Topics in Physics: Problem Set #1 (TA version)

Topics: physical units, mathematical models, estimation, vectors

General TA instructions

- Give students the estimated time for each section, and plan to spend about 50% of the estimated time going through answers.
- Try not to go over time for the earlier sections, since many of the more interesting activities are at the end of the assignment.
- Ask for student volunteers to explain their answers and discuss discrepancies.
- Be stingy with hints, but if a student seems legitimately stuck or won't finish the section in time, try to walk them through how to solve the problem without giving them the answer. You could also have them discuss with any students who have finished the section.

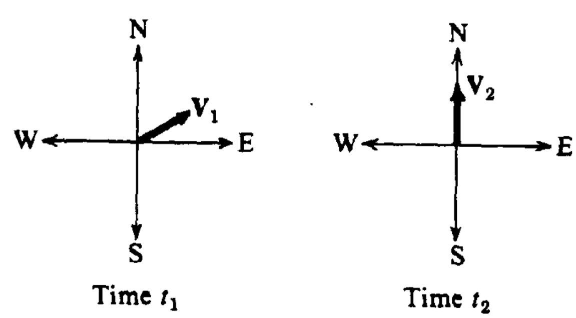
Practice Problems (approx. 40 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. The world's top 100 circuses team up in the most stunning acrobatic feat in history: all of the performers stand on each others shoulders, making a human tower 1 person wide! How tall is the tower? How much weight is the bottom person carrying?

   (a) Each circus has, say, 30 performers. If they are all about 2 meters tall, then the tower is $30 \times 2\text{m} \times 100 = 6\text{km}$. If each person weights 70kg, the bottom person is carrying 210,000 kg.

2. Vectors $\vec{v_1}$ and $\vec{v_2}$ shown below have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. Draw the direction of the average acceleration vector (ignoring overall magnitude).



(a) Vector should be pointing northwest and should be attached to the origin, not the tip of v_1 or v_2.

3. A ball is thrown with an initial velocity of 70 m/s, at an angle of 35°from horizontal. Find the vertical and horizontal components of the velocity.

   (a) $\vec{v} = 70 \cdot \cos(35\,\text{deg})\,\hat{x} + 70 \cdot \sin(35\,\text{deg})\,\hat{y} \approx 57.34\hat{x} + 40.15\hat{y}$.

4. Approximately how many meters per year does a snail travel on average? How many hydrogen atom diameters per second?

   (a) A snail might travel 1mm per second (10mm per second is also reasonable). That's 31.5 km per year or 10 million hydrogen atom radii (0.1nm) per second.

5. A bicycle tire with radius 0.4m rolls along the ground through three quarters of a revolution. Consider the point on the tire that was originally touching the ground. What is its displacement from its starting position?

   (a) The horizontal displacement is 3/4 of the circumference plus 1 radius, since the point went from 6 o'clock to 3 o'clock. The vertical displacement is one radius. Thus, the displacement vector is $\left(\frac{3}{4} \cdot 2\pi r + r\right)\hat{x} + r\hat{y} = 2.28\hat{x} + 0.4\hat{y}$. This has a magnitude of 2.31 meters, which is fine as an answer, but you should remind students that displacement is a vector, not a scalar.

6. A person is swimming north across a river with velocity $\vec{v}_{swim} = (0\hat{x} + 1\hat{y})\,\text{m/s}$. The river is flowing east with velocity $\vec{v}_{river} = (0.5\hat{x} + 0\hat{y})\,\text{m/s}$.

   (a) What speed would the person be swimming at if they were in still water?

      i. In still water, you add the vectors so $\vec{v} = (-.5\hat{x} + 1\hat{y})$, which has magnitude of 1.11m/s.

   (b) What angle is the person swimming at relative to the flow of the river? (Hint: recall $\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos\theta$)

      i. The person is swimming at $\vec{v} = (-.5\hat{x} + 1\hat{y})$ relative to the river's flow of $\vec{v}_{river}\,(0.5\hat{x} + 0\hat{y})$. So $\vec{v} \cdot \vec{v}_{river} = (-.5, 1) \cdot (0.5, 0) = -0.25$. Since $\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos\theta$, then $\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\left\|\vec{b}\right\|}\right) = \cos^{-1}\left(\frac{-0.25}{\sqrt{.5^2+1^2} \cdot \sqrt{.5^2}}\right) = 2.03\text{rad} = 116°$.

