Housing and Fertility*

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Abstract

I build a dynamic equilibrium model of household behavior with unobserved heterogeneity in the desired number of children, to examine how policies targeting the housing market affect choices of fertility, location, and house size of young households. I estimate the model's structural parameters using data from Hungary, to evaluate the dynamic effects of the Family Housing Allowance policy, which provided a large lump-sum subsidy for house purchases, with built-in commitment regarding the number of children the family would have. The model suggests that the combination of lower interest rates and the allowance increases house prices substantially compared to the baseline, which for poorer households counteract some of the positive welfare effects of the policy. While according to the model completed fertility increases due to the policy by around 5-10% on average, mainly driven by poorer households, their housing conditions worsen in the long run due to the elevated house prices.

Keywords: Housing Demand, Fertility, Housing Policy, Residential Choice

JEL Classification: J11, J13, J61, H31, R21

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1 Introduction

Housing plays a central role in the life of a typical household. In the United States and other developed countries, housing wealth has accounted for around 90% of young and poor households' assets in the last few decades, and 35% of all household assets (Davis and Van Nieuwerburgh, 2015). Jordà et al. (2019) documents that housing has provided historically the largest part of capital stock, with returns comparable to equity's. Furthermore, housing expenses has constituted around 20-25% of the total household expenses, a consistent finding for different time periods, and locations (Ortalo-Magné and Rady, 2006; Davis and Ortalo-Magné, 2011; Piazzesi and Schneider, 2016). Housing also contributed significantly to the Great Recession of 2008 (Mian et al., 2013; Famiglietti et al., 2020), and is understood to be a significant channel of monetary policy (Di Maggio et al., 2017). Real estate differs from other assets in several aspects. Houses are illiquid due to search frictions (Merlo et al., 2015), they are indivisible and relatively expensive, so most households must borrow in order to purchase one. Households usually end up owning only one house, where they reside, and search for employment in its proximity (Davis and Van Nieuwerburgh, 2015). Compared to generic goods and services, housing is also special as its services must be consumed by its own residents. Furthermore, real estate functions as collateral to other types of borrowing, providing a channel through which household consumption is connected to the housing market (Attanasio et al., 2012; Di Maggio et al., 2017; Cloyne et al., 2019).

The last few decades housing conditions of young households have been deteriorating. House ownership amongst young people has declined, or been postponed notably in the developed world (Fisher and Gervais, 2011; Green and Lee, 2016; Flynn, 2020). Due to their high price, most households normally would use mortgages to purchase houses, and these mortgages are naturally issued mostly to young households who decide to suppress consumption in their early earning years to be able to afford the down payment for the house (Chan et al., 2015). This is however by necessity: most renters otherwise cannot afford a house. Therefore down payment, ranging between 20%-50% of the total price, is the dominant obstacle for young households' home ownership (Davis and Van Nieuwerburgh, 2015). Down payment depends on house prices, which means that escalating house prices could contribute largely to young households' inability to get a mortgage (Barakova et al., 2014). Moreover, evidence shows that housing is deeply intertwined with fertility decisions, and the choice of residential location. Difficult housing conditions in terms of prices or security are documented to decrease or delay fertility amongst young households who look for their first home ownership, a wide-spread finding in the literature (Ermisch, 1999; Ström, 2010; Öst, 2012; Vignoli et al., 2013; Kulu and Steele, 2013; Dettling and Kearney, 2014; Day and Guest, 2016; Lin et al., 2016; Öst and Wilhelmsson, 2019)[1]. Young couples are also found to adjust their housing status to budgetary circumstances, credit constraints, and to anticipated fertility (Ortalo-Magné and Rady, 1999; Mulder and Lauster, 2010; Fisher and Gervais, 2011; Ermisch and Steele, 2016; Vidal et al., 2017; Mulder, 2018). This channel, alongside important changes in the female role of the household (Esping-Andersen and Billari, 2015), could have contributed to low fertility rates in developed countries (Billari and Kohler, 2004), as high home ownership along with low access to mortgages is associated with the lowest fertility rates (Mulder and Billari, 2010). An unfriendly housing environment for young couples then could contribute to potential future difficulties in the social security systems of European welfare states (Flynn, 2017; Zeman et al., 2018).

In this paper I study how fertility and the housing market interplay with the residential choices of young households, and how policy interventions that target housing conditions could be effective in improving fertility outcomes. To address this question, I build a life-cycle model of forward-looking households choosing fertility, house size, ownership, and location, with unobserved heterogeneity in their desired number of children. I apply the model to analyze the effects of a government program in Hungary running since 2014/2015, called the Family Housing Allowance (further referred to as FHA)[2], which aimed to ease the borrowing constraints of young households at home purchases or mortgage down payments, and to revitalize the housing market of the country. The policy provides a substantial non-refundable lump sum at house purchases to families if they commit to or already have at least three children.[3] The overall effects are not straightforward to assess, neither in

[1] At the same time, increasing house prices provide one of the most important source of asset value growth for households (Jordà et al., 2019).

[2] In Hungarian 'Családi Otthonteremtési Kedvezmény', abbreviated as CSOK

[3] The program since has been extended to two children, with a substantially lower amount per child.

the long run or short run (Banai et al., 2019; HNB, 2019). On the one hand, addressing the credit constraints of young households should lead to better housing conditions and fertility outcomes, especially considering direct incentives in this case. On the other, in an environment with inelastic housing supply, the policy could result in a substantial increase of house prices which could force some families out of certain real estate markets decreasing their welfare, and might result in redistribution of resources between different types of households.

Due to the long-term nature of both mortgage and fertility decisions, dynamic models are the natural choice in the economic literature to analyze them, as fertility decisions and residential choices are jointly determined, possibly evolving with the housing markets in an endogenous manner. As mentioned previously, the housing market is documented to influence household welfare and decisions regarding fertility. At the same time, the other direction of causality is also present, as activity on the housing market is similarly affected by fertility decisions. Relatively higher mobility in the spatial dimension is well-documented for younger households, in which child birth itself plays an instrumental role. Families look for areas with better amenities (Gambaro et al., 2017), which could indeed result in better life outcomes for small children (Chetty et al., 2016). Moving to agglomerations of larger cities can provide better amenities and labor market conditions, while it might increase the costs of housing, transportation and consumption (Davis et al., 2014; Combes et al., 2019). It is also documented that families with different fertility choices can be observed to sort into different types of housing, more spacious dwellings giving home to families with more children (Kulu and Vikat, 2007; Kulu and Steele, 2013; Chudnovskaya, 2019). Capturing endogenous relations of fertility and housing with dynamic structural models is even more important when examining policies such as the Family Housing Allowance, as the short-run reduced form approach would not be able to capture long-term unintended consequences, which might turn out to be substantial. An example of this issue, as studied by Parent and Wang (2007), is a benevolent expansion of the Canadian family tax exemption, which seemed to increase fertility in the short run, but the cohort-level long-run analysis shows that completed fertility remained unchanged, so the actual effect of the policy appeared only in the timing of births.

According to the model developed in this paper, the Family Housing Allowance (FHA) itself would not have increased house prices in the realized magnitude, if the earlier somewhat higher interest rate conditions stayed intact. However, I find that the combination of low interest rates and the housing allowance would result in an around 70-90% increase in house prices in the medium-run (4-6 years) both in the urban centre, and the rural areas. So while the FHA by itself would have resulted in higher ownership rates and fertility compared to the baseline, due to elevated house prices home ownership declines slightly amongst the poorer households, and they are forced more often to rent smaller, central location apartments despite its negative welfare effects (according to the model specification). Richer families' housing conditions are not affected largely by the policy. Regarding completed fertility, I find that the policy changes the timing and completed fertility as well: births occur earlier in the female life-cycle, and an around 5-10% increase in completed births is implied by the model, which seems to be high considering past results on the effect of welfare policies in Hungary (Spéder et al., 2020). The fertility effects of the policy are driven by families with lower education level (of around 6-8%), however the model indicates an around 2.5% increase in births for families of higher education level as well. So overall the policy seems to be effective in its goals regarding fertility, however it seems to damage the prospects of poorer households regarding housing welfare.

This paper contributes to the literature primarily by constructing a life-cycle model of households' joint decision over fertility, housing size, and residential location with unobserved heterogeneity regarding the desired number of children, importantly with the endogenous evolution of house prices. This is a first attempt as far as I know. In this effort, I attempt to synthesise the relevant findings of macro, urban and labor economic literature along with demography on the topic of fertility, housing, and location choice. In the macroeconomic literature, authors mostly focused on studying houses as special assets within the frames of optimal portfolio decisions, treating fertility as an issue of mostly exogenous consumption commitment (Cocco, 2005; Yao and Zhang, 2005; Love, 2010; Li et al., 2016; Fischer and Khorunzhina, 2019), endogenous fertility appears rarely and not in the context of housing (Sommer, 2016). The effect of housing on the life-cycle of young households has been studied in the past (Li and Yao, 2007; Attanasio et al., 2012), along with how migration and home ownership interacts impacting welfare of households (Oswald, 2019), but these pieces did not consider the role of endogenous fertility in housing choices. In urban economics work has been done on the joint modeling of housing services and location choice, (Ortalo-Magné and Rady, 2006; Ortalo-Magné and Prat, 2016), along with how demographic changes interplay with urban costs (Combes et al., 2019). Similarly to the macroeconomic literature, fertility has been a less studied factor in urban economics. However, in labor economics endogenous fertility plays a central role in the study of female labor supply (Becker, 1991), with important consequences for long-term labor market outcomes for women (Adda et al., 2017; Eckstein et al., 2019). I have not found an example for the use of housing aspects as choice variables. And finally, in works of demography and reduced-form econometrics many stylized facts and descriptive evidence have been gathered over the years, suggesting strong interconnectedness between fertility, housing type and location choice (Ermisch, 1999; Kulu and Vikat, 2007; Öst, 2012; Vignoli et al., 2013; Mulder, 2013; Dettling and Kearney, 2014; Day and Guest, 2016; Chudnovskaya, 2019). Furthermore, the model enables the study of the effect of such policies in the medium and long run, which would not be possible in a reduced form setting so close in time to the start of the policy, while also connecting to the literature of combining structural models to study the effects of policy shocks, using the housing market as an ex-post model validation relating to the idea of Todd and Wolpin (2006).

The structure of the paper is as follows. First, I describe the policy context of the question in Hungary. Then I introduce the model in detail. Afterwards, I turn to the empirical strategy of the paper, I describe the estimation and calibration of the parameters. And finally, I use the model to run counterfactual policy experiments to evaluate the effects on fertility and housing.

2 Context: the Family Housing Allowance program of Hungary

Since 2010, the Hungarian government has introduced several new policies targeting directly or indirectly the fertility decisions of young households (Makay, 2020), one of them is the Family Housing Allowance announced in 2014 (further on: FHA)[4]. The allowance in its

[4] Legislated by Government Decrees No. 16/2016 and No. 17/2016 (Hungarian Government, 2016a,b)

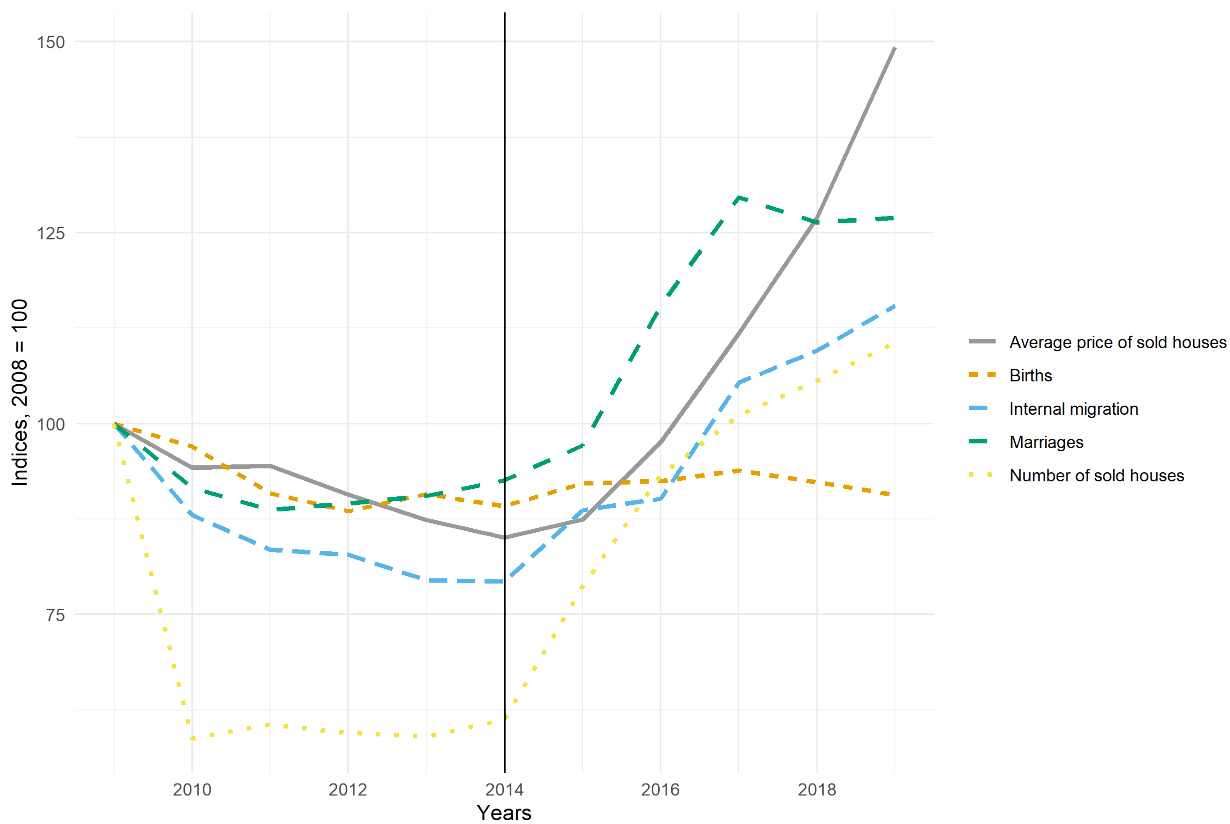
original form provided families that commit to having three children in total, with a 10 million HUF (∼30,000 EUR) lump sum for the purchase of a newly constructed house that satisfies certain minimum quality requirements. At the same time another 10 million HUF interest-subsidized mortgage was also introduced to go along with the allowance. Later on the policy was extended also to purchasing owner-occupied dwellings, and to families that plan to have one or two children as well, but the latter only with a substantially lower sum.

