Day 2 - Newton's laws, kinematics, momentum + energy, collisions

Survey:
- final projects individual / groups of 1-3
- group projects expected to be more involved

Finish calculus from last lecture (~50 min)

Kinematics

Describing the motions of bodies without concerning about the forces causing the motion

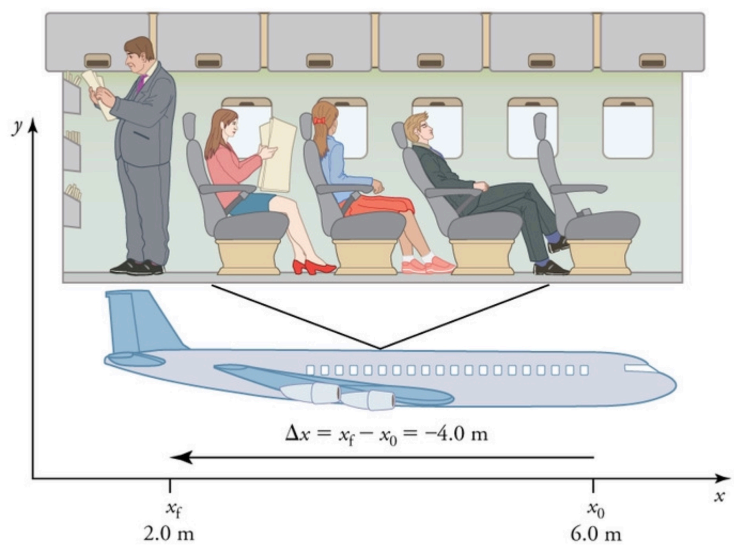
Basically only quantities you'll encounter are:

x, v, a, t

Reference frames (10 min)

key concept in all of physics, shows up a lot in relativity

a reference frame is a coordinate system and a set of physical ref. points that uniquely determine system.



Displacement in Terms of Frame of Reference: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far).

in n dimensions, $n+1$ points uniquely determine ref frame.

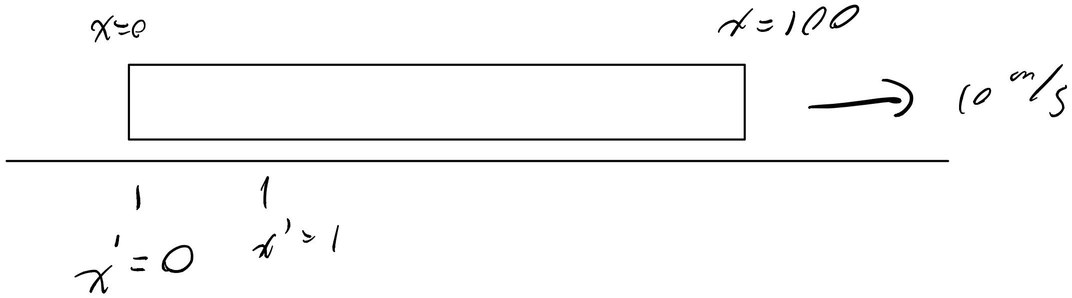
1D ref. frame examples

train moving at 10 m/s

↳ zero point (origin) — caboose

direction & scale of units?

↳ x = +100 point — engine



bystander ref frame

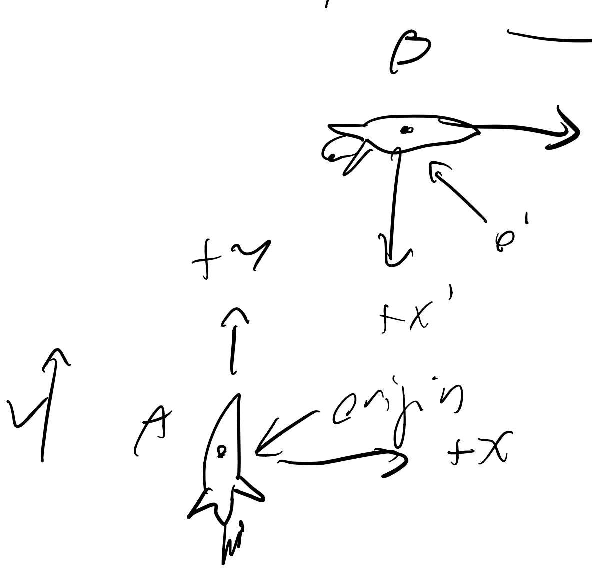
origin: bystander position

direction: + is right

translate between reference frames:

