Thermodynamics + Entropy

~~(Star)t with a question:~~

~~Stars have different colors:~~

~~Red ... Orange ... yellow ... white ... blue~~

~~why? ⟶ Temperature~~

~~So why are there no green stars?~~

Thermodynamics (10m)

branch of physics dealing with heat, temperature, and disorder, and their relation to energy, radiation, and properties of matter

thermodynamics developed originally to increase efficiency of steam engines

↳ Sadi Carnot (1824) ⟶ Carnot cycle, carnot efficiency

Steam engines responsible for basically the entire industrial revolution

↳ what is an engine?

Engines (10m)

an <u>engine</u> is a device to convert thermal energy to mechanical energy

Stirling engine on coffee mug

engine moves hot → cold and extracts work in the process

↳ can't just be hot: has to have a temperature gradient

↳ a Dam built on a high lake can only extract energy from Δh

Examples of engines

car
steam engine
power stations
Stirling engine
earth's atmosphere ↗ moves solar heat around ↳ work extractable as hydroelectric, wind, etc.
refrigerators (reverse)

Ideal gas law (10m)

$PV = nRT$ ideal gas constant

$R \approx 8.314 \frac{J}{K \cdot mol}$

$R = k_B N_A$

Boltzmann constant

$k_B \approx 1.38E{-23} \frac{J}{K}$

relates KE to T:

$KE \propto k_B T$

derived empirically at first

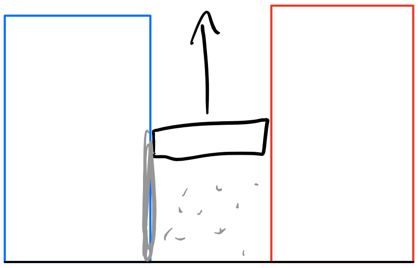
$$PV = N k_B T$$

Do water jug demo

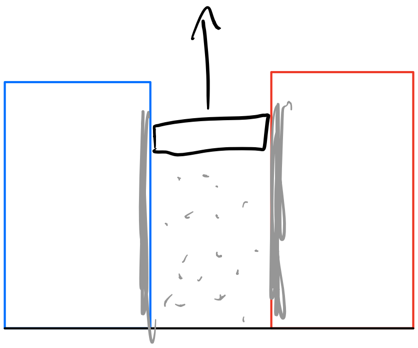
Expansion / contraction of ideal gases is what powers ideal engine ... carnot cycle

Carnot cycle (13m)

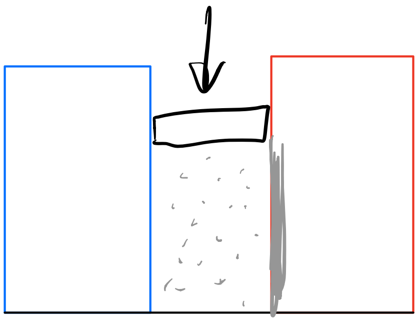
engines generally operate in 4 stages:

1. 

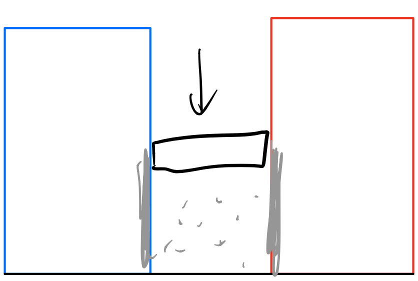
expansion of gas (isothermal)
↳ piston goes up, does work

2. 

continued expansion of gas (isentropic)
↳ piston goes up until pressure balanced, does work, gas cools

3. 

contraction of gas (isothermal)
↳ push piston down at same rate that gas cools, keeping T constant
↳ environment does work on engine!

4. 

compression of gas (isentropic)
↳ piston pushes down until pressure balanced, receives work, heats up

What is max efficiency of an engine?

↳ Carnot efficiency $\eta_{max} = 1 - \frac{T_C}{T_H}$

why? need to understand more thermodynamics first...