There were several restrictions and penalties built into the policy. Only couples where one of the parties is below 40 years of age could apply. In case a family does not fulfill the requirement of the number of children they committed to (except for medical reasons), they must pay a penalty. If they have two instead of three children, they have to pay 7,400,000 HUF (∼20,000 EUR), if less than two, they have to pay back the whole 10,000,000 HUF (∼30,000 EUR), with an interest at a yearly rate of five times the size of the central bank's policy rate (set at 0.9%). The 'schedule' of promised children depend on the number of children the family has at the time of applying for the subsidy. In case they have two children (so they only promise one extra child), the couple has four years to fulfill the commitment, for two extra children it is eight years, for three it is ten. Another restriction is that a dwelling purchased with the subsidy cannot be resold for 10 years.

The Hungarian Government declared two objectives (Hungarian Government, 2016a; Sági et al., 2017). On the one hand, she aimed to support families in raising children and purchasing new homes. Survey evidence shows that in Hungary house ownership is perceived as a necessary condition for raising children (Szalma and Takács, 2015). Other survey evidence by Kapitány (2016) suggests that when young adults were asked to rank several obstacles to have children, housing was named a major, but actually not a top issue. However, two demographic groups were found for whom housing was mentioned as a more important aspect. One group includes those who do not plan to have a child in the short run, but do in the long run, and the other group consists of those, who do not want more children exactly due to bad housing conditions. We can think of these groups as those that the government might 'intend-to-treat'. The other declared aim of the legislation to revitalize a struggling housing market with a demand push by young households, as it was unable to reach the pre-crisis output levels of the 2000s. The report of the Hungarian National Bank (HNB, 2019) shows that in 2007 the number of newly built dwellings stood around 35,000, while in 2015 only at around 8,000, and did not show signs of recovering at that point in time.

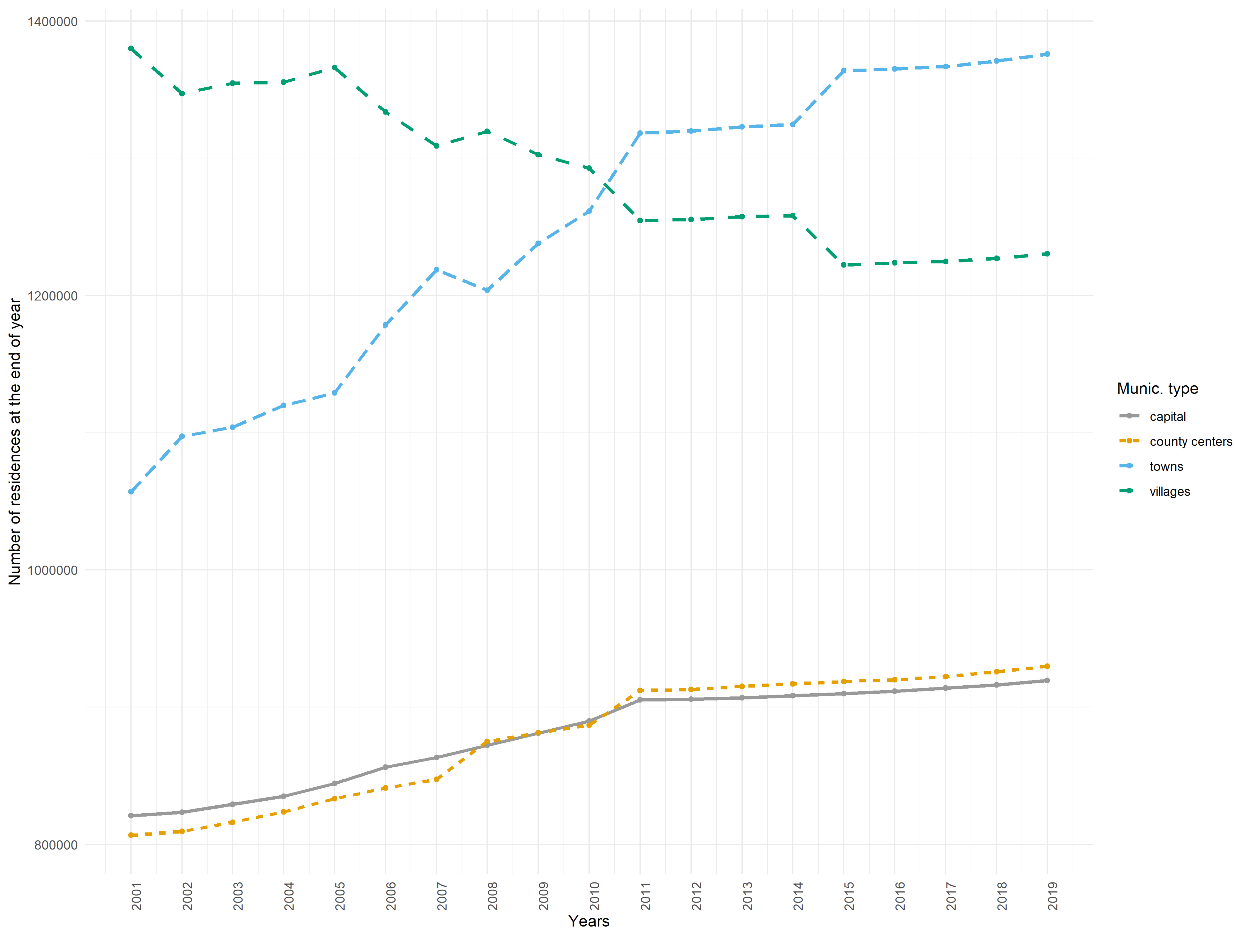
As illustrated by Figure 1, after 2014 and 2015 several macroeconomic indicators connected to the policy seem to have been perturbed. The average price and number of sold occupied houses increased dramatically, along with the number of marriages and internal migration, while the number of births seemed to experience a more modest growth. Figure 2 displays the total number of houses at the end of calendar years by type of municipalities, which shows that while in rural towns the number of houses seemed to increase after the policy, the rate of change for other municipality types does not seem to be substantially affected, at least in the short run.

Figure 1: Prices and number of sold houses, marriages, internal migration, and live births, 2008-2018



Note: The yearly time series are based on the publicly available settlement level data of the HCSO, accessed 16/02/2020. The vertical line at 2014 indicates the announcement of the Family Housing Allowance program.

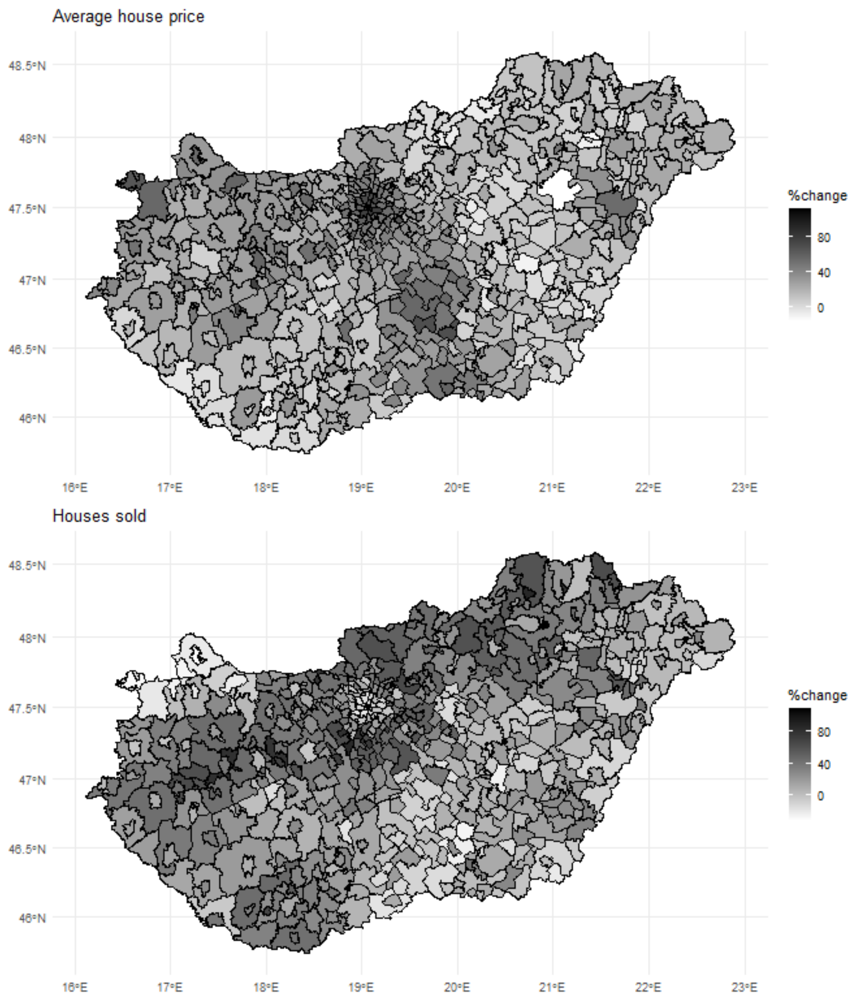
Figure 2: Total number of houses by municipality type



Note: The yearly time series are based on the publicly available settlement level data of the HCSO, accessed 16/02/2020.

There is also substantial heterogeneity within the country regarding changes in prices and the number of sold houses, comparing the periods 2015-2019 and 2008-2014. Figure 3 shows the percentage change on the municipality level, with smaller municipalities aggregated to their next administrative unit ('járás'). Prices increased the most in Budapest and closer towns (by more than 50%, reaching levels of 80%), while at the bottom of the distribution we find the eastern areas of the country. However, the change in quantities sold correlates negatively with price changes: Budapest districts experienced a decline in the number of houses sold, while in rural areas higher numbers were sold than before, also reflected in increasing immigration to rural areas, and decreasing immigration to the center, Budapest.

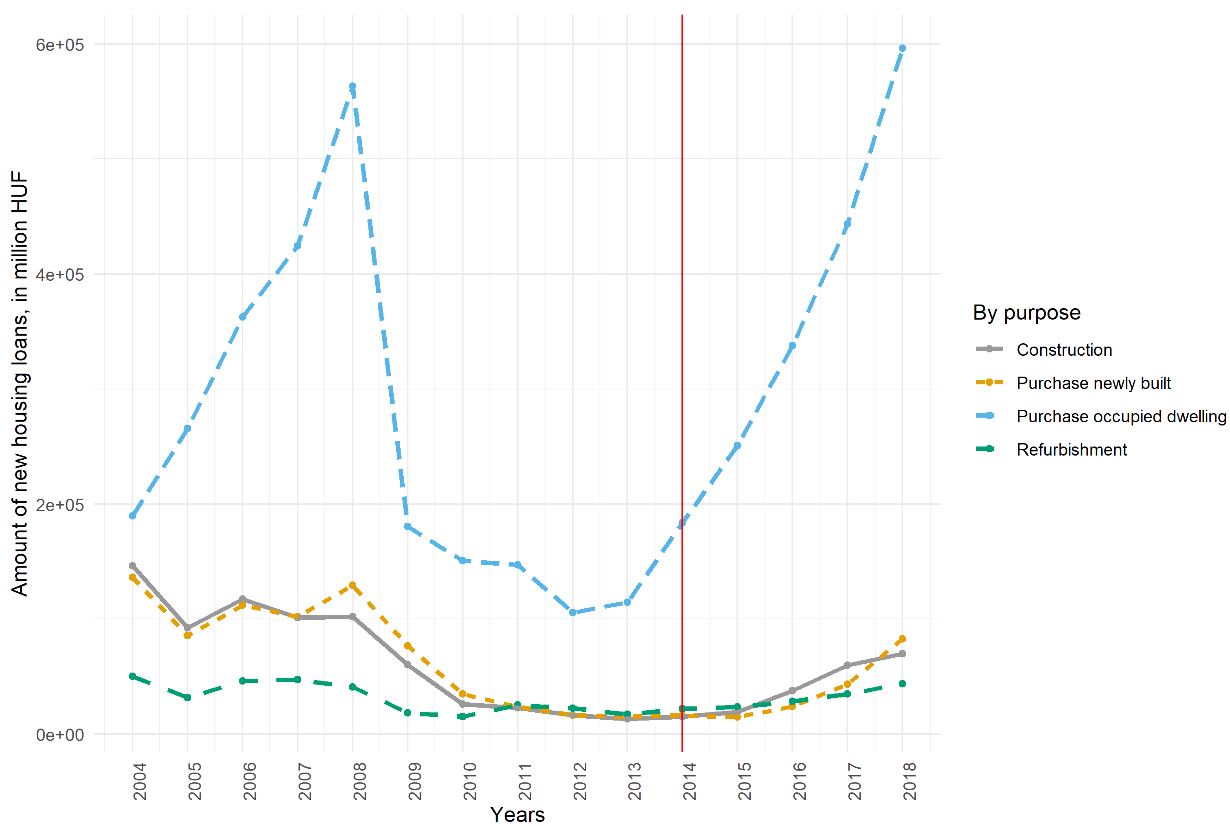
Figure 3: Change in average prices and number of houses sold in Hungary, 2015-2019 vs. 2008-2014



Note: based on the publicly available municipality level data of the HCSO, accessed 16/02/2020. The time windows reflect the state of the housing market of Hungary, before and after the introduction of the Family Housing Allowance.

The effects of the policy are far from trivial (HNB, 2021). On housing demand the impact seems to be clearly substantial, but not necessarily positive overall. Since 2016 around 20% of all mortgage contracts are connected to the FHA subsidy, but at the same time due to the surging house prices the accessibility of purchasing a house has been shrinking (HNB, 2019). That is especially true for the capital Budapest, which experienced the largest growth of house prices within the last decade in the European Union, reaching by the second quarter of 2019 around 230% of the 2009 price level. The supply side effect also manifests in the growing number of permits issued starting from 2015, but the number of newly built dwellings still remain at a low 12,000 a year, less than half of the 2007 level. Figure 4 displays the amount of newly issued housing loans, which shows that the growth in the related outstanding debt can be mainly attributed to purchasing occupied dwellings. A debate whether the FHA policy could have affected the number of births in Hungary is also ongoing. Kapitány and Spéder (2020) suggests that the increasing number of live births fit into the regional trends of recovering fertility levels, and might have happened regardless of the family policies. At the same time, Sági et al. (2017) show indicative evidence that some of the growth could be attributed to the policy introduced.