$$x' = x + vt$$

2D example:



$$x = y' + v't$$
$$y = x' + vt$$

Inertial reference frames (5 min)

reference frame where a body w/ zero net force acting on it does not accelerate

↳ is classroom inertial ref. fra.? <u>No</u>

↳ outer space is approx. inertial

↳ <u>not</u> a spinning rocket in space

1D Kinematics (15 min)

derive some important equations

x - position $\qquad x_0, v_0, a_0$ - initial cond.

$v \equiv \frac{dx}{dt}$ $\qquad$ constant of integration:

$a \equiv \frac{dv}{dt}$ $\qquad \int \frac{df}{dt} dt = f + \underline{\underline{C}}$

Uniform velocity:

$x = \int \frac{dx}{dt} dt = \int v\,dt \rightarrow$ if $v = \text{const} = v_0$

$$x = v_0 t \underbrace{+ x_0}_{\text{const of int.}}$$

uniform acceleration:

$v = \int \frac{dv}{dt} dt = \int a\,dt \rightarrow v = at + v_0$

$x = \int v\,dt = \int (at + v_0)\,dt = \frac{1}{2}at^2 + v_0 t + x_0$

What if we only can measure x, v, a but not t?

$$\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx} = a \cdot \frac{1}{v}$$

$$\frac{dv}{dx} = a \cdot \frac{1}{v} \rightarrow \quad v\,dv = a\,dx$$

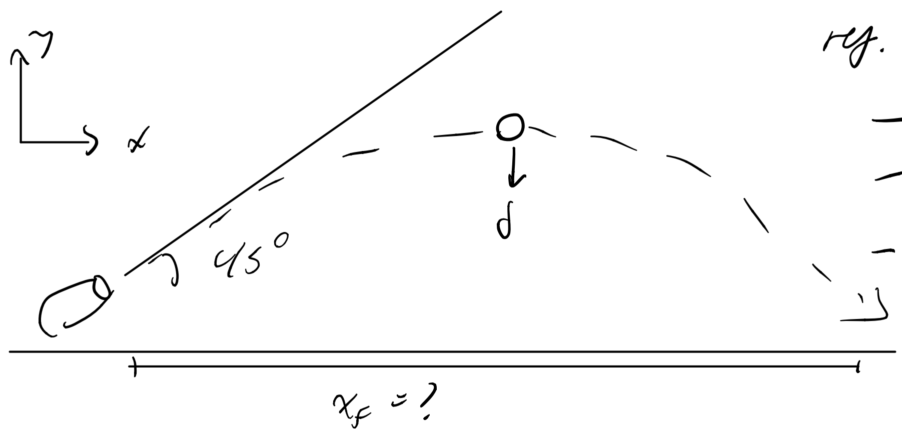
$$\int_{v_i}^{v_f} v\,dv = \int_{x_i}^{x_f} a\,dx$$

$$\frac{1}{2}(v_f^2 - v_i^2) = a(x_f - x_i)$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

2D kinematics example (5 min)

a cannonball is launched at $\theta = 45°$ at $v = 100\,m/s$.
how far does it go?



ref. frame:
- cannon is origin
- up is $+y$ ($g = -y$)
- right is $+x$

time to peak: $0 = 100/\sqrt{2} + -10 \cdot t$

$t = 10/\sqrt{2}$

time total $t_{tot} = 20/\sqrt{2}$

distance $= 100/\sqrt{2}\,(t_{tot}) = 2000/2 = 1000\,m$

Newton's laws of motion (10 min)

Laid foundations for classical physics

Law 1: "what is a force?"

in inertial ref. frame, an object moves with constant velocity unless acted upon by a force

$$\sum_n \vec{F}_n = \vec{0} \iff \frac{dv}{dt} = 0$$

Law 2: "how do you measure a force?"