Laws of Thermodynamics (10m)

Law 0: if two systems are each in thermal equilibrium with a third, they are in thermal equilibrium with each other

no perpetual motion machines!

Law 1: energy is conserved

$\Delta U = Q - W$

Q: heat supplied to system

W: work done by system

↳ $\Delta U_{\text{full cycle}} = 0$

Law 2: in any thermodynamic process, the total entropy of all interacting systems never decreases

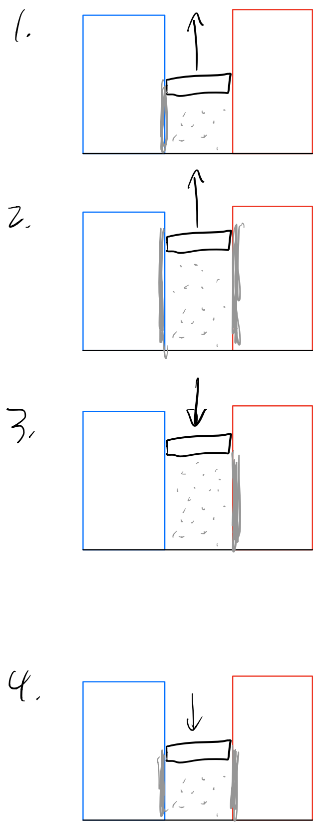
talk in a moment

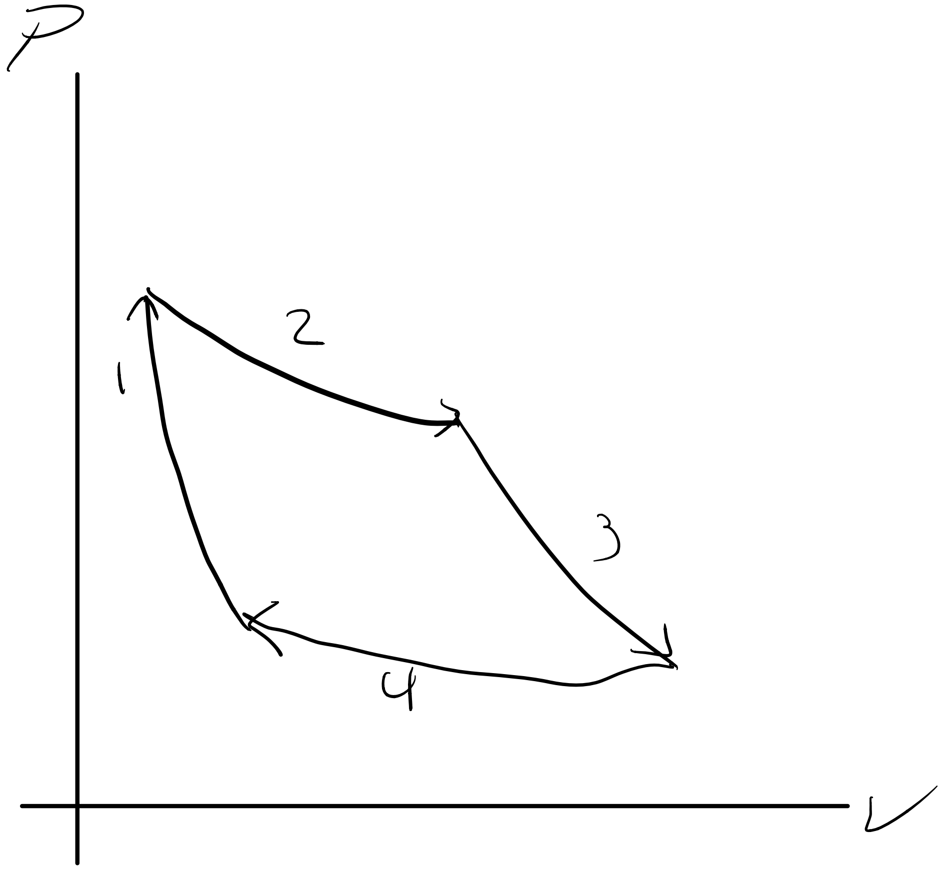
$$\sum_i \Delta S_i \geq 0$$

"systems evolve from less probable states to more probable states"