Figure 4: The amount of new housing loans issued in million HUF by purpose, 2004-2018



Note: based on the publicly available data of the Hungarian Central Statistical Office.

There are also other important stylized facts to consider, which caution against overly simplistic interpretations of the data points, and show important limitations to the model this paper presents. Based on the survey published by the Hungarian National Bank HNB (2019), between 2015 and 2019 more than 40% of all house purchases in Budapest could be categorized as 'with investment purpose', while the fraction of 'first home purchases' has fallen from around 35% to 20% of all transactions. It is also worth noting that while the FHA policy aims at buying newly constructed houses, their magnitude is small compared to the around 160,000 house market transactions a year, as shown by Figure A9 (even though the entire market might be affected due to substitution effects). So it might be important to disentangle in the future the effect of speculative housing investment on the housing life-cycle of young households, and the how the demand of young households themselves affect the market.

3 The model

3.1 Demand side: the problem of the household

The demand side of the model examines how fertility, residence size, and location choice of young households evolve over the life-cycle, with finite number of periods. Household formation and divorce are not included in the model, the couple have already finished their education, and the female member is 25 years old. Each period, they make a forward-looking decision to have an additional child, and about the size of the house, ownership, location, and choosing between non-durable consumption and savings. They can also apply for a government subsidy representing the Hungarian government's Family Housing Allowance (FHA), providing a one-time, lump-sum subsidy in case of a house purchase with the additional commitment to having three children. Each household i is characterized by their desired number of children (ν_i), the education of the male (ed_i^M), and the education of the female (ed_i^F). While education is observed, desired number of children remains unobserved, inducing unobserved heterogeneity in the model. However, the population distribution of desired children can be estimated, which allows for assigning households a value drawn from this distribution. Kapitány and Spéder (2020) also reports that these preferences stayed stable during the last decades. This allows the model to avoid assuming an error term distribution driven by the inference strategy itself as we can treat this distribution known. Households also face idiosyncratic uncertainty due to unemployment shocks, infertility shocks, and global uncertainty due to changing house prices.

The household's utility function $u(.)$ depends on the non-durable period consumption, and relevant state variables: the number of children in the household, the size of the residence, its location, and ownership status. Novelty in this paper's approach is that each household has their desired number of children, compared to the usual way of treating children in the utility as a purely 'normal goods' such as consumption. This means that more than the desired number will decrease utility ceteris paribus, so households would not opt to have infinitely many children, even with unbounded resources. However, the housing allowance itself might induce households to end up with more than their desired number of children.

Another feature is that the concept of housing services receives intuitive meaning in the model, as the household suffers disutility from crowdedness of the dwelling in the form of having the number of children divided by the house size in the utility function, providing a natural mechanism for larger families sorting into larger houses. House sizes are discretized instead of continuous 'housing services', and households derive extra utility from ownership, both aspects featured also in Attanasio et al. (2012).

Regarding locations, households suffer disutility (or gain utility), from being further away from the central location, resembling one version of the standard monocentric city model of urban economics (Duranton and Puga, 2015). It represents amenity or disamenity effects of living in the central area of the economy, an idea appearing for instance in Brueckner and Zenou (1999) or Combes et al. (2019). In the model I introduce two locations representing the center and rural areas, each of which has a separate housing market supply, connected by the demand for housing from households concerning both locations. Wages received by the agents also differ according to the location of the household, calibrated to the Hungarian case: wages are higher in the central location, but not reacting endogenously to household decisions.

I introduce two house sizes. Size 0 represents a 50 m^2 apartment (approximately 1-2 rooms), while Size 1 represents the ownership of a 100 m^2 house (approximately 3+ rooms). The price of housing can vary by locations but is given in m^2 prices, hence the price of 50

m^2 residence is exactly half of a 100 m^2 one[5]. Table 3.1 shows the empirical counterparts of these idealized concepts based on the 2011 Census of Hungary. Looking at all residences in Hungary, around 2 million of them has 1-2 rooms, and around 2 million of them has 3 or more, with median sizes of 54 and 90 m^2, which types of housing are approximately represented in the model. Not being exposed to costs of owned housing is another simplification, which captures the idea of imputed rents, found to benefit house owners substantially (Kilgarriff et al., 2019). Households can choose between renting a house and purchasing one, while also choosing the size of the house. In case the residence of the household is not owned, the household must pay rents, set at 5% of the house price corresponding to a Hungarian context (and also similarly to Attanasio et al. (2012)).

Table 1: Distribution of house sizes in Hungary, 2011

No. of rooms	No. of houses	Median size (m2)	Mean size (m2)	Std. Dev.
1-2	2,135,857	54	56.93	19.41
3+	2,233,623	90	93.99	29.14

Note: the author's calculation based on the 2011 Census of Hungary.

I incorporate the Family Housing Allowance into the model the following way. As the policy in its entirety is quite intricate and would require too many new state variables, I refrain from including all details. First, I only consider the original version of the allowance with the condition of three children, as the size of the subsidy in that case is considerably larger than for two children (10 million HUF compared to 2.6 million HUF). Second, the fertility commitments of the households are checked only at the terminal period of the household, representing the end of fecundity, while the actual regulation gives couples four years to have one additional child, eight years for two, and ten years for three children. This way I avoid having to track the birth years of the children, reducing computational needs significantly. Third, the dwelling purchased using the FHA subsidy cannot be sold in the model, even though in reality selling is forbidden only for ten years; this simplifies the

[5]The ratio of the two house prices could have been set as a parameter such as Attanasio et al. (2012), here I simplify to have fewer parameters by assuming that house prices scale linearly with size

state space of the model. Fourth, the subsidy can only be applied for when the households purchase a large house, a 100 m^2 in the model, representing that the policy has a minimum 60 m^2 built-in threshold regarding size[6]. And finally, only those households are eligible, for which the age of the household (age of the female) is lower than the maximum age for which birth is possible, which is set at 40 years in the model.

The setup of the problem builds on the standard macroeconomic models for mortgages, presented in Davis and Van Nieuwerburgh (2015). Households are modeled to be alive for a finite number of periods, making decisions from 25 until 45 years of age of the female (10 periods, or 20 years), and living an additional 20 years [7]. When they remain both in the same type of house and the same location, they are considered not to move, otherwise they are always assumed to be able to sell or buy for the market price of the house. In case they move to a residence of a different size, or to a different location, they suffer a cost as in Attanasio et al. (2012), set at 10% of the price[8], representing transportation and legal costs of the choice. Houses are expensive (prices are calibrated to reflect the property prices of Hungary), and households can use savings, their property, and mortgages to purchase them. If they do not own a house, they are required to rent an apartment. Outside of mortgage loans, households are only allowed to have positive savings, which they could also use for consumption smoothing. The repayment schedule of mortgages are not specified, providing flexibility and inclusion of several regimes, although different institutional contexts are found to affect household behavior (Chambers et al., 2009).

3.1.1 Choice variables

Choice variables include non-durable consumption ($c_{i,t} \geq 0$) set simultaneously with next period's savings ($S_{i,t+1} \in \mathbb{R}$), the willingness to have an additional child ($n_{i,t} \in \{0,1\}$), house ownership ($o_{i,t} \in \{0,1\}$), size ($h_{i,t} \in \{0,1\}$) and location ($l_{i,t} \in \{0,1\}$), the choice of taking out a mortgage ($m_{i,t} \in \{0,1\}$), and the choice of applying for the government subsidy Family

[6]Note, that a study of the Hungarian National Bank (HNB, 2019) finds that this hard threshold itself might have induced a distribution change of size among newly built apartments such that instead of the 50 m^2 apartments 60 m^2 ones have started to be constructed.

[7]About the impact of choosing finite vs. infinite number of periods, see Hedlund (2018).

[8]Compared to Attanasio et al. (2012) which uses 5% for the United Kingdom, I implement a 10% cost to reflect the considerably lower levels of real estate prices in Hungary

Housing Allowance ($f_{i,t} \in \{0,1\}$).

The rules for the choices are as follows. Each household can own up to one residence. Savings can be used for consumption smoothing or mortgage repayments, however the only case a household can get indebted is when they take out a mortgage. This restriction is in accord with evidence that historically mortgage debt has constituted around 70% of all household debt (Piazzesi and Schneider, 2016). Following Attanasio et al. (2012), I allow households to take out a mortgage even if they choose not to move (borrow against their property). This follows the results of Cloyne et al. (2019), who find that house prices are the drivers of households' ability to borrow via collateral effects. Households are however not allowed to have more than one mortgage contracts at the same time, but they are allowed to apply for a new one after they have repaid the previous one. They are also forbidden to sell their house if they have a mortgage against it. They are also forbidden to sell if they purchased it using the FHA subsidy. In case they are indebted in one period, they must weakly increase their position over time. Children can only be conceived until 15 years into the model, representing the end of the female fertility cycle at around 40 years. They are also forbidden to apply for the government subsidy after that time, which condition is also present in the regulation.

3.1.2 State variables

Households are assigned at the beginning their desired number of children ($\nu_i \in \{0,1,2,3\}$), and the education of male and female adults ($\mathrm{ed}_i^M \in \{0,1\}, \mathrm{ed}_i^F \in \{0,1\}$, 1 representing completed tertiary education). $\Omega_{i,t}$ collects the state variables for household i in period t, consisting of the following: house prices ($p_t^H(h_{i,t}, l_{i,t}), \forall h \in \mathcal{H}, l \in \mathcal{L}$), employment status ($\mathrm{e}_{i,t}^M \in \{0,1\}, \mathrm{e}_{i,t}^F \in \{0,1\}$), number of children ($N_{i,t} \in \{0,1,2,3\}$), current house type and location ($H_{i,t} \in \mathcal{H} = \{0,1\}, L_{i,t} \in \mathcal{L} = \{0,1\}$), and the savings of the household ($S_{i,t} \in \mathbb{R}$), with additional states tracking the household's status status regarding mortgage ($M_{i,t} \in \{0,1\}$), ownership status ($O_{i,t} \in \{0,1\}$) and the government subsidy ($\mathrm{F}_{i,t} \in \{0,1\}$ for Family Housing Allowance). Lastly, households face infertility shocks ($\iota_{i,t} \in \{0,1\}$) which are realized only after making the decision of having an additional child or not. The distribution of infertility shocks are dependent on the female's age and education, and is considered to be known by the households.

The number of children, the house size and location, savings, mortgage and government subsidy are determined in the previous period endogenously by the optimal choices of the households, taking the infertility shocks into account. I do not introduce uncertainty into the survival of the household, examined for instance for the case of divorce by Fischer and Khorunzhina (2019), or death. It is plausible that the policy affects couple formation and separation as well, however these aspects are beyond the scope of this paper.

3.1.3 Stochastic processes

There are five stochastic processes in the model: the employment status of the male and of the female adults of the households separately, infertility shocks depending on the age and education of the female, and the house prices of the central and the rural location. Unemployment shocks are realized before the choices of each period, house prices are determined jointly with household decisions, while infertility shocks are realized after choices are made, but the distribution of infertility shocks are known to households.

The employment status of the male and the female adults follow separate, two-state Markov processes, which depend on the level of education[9]. Households form correct beliefs over the transition probabilities of employment status. These probabilities are set such that the stationary state unemployment rate corresponds to the long-run unemployment rates in Hungary conditional on education for the relevant time period, which are around 9% for lower than tertiary, and 3% for tertiary educated.[10].

House prices are calculated as feasible 'pseudo-equilibrium' outcomes which concept I introduce later in detail. Households are assumed to be price takers, and they form their expectations in a naive way over future m^2-prices such that $\mathbb{E}_t[p^H_{l,t+1}|\Omega_{i,t}] = p^H_{l,t}$, for $l \in \{0,1\}$. This can be also seen as households falsely believing that the time series of house prices follow a simple random walk process. Even if within the frames of the model this belief is false, there is evidence such that real house price time series are indeed random walks (Holly

[9]Although there could be good reasons to consider location dependent employment transitions, a possible future improvement of the model.

[10]Based on the data of the Databank of the Institute of Economics, available as of 03/02/2021 here, and here.

et al., 2010). It is also worth noting, that more sophisticated expectation formation could be introduced to the model, which is a possibility for a future extension.

Infertility shocks are realized after households have made a decision to have an additional child, while they are also aware that having a child at different points in their life-cycle implies different risks of infertility. I estimate these distributions conditionally on female age and education[11], as the fraction of miscarriages of births plus miscarriages, for the period 2004-2014, using the complete individual level administrative data collected by the Hungarian Central Statistical Office. Note that due to miscarriage events often not being discovered or reported I probably underestimate this probability. Also note, that I do not consider abortion in the model, or undesired pregnancies.

3.1.4 Initial and terminal conditions

Households are assigned an inheritance in the form of home ownership, drawn from the distribution estimated from the available Household Budgetary Survey of Hungary for the period 2004-2014 (introduced later in more detail), along with house size, mortgage and location distributions at the age of 25 of the female. Only those households can have mortgages which are simultaneously assigned home ownership at the start as well, in accordance with the model's logic. These households also start out with negative savings corresponding to the down payment for the owned house, set at 50% of the value of the house priced at the 2004-2014 levels, with a hard cap of 10 million HUF to avoid poorer households not being able to repay their debt.

Households start out employed, with their education and their desired number of children fixed. The education group along with the number of initial children of the household (at the age of the female at 25) are drawn from their joint distribution estimated from the 2011 Census ($\mathbb{P}^{\text{ed}^{M,F},N}$), which then captures assortative mating and selection into education tracks due to preferences, studied for instance in Adda et al. (2017). The desired number of children is drawn from population distribution $\mathbb{P}^{\nu}$, independently from the educational attainment of the parents, as this piece of information is not available.

[11] I also estimated the distributions w.r.t. child parity, but it did not change the probabilities significantly, so I kept the simpler version.

