# Topics in Physics: Problem Set #1

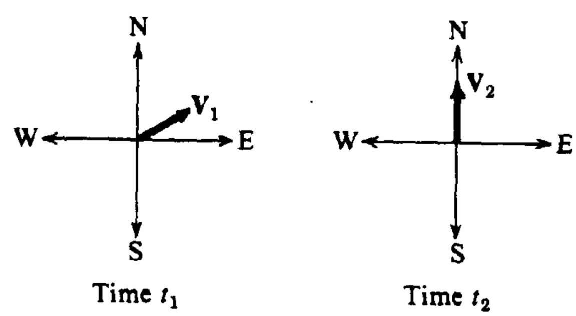
**Topics: physical units, mathematical models, estimation, vectors**

## Practice Problems (approx. 40 min)

You should try to do these problems individually. None of them should take very long to solve; if you get stuck, ask a TA for help!

1. The world's top 100 circuses team up in the most stunning acrobatic feat in history: all of the performers stand on each others shoulders, making a human tower 1 person wide! How tall is the tower? About how much weight is the bottom person carrying?

2. Vectors $\vec{v_1}$ and $\vec{v_2}$ shown below have equal magnitudes. The vectors represent the velocities of an object at times $t_1$, and $t_2$, respectively. Draw the direction of the average acceleration vector (ignoring overall magnitude).




3. A ball is thrown with an initial velocity of 70 m/s, at an angle of 35°from horizontal. Find the vertical and horizontal components of the initial velocity.

4. Approximately how many meters per year does a snail travel on average? How many lightyears per hour? How many hydrogen atom diameters per second?

5. A bicycle tire with radius 0.4m rolls along the ground through three quarters of a revolution. Consider the point on the tire that was originally touching the ground. What is its displacement from its starting position?

6. A person is swimming north across a river with velocity $\vec{v}_{\text{swim}} = (0\hat{x} + 1\hat{y})\,\text{m/s}$. The river is flowing east with velocity $\vec{v}_{\text{river}} = (0.5\hat{x} + 0\hat{y})\,\text{m/s}$.

   (a) What speed would the person be swimming at if they were in still water?

   (b) What angle is the person swimming at *relative to the flow of the river*? (Hint: recall $\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos\theta$)

7. As we discussed in class, the cross product of two vectors $\vec{a}$ and $\vec{b}$ yields a vector $\vec{c}$ which is orthogonal (perpendicular) to both $\vec{a}$ and $\vec{b}$; the cross product is defined by

$$\vec{a} \times \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \sin(\theta)\, \hat{n},$$

where $\hat{n}$ is a unit vector determined by the right-hand rule and $\|\vec{a}\| \left\|\vec{b}\right\| \sin(\theta)$ is the magnitude of the resulting vector. <u>**NOTE**</u>: for this problem only, you shouldn't use WolframAlpha.

   (a) Let $\hat{x}$, $\hat{y}$, and $\hat{z}$ be the standard 3D Cartesian unit basis vectors. What is:

      i. $\hat{x} \times \hat{y} = ?$

      ii. $\hat{x} \times -\hat{y} = ?$

      iii. $-\hat{x} \times \hat{y} = ?$

      iv. $\hat{y} \times \hat{z} = ?$

      v. $\hat{z} \times \hat{x} = ?$

      vi. $\hat{x} \times \hat{x} = ?$

   (b) Find the cross product of $(1, 1, 0) \times (-2, 2, 0)$.

   (c) Find the cross product of $(1, 1, 0) \times (-2, -2, 0)$.

8. Earth has a mass of $m$, and its displacement from the sun can be written as a vector $\vec{r}$, and its velocity around its orbit can be written as a vector $\vec{v}$. Let $\vec{L} = m\,(\vec{r} \times \vec{v})$. What direction is $\vec{L}$ pointed in? What is the magnitude $\|\vec{L}\|$? What are its units?[1]

[1] The quantity $\vec{L} = m\,(\vec{r} \times \vec{v})$ is called the *angular momentum* of an object. We'll cover this in a few days when we do angular dynamics.

# Challenge Problems (approx. 20 min)

You may work in small groups to solve these problems, but each student should submit and understand their own answer. These problems are challenging but not impossible to solve. If you get stuck, ask another student or a TA how to approach the problem, and if you are helping another student, try to explain so they understand how to solve the problem (don't just give them the answer). Show all your work and walk the reader through the solution; you may get feedback on both the approach and the clarity of your solutions.