Law 3: the entropy of a perfect crystal at absolute zero is $S = 0$

Law 1: $\Delta U = Q - W$ (10m)

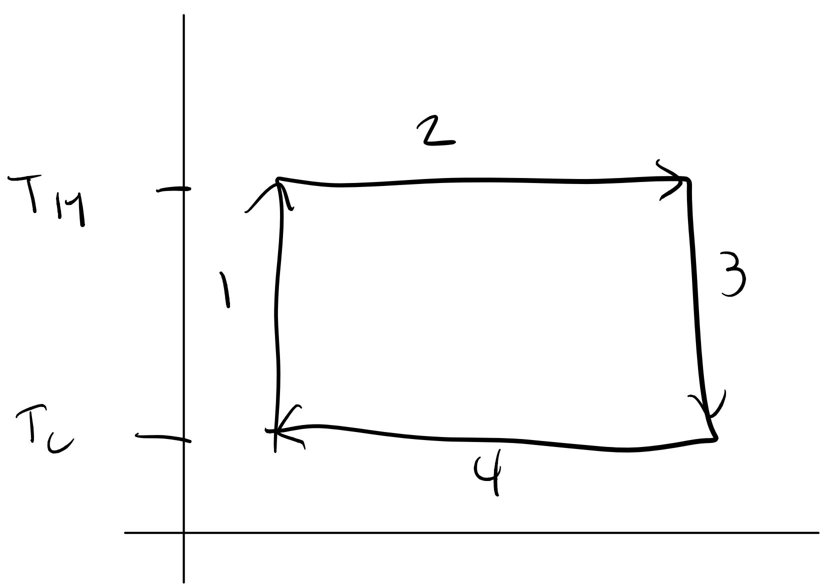




Units of PV? $\frac{N}{m^2} m^3 = Nm = J$

Can also plot this on TS $\rightarrow$ Q

$\rightarrow$ S, entropy, units J/K



units of TS? energy!

Fundamental thermodynamic relation

$$\Delta U = Q - W$$

$$\Downarrow$$

$$dU = T\,dS - P\,dV$$

What is entropy?

→ Cosmology: heat death of universe

A history (10m)

1854

Rudolf Clausius: "the internal property which changes as heat moves around in a system"

$$\Delta S = \frac{\Delta Q}{T} \rightarrow \frac{\text{heat moving into/out of reservoir}}{\text{temperature of reservoir}}$$

- for a Carnot cycle, $\Delta S = 0$
- anything less efficient, $\Delta S > 0$
- an increase in temperature means heat reservoirs
- approaching same temperature, reducing capacity to do useful work
- measure of how evenly spread out energy is

1877

Ludwig Boltzmann: Statistical mechanics

↳ Kinetic theory of gases
explained thermodynamic behavior as result of microscopic particles under Newton's laws of motion

Statistical mechanics (5m)

Fundamental postulate:

"For a system with a given set of macroscopic properties, every possible configuration of particles that could yield these properties is equally likely"

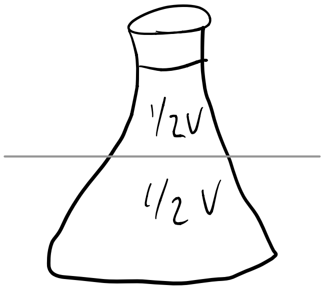
macrostate:
Temp, pressure, volume, etc..

microstate:
for each particle,
position, velocity, etc

Dye in flask demo

~80min in?