The terminal period savings are also impacted by the household's government subsidy uptake. They are allowed to violate the children commitment to the government, but it comes with a penalty according to the legislation. If they decide not to respect the requirement of having the number of children they committed to, they are forced to pay back the remaining corresponding amount of subsidy to the government as a lump sum at the end. If they have two instead of three children, they have to pay 7,400,000 HUF (∼20,000 EUR), if less than two, they have to pay 10,000,000 HUF (∼30,000 EUR)[12]. Defaulting on the debt is not allowed in the model.

The terminal value $V_{i,T+1}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}$ captures an additional 20 years of life with the same housing conditions and the number of children they have at time T. The non-durable consumption value $\underline{\mathrm{c}}_i$ is imputed into the post-terminal utility function, which is calculated the following way. Households are required to have a non-negative asset position by the end of the time horizon, set at 20 years. After having checked the conditions for the number of children due to the government subsidy, the residual savings after calculating any penalties are checked to be non-negative. These savings are then consumed in equal shares for the following twenty years[13]. The net household income is also assumed to stay constant for the future periods, and the probability of unemployment is set to 0. The choice of specifying a terminal value is necessary from a modeling perspective as households would often sell their residence in the last period if they see no further utility in keeping it. It is equivalent to a particular type of bequest where the utility derived from the additional 20 years of owning the residence is providing the bequest incentive.

[12] I abstract away from the penalty interest paid, as it does not change the magnitudes of the penalty even if they might be large, but would introduce unnecessary complexity in the computations.

[13] There could be a future point of improvement, by setting consumption to be based on the consumption Euler equation. In this iteration of the project, I stayed with this simpler setup

3.1.5 Period utility function

Households derive instantaneous utility from non-durable consumption, the number of children, the crowdedness of the house, location, and home ownership the following way:

$$u_{\text{ed}^F}(c_{i,t}, N_{i,t}, H_{i,t}, L_{i,t}, O_{i,t}) =$$
$$\frac{(c_{i,t}-1)^{1-\gamma_{\text{ed}^F}}-1}{1-\gamma_{\text{ed}^F}} \exp\Big(- w^N_{\text{ed}^F}(N_{i,t}-\nu_i)^2 - w^H \frac{N_{i,t}+1}{I[H_{i,t}>0]+1} - w^L L^2_{i,t} + w^O I[O_{i,t}>0]\Big)$$

where $I[.]$ denotes the indicator function. The utility function is multiplicative reflecting complementarity between the components of the function, often used in the literature. As mentioned earlier, ν_i is the desired number of children, which is the source of unobserved heterogeneity that might induce different choices for observationally identical households. However, I censor the distribution to include only 0-3 preferred children, with values above three incorporated into the highest category. I also let agents to have three children at maximum. Notice that households with lower and higher educated females are allowed to have different risk aversion parameters (γ_{ed^F}) and different preferences for children ($w^N_{\text{ed}^F}$), the latter represented as suffering disutility from being further away from the desired number. Allowing for education-dependent parameters captures previous self-selection into different education and career tracks according to preferences about children and consumption, as indicated by Adda et al. (2017). Consumption is constrained from below by 1 with a price of p^c, which denotes the living costs corresponding to the subsistence level based on the calculations of the Hungarian Central Statistical Office (HCSO, 2016)[14]. House size $H_{i,t}$ is coded as binary variable, representing 50 and 100 m^2 apartment sizes as 0 and 1. However, the number of children also affects the crowdedness component (third term) causing disutility, calculated as the number of children over the house size, where rented or owned apartments are half the size of a rented or owned house or large apartment[15]. The household also derives utility/disutility from being further away from the city centre in terms of a squared distance representing city center or capital amenities. Ownership status also provides positive utility,

[14] I do not normalize by this value in order to impute nominal prices more easily during the model development.

[15] Differently from Attanasio et al. (2012), renting larger houses is allowed for the households.

as it is generally assumed, and found in the literature (Davis and Van Nieuwerburgh, 2015).

3.1.6 Budget constraint

$$p^c\Big(1+\frac{N}{3}\Big)c_{i,t} + S_{i,t+1} + \kappa(n_{i,t}, N_{i,t}) + p_t^H(h_{i,t}, l_{i,t}) + \mu(H_{i,t}, L_{i,t}, h_{i,t}, l_{i,t}, \mathrm{F}_{i,t}, M_{i,t}) +$$
$$\rho(H_{i,t}, L_{i,t}) \leq NW(W_{i,t}^M, W_{i,t}^F) + (1+ir)S_{i,t} + \mathrm{FHA}(f_{i,t}) + p_t^H(H_{i,t}, L_{i,t}, O_{i,t})$$

The budget constraint builds on the baseline setup of macroeconomic models with mortgage decisions (Davis and Van Nieuwerburgh, 2015). The first term is the value of the total household consumption $p^c\Big(1+\frac{N}{3}\Big)c_{i,t}$, with a child's consumption corresponding to about one third of an adult couple, in accordance with the estimates of the Hungarian Central Statistical Office (HCSO, 2016). This term also introduces the flow cost of children in terms of consumption such that higher consumption of adults in the household also induces higher spending on children in a linear way. Therefore we implicitly assume a structure of preferences regarding child quality, which is omitted from this model (discussed for instance in Sommer, 2016). Next period's savings are denoted by $S_{i,t+1}$, as discussed earlier. The function $\kappa(n_{i,t}, N_{i,t})$ denotes the costs and benefits associated with children in the household. It includes child benefits, however also the out-of-pocket spending for giving birth in Hungary, which might be substantial, even if not reported officially[16]. The function $p_t^H(h_{i,t}, l_{i,t})$ assigns a price to a house with house type $h_{i,t}$ and location $l_{i,t}$, while $\rho(H_{i,t}, L_{i,t}, O_{i,t})$ gives the rental cost, or house service costs associated with the house type at t (the latter assumed to be 0 for now). Finally the function $\mu(.)$ introduces moving costs, which gives incentives for agents to remain sedentary (as often used in the literature, per Davis and Van Nieuwerburgh, 2015). The moving costs are set up such that houses under mortgage commitment cannot be sold, while being 10% of the house price in case of selling.

The revenue side of the budget is the following. The net income of the household, $NW(.)$, is the function of the gross incomes of the male and female in the household[17]. Unemployment

[16]Based on the blog of the Hungarian National Bank, in Hungarian: https://novekedes.hu/elemzesek/ne-kamuzzunk-sulyos-szazezrekbe-kerul-magyarorszagon-az-ingyenes-szules

[17]Since 2011 the number of children plays a much larger role in personal income taxation, however for simplicity in the model I set taxes at 42%, consisting of a 15% personal income tax, and a 27% social contributions, reflecting the taxation of the era.

spells are set to one year, during which the unemployed person receives lower wages[18]. In case of a new child, the salary of the woman is accounted for as two thirds approximating the maternity policies of the government in Hungary (Makay, 2020).

Gross income itself is given by a Mincerian reduced form equation of education, employment status, location choice, and experience (here measured by age of the household). Women face less experience due to their number of children to address the opportunity cost of child birth, and each child counting as two years less experience[19]. The equations are the following, where $e_{i,t} \in \{0,1\}$ represents employment, and $W_{\min}$ denotes the amount representing unemployment benefit:

$$W_{i,t}^{M} = \mathrm{e}_{i,t} \exp(\beta_0^{W,M} + \beta_1^{W,M} X_{i,t} + \beta_2^{W,M} X_{i,t}^2 + \beta_3^{W,M} \mathrm{educ}_i^M + \beta_4^{W,M} L_{i,t}) + (1 - \mathrm{e}_{i,t}) W_{\min}$$

$$W_{i,t}^{F} = \mathrm{e}_{i,t} \exp(\beta_0^{W,F} + \beta_1^{W,F} \min\{X_{i,t} - 2N_{i,t}, 0\} + \beta_2^{W,F} \min\{X_{i,t} - 2N_{i,t}, 0\}^2 + \beta_3^{W,F} \mathrm{educ}_i^F + \beta_4^{W,F} L_{i,t}) + (1 - \mathrm{e}_{i,t}) W_{\min}$$

The parameters are estimated by regressing log of gross wages on experience, education and location, separately for the two genders[20]. Note that these parameters should not be interpreted as causal effects of the variables, they should only be interpreted as coefficients of the best linear prediction.

[18] Calibrated to the minimum wage as of 2011 at around 80,000 HUF a month

[19] In accordance with maternity benefits design of Hungary, Makay (2020)

[20] I omitted here the variance parameter of the log-normal distribution which would be the appropriate form for expected value.

3.1.7 Recursive form

The household problem can be summarized in a recursive form the following way:

$$V_{i,t}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}(\Omega_{i,t}) = \max_{\substack{c_{it}\geq 1, n_{i,t}\in\{0,1\},\\ h_{i,t}\in\mathcal{H}, l_{i,t}\in\mathcal{L}, o_{i,t}\in\mathcal{O}\\ m_{i,t}\in\{0,1\}, f_{i,t}\in\mathcal{F}_{i,t},\\ S_{i,t+1}\geq S_{i,t}I[S_{i,t}<0]+\\ -m_{i,t}(1-\delta)p^H(h_{i,t},l_{i,t})I[S_{i,t}\geq 0]}} u_{\mathrm{ed}^F}(X_{i,t}) + \beta\mathbb{E}_{\sigma_{i,t+1}}[V_{i,t+1}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}(\Omega_{i,t+1})|\Omega_{i,t}]$$

$$X_{i,t} = (c_{i,t}, N_{i,t}, H_{i,t}, L_{i,t}, O_{i,t})$$

$$\Omega_{i,t} = (\sigma_{i,t}, H_{i,t}, L_{i,t}, O_{i,t}, S_{i,t}, M_{i,t}, \mathrm{F}_{i,t})$$

subj. to:

Budget constraint:

$$p^c\Big(1+\frac{N}{3}\Big)c_{i,t} + S_{i,t+1} + \kappa(n_{i,t}, N_{i,t}) + p_t^H(h_{i,t}, l_{i,t}) + \mu(H_{i,t}, L_{i,t}, h_{i,t}, l_{i,t}, \mathrm{F}_{i,t}, M_{i,t}) +$$
$$\rho(H_{i,t}, L_{i,t}) \leq NW(W_{i,t}^M, W_{i,t}^F) + (1+ir)S_{i,t} + \mathrm{FHA}(f_{i,t}) + p_t^H(H_{i,t}, L_{i,t})$$

State transitions:

$$N_{i,t+1} = N_{i,t} + n_{i,t}\cdot\iota_{i,t}, L_{i,t+1} = l_{i,t}, H_{i,t+1} = h_{i,t}, F_{i,t+1} = F_{i,t} + f_{i,t}$$

$$M_{i,t+1} = \begin{cases} 0, & \text{if } M_{i,t}=1 \text{ and } S_{i,t+1}\geq 0 \\ M_{i,t}+m_{i,t}, & \text{otherwise} \end{cases}$$

Initial conditions:

$$\nu_i \sim \mathbb{P}^{\mathrm{nu}}, \iota_{i,t} \sim \mathbb{P}_t^{\mathrm{ed}_i^F},$$

$$(\mathrm{ed}_i^M, \mathrm{ed}_i^F, N_{i,0}) \sim \mathbb{P}^{\mathrm{ed}^{M,F},N}, (\mathrm{e}_{i,0}^M, \mathrm{e}_{i,0}^F) = (1,1),$$

$$H_{i,0} \sim \mathbb{P}^H, L_{i,0} \sim \mathbb{P}^L, O_{i,0} \sim \mathbb{P}^O, M_{i,0} \sim \mathbb{P}^M$$

$$\textit{Terminal value: } V_{i,T+1}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}(\Omega_{i,T+1}) = \sum_{s=0}^{19}\beta^s u_{\mathrm{ed}^F}(\underline{\mathrm{c}}_i, N_{i,T+1}, H_{i,T+1}, L_{i,T+1})$$

where i denotes a household, t denotes time periods which at this point correspond to years of age for the female adult of the household, $\sigma_{i,t}$ collects the random states of employment, house prices, and number of children, that the agents take expectation over. $u(.)$ denotes the utility or period utility function, β denotes the discount factor. Interest rate (ir) is set as a baseline at 5% a year[21] reflecting the 2013-2019 mortgage interest rates of Hungary (HNB, 2019), and $\beta = \frac{1}{1+ir}$ is set accordingly. The down payment ratio (δ) is also fixed, as a baseline at 0.5[22]. Using fixed interest rates implicitly assumes fixed-rate mortgages (vs. adjustable rate mortgages). This type of mortgage contracts has been the most prominent option in Hungary in the last decade (HNB, 2019), and it constitutes the majority in the U.S. as well (Piazzesi and Schneider, 2016).

3.2 House supply

House supply introduces several complications into the analysis both analytically and computationally, hence many in the literature treat house prices as exogenous to the household problem (Duranton and Puga, 2015). The price of housing is composed of two elements: the price of the land, and the price of the structure itself. Piazzesi and Schneider (2016) documents that the overwhelming majority of historic variation of house prices can be attributed to land prices. It is easy to see, that rising land prices can be the result of a sudden increase in demand, which an inelastic supply cannot follow fast enough. In my paper I focus on this phenomenon, which requires some level of equilibrium responses of prices. There are instances in the literature where supply is explicitly modelled such as Glaeser et al. (2008), however their model would not be feasible in this setting.