7. As we discussed (briefly!) in class, the cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined by

$$\vec{a} \times \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \sin(\theta)\,\hat{n},$$

   where $\hat{n}$ is a vector given by the right-hand rule.

   (a) Let $\hat{x}$, $\hat{y}$, and $\hat{z}$ be the standard 3D Cartesian basis vectors. What is:

      i. $\hat{x} \times \hat{y} = \hat{z}$
      ii. $\hat{x} \times -\hat{y} = -\hat{z}$
      iii. $-\hat{x} \times \hat{y} = -\hat{z}$
      iv. $\hat{y} \times \hat{z} = \hat{x}$
      v. $\hat{z} \times \hat{x} = \hat{y}$
      vi. $\hat{x} \times \hat{x} = \vec{0} = (0, 0, 0)$ (explain this if they don't understand)

   (b) Find the cross product of $(1, 1, 0) \times (-2, 2, 0)$.

      i. $(0, 0, 4)$. I didn't have time to go over determinant method of finding 3D cross products, but they know the magnitude should be $\sqrt{2} \cdot \sqrt{8}$, and if they draw the vectors they should see that they are perpendicular, so they know $\|\vec{a}\| \left\|\vec{b}\right\| \sin(\theta) = \sqrt{2} \cdot \sqrt{8} \cdot \sin(90°) = 4$. Use the right hand rule to get the direction as $+\hat{z}$ and you get $(0, 0, 4)$.

   (c) Find the cross product of $(1, 1, 0) \times (-2, -2, 0)$.

i. The vectors are scalar multiples of each other so they have $\theta = 180°$, so $\sin\theta = 0$, so the result is $(0, 0, 0)$.

8. Earth has a mass of m, and its displacement from the sun can be written as a vector $\vec{r}$, and its velocity around its orbit can be written as a vector $\vec{v}$. Let $\vec{L} = m(\vec{r} \times \vec{v})$. What direction is $\vec{L}$ pointed in? What is the magnitude $\|\vec{L}\|$? What are its units?[1]

   (a) $\vec{L}$ is pointed upward out of the plane of the solar system. Putting `earth mass*earth orbital radius*earth velo` into WolframAlpha gives $2.66 \times 10^{40} \mathrm{J} \cdot \mathrm{s}$. This is units of Joule-seconds, or angular momentum.

Challenge Problems (approx. 20 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

Problem 1: Conical reservoir

1. Suppose Stanford University decides to build a large water reservoir in the shape of an inverted cone. The cone's base (which is the surface of the reservoir) has a radius of 30 meters and the depth in the center is 5 meters. The lead engineer wants to model the rate of water evaporation, which will depend somehow on the amount of water in the reservoir. Develop a model for the rate of evaporation, R, which depends only on the depth, z, of the water. After building the reservoir, the engineer adds $z = 3$m of water and measures the rate of evaporation to be 100 liters per hour. According to your model, what will be the rate of evaporation when the reservoir is full?

   (a) The evaporation rate will depend on the surface area of the water, $S = \pi r^2$ and $r = \frac{300}{50} z = 6z$. So $R = kS = 36k\pi z^2$ where k is an unknown constant. Since the engineer measures $R = 1$ liter per hour when $z = 3$m, $k = 0.098$ (in units of liters per hour per square meter, which is length $\times$ time^{-1}). So when $z = 5$, the evaporation rate will be 277 liters per hour.

Physics "Tech Tree" (approx. 45 min)

Team up into groups of 3-5 people and make a map of physics equations! Your goal is to incrementally build a "tech tree" which directly or indirectly relates as many physics equations to base SI units as possible. By building up formulae for various physical quantities, you can "unlock" increasingly complex equations. The rules are as follows:

1. Each team gets a sheet of large easel paper (you can get another if you mess up), a set of markers, and a bunch of sticky notes.

2. Look up the figure of the new SI units that was discussed in lecture (it's the last figure on https://en.wikipedia.org/wiki/S Write the seven base units and the quantities they measure in marker around the edge of the paper and include the constants that are used to define them. You will fill in the center of the paper, so give yourself space to work!