Microstates:



N particles of dye
can be on either top or bottom of flask
2^N possible microstates

(20m)

microstate	# microstates, $N=10$	# microstates, $N=1000$
(all possible states)	$2^{10} = 1024$	$2^{1000} \approx 1.07 \times 10^{300}$
"all dye in top half"	1 low entropy	1 low
"50% of dye in each half"	$\binom{10}{5} = 252$ high entropy	$\binom{1000}{500} \approx 2.70 \times 10^{299}$ high

larger N = less likely to end up in low entropy arrangement

Chance of having a room full of 10^{26} particles of air on one side? $\frac{1}{2^{10^{26}}} \approx 0$

if you have a system alone, states will randomly evolve, but vast majority of states are at classical values we expect:

<u>thermal equilibrium</u>

2^{nd} law: "systems evolve from less probable states to more probable states"

classical mechanics is just carryover of quantum mechanical behavior as $N \to$ big

Relation between entropy & macroscopic properties?

Boltzmann equation

$$S = k_B \log \Omega$$

entropy

Boltzmann constant

microstates

Thermodynamic equilibrium: $\max S = \max \Omega$

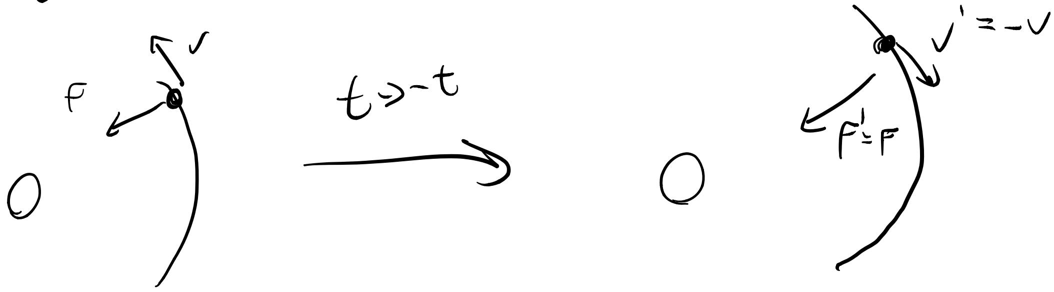
Arrow of time

(10min)

Most laws of physics don't care about direction of time

$t \to -t$ leave behavior unchanged

ex)



How do we distinguish past from future?

ENTROPY

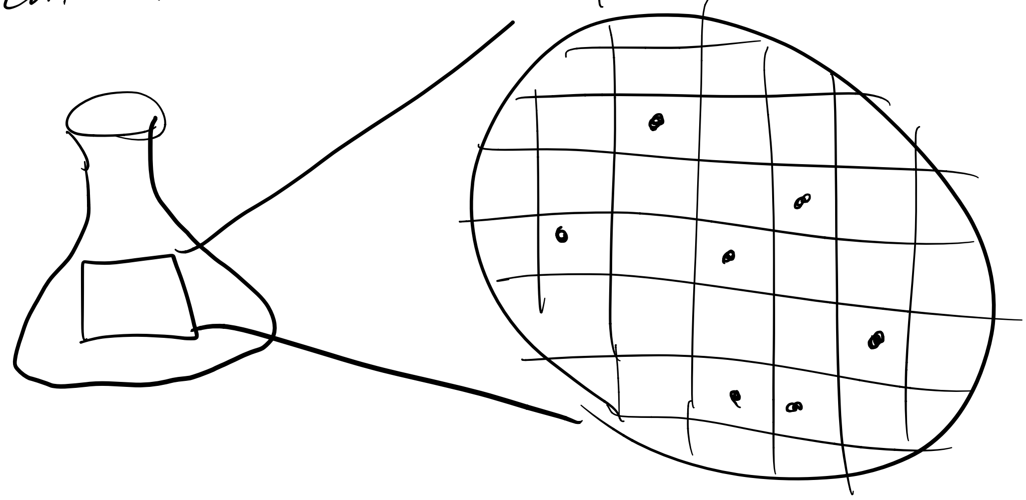
The only* quantity in physics which requires a direction of time entropy is a measure of ignorance