$\vec{F} = m\vec{a}$

the (vector) sum of forces on an object is mass x net acceleration

$$\sum \vec{F} = m \sum \vec{a} = m\vec{a}_{net}, \quad \vec{F} = \frac{d\vec{p}}{dt}$$

Law 3: "a single, isolated force doesn't exist"

$F_A = -F_B$

"for every action ∃ equal & opposite reaction"

↳ $\vec{p} = m\vec{v}$

$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$\vec{F} = \frac{d\vec{p}}{dt}$ → $F_A + F_B = 0$

$\frac{d\vec{p}}{dt} = 0$ ← conservation of momentum!

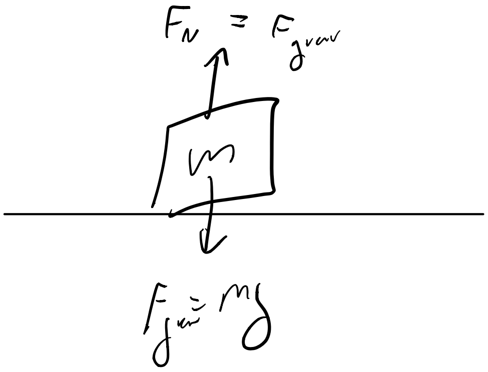
Ball toss activity (5-10 min)

Free body diagrams (10 min)

Super important for classical mech!

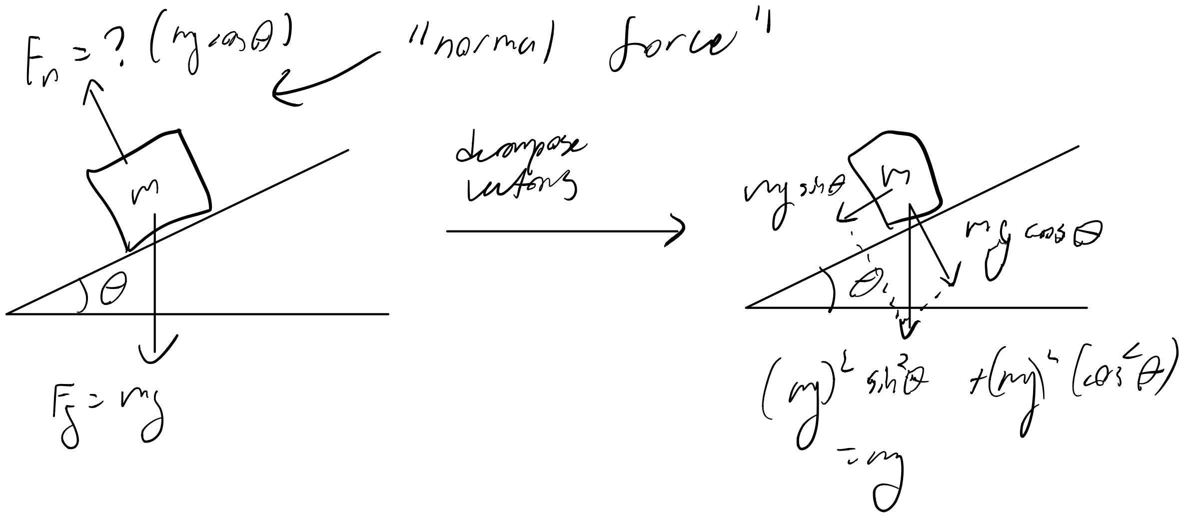
- helps solve statics + dynamics problems
- includes all forces acting on an object