3. For each sticky note, write down a physics equation and what it means. You don't need to know th equations by heart; you can look them up on Google or Wikipedia, and physics.info/equations is also a helpful listing of many common formulas. You will relate the equations to the SI base units,

[1]The quantity $\vec{L} = m(\vec{r} \times \vec{v})$ is called the *angular momentum* of an object. We'll cover this in a few days when we do angular dynamics.

and you can build definitions from other definitions. At the end of the activity you will draw marker lines connecting all the concepts, but save this for the last few minutes so you can reposition notes as needed.

(a) Include a short verbal description on the sticky note for each equation, (e.g. "$E = mc^2$: rest energy is mass times speed of light squared"), and include the name of the equation if it has one (e.g. "Coulomb's law").

(b) If the equation depends on another physics concept which isn't one of the base units, you need to define that concept too.

   i. For example, acceleration is $a = \frac{dv}{dt}$, which depends on velocity. To put down acceleration, you also need to define velocity, which is $v = \frac{dx}{dt}$. Since x (distance) and t (time) are both base units, you can stop there.

(c) If the equation depends on a constant that is already on the paper, you should connect it to that constant, along with any other units in the formula. For example, $E = mc^2$ would be connected to mass (kg) and to c.

(d) If the equation depends on a constant that is not already on the paper, write it along the edge of the paper with the other constants. Include the symbol, the name of the constant, and the base units of the constant. (You can type any constant into WolframAlpha to get a basic unit decomposition.) You won't need to draw lines connect constants to existing units.

   i. For example, if you put down Newton's law of gravity, $F_G = -G\frac{m_1 m_2}{r^2}$, it would be connected to mass, length, and your definition of G. To define G, draw a circle somewhere near mass and length, and fill out "G: gravitational constant, mass^{-1}length^{3}time^{-2}".

4. Try to get as many equations as you can! For bonus points, try to use each base unit at least once. The TAs will let you know when time is almost up; once they do, figure out where you want to place all the notes and draw lines connecting them to each other and to the base units and constants.

An example map is shown below. You've got about 45 minutes to work on your map, so you can probably get more equations than I did, but use this example to organize yours. When you're done, spend a few minutes admiring other groups' maps! Which equations did you have in common? Which ones are unique to a team?