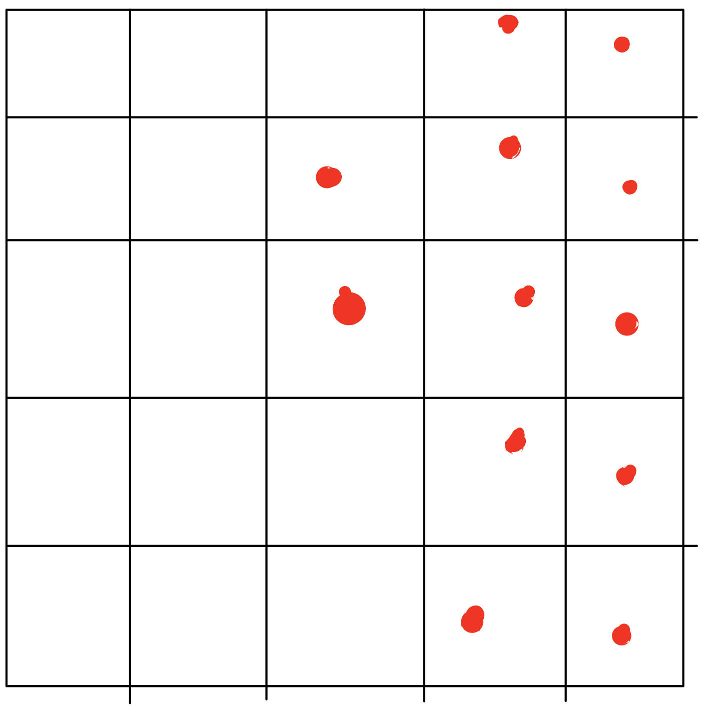
measures how much information we don't have about a system

(10 min)

Another analogy

Let's divide flask-ink system into "pixels" which can either have a particle of dye or not





5x5 grid = 25 pixels in space

Suppose we have 12 dye particles

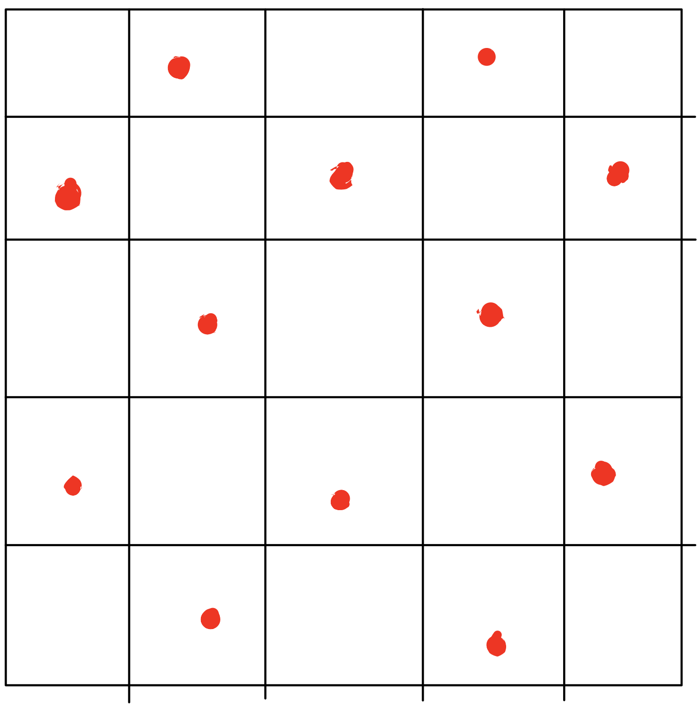
There are $\binom{25}{12} = 5\,200\,300$ possible states

Suppose all particles is right half of grid

There are $\binom{5}{2} = 10$ possible states

↳ close to minimum entropy

What about this config?



Low or high entropy?

might think, it's low entropy because $\Omega = 1$

but particles are evenly spread out, so how do you extract energy?

Thermodynamic entropy is related to amount of hidden information based on macroscopic thermodynamic knowledge only!