## Problem 1: Conical reservoir

Suppose Stanford University decides to build a large water reservoir in the shape of an inverted cone. The cone's base (which is the surface of the reservoir) has a radius of 30 meters and the depth in the center is 5 meters. The lead engineer wants to model the rate of water evaporation, which will depend somehow on the amount of water in the reservoir. Develop a model for the rate of evaporation, $R$, which depends only on the depth, $z$, of the water. After building the reservoir, the engineer adds $z = 3$m of water and measures the rate of evaporation to be 100 liters per hour. According to your model, what will be the rate of evaporation when the reservoir is full?

# Physics "Tech Tree" (approx. 45 min)

Team up into groups of 3-5 people and make a map of physics equations! Your goal is to incrementally build a "tech tree" which directly or indirectly relates as many physics equations to base SI units as possible. By building up formulae for various physical quantities, you can "unlock" increasingly complex equations. The rules are as follows:

1. Each team gets a sheet of large easel paper (you can get another one if you mess up), a set of markers, and a bunch of sticky notes.

2. Look up the figure of the new SI units that was discussed in lecture (it's the last figure on the Wikipedia SI base units page). Write the seven base units and the quantities they measure in marker around the edge of the paper and include the constants that are used to define them. You will fill in the center of the paper, so give yourself space to work!

3. For each sticky note, write down a physics equation and what it means. You don't need to know th equations by heart; you can look them up on Google or Wikipedia, and `physics.info/equations` also has a helpful listing of many common formulas. You will relate the equations to the SI base units, and you can build definitions from other definitions. At the end of the activity you will draw marker lines connecting all the concepts, but save this for the last few minutes so you can reposition the notes as needed.

   (a) Include a short verbal description on the sticky note for each equation, (e.g. "$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$: force between charges is proportional to product of charges over separation squared"), and include the name of the equation if it has one (e.g. "Coulomb's law").

   (b) If the equation depends on another physics concept which isn't one of the base units, you need to define that concept too.

      i. For example, acceleration is $a = \frac{dv}{dt}$, which depends on velocity. To put down acceleration, you also need to define velocity, which is $v = \frac{dx}{dt}$. Since $x$ (distance) and $t$ (time) are both base units, you can stop there.

   (c) If the equation depends on a constant that is already on the paper, you should connect it to that constant, along with any other units in the formula. For example, $E = mc^2$ would be connected to mass (kg) and to $c$.

   (d) If the equation depends on a constant that is not already on the paper (for example, vacuum permittivity$\varepsilon_0$), write it along the edge of the paper with the other constants. Include the symbol, the name of the constant, and the base units of the constant. (You can type any constant into WolframAlpha to get a basic unit decomposition.) You won't need to draw lines connect constants to existing units.

      i. For example, if you put down Newton's law of gravity, $F_G = -G\frac{m_1 m_2}{r^2}$, it would be connected to mass, length, and your definition of $G$. To define $G$, draw a circle somewhere near mass and length, and fill out "$G$: gravitational constant, units: mass$^{-1}$length$^3$time$^{-2}$".

4. Try to get as many equations as you can! For bonus points, try to use each base unit at least once. The TAs will let you know when time is almost up; once they do, figure out where you want to place all the notes and draw lines connecting them to each other and to the base units and constants.

5. Take a picture when you're finished!

An example map is shown below. You've got about 45 minutes to work on your map, so you can probably get more equations than I did, but use this example to organize yours. When you're done, spend a few minutes admiring other groups' maps! Which equations did you have in common? Which ones are unique to a team?