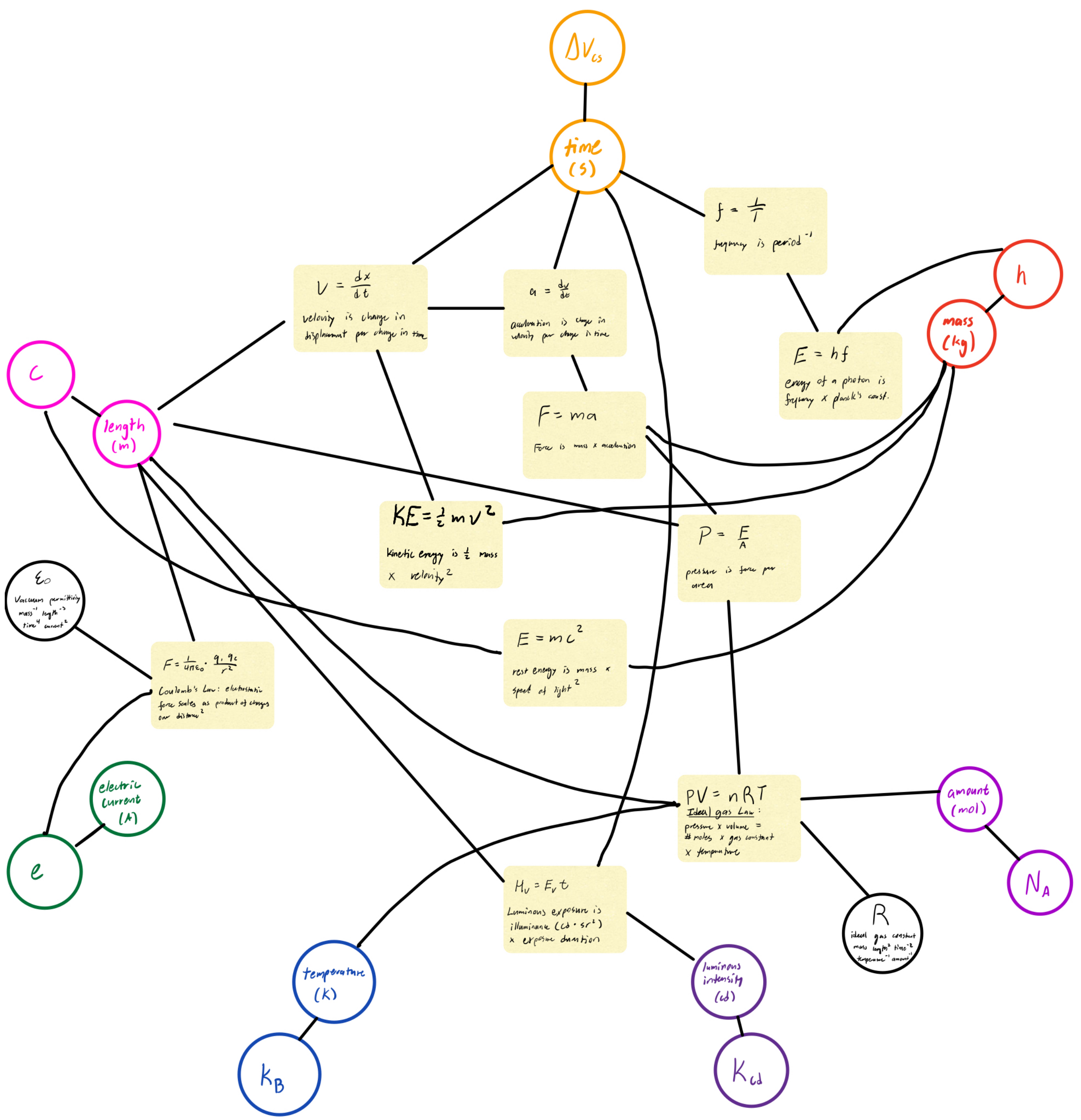


Figure 1: Example physics "tech tree".

The Fermi Question Contest (approx. 80 min)

Form teams of 3-4 students and come up with a team name! There will be 5 rounds (each lasting about 15 minutes), which proceed as follows:

1. The question is read aloud (or written where everyone can see), and the teams have 5 minutes to discuss/solve it (set a timer!). At the end of the 5 minute period, each team should submit a single piece of paper with a circled answer. Include units in your answer if there are any. The TAs will give a 30 second warning before the time expires, then collect the papers.

2. Each team may now ask for one piece of information from the TAs. They will send a representative to privately ask the question and receive an answer. The information is not shared between the teams. The TAs may have to look up the answers online.
   (a) The TAs have the prerogative to give partial answers to questions which aim to skip over multiple steps. For example, rather than answering "what is the volume of the atmosphere", the TA might just give the altitude of the edge of the atmosphere.
3. Each team then has 3 more minutes resubmit their answer. It doesn't necessarily have to change. Again, they should write their answer on a sheet of paper which will be collected after 3 minutes (give a 10 second warning).
4. The TAs will then reveal the solution and award points. The teams score 10 points minus 2 points for every order of magnitude of error on their first guess. Their second answer is worth 5 points minus 2 for every order of magnitude error.
   (a) Example: If the correct answer is 1.5×10^6 and the student answers first 2×10^5 and then 1×10^6, they get full credit (15 points). If their first answer is 1×10^4, and their second is 1×10^5, they get $10 - 4 + 5 - 2 = 9$ points.
5. Take about 5 minutes (as needed) to discuss the solution. Ask for the students' solutions first, then give my versions if they are different.

Questions + answers

Use the answers on this sheet as a guideline. If students arrive at a different but reasonable answer, you can award points at your discretion.

NOTE: I've adjusted the time for the sections on this problem set, so you might not finish all of the questions; that's fine.