↳ Defines how far a system is from thermal equilibrium & also

↳ defines how much energy can be extracted as you move to equilibrium

$$\Delta S = \frac{\Delta Q}{T} \rightarrow \Delta Q = T\Delta S$$

Thermodynamic entropy is low if there are differences in average thermodynamic properties from one region to another

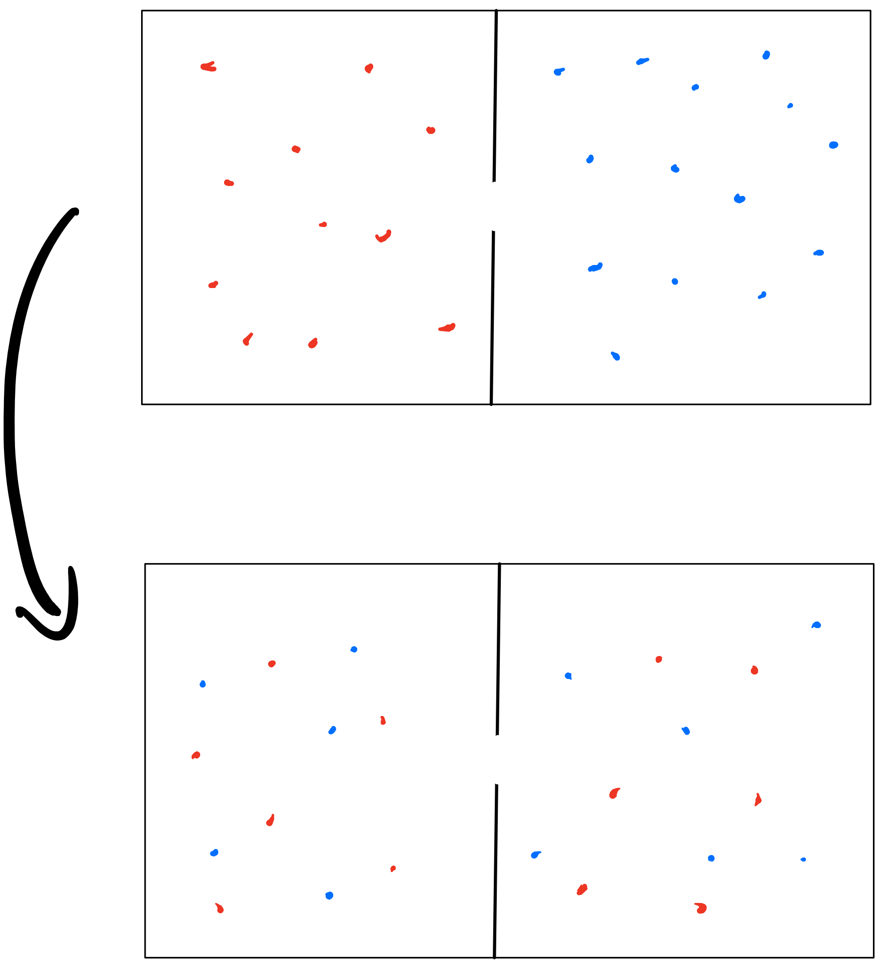
But Ω is still 1, so S should be low? What gives?

High entropy states can be transformed to low entropy states if you gain information about the system

