Topics in Physics: Problem Set #8 (TA version)

Topics: thermodynamics, entropy

Half-length problem set

This problem set is designed to take about half of the afternoon session to complete. Plan to spend the remaining time in the session working on your final projects.

General TA instructions

- Give students the estimated time for each section, and plan to spend about 50% of the estimated time going through answers.
- Try not to go over time for the earlier sections, since many of the more interesting activities are at the end of the assignment.
- Ask for student volunteers to explain their answers and discuss discrepancies.
- Be stingy with hints, but if a student seems legitimately stuck or won't finish the section in time, try to walk them through how to solve the problem without giving them the answer. You could also have them discuss with any students who have finished the section.

Challenge Problems (approx. 90 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

1. Macrostate 1 could be any of Ω microstates. Macrostate 2 could be any of 2Ω microstates. Macrostate 3 could be any of Ω^2 microstates. Ω is a *very* large number. Compare the entropies of each macrostate to each other macrostate.
   (a) You could do this by plugging in numbers. Or you can use a handy formula for logarithms: $\log(2\Omega) = \log\Omega + \log 2$. If Ω is very big, the $\log 2$ part is negligible. So $S_2 = \log(2\Omega) \approx \log\Omega = S_1$. Using $\log\left(\Omega^2\right) = 2\log\Omega$, we get that $S_3 = 2S_1$. Conclusion: doubling the number of micro states doesn't make much of a difference. But squaring the number does!
2. A pressure cooker slowly heats a constant volume of air. If the air pressure was 1 atmosphere when the pressure cooker was first turned on (at room temperature) and is now 1.5 atmospheres, how hot is the pressure cooker? (use the ideal gas law!)

(a) If P increases while V stays constant, T must increase proportionally. So T=298 K$\rightarrow$ 1.5$\times$298 K=447 K=174 degrees Celsius

3. Look up the performance of your computer's CPU online to get an estimate of the number of FLOPS (floating point operations per second) it can perform. If you can't find a direct statistic for FLOPS, you can estimate the number by [number of cores $\times$ clock speed of CPU (cycles / sec) $\times$ FLOPs / cycle]. (Typically this last value is either 8 or 16; you can probably find it on the Wikipedia page for FLOPS.) If your CPU uses double-precision (DP) FLOPS, there are 64 bits per floating point operation. Calculate the effective information processing speed of your CPU in bits/sec. If you assume that all information the CPU computes is quickly overwritten (this is a good assumption as most of it is stored in volatile L0-L3 cache memory), what is the theoretical minimum power cost to run your CPU at max capacity at room temperature? Look up the power consumption of your CPU. (This page may be helpful: `https://en.wikipedia.org/wiki/List_of_CPU_power_dissipation_figures`) How far from optimal efficiency is it? (Note: if you get stuck trying to find statistics for your CPU, don't worry too much about it. You can pick a common CPU like a core i7 and just use stats for that. This is an order of magnitude calculation, so specific numbers don't matter too much.)

   (a) For my MacBook Pro (Intel Core i7, 8 cores, 2.9GHz, 16 DP FLOPS/cycle), information processing rate is $8 \cdot 2.9 \times 10^9 \cdot 16 \cdot 64 = 2.2 \times 10^{13}$ bits/sec. Running at room temperature (300K), minimum power requirement is $k_B T \log 2 \times 2.2 \times 10^{13} = 6.3 \times 10^{-8}$Watts. Thermal power dissipation for my CPU is 37W, so it uses about 5.8×10^8 times as much energy as the theoretical minimum.

4. On the wikipedia page for "Atmosphere of Jupiter", you'll find a plot which shows the pressure and temperature of Jupiter's atmosphere at various altitudes relative to the clouds. Use the ideal gas law to estimate the volume of 1 mole of gas at 100km, 320km, 50km, and -132km.

   (a) At 50km, for example, $P = 10^4$ Pa and $T = 110$K, so $V = Nk_BT/P = 91$ liters. At -132 km, $P = 2 \times 10^5$ and $T = 410$K, so $V = Nk_BT/P = 17$ liters

5. If the exhaust gas from a car engine has a temperature of 500 degrees Celsius and the temperature of the gas just after combustion is 1500 degrees Celsius, what is the maximum theoretical efficiency of the engine? If the car gets 30 miles per gallon at this efficiency, how much money would you save each year if the combustion temperature were 1700 Celsius (you'll have to make some reasonable assumptions).

   (a) Efficiency: $1 - \frac{T_C}{T_H} = 0.56$. New efficiency: 0.61. So 30mpg becomes $30 \times 0.61/0.56 = 32.7$mpg Miles driven per year: 13500 miles. Gallons per year saved: $13500(1/30 - 1/32.7) = 37$. Price per gallon: \$3.50. Total savings: \$130.

6. Advanced alien races may efficiently harness the energy of their home star by building a Dyson sphere - an enormous structure which completely encloses a star and absorbs its heat. If we built a Dyson sphere around the sun, what would its maximum efficiency be? (Hint: T_H would be the temperature of the sun, and T_C would be the temperature of space.) What would be its maximum possible power output?

   (a) efficiency $= 1 - \frac{T_C}{T_H} 1 - \frac{2.7}{5772} = 99.95\%$. (2.7K is the temperature of deep space, but there are other reasonable assumptions to make... things are slightly warmer in the solar system).

   (b) Power output of sun: 3.828×10^{26} Watts. Maximum output of Dyson sphere: 3.826×10^{26} Watts

7. Look at designs for perpetual motion machines. Choose one that looks particularly interesting to you and describe how it's supposed to work. Also describe why it doesn't actually work.