ex.1 block on ground



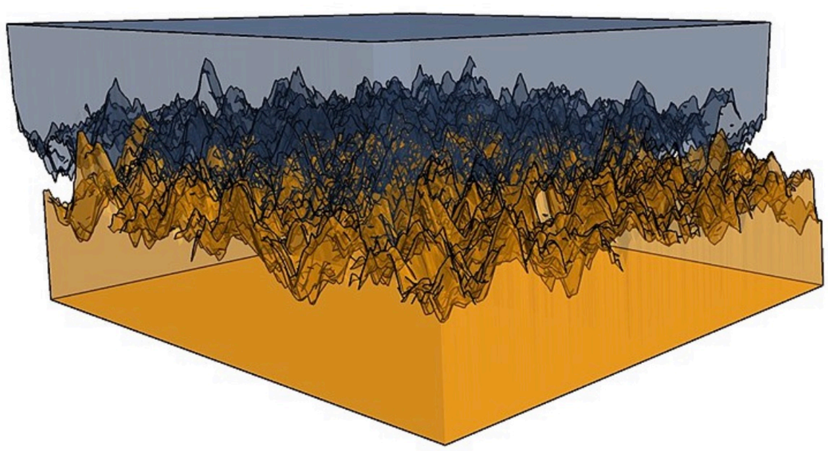
$\boxed{\text{review trig if necessary}}$ (10 min)

ex 2 block on frictionless ramp

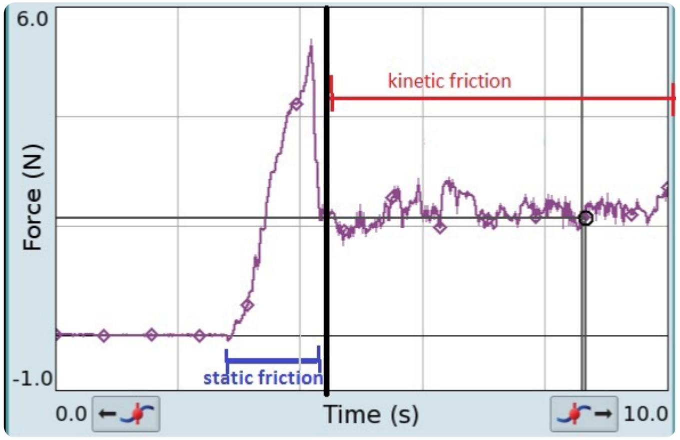


Friction (15 min)

- force resisting relative motion of surfaces
- lots of types:
  - dry ☆
  - fluid – between fluid layers
  - lubricated — solid – fluid – solid
  - skin – solid on liquid
  - internal – deformation
- dry friction
  - arises from inter-surface adhesion



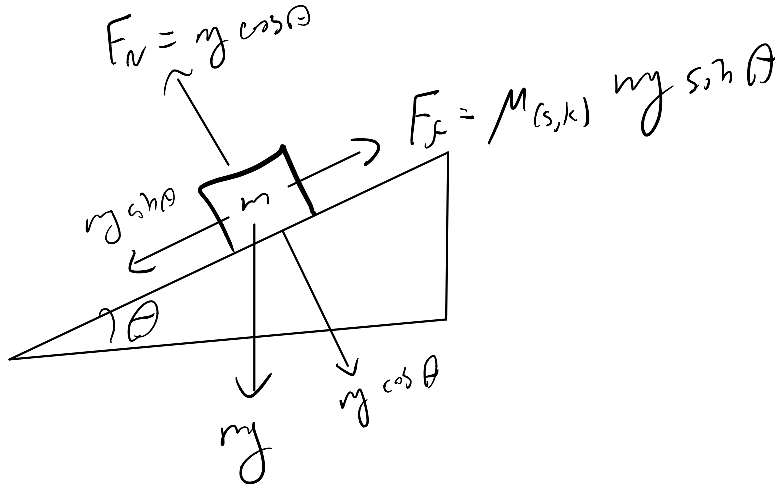
- static & kinetic friction
  - fill examples
  - force ⟶