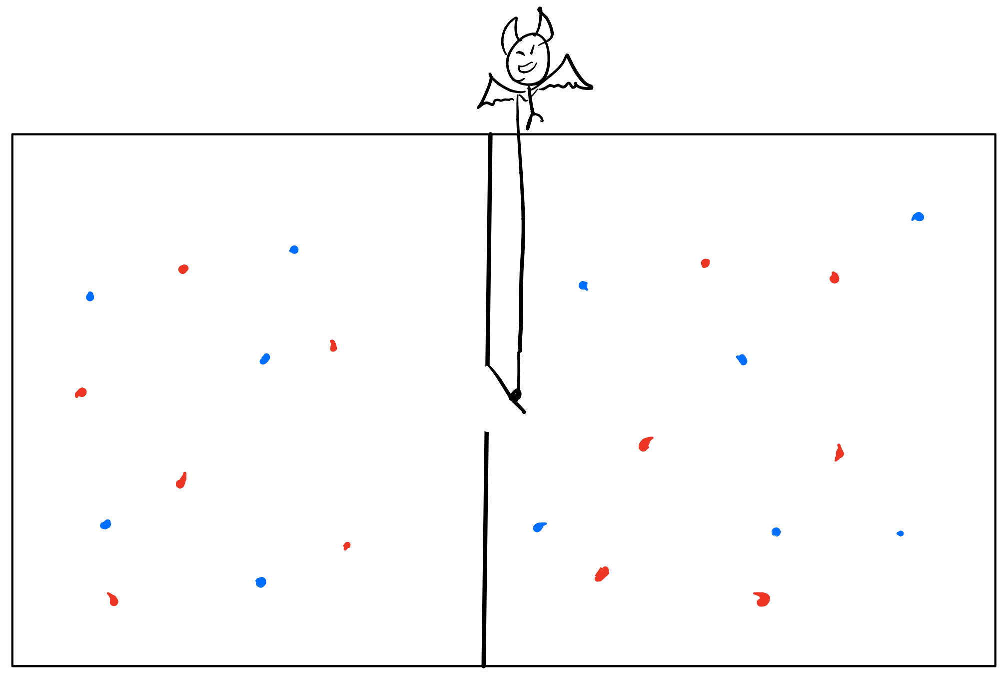
~120 min Break

Maxwell's demon (15 min)

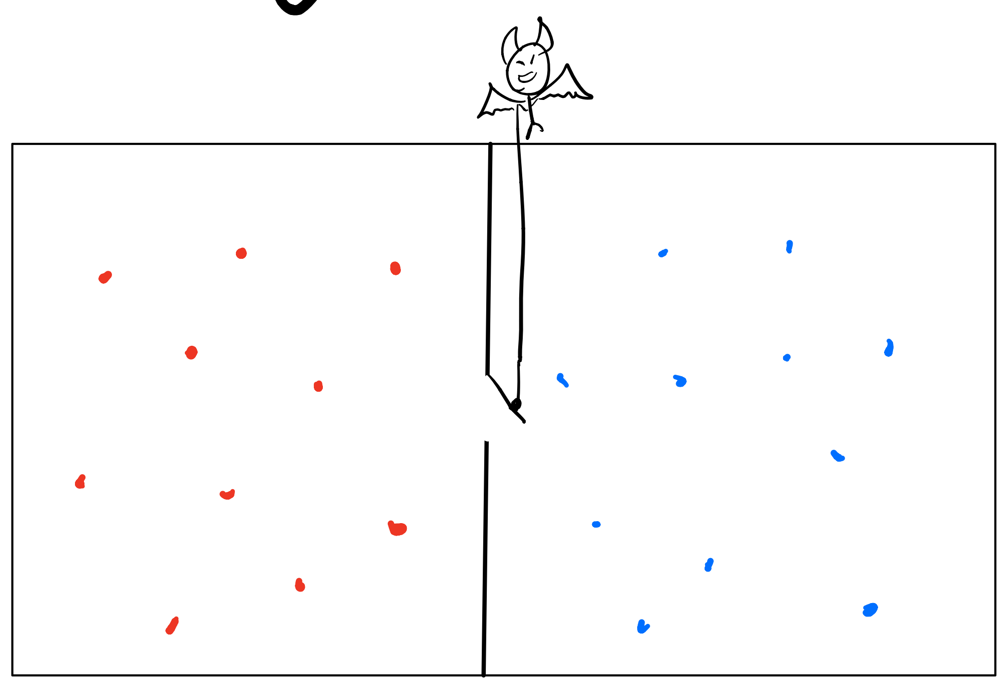
James Clerk Maxwell → maxwell's equations
→ cofounder of Stat mech



Maxwell experiment



Eventually ...



Problem 1: $\Delta S < 0$ and no energy was exchanged with outside universe

Problem 2: Left side is hot, right side is cold!

This temp. diff. can be used to drive an cycle, violating energy conservation!

Seen as a serious problem in physics for over a century!

What is the resolution? we're missing entropy somewhere...

