \cdot 15\text{m} = 9\text{m}$ long.

(d) Does the runner believe the pole is entirely inside the barn when its front end hits the end of the barn? Explain.

ANSWER: The runner sees the front of the pole hit the barn, followed by the back door closing some time later. The events are spacelike separated, so they can occur in this order in the runner's frame, in the opposite order in the barn's frame, and simultaneously at some slower-moving intermediate frame.[1]

(e) After the collision, the pole and runner come to rest in the earth's frame. The 20 m pole is now inside a 15 m barn, since the barn door was shut before the pole stopped. How is this possible? From the runner's point of view, the collision should have stopped the pole before the door closed, so the door could not be closed at all. Was or was not the door closed with the pole inside? Explain.

ANSWER: In the runner's frame, the pole does not decelerate simultaneously; the deceleration propagates down the body of the pole at some speed $v_{acc} < c$. In the barn's frame, it would look as though the 12m pole enters the 15m barn, then the door is closed, then the front of the pole hits the back wall, then the pole expands from front to back to a 15m distance. In the runner's frame accounting for shifts in simultaneity but not speed of light "sight" delays, it would seem as though the front of the pole hits the barn, then the pole begins accelerating backwards and contracting from front to back, then the barn door closes once the pole is 15m long. In both frames, some of the kinetic energy of the pole has gone into compressing the pole, and in both frames, the pole has ended stationary in the barn with the door closed.

Problem 2 (BONUS): why Rick and Morty's portal gun breaks physics

<u>**NOTE:**</u> This problem is pretty involved; only attempt it if you are looking for a challenge! (However, you can read over the problem without solving it to understand most of the solution, which will be posted at 6pm.)

Suppose Rick (A) and Morty (B) and are each traveling along their world lines, $x_A^\mu(\tau)$ and $x_B^\mu(\tau)$, respectively, where τ represents proper times as measured by each person, and $x_i^\mu = (t_i, x_i)$,as shown in the first panel of Figure 1.[2]

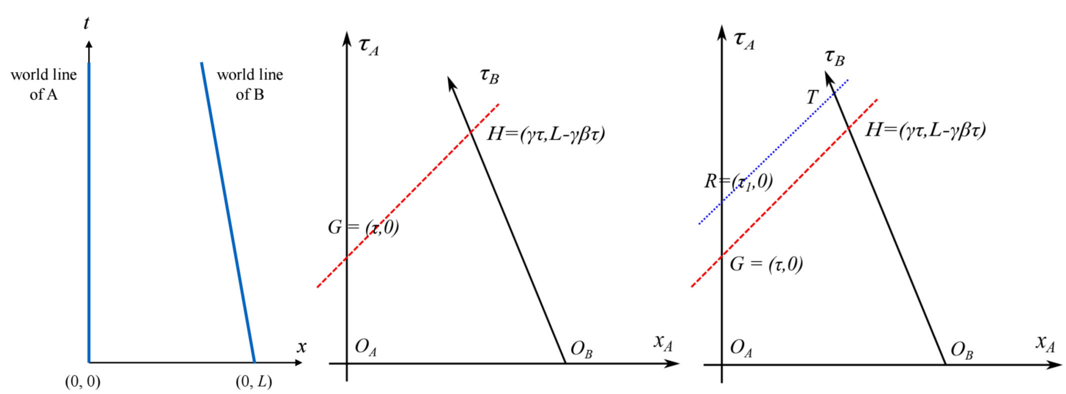


Figure 1: Rick and Morty's world lines.

Consider the frame where Rick is at rest and Morty is moving toward Rick at $\beta = \frac{v}{c}$. At $t = 0$, both people set their clocks to $\tau = 0$; Rick is at $(0, 0)$ and Morty is at $(0, L)$. Since Rick is at rest, $x_A^\mu(\tau) = (\tau, 0)$, and using our Lorentz transformations, we can write Morty's motion in Rick's frame as $x_B^\mu(\tau) = (\gamma\tau, L - \gamma\beta\tau)$.

Now consider a time τ_0 where Rick and Morty have an invariant separation $\Delta s = 0$ which is light-like, such that a light pulse from Rick's point at his time τ_0 will reach Morty's point at Morty's time τ_0, which is $\gamma\tau_0$

[1] The classic version of this problem has the pole moving at a slower speed such that in the barn's frame, the pole hits the wall and the door closes at the same time.

[2] The notation x^μ is a vector with elements indexed by μ which describe the time and space components of the vector. (So Rick's line, $x_A^\mu = (ct, 0)$ describes him moving through time at speed ct and through space at speed 0.) The proper time τ is just a way of saying the time that a clock following that world line would read.

in Rick's frame. This is shown as the red line in the second panel of Figure 1. Then:

$$\Delta s^2 = \Delta t^2 - \Delta x^2 = (\gamma - 1)^2 \tau_0^2 - (L - \gamma\beta\tau_0)^2 = 0.$$

(a) Find an expression for τ_0 in terms of L, γ, and β.

ANSWER: $\tau_0 = \frac{L}{\gamma(1+\beta)-1}$ satisfies that $\Delta s = 0$. (Just algebra to get here.)

Rick and Morty's portal gun allows them to teleport objects instantaneously between $x_A(\tau)$ and $x_B(\tau)$. In other words, the portal gun sends an object from Morty's location at a proper time τ_1 to Rick's location with the same proper time τ_1 (the same value of τ_1, but in Rick's frame).

At time $\tau_1 = \tau_0 + \varepsilon$, Morty teleports an alien to Rick. If Rick receives a Gazorpian, the Gazorpian starts attacking Rick and he sends a light pulse as a distress signal to Morty (shown as the blue line in the third panel of Figure 1). If he receives a Meeseeks, the Meeseeks can defend Rick and he will never send the light signal.

If at any point Morty sees a distress signal from Rick, he teleports a Meeseeks to help Rick out!

Suppose at time τ_1Rick gets a Gazorpian and immediately sends the light pulse (blue) to Morty, shown in Figure 1. Using Lorentz transformations, we can see that the time in Rick's frame where the light pulse reaches Morty is $\tilde{\tau}_A = \tau_1 + \frac{L-\beta\tau_1}{1+\beta}$, and in Morty's frame, the proper time is $\tilde{\tau}_B$.

(b) Find an expression for $\tilde{\tau}_B$ in terms of τ_1, γ, L, and β. Is this before or after τ_1?

ANSWER:$\tilde{\tau}_B = \frac{1}{\gamma}\tilde{\tau}_A = \frac{\tau_1}{\gamma} + \frac{L-\beta\tau_1}{\gamma(1+\beta)}$, which is before τ_1. (We know $\frac{\tau_1}{\gamma} + \frac{L-\beta\tau_1}{\gamma(1+\beta)} < \tau_1$ because, since τ_1 is before Morty reaches $x = 0$, then $L < (\gamma\beta + \gamma - 1)\,\tau_1$.)

So Morty's alien arrives back to Rick **before** he sends the light pulse to him! This poses an obvious paradox:

- At time τ_1, a Gazorpian pops out of Rick's end of the portal and Rick sends a distress signal to Morty.
- Morty sees Rick's distress signal at τ_1 as measured in his frame and teleports a Meeseeks to Rick, who will help him out.
- Morty's Meeseeks arrives at Rick's location at $\tilde{\tau}_B < \tau_1$, so he has someone to defend him when the Gazorpian arrives a short time later at time τ_1.
- But then Rick never sent the distress signal to Morty, so he never received the Meeseeks!