1. How much money is in a briefcase stuffed full of hundred dollar bills?
   (a) Answer: \$2 million
   (b) Info:
      i. bill thickness: 0.1mm = 0.004 inch
      ii. bill area: 2 inches $\times$ 6 inches
      iii. briefcase dimensions: 1.5 feet $\times$ 1 foot $\times$ 6 inches
      iv. So briefcase volume/bill volume $\approx$ 20,000 total = 20,000 $\times$ \$100 = \$2 million
2. Saturn's rings weigh about 3×10^{19}kg. Assume they are made entirely of ice particles (density: 1g/cm^3) which are spread out so the rings are 97% empty space. Saturn's diameter is 100 million kilometers. How thick are the rings?
   (a) Answer: 20m
   (b) Info:
      i. Ring inner edge: 1.5×10^8km
      ii. Ring outer edge: 2×10^8km
      iii. Volume of rings is mass / density / %filled space
         A. (Not a hint) $V = 3 \times 10^{19}\text{kg} \cdot \left(1\text{g/cm}^3\right)^{-1} \times (1 - 0.97)^{-1} \approx 10^{18}\text{m}^3$
      iv. Area $= \pi r_{out}^2 - \pi r_{in}^2 = 5 \times 10^{16} m^2$
      v. thickness is volume/area = 20m

3. If a gallon (3.8 liters) of paint were used to make a 1 atom-thick line all along the equator of the earth (pretend it's possible to paint the ocean), how wide would the line be? (Hint: the United States is about 3000 miles across and has 3 time zones.)

   (a) Answer: 1 meter

   (b) Info:

      i. 1L = $0.001m^3$
      ii. radius of earth: 6000km
      iii. (Not a hint) circumference of earth: $2\pi r \approx 40000$km
      iv. diameter of an atom: ~0.1nm
      v. So $3.8L \times 0.001m^3/L \times \frac{1}{10^{-10}m} \times \frac{1}{4\times10^7m} \approx 1m$. This is actually a bit of a trick question, since paint color is often due to molecules which are much larger than the size of an atom.

4. Democritus, a greek philosopher living around 400BC, postulated that the universe was made out of indivisible units he called "atoms". About how many lungfuls of air must we breath before we are likely to have inhaled at least one atom that was present in Democritus' lungs when he took his last breath? Hint: one mole of air (6×10^{23} atoms) has a volume of 22.4 liters.

   (a) Answer: 0.01 lungfuls

   (b) Info:

      i. column height of atmosphere (the height that gives the volume of the atmosphere if it had uniform vertical density): 20km
      ii. Radius of earth: 6000km
      iii. surface area of earth:$4\pi \times (6000km)^2 = 5 \times 10^{14}m^2$
      iv. (Not a hint) Volume of atmosphere: $S \times h \approx 1 \times 10^{19}m^3$
      v. volume of lungs: 6L (1liter=1000mL=1000cm^3 =.001m^3)
      vi. atoms per liter: $6 \times 10^{23}/22.4$
      vii. (Not a hint) lungfuls of air in atmosphere: (Volume of atmosphere/lungful air) =10^{21}
      viii. atoms of air per lungful: (22.4 L/mol) / (lungful air)=10^{23}
      ix. So there are about 10^{23}atoms from Democritus' last breath in the atmosphere. We can assume they are evenly distributed among the 10^{21} lungfuls of air in the atmosphere. So the number of atoms from Democritus' last breath in one lungful of air is $10^{23}/10^{21} = 100$. So the answer is 0.01 lungfuls! Most people find this result surprising. The key insight is that there are many more atoms in a lungful of air than lungfuls of air in the atmosphere.

5. The world power consumption is currently 17 terawatts. Suppose Stanford wanted to research alternative energy sources and decided to launch a massive disk-shaped satellite covered in solar panels to orbit the sun and beam back energy to earth. The satellite will have an orbital radius half that of Earth's. What would the diameter of the satellite need to have? [Hint: modern solar panels have an efficiency of about 25%, and assume beaming energy back is 100% efficient]

   (a) Answer: ~1000kg

   (b) Info:

      i. Solar power density on earth: $1kW/m^2$
      ii. (Not hint) Solar power density at half earth's radius: $1kW/m^2/(1/2)^2 = 4kW/m^2$
      iii. (Not hint) Solar power absorbed in orbit: 25
      iv. (Not hint) Surface area needed: $17 \times 10^{12}W/(1 \times 10^3W/m^2) = 17 \times 10^9m^2$
      v. So the radius of the satellite is $\pi r^2 = S \to r = \sqrt{S/\pi} \approx 73.5$km.