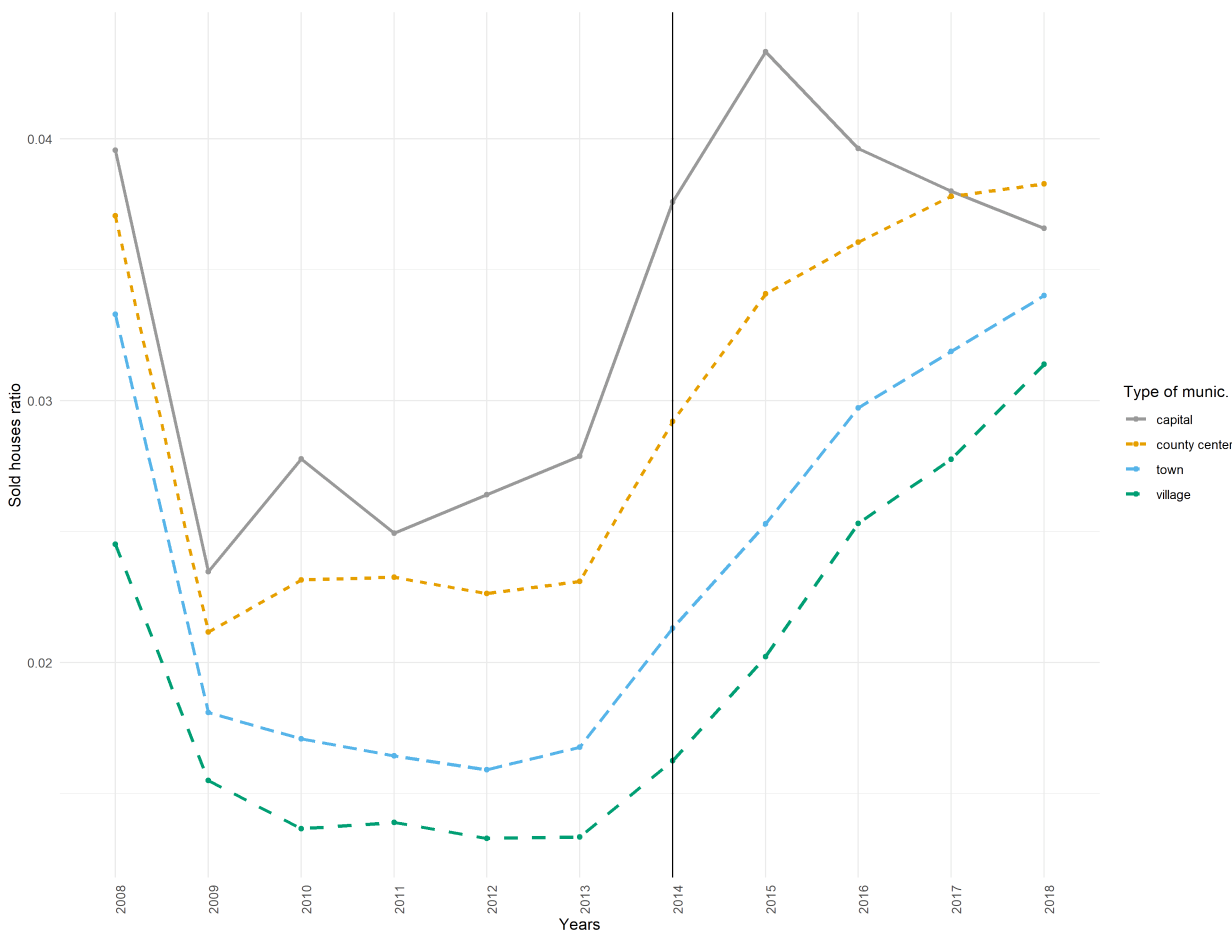
In my model the supply of housing has three components. First there is a constant per period number of available m^2 space for newly built houses, governed by calibration parameters for each location denoted as (α^0, α^1), representing a 'fixed stream of housing services' (Piazzesi and Schneider, 2016) often used in the literature. The revenue generated by selling these houses are not contributing to the household budgets, and I do not consider this income in the model. The available external supply of housing measured in m^2 is then calculated the following way for location l: $H_l^S = \alpha^l \cdot |\mathcal{I}| \cdot 50$ where $|\mathcal{I}|$ denotes the number of households in a cohort, in the model. So the parameter captures the relation between the amount of newly available space and the number of new households entering the housing

[21] In case of biannual periods, it is set as $ir = (1.05)^2 - 1$

[22] According to the calculations of the website Bankmonitor.hu, specialising in mortgage loans, banks typically require 0.4-0.5 down payment ratio in Hungary. Source in Hungarian:, accessed 29/07/2020.

market on the demand size. This component has the most relevance in this policy context as the Family Housing Allowance originally aimed at purchasing newly constructed dwellings. In this version of the model, this component is not reacting to price changes corresponding to a fully inelastic housing supply; experimenting with additional elements to the supply side could provide valuable model extensions. However, as Figure 5 shows, the number of real estate transactions is less than 5% of the total available houses for all municipality types, suggesting that even with sharply increasing house prices, an overwhelming fraction of owners do not sell their property, but at the same time as we have seen earlier, house transactions dominantly occur in the domain of owner-occupied properties.

Figure 5: Fraction of houses sold of total, by type of municipality, 2007-2018



Note: the author's calculations based the publicly available data of the Hungarian Central Statistical Office.

Second, I introduce a set of 'old' households starting with house ownership, and maximizing expected utility similarly to the young households in a simplified model: without the ability to have children, or to take up a mortgage or the government benefit. They react to changes in prices, and present a finite stock of housing available on the housing market reacting to changing prices, decreasing weakly as we move forward in time. However, in other applications of the model this part of the supply could play a significantly more important role. And third, young households also constitute part of the housing supply in case they change their residence. It is important to mention, that I abstract away from several aspects which could play a role, such as quality of the apartments, optimal portfolio decisions, AirBnB, and most crucially, speculative or investment focused house purchases are not considered here. These are all parts of that might be worth investigating further in the future.

3.3 'Pseudo-equilibrium' in the housing market

House prices are present in the model as state variables, which due to constraints of feasibility must have a small number of discrete levels. That setup does not allow for an actual temporary equilibrium price discussed in Piazzesi and Schneider (2016), which would clear the housing market for each locations in each time period. Instead, I employ the following algorithm, that enables period-by-period 'pseudo-equilibrium' prices and allocations to emerge:

1. Fix period t
2. For each combination of possible location-m^2-prices collected into vector $p^H \in \mathbb{R}^2_+$, we calculate for each household i and for each location l the net demand for total space, denoted by $h^D_{i,l}(p^H)$
3. For each combination of location-m^2-prices, we also calculate the external net supply for each location l as a sum of fixed external stream of housing, added to the housing sold by the old households, denoted jointly by $h^S_{j,l}(p^H)$, j indicating different sources of supply

4. We sum up the net demand for space of the households, and the net supply from different sources, for each locations

5. We calculate the squared difference of the total net demand from the total net supply, and select the m^2-price yielding the least distance as the 'pseudo-equilibrium' price

So the temporary 'pseudo-equilibrium' price vectors for the housing markets can be defined the following way:

$$(p^H)^* \in \text{argmin}_{p^H \in \mathbb{R}^2_+} \sum_{l \in \{0,1\}} \omega^l \left(\sum_{i \in \mathcal{I}} h^D_{i,l}(p^H) - \sum_{j \in \mathcal{J}} h^S_{j,l}(p^H) \right)^2$$

where $p^H \in \mathbb{R}^2_+$ denotes the m^2-price vectors, $h^D_{i,l}(p^H)$ denotes the net housing space demand of household i in location l given the m^2-price vector, and $h^S_{j,l}(p^H)$ denotes the net housing space supply provided by either an 'old' household, or provided externally. The ω^l parameters govern how much weight we put on each location's deviation from its market clearing, which could be helpful in case we wanted to have asymmetric approximate market clearing conditions for different housing markets.

This algorithm would yield a price vector for each time period and each location for the housing market, such that it would close the gap between demand and supply as much as the discretization allows for. Note, that this mechanism plays the role of the auctioneer in the Walrasian setting of equilibrium price, and in case there exists a temporary equilibrium price vector such that actual market clearing occurs for each market, with sufficiently fine price grids this algorithm would find the equilibrium prices. Uniqueness is generally not guaranteed (Duranton and Puga, 2015), which stands for this algorithm as well.

3.4 Model solution

The model can be solved by backward induction, yielding a unique optimum due to the strict concavity of the utility function in non-durable consumption. First, I solve the model for young and old households, which results in generating value and policy functions. In this model prices are in temporary 'pseudo-equilibrium', meaning that they are allowed to react to the exogenously given inelastic supply of housing, interacting with the house demand of young households following its life-cycle optimization program. Therefore it is necessary that we simulate the entire histories of all housing markets jointly over the life-cycle of the agents, somewhat similarly to how it is treated in the literature when exogenous price processes are separately estimated such as Attanasio et al. (2012).

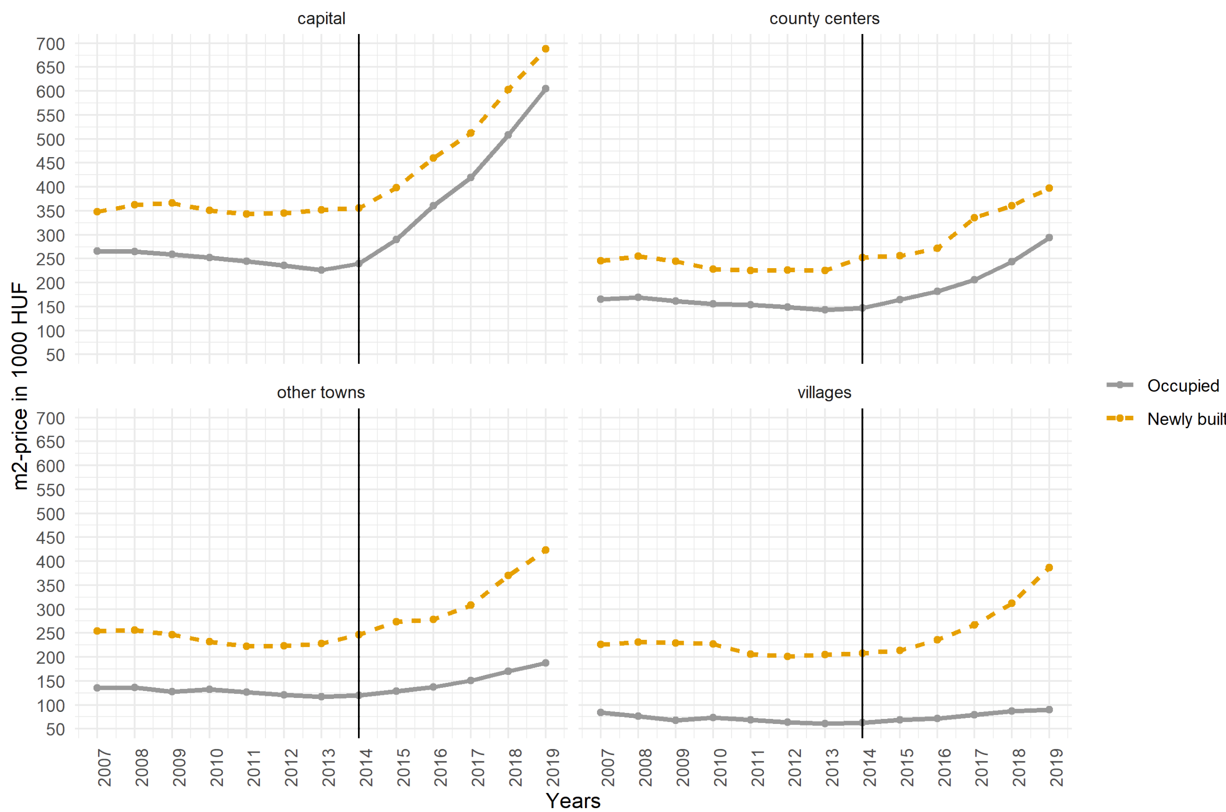
In the simulation I include two cohorts of 150 'young' households (called Cohort-0, and Cohort-1), and 150 'old' households, which provide price-sensitive supply. Cohort-0 households start at the beginning of historic time of the simulation at age 25, and make optimal decisions according to the prescriptions of their policy functions regarding their age corresponding to the historic time, in a price-taking manner. Cohort-1 households enter historic time 2 periods later also at age 25, and make optimal decisions, however they are lagging behind in their life-cycle. This modeling choice allows for distinguishing between the effects of the policy on the first impacted cohort, vs. future generations who already has to face the price impacts of the policy at an earlier stage in their life. This is the representation of the assessment of the Hungarian National Bank's comment on how the increasing house prices (if indeed the model generates that dynamic) might undermine the accessibility of housing for younger generations.

The m^2-price grids include six-six possibilities (Table 2) for each of the central and the rural locations, based on the historic price dynamics displayed in Figure **??**, in the Hungarian context (HNB, 2019). We can see, that for our estimation period the central location (capital) prices for newly built dwellings are around 350,000 HUF, while for the other locations they are around 250,000 HUF. As I describe later, I use these m^2-prices to estimate the demand parameters for the households.

Table 2: Possible m^2-prices of houses

Central location	200	300	400	500	600	700
Rural location	50	100	150	200	250	300

Figure 6: Yearly average m^2-prices by type of municipality and age of house, 2007-2019



Note: based on the house price estimates of the Hungarian Central Statistical Office.

The savings grids differ for each education group, which allows to represent the different credit limits given by their different earning powers, and also that I could set the savings grids finer for the lower educated group. Each grid contains around 20 grid points. As I use the biannual model version instead of the annual, the crudity of the savings grids plays an even less important role. As having mortgages could potentially result in large indebtedness, it is important to have a wide range of grid points for the savings accounts of households. Households are bound to choose a savings level from the grid point.

4 Estimation

I combine Hungarian survey and administrative datasets to estimate the model parameters using the 2004-2014 time period, when real estate prices did not experience large turbulences.[23] I employ Simulated Method of Moments to estimate the preference parameters in the utility function, while I use the aforementioned reduced form Mincerian regressions to estimate the wage regression parameters, which I treat simply as projection coefficients without any causal interpretation. I also estimate the probability distributions concerning the initial values of the state variables using the available information about households with cohabiting adults where females are of 25-26 years old. The supply parameters (α parameters) are then calibrated in a second step the following way. Given the utility function parameter estimates, I simulate the model with endogenous prices and without the subsidy using different (α^0, α^1) supply parameters. Then I select the ones that produce the smallest squared deviation from the (350,250) prices for the central and rural locations respectively that could be considered as closest to the equilibrium prices for the 2004-2014 period. Figure 6 shows the average m^2-price levels for newly built and older houses for reference: we can see that the target prices for the second step is slightly higher for the central (Budapest, capital) location then they were historically, however due to feasibility the price grids could not be extended further.

