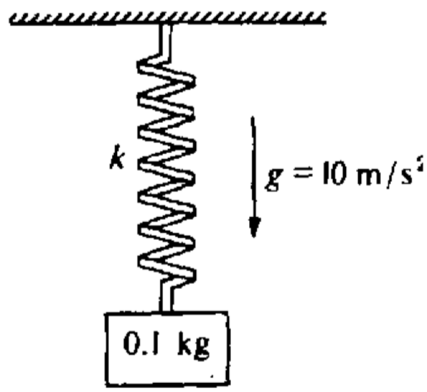
Topics in Physics: Problem Set #5

Topics: springs, oscillations, waves

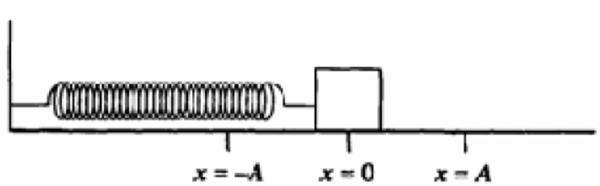
Practice Problems (approx. 30 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. A 0.1 kilogram block is attached to an initially unstretched spring of force constant $k = 40$ Newtons per meter, as shown below. The block is released from rest at time $t = 0$. What are the amplitude and period of the resulting oscillation?



2. A block on a horizontal frictionless plane is attached to a spring, as shown below. The block oscillates along the x-axis with simple harmonic motion of amplitude A.



Indicate where the block will have a maximum magnitude of:

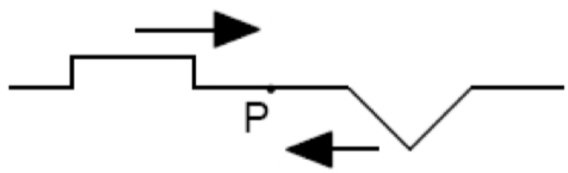
(a) Displacement
(b) Velocity
(c) Acceleration

(d) Kinetic energy

(e) Potential energy

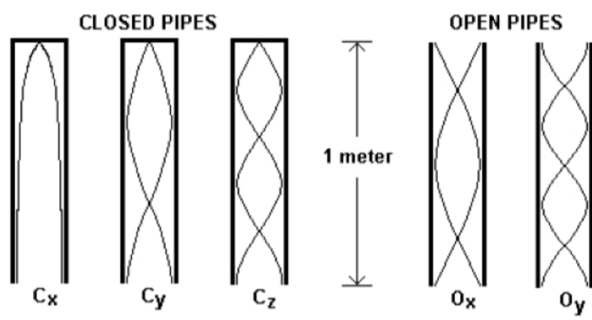
(f) Total energy

3. Two waves pulses approach each other as seen in the figure. The wave pulses overlap at point P.



Draw the appearance of the waves (i) as their centers overlap at P, and (ii) after they leave point P.

4. The diagrams below represent 5 different standing sound waves set up inside of a set of organ pipes 1 meter long Which of the following statements correctly relates the frequencies of the organ pipes shown? Choose all correct answers.



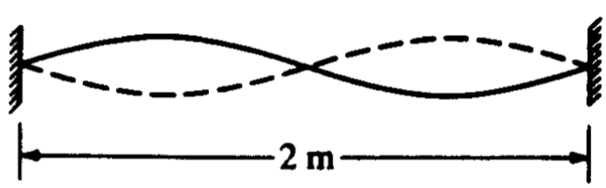
(a) C_y is twice the frequency of C_x

(b) C_z is five times the frequency of C_x

(c) O_y is twice the frequency of O_x

(d) O_x is twice the frequency of C_x

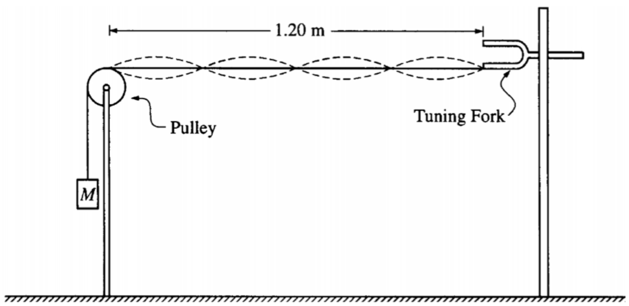
5. A standing wave of frequency $f = 5$Hz is set up on a string 2 meters long with nodes at both ends and in the center, as shown below.



(a) What is the speed at which a wave propagates along the string?

(b) What is the fundamental frequency of vibration in the string?

6. To demonstrate standing waves, one end of a string is attached to a tuning fork with frequency 120 Hz. The other end of the string passes over a pulley and is connected to a suspended mass M as shown in the figure below. The value of M is such that the standing wave pattern has four "loops." The length of the string from the tuning fork to the point where the string touches the top of the pulley is 1.20 m.