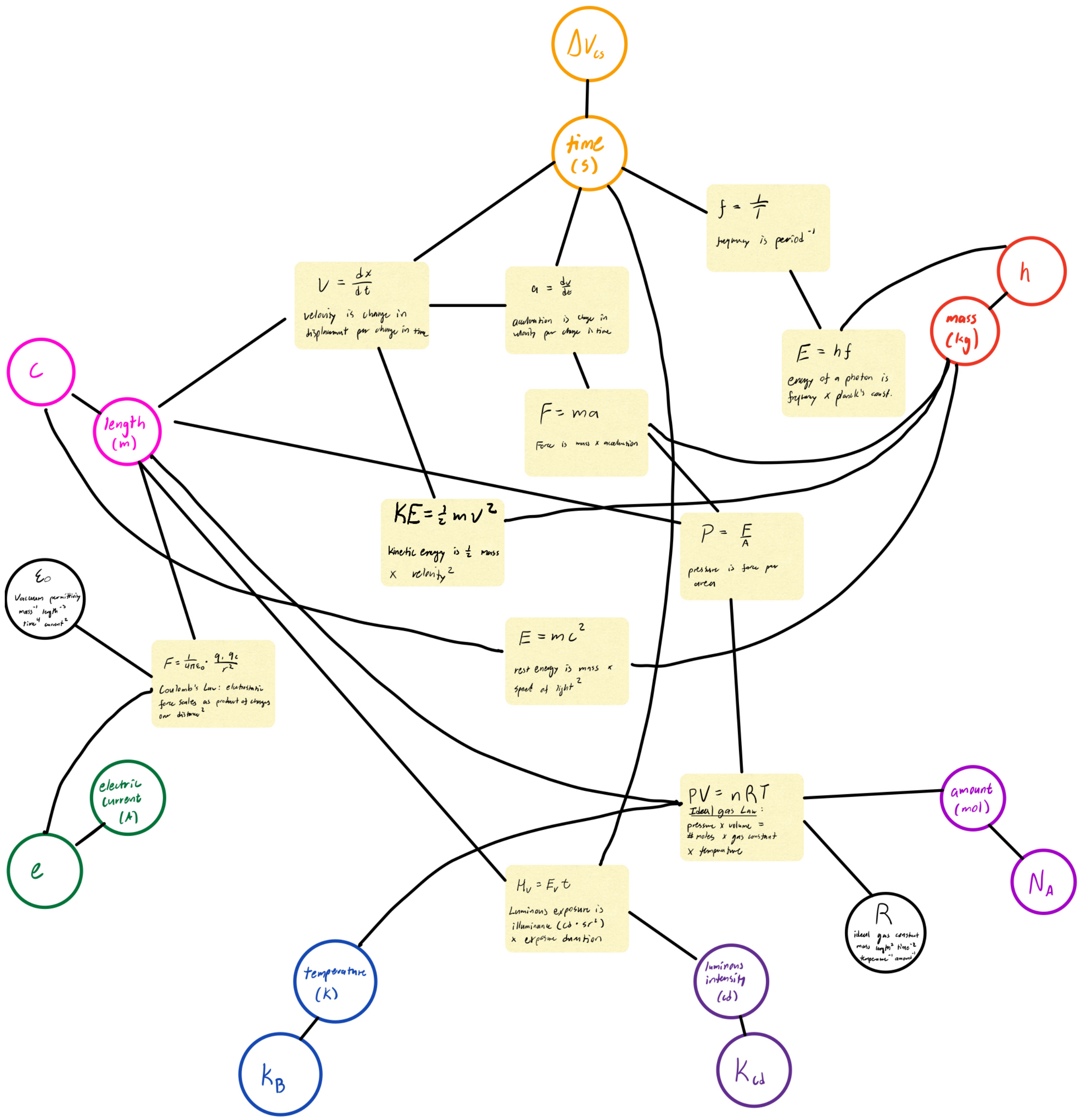



Figure 1: Example physics “tech tree”.

# The Fermi Question Contest (approx. 60 min)

Form teams of 3-4 students and come up with a team name! There will be 5 rounds (or as many as time allows), each lasting about 15 minutes, which proceed as follows:

1. The question is read aloud (or written where everyone can see), and the teams have 5 minutes to discuss/solve it (set a timer!). At the end of the 5 minute period, each team should submit a single piece of paper with a circled answer. Include units in your answer if there are any. The TAs will give a 30 second warning before the time expires, then collect the papers.
2. Each team may now ask for one piece of information from the TAs. They will send a representative to privately ask the question and receive an answer. The information is not shared between the teams. The TAs may have to look up the answers online.
   (a) The TAs have the prerogative to give partial answers to questions which aim to skip over multiple steps. For example, rather than answering "what is the volume of the atmosphere", the TA might just give the altitude of the edge of the atmosphere.
3. Each team then has 3 more minutes resubmit their answer. It doesn't necessarily have to change. Again, they should write their answer on a sheet of paper which will be collected after 3 minutes (give a 10 second warning).
4. The TAs will then reveal the solution and award points. The teams score 10 points minus 2 points for every order of magnitude of error on their first guess. Their second answer is worth 5 points minus 2 for every order of magnitude error.
   (a) Example: If the correct answer is $1.5 \times 10^6$ and the student answers first $2 \times 10^5$ and then $1 \times 10^6$, they get full credit (15 points). If their first answer is $1 \times 10^4$, and their second is $1 \times 10^5$, they get $10 - 4 + 5 - 2 = 9$ points.
5. Take a few minutes (as needed) to discuss the solution. Ask for the students' solutions first, then give my versions if they are different.