- coefficient of friction
  - ratio of frictional force between two objects & forces pressing together

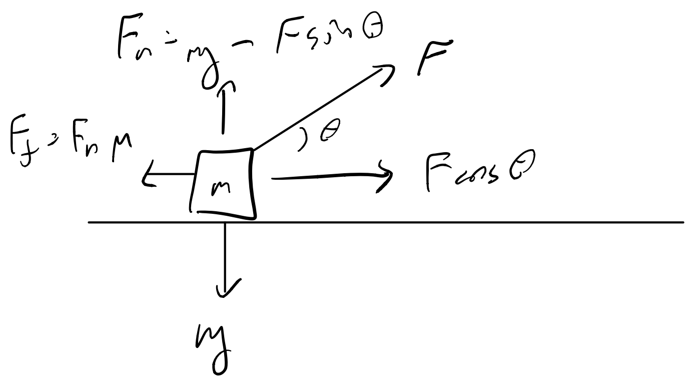
~ denoted by μ

- $F_f = \mu F_N$
- μ_s - static, μ_k - kinetic



Problem

pull block w/ rope @ angle θ, what is needed F?



$$F\cos\theta \geq F_N \mu_s$$

$$F\cos\theta \geq \mu_s (mg - F\sin\theta)$$

$$F\cos\theta + \mu_s F\sin\theta \geq \mu_s mg$$

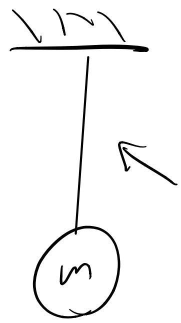
$$F \geq \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Tension (15 min)

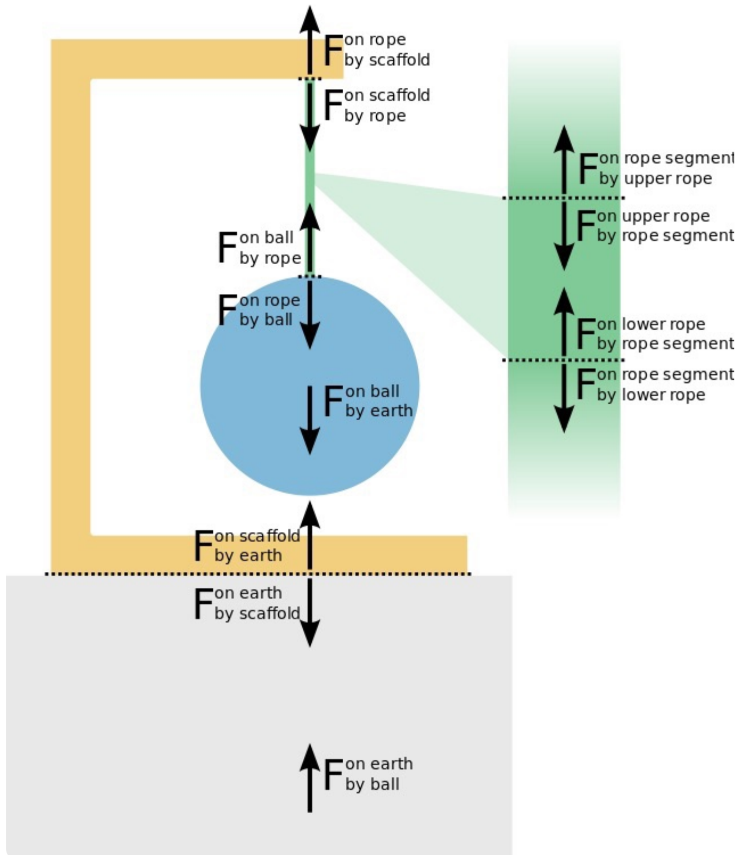
pulling force exerted by 1D system, i.e. rope / chain / cable