(a) Determine the wavelength of the standing wave.

(b) Determine the speed of transverse waves along the string.

(c) The speed of waves along the string increases with increasing tension in the string. Should the value of M should be increased or decreased in order to double the number of loops in the standing wave pattern? Why?

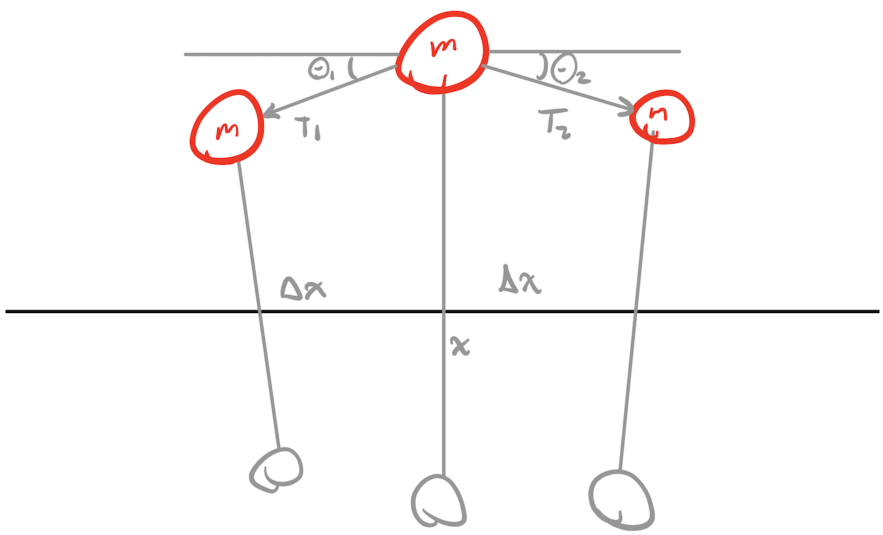
(d) If a point on the string at an antinode moves a total vertical distance of 4 cm during one complete cycle, what is the amplitude of the standing wave?

Challenge Problem (approx. 30 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

Problem 1: deriving the 1D wave equation on the torsional wave machine

Consider the torsional wave machine we made in class. The position along the length of the machine is denoted by x, the spacing between sticks is Δx, each pair of candies and stick(which we will treat as one body) has mass m, and the vertical displacement of the wave as a function of position and time is $u(x,t)$.



1. Consider the forces acting on a single stick-candy system. The tension of the duct tape results in a pair of forces T_1 and T_2 pushing the candy to the neighboring candies which are directed at angles $\theta_1$and θ_2, as shown above.
   (a) What is the the net vertical force acting on the candy?
   (b) Using $F = ma$ and $a = \frac{\partial^2 u}{\partial t^2}$, write an equation relating $T_1, T_2, \theta_1, \theta_2, m$, and u.
2. Because the candies do not move horizontally, the horizontal forces are approximately zero for small displacements, so $T_1 \cos\theta_1 \approx T_2 \cos\theta_2 \approx T$. Therefore,
$$-\frac{m}{T}\frac{\partial^2 u}{\partial t^2} \approx \frac{T_1 \sin\theta_1 + T_2 \sin\theta_2}{T} = \frac{T_1 \sin\theta_1}{T} + \frac{T_2 \sin\theta_2}{T} \approx \frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} + \frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \tan\theta_1 + \tan\theta_2.$$
   Geometrically, if $\frac{\partial u}{\partial x}$ is the slope of the wave at x, and $\Delta\frac{\partial u}{\partial x}$ is the difference in slopes between the candy at x and the neighboring candy at $x + \Delta x$, then $\tan\theta_1 + \tan\theta_2 = -\Delta\frac{\partial u}{\partial x}$.
   (a) Substitute this into your equation relating $T_1, T_2, \theta_1, \theta_2, m$, and u.
3. The linear mass density μ of the machine is the total mass over the length of the machine: $\mu = \frac{\sum m_i}{x_{\max} - x_{\min}}$. If the candies are evenly spaced at intervals Δx, then $\mu = \frac{m}{\Delta x}$.
   (a) What are the units of μ?

(b) Substitute $m = \mu \Delta x$ in your equation.

(c) Divide out Δx to obtain an expression which relates $\frac{\partial^2 u}{\partial t^2}$ to $\frac{\Delta \frac{\partial u}{\partial x}}{\Delta x}$.

4. Finally, suppose we turn the machine into a continuous candy rope, with the same mass density μ, but with $m \to 0$ and $\Delta x \to 0$.