4.1 Data

The primary data source of the exercise is the Hungarian Household Budget Survey[24] , which is a yearly survey of private households, representative at the country level. The Hungarian Central Statistical Office collects detailed information on the consumption, income, housing, and several other demographic features of the population, which is then used to calculate the product weights of the consumer price index, the information also contributes to national account calculations. The structure of the surveys have changed over the relevant period of

[23]I am grateful for the help of the Databank at the Institute of Economics, for providing access to the datasets used in this paper.

[24]In Hungarian: "Háztartási Költségvetési és Életkörülmény Adatfelvétel" abbreviated as HKÉF, English description is available at
http://www.ksh.hu/apps/meta.objektum?p_lang=EN&p_menu_id=110&p_ot_id=100&p_obj_id=AEAA

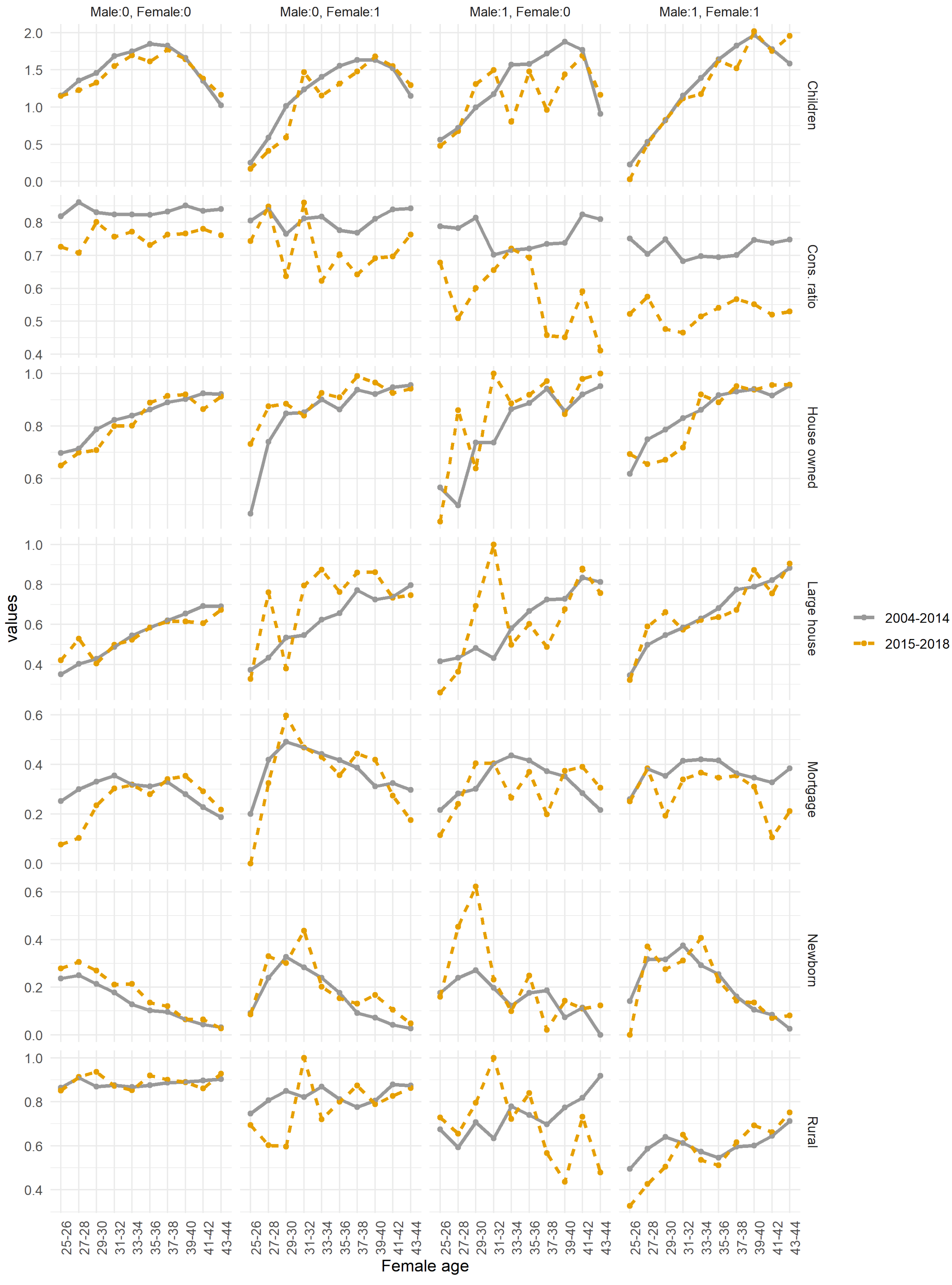
2004 and 2018, which required to maintain simple definitions of the variables that the model could map to. Most importantly, since 2010 the survey has not included detailed nominal household income information, I used the Hungarian Wage Survey data to impute expected net household wages conditional on the calendar year, age, gender, education, administrative region, and the type of municipality.

For each survey of 2004-2018, I kept in the sample those households which included two adults identified as the male and female members of a cohabiting couple, not necessarily in marriage. Then I created the following household level variables for the estimation of the model: consumption ratio (consumption as fraction of net household wage income), house size category where houses with 3 or more rooms are categorized as large, location category based on residing in the capital (Budapest) as a proxy for the central location in the model, house ownership, mortgage, newborn children (children less than 1 year old) and total number of children (children family status and less than 19 years of age). Finally, I pooled the pre-policy period of 2004-2014, and the post-policy period of 2015-2018 to increase the number of observations, as individually each survey contained a only a small number of households after conditioning on education and female age.

A major deficiency of the dataset is that it was not possible to reconstruct net savings position of the households, hence moments connected to this information cannot be used in the estimation. Another potential problem is connected to selection into being a couple, which is not addressed by neither the model, nor the sampling. The latter issue appears both regarding calendar time with changing family structures over time, and due to the actual effect of the policy on family formation itself. Addressing this selection problem could be a potentially important addition to the present model.

The following Figure 7 shows by education categories and periods, the averages of the main choice and state variables as a function of the age of the female. We can immediately notice the larger variance of the time series for the shorter period due to the smaller time window. Otherwise, we can see that most variables, such as the total number of children evolves over the life-cycle more or less similarly in both periods on average. Consumption ratio seems to be lower, which could be an unfortunate artefact of the wage imputation. The evolution of home ownership also seems to be similar starting from a high level, however we can notice a slight uptake in the ratio of larger houses in the latter period for younger households. Another thing to note is that at the older age of females, children start to leave the household resulting in a decrease in the number of children at the end, which is impossible in the model (hence I use the life-cycle maximum of the average, instead of the terminal value).

Figure 7: Average values for the main choice and state variables, 2004-2014 vs. 2015-2018



For the parameters in the Mincerian wage regressions of the model, I used the data from the Hungarian Wage Survey, which contains demographic and salary information about the labor force of firms with more than 5 employees. I regressed gross monthly salaries on a constant, age, age squared, and dummies for higher education and rural location for each gender, for each year. Figure 8 shows the point estimates with their 95% confidence intervals, we can see that over the periods there is substantial variance in the estimates of some parameters such as age. I use the 2011 estimates as parameter inputs for the model simulations.

Figure 8: Parameter estimates for the wage equation with 95% CI

Lastly, I estimated the distributions for the initial conditions of the state variables the following way. I used the 2011 Census data of Hungary to estimate the joint probability distribution of households where the female's age is 25-26 by male education, female education, and the number of children. For the other state variables (home ownership, large house, location, mortgage) I used the univariate distributions estimated from the Household Budget Survey, as they would have been overly saturated for a multivariate estimation, and the Census data held no information on mortgages.[25] Table 3 presents the household counts, and the initial moments for households by education group and period which the imputed distributions are based on.

Table 3: Averages of initial state variables by education and period, Household Budget Survey, 2004-2018

Female age	Male high educ.	Female high educ.	Period	Household count	Large house	Owned house	Rural	Mortgage	Children
25-26	0	0	2004-2014	661	0.35	0.70	0.86	0.25	1.15
25-26	0	0	2015-2018	110	0.42	0.65	0.85	0.08	1.15
25-26	0	1	2004-2014	119	0.37	0.47	0.75	0.20	0.25
25-26	0	1	2015-2018	6	0.33	0.73	0.69	0.00	0.17
25-26	1	0	2004-2014	56	0.42	0.57	0.68	0.22	0.56
25-26	1	0	2015-2018	15	0.26	0.44	0.73	0.11	0.48
25-26	1	1	2004-2014	138	0.34	0.62	0.49	0.26	0.23
25-26	1	1	2015-2018	16	0.32	0.69	0.33	0.25	0.03

Note: the author's calculation based on the Hungarian Household Budget Survey, 2004-2018.

In the model, the unobserved heterogeneity originates from the latent desired number of children, for which the distribution is estimated in the demographic literature (Kapitány and Spéder, 2015). This distribution is displayed in Table 4.

Table 4: Estimated distribution of the desired number of children

k	0	1	2	3
$\mathbb{P}^{\nu}(k)$	0.02	0.12	0.65	0.21

Note: based on (Kapitány and Spéder, 2015).

[25] It is possible however to use the Census for joint estimation regarding the other variables besides mortgage, although for some cells the number of observations would be low even for the entire population. I decided against this option because according to the 2011 Census, around 80% of the households with 25-26 year old females live in a house owned by them, which seems overestimated compared to the 46-70% ratios found in the Household Budget Survey. One possible explanation is that in the census cohabiting couples might not appear as officially many of them could have their permanent residence at their parents, so some cohabiting couples who live in a rented apartment might be accounted for as 'children' in the Census.

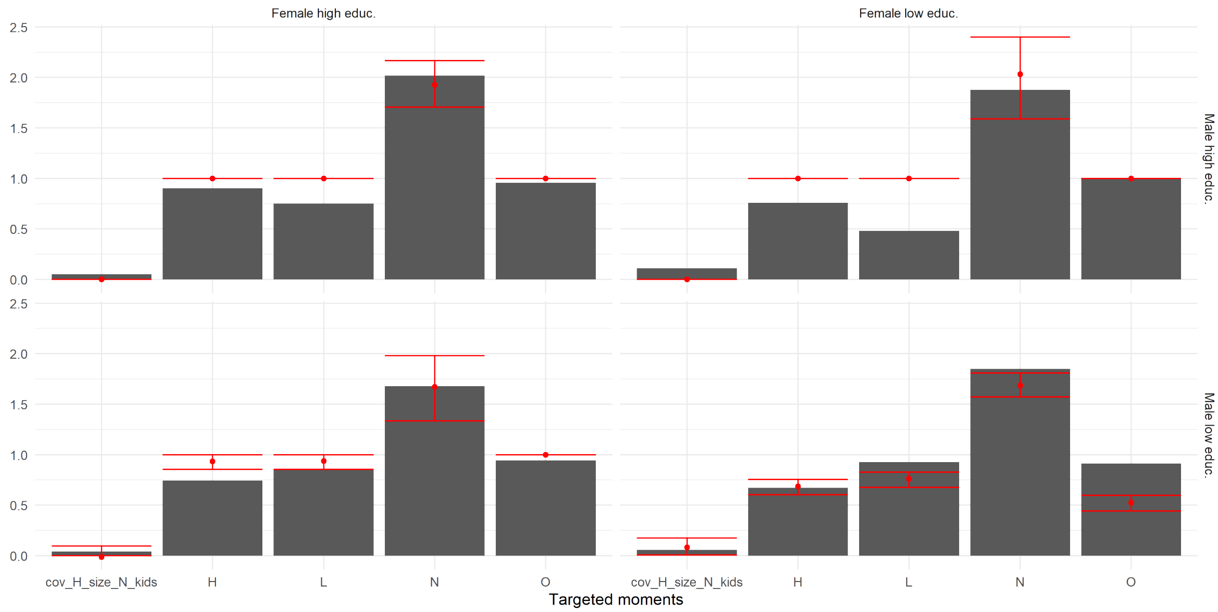
4.2 Model fit

In this subsection I examine the model fit diagnostics. To estimate the parameters, I use Simulated Method of Moments (SMM) to match on a set of moment conditions comprising of conditional expected values and covariances, for which the targets are derived from the Hungarian Household Budget Survey, mentioned earlier. In the loss function, I use simple squared loss function, without weighting the moments. I implemented the Cyclic Coordinate Search Algorithm (following Oswald, 2019), to estimate the demand parameters of the model. The algorithm fixes the previous guess of the parameter vector, and then converges to a minimum changing only one of the parameters.

I used the following terminal period moments conditional on each combination of parental education of the first cohort: home ownership rate, large-size house rate, number of children, rural location rate, and covariance between house size and number of children. For the model fit I used 200 Cohort-0 households without any other cohorts in 150 simulations, as I fix the house prices at 350,000 HUF and 250,000 HUF for the central and the rural location respectively, corresponding to the long-run average as we have seen in Figure 6 earlier.

The following Figure 9 displays the in-sample fit of the model, the grey columns show the targeted data moments, while the red error bar shows the mean simulated moment with the 5th and 95th percentile, resulting from 150 simulations. We can see that the model has varying success across the moments: while fertility seems to be well-captured, other aspects often run into corner solutions and overshoot (house size, or location for households of highly educated males), or in some other cases undershoot (low education home ownership) the target.