at atomic level, comes from inter-molecular restoring forces

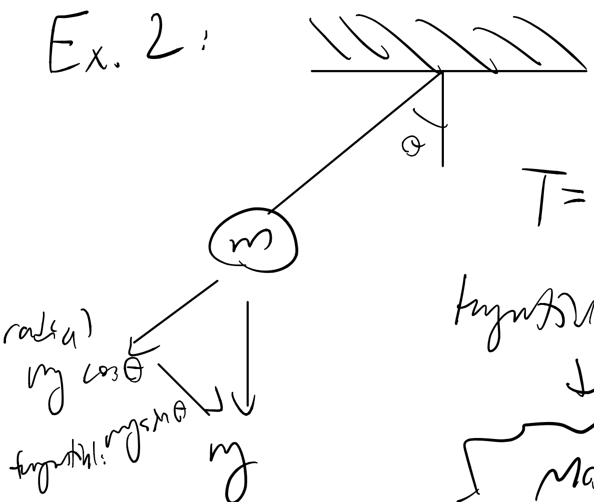
Ex. 1:



T? = mg



Ex. 2:

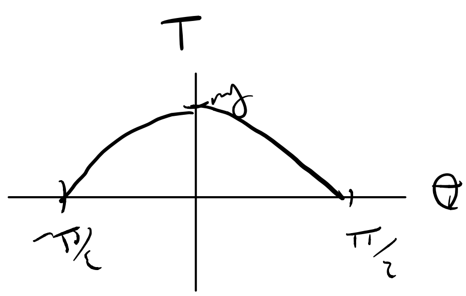


balanced

$T = mg\cos\theta$

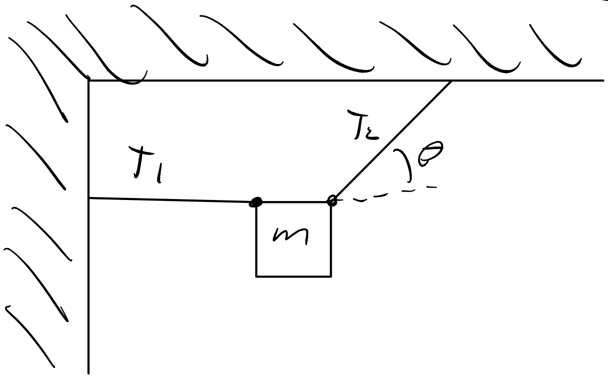
tangential forces are unbalanced!

MOTION



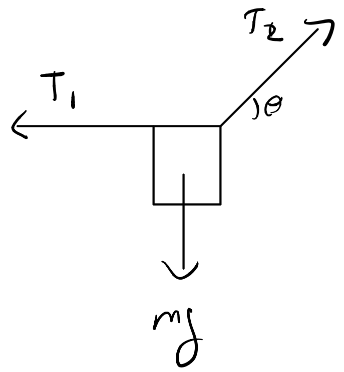
Ex. 3 — solving 2D statics problems

statics = no motion

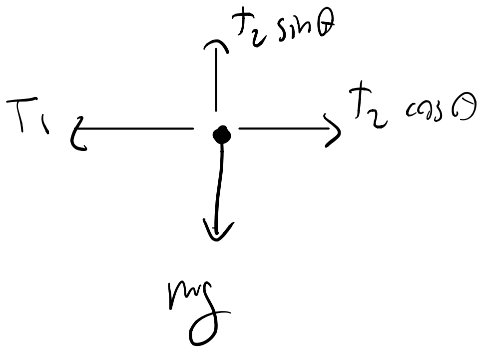


find T_1 & T_2!

Step 2: write a free body diagram



Step 2: break into x/y component vectors



Step 3: if static, forces must be balanced

$$T_2 \sin\theta = mg \rightarrow T_2 = \frac{mg}{\sin\theta}$$

$$T_1 = T_2 \cos\theta \rightarrow T_1 = \frac{mg}{\sin\theta}\cos\theta = mg\cot\theta$$