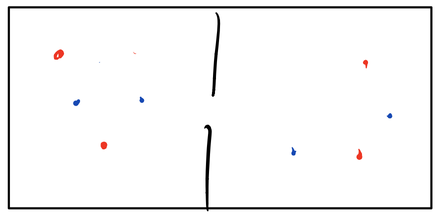
entropy is transferred into the memory of the demon!

Suppose demon $\rightarrow$ computer. There are (15m)

N particles and computer has N bits of memory

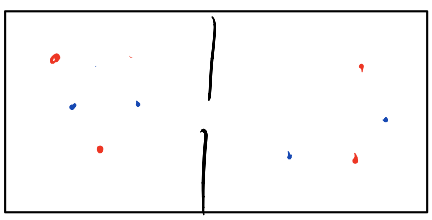
every time computer sees a particle, has to decide to open door (1) or leave closed (0)

memory starts in known, predictable state, altered by interaction with a particle into an unknown state



0	0	0	0	0	0	0	0

Initial



$\Omega = \binom{4}{2}\binom{4}{2} = 36$

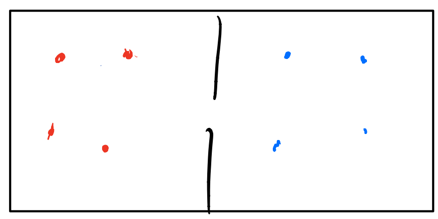
$S \approx 5.17$

0	0	0	0	0	0	0	0

$\Omega = 1$

$S = \log 1 = 0$

Final



$\Omega = 1$

$S = 0$

0	1	1	1	0	1	0	0

$\Omega = \binom{8}{4} = 70$

$S \approx 6.13$

$$\Delta S > 0!$$

<u>information entropy</u> is physical!

This solves problem #1

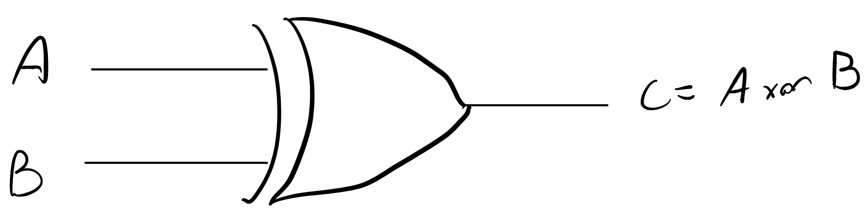
daemon processes named for this

What about problem #2?

Demon has finite memory & eventually must overwrite (erase) information.

How much energy to erase 1 bit? (15 min)

consider a XOR gate



A	B	C
0	0	0
1	0	1
0	1	1
1	1	0

This erases a bit of information!

$(A, B) \to (C)$

Ω for A, B: $2^2 = 4 \to S = \log 4 = 2 \log 2$

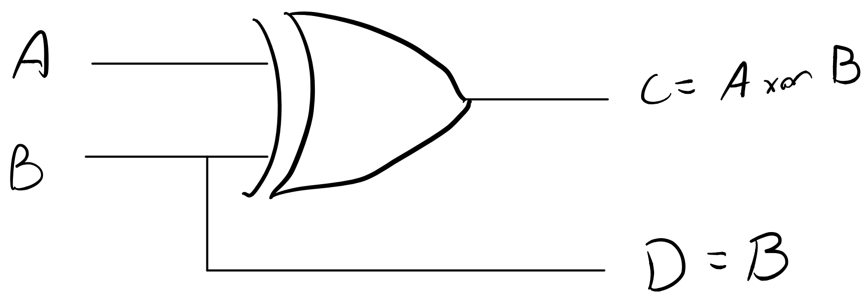
Ω for C: $2^1 = 2 \to S = \log 2$

$\Delta S = \log 2$, so

$$\Delta E = k_B T \log 2$$

Landauer limit!

Reversible computing (10 min)



Can reconstruct inputs from outputs
↳ no information lost!

Reversible computers require no thermal energy cost!

To summarize: information ⟷ energy

entropy

perfect knowledge of every particle in the box allows you to turn information into work or into structure.