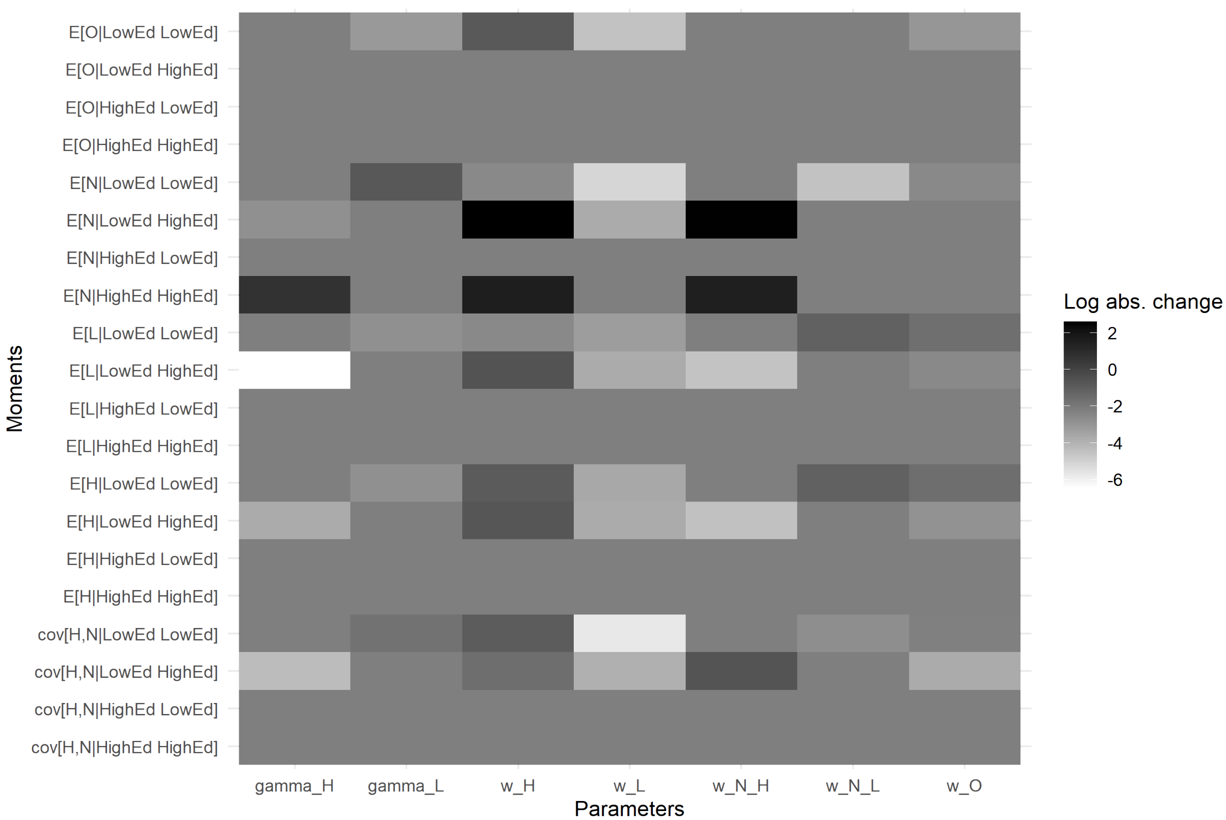
Figure 9: Fit of targeted moments



Note: model fit using 150 simulations of 200 cohort-0 households, grey columns show the target, while the red error bars show the simulated moment with its 5th and 95th percentile.

Identification of the demand parameters can be summarized by the following heatmap of Figure 10, showing the intensity of change in moments responding to a marginal change in parameter values. The results are not surprising. We can see that the CRRA-parameters (γ_{ed^F}) and the children parameters ($w^N_{\text{ed}^F}$) react mostly to the expected number of children, house crowdedness parameter w^H to house size and the number of children, while location w^L and ownership w^O affect the moments more weakly, but more uniformly.

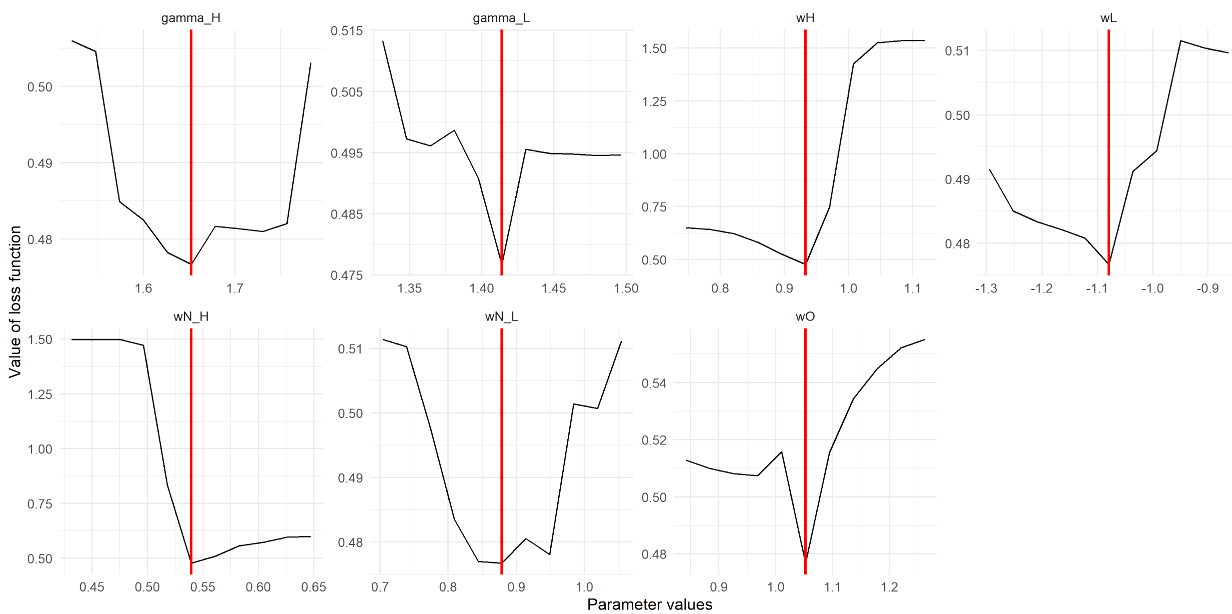
Figure 10: Identification of demand parameters



Note: using 150 simulations of 200 cohort-0 households.

Finally Figure 11 shows the values of the loss function as a function of the parameters, indicating that indeed we have reached a local minimum with the combination of the parameter estimates, in a considerable neighborhood.

Figure 11: Loss function by parameters



Note: using 150 simulations of 200 cohort-0 households.

4.3 Parameter estimates

Table 5 presents the parameter estimates and calibrations used for the model simulations. The parameters in the utility function are estimated by simulated method of moments discussed earlier, while the wage regression parameters are estimated using reduced form regressions. As discussed earlier, the supply parameters are chosen to minimize the distance from the price levels treated as equilibrium prices under the flexible price regime.

We can see that the γ parameters of the CRRA-function are estimated to be close to the value common in the literature (as discussed by Attanasio et al. (2012), around 1.5), and found to higher for households of females with higher education. The w^N-parameters governing the disutility of being distant from the desired number of children is found to be higher for females with lower education, consistent with the findings of Adda et al. (2017) that early career choices incorporate future fertility plans for females. The preference parameters regarding housing conditions indicate that given the model is correct, crowdedness (w^H), location (w^L) and home ownership (w^O) all play an important roles in the life-cycle behavior of households, however the estimate for the location parameter is not statistically significant. Households are also found to appreciate ownership, and dislike crowdedness (in accordance with intuition, and the literature).

The regression parameters of the wage regressions show that wage profiles by age follow the usual quadratic shape on average for both genders, while there is a substantial premium for higher education, and also a substantial premium for being located in the central area[26].

[26]Note that we abstract away from the fact of commuting, which could potentially be very important, here the estimates refer to the location of the firm sites.

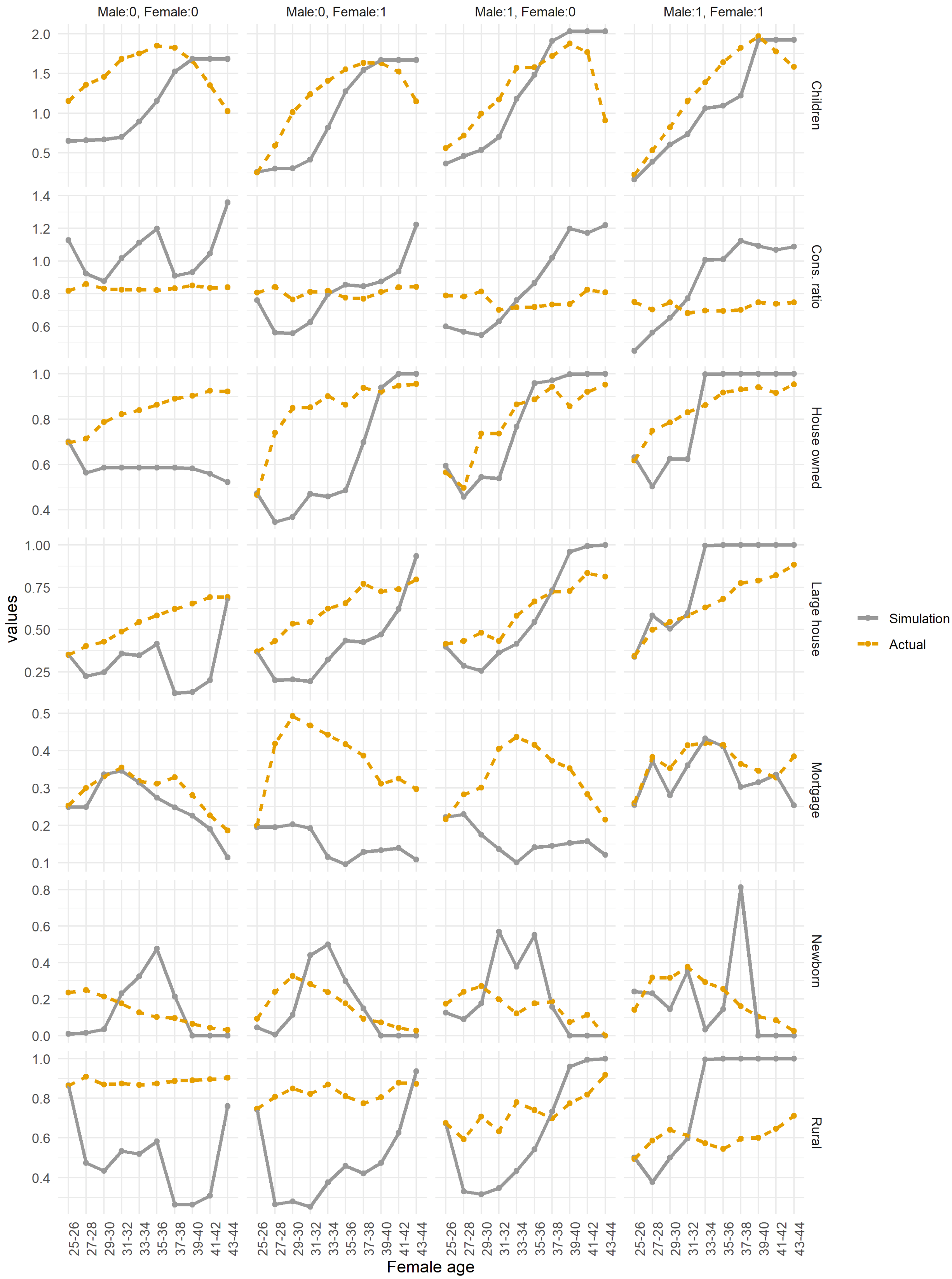
Table 5: Parameter estimates and calibrations

Parameter	Value, Estimate (S.E.)	Source
Utility function		
β	0.91	Set manually (biannual case)
γ_l	1.4140 (0.0599)	Estimated (SMM)
γ_h	1.6524 (0.0299)	Estimated (SMM)
w_l^N	0.8790 (0.1021)	Estimated (SMM)
w_h^N	0.5394 (0.0225)	Estimated (SMM)
w^H	0.9330 (0.0212)	Estimated (SMM)
w^L	-1.0790 (0.8978)	Estimated (SMM)
w^O	1.0522 (0.1242)	Estimated (SMM)
Budget constraint		
ir	0.10	HNB (2019) (biannual case)
δ	0.50	Set manually
ρ	0.05	Set manually
$\beta_0^{W,M}$	11.78 (0.0062)	Estimated (reduced form)
$\beta_1^{W,M}$	0.0219 (0.0005)	Estimated (reduced form)
$\beta_2^{W,M}$	-0.0005 (1.54e-05)	Estimated (reduced form)
$\beta_3^{W,M}$	0.913 (0.0071)	Estimated (reduced form)
$\beta_4^{W,M}$	-0.101 (0.0057)	Estimated (reduced form)
$\beta_0^{W,F}$	11.77 (0.0066)	Estimated (reduced form)
$\beta_1^{W,F}$	0.0170 (0.0006)	Estimated (reduced form)
$\beta_2^{W,F}$	-0.0004 (1.60e-05)	Estimated (reduced form)
$\beta_3^{W,F}$	0.772 (0.0071)	Estimated (reduced form)
$\beta_4^{W,F}$	-0.183 (0.0062)	Estimated (reduced form)
p_c	1,552.5 (in 1,000 HUF)	HCSO (2016)[27]
$W_{\min}$	960 (in 1,000 HUF)	Based on minimum wage in Hungary
Supply parameters		
α^0	0.0274	Calibrated to 2004-2014 house prices
α^1	0.0397	Calibrated to 2004-2014 house prices

4.4 Model validation

As we fit the model on the moments of the terminal values of the life-cycle, we can use the previous periods as validation to evaluate how well the model is capturing the evolution of the key variables over the life-cycle. Figure 12 displays how our main variables compare with their actual counterparts. We can see that there are some weaknesses, most importantly concerning rural vs. central location choice, which could explain why the parameter estimate governing that aspect is not significant statistically. While in actuality highly educated households choose to reside in the central location more than lower education households, in this model we get the opposite, as the central location provides higher earnings that lower education households need comparatively more, while also provides disutility to households according to the parameter estimates. This results in households with higher education choosing to reside in the rural area without exception in the model.

Figure 12: Evolution of key variables over the life cycle, simulation vs. actual



Note: using 150 simulations of 200 cohort-0 households, and values based on the HKÉF data of 2004-2014.