   (a) This corresponds to taking $\frac{\Delta}{\Delta x} \to \frac{\partial}{\partial x}$. Substitute this into your equation.

   (b) Finally, write your equation in the form of a wave equation: $\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$. What is the velocity that the wave will travel down the machine at? Verify the units are correct.

Experiment: soda bottle spectroscopy (approx. 30 min)

In this exercise, we'll measure the volume of air in a bottle by looking at the frequency it produces when you blow into it.

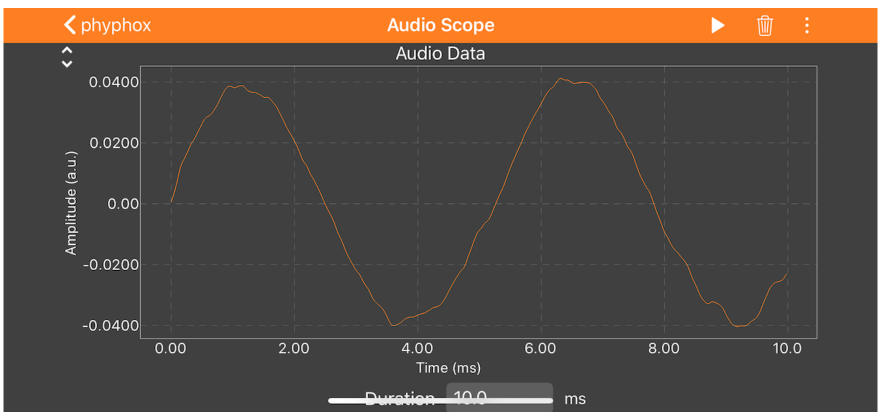
Helmholtz resonance, or "wind throb", is the phenomenon of air resonance in a cavity, such as when one blows across the top of an empty bottle. The name comes from a device created in the 1850s by Hermann von Helmholtz, the Helmholtz resonator, which he used to identify the various frequencies or musical pitches present in music and other complex sounds. The resonant frequency of a Helmholtz resonator is given by:

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{LV}},$$

where v is the speed of sound in the gas, A is the cross-sectional area of the neck, L is the length of the neck, and V is the volume of the resonator.

First, find an empty bottle. A 2-liter soda bottle works best for this experiment, but anything that you can get to produce a consistent tone by blowing over it will work. Partially fill the bottle with some amount of water, and determine the length L and cross-sectional area A of the neck.

Using the phyphox app, go the Audio Scope experiment. Record yourself blowing over the neck of the bottle, and stop the recording while you are still blowing. You should get data which looks sort of like this:



Isolate the primary frequency of the wave you are producing. Use this to determine the volume of air V in the soda bottle. Now measure the amount of air in the bottle by subtracting the amount of water in the bottle from the total volume of the bottle. How does your calculated V compare to the actual volume of air?

Experiment: two-speaker interference (approx. 45 min)

In this activity, you'll see how interference between two in-phase speakers can create a 2D map of nodes and antinodes in a room.

Constructive interference between two sound (or light) sources occurs with the difference in path length ΔL is an integer multiple of the wavelength λ: $\Delta L = n\lambda$ for $n \in \mathbb{Z}$. Destructive interference occurs when ΔL is a half-integer multiple of the wavelength, $\Delta L = \left(n + \frac{1}{2}\right)\lambda$. Take a moment to think about this and convince yourself of why this is true.

Set up two speakers a distance $d = 1$m apart in a carpeted room (carpet suppresses echoes which disrupt this experiment). Go to `http://onlinetonegenerator.com/` and have the speakers output a sine wave of middle C (440 Hz). Determine what wavelength λ this tone corresponds to.

Walk around the room: you should notice there are areas with acoustic nodes (minimal sound) and antinodes (maximal sound). If you're having trouble finding the nodes, you can use the decibel meter in the course supplies, or use your phone's microphone to measure the sound amplitude in the "Audio Amplitude" experiment in phyphox.

You may notice that at the acoustic nodes, the sound doesn't completely cancel out. Why do you think this is?

Using two colors of sticky notes (or masking tape), mark the location of some of the nodes and antinodes on the ground. Try to find at least 5 nodes/antinodes. For each marked location, use a tape measure to determine the difference in path lengths between the two speakers. (Methodology is important here! If you found the acoustic node using your ears, you should measure the distance to each of the speakers from ear height. If you used your phone, measure it from the height you held your phone at.)

Divide your path length by λ and see how your measured node locations correspond to where the nodes should occur based on the path length conditions for constructive and destructive interference.

Try repeating this experiment for a few other tone frequencies. How does the spacing and the density of the nodes/antinodes change? Is there a maximum or minimum frequency that you can use and still obtain results which agree with theory?