5 Results of the policy simulations

Using the parameter estimates I simulated the model for six different scenarios 150 times, with 150 Cohort-0, Cohort-1, and older households over 10 biannual periods. House prices are endogenously evolving in all of them, possibly counteracting the intended policy effects. Then I study how fertility and housing variables evolve under these scenarios, and show comparisons of household welfare, in a partial equilibrium setting. The scenarios were the following:

1. No allowance (baseline)
2. Allowance available (Family Housing Allowance for larger houses)
3. Extra supply in housing without allowance (+200% in the α parameters)
4. Lower interest rate without allowance (3.5% instead of 5%)
5. Lower down payment requirement without allowance (40% instead of 50% required)
6. Lower interest rate with allowance (3.5% instead of 5% rate, with Family Housing Allowance)

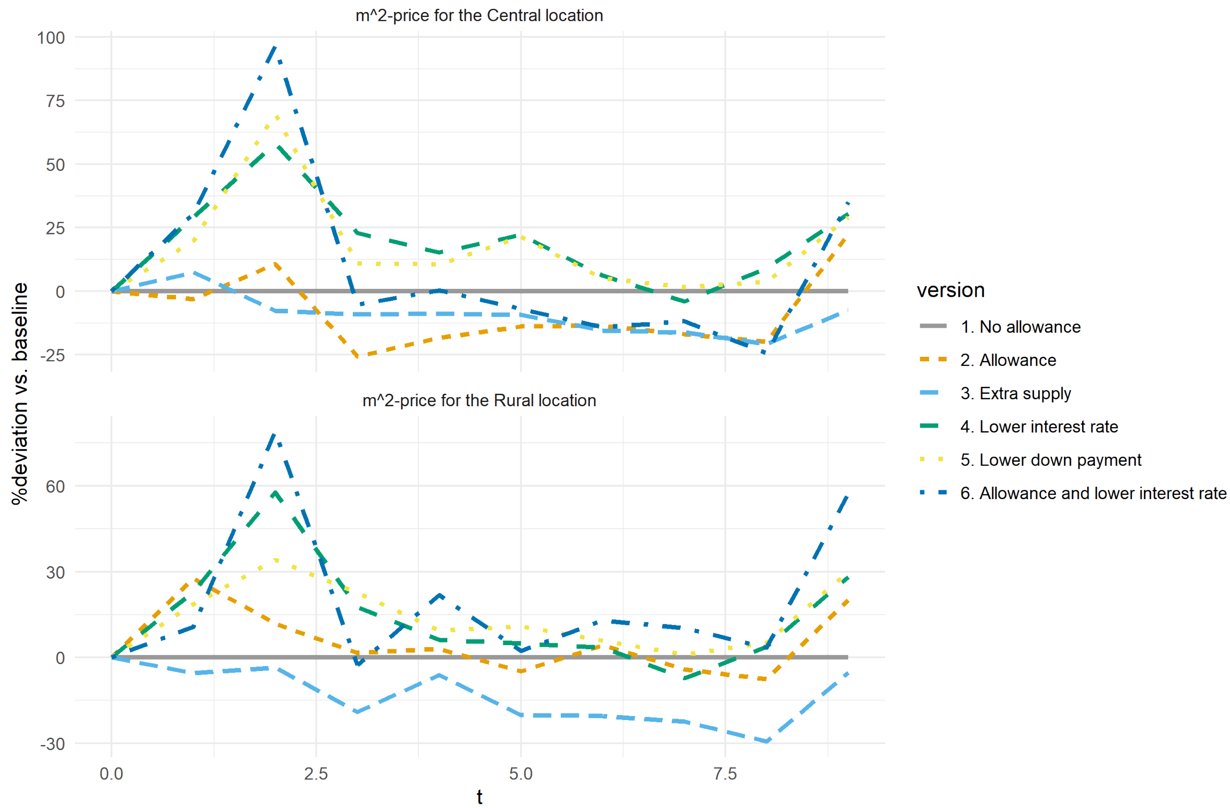
I included a combined monetary and fiscal policy scenario 'Lower interest rate with allowance' to reflect that the Hungarian National Bank decreased interest rates substantially starting from 2013, so instead of the 5% interest rate assumed during the model estimation more relevant for the pre-2014 period, I use 3.5% representing the post-2014 period more accurately. Scenarios 'Allowance', and 'Lower interest rate' with and without the government allowance enables us to disentangle the potential effects of the policy on housing, and on fertility as well. The scenarios 'Extra supply' and 'Lower down payment' represent alternative housing market policies that directly target the housing market without any fertility incentives, that allow us to examine the effects if simply alleviating housing-related constraints on the households.

5.1 Effect on the housing market

In this exercise I am going to assess the evolution of house prices compared to the baseline scenario. Note, that even in the baseline scenario we cannot reproduce constant prices at the target values described earlier with only two parameters (α), only the average over the 10-period (20-year) frame will be close to those values, otherwise there are fluctuations driven by the young households' life-cycle (in Appendix, Figure A8). However, these life-cycle fluctuations are the same in all scenarios, hence the differences across the scenarios can be interpreted as the effect of the policies we examine.

First, let us consider the evolution of house prices under the proposed scenarios in Figure 13, showing %-deviation compared to the baseline. We can immediately observe that lower interest rates and lower down payment ratios will result in higher house prices both in the central and the rural location (50-80% for the central, and 30-60% for the rural) for the first 4 years (2 periods). More importantly, we can see that other than an around 30% short-term increase in rural area house prices, the government allowance by itself does not induce such large changes close to what we would observe in the actual data. But, the allowance combined with a lower interest rate does produce around 100% increase in house prices for the central, and an around 75% for the rural areas, which are somewhat close to the actual datapoints. In the longer run however after 3-4 periods, house prices will not differ much from the baseline scenario values, so the deviation is only temporary in this model (possibly due to only being able to simulate two cohorts, implicitly enforcing this adjustment mechanism). On the contrary, with the extra supply scenario we naturally can drive down the prices even in the longer run, but even an around 200% permanent increase in new housing decreases house prices by only 20-30%.

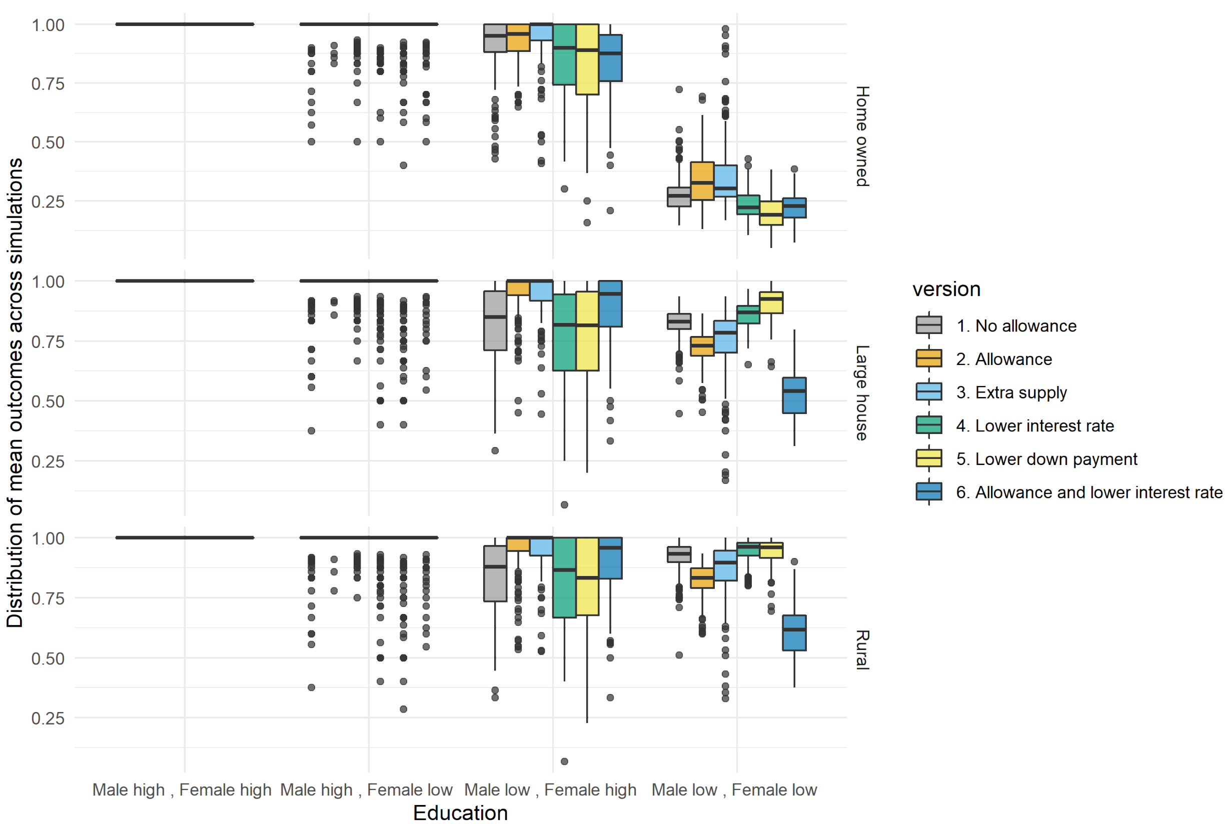
Figure 13: Evolution of house prices compared to the baseline scenario



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households. One period on the horizontal axis represents two years.

The housing-related final outcomes of households are displayed in Figure 14 (the evolution of the variables can be found in the Appendix, Figure A5). It is easy to see that there is no effect for households where the male education is high, as we run into a corner solution for most of these households. However it suggests that housing outcomes in this model are related to household income primarily driven by male income. For households with low male education the results are mixed, I am going to focus on the most numerous low male and low female education group. Regarding ownership, we can see that home ownership ratio does not change much due to the scenarios: while with allowance and extra supply it is slightly higher, in the other scenarios it is slightly lower. The fraction living in a large house does change substantially due to the policies. The Housing Allowance counter-intuitively seem to decrease the fraction of these poorer households living in a larger house, especially if interest rates are lower which induces the most increase in house prices. Living in a large house is higher compared to the baseline if down payment requirements are lower, while they are around the same for the other scenarios. The fraction living in the rural location also decreases due to the allowance, even more under lower interest rates. This is the reflection of the elevated house prices, and being able to afford only smaller houses, while moving into the central location where salaries are higher.

Figure 14: Effect of scenarios on end-of-life-cycle housing outcomes



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, based on the results of Cohort-0, the results for Cohort-0.

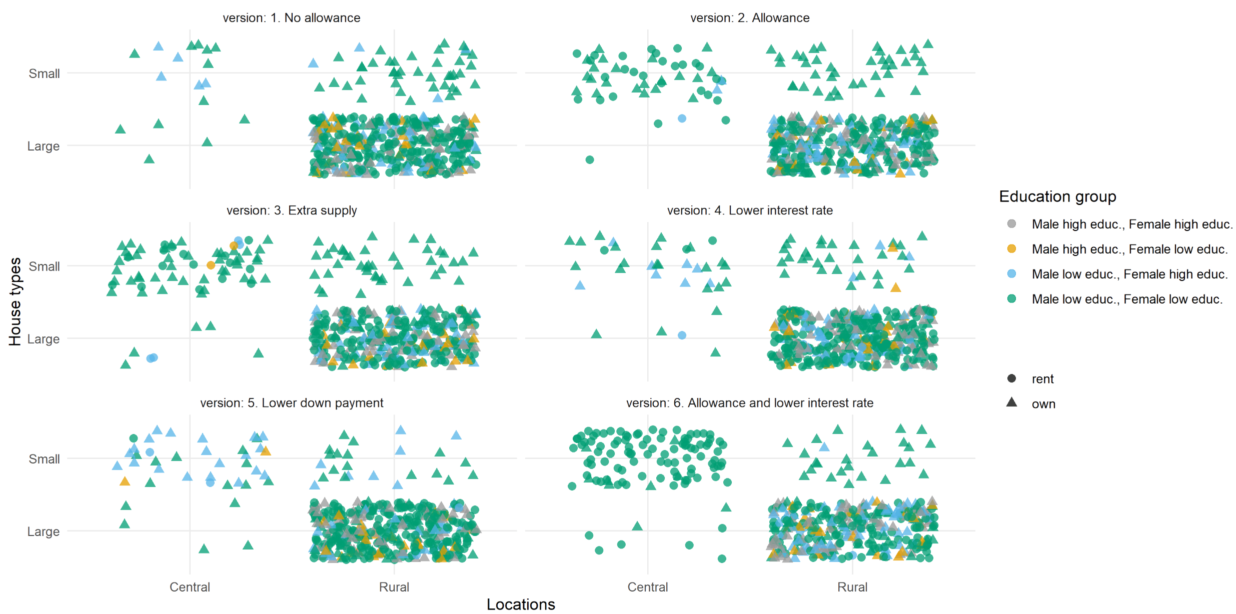
We can examine the sorting into house sizes and locations of households under our scenarios, by education groups. Table 6 shows how households are distributed on average within their education group across sizes and locations, while Figure 15 illustrates the distribution of households with information on ownership status as well. We can again see that high male education households live dominantly in large rural houses under any scenarios, while the distribution of low male education households vary by the circumstances. Most importantly, under the 'Allowance with lower interest rate' scenario, many more of the low male and low female education households live in small central area houses than under the other scenarios, which spot is the least desired according to the estimated preference parameters. At the same time, our figure shows that these households actually rent and not own their apartments in the centre, which then suggests the the Allowance policy combined with a low interest rate policy end up harming the housing conditions of the poorest households, while by themselves neither would have such an impact.

Table 6: Household sorting into house size and location, by education

Size	Location	Education	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Large	Rural	Male high educ., Female high educ.	1.00	1.00	1.00	1.00	1.00	1.00
Large	Central	Male high educ., Female low educ.	0.00	0.00	0.00	0.01	0.01	0.01
Large	Rural		0.97	1.00	0.98	0.94	0.94	0.97
Small	Central		0.03	0.00	0.01	0.04	0.04	0.02
Small	Rural		0.00	0.00	0.01	0.01	0.01	0.00
Large	Central	Male low educ., Female high educ.	0.01	0.02	0.01	0.01	0.02	0.03
Large	Rural		0.81	0.92	0.93	0.76	0.75	0.86
Small	Central		0.16	0.04	0.04	0.17	0.17	0.08
Small	Rural		0.03	0.02	0.01	0.06	0.06	0.04
Large	Central	Male low educ., Female low educ.	0.01	0.01	0.01	0.01	0.01	0.01
Large	Rural		0.80	0.71	0.73	0.85	0.89	0.52
Small	Central		0.07	0.17	0.13	0.05	0.05	0.38
Small	Rural		0.12	0.11	0.13	0.10	0.05	0.09

Note: the author's calculation based on 150 simulations of the model Cohort-0.

Figure 15: Household sorting into house size, location by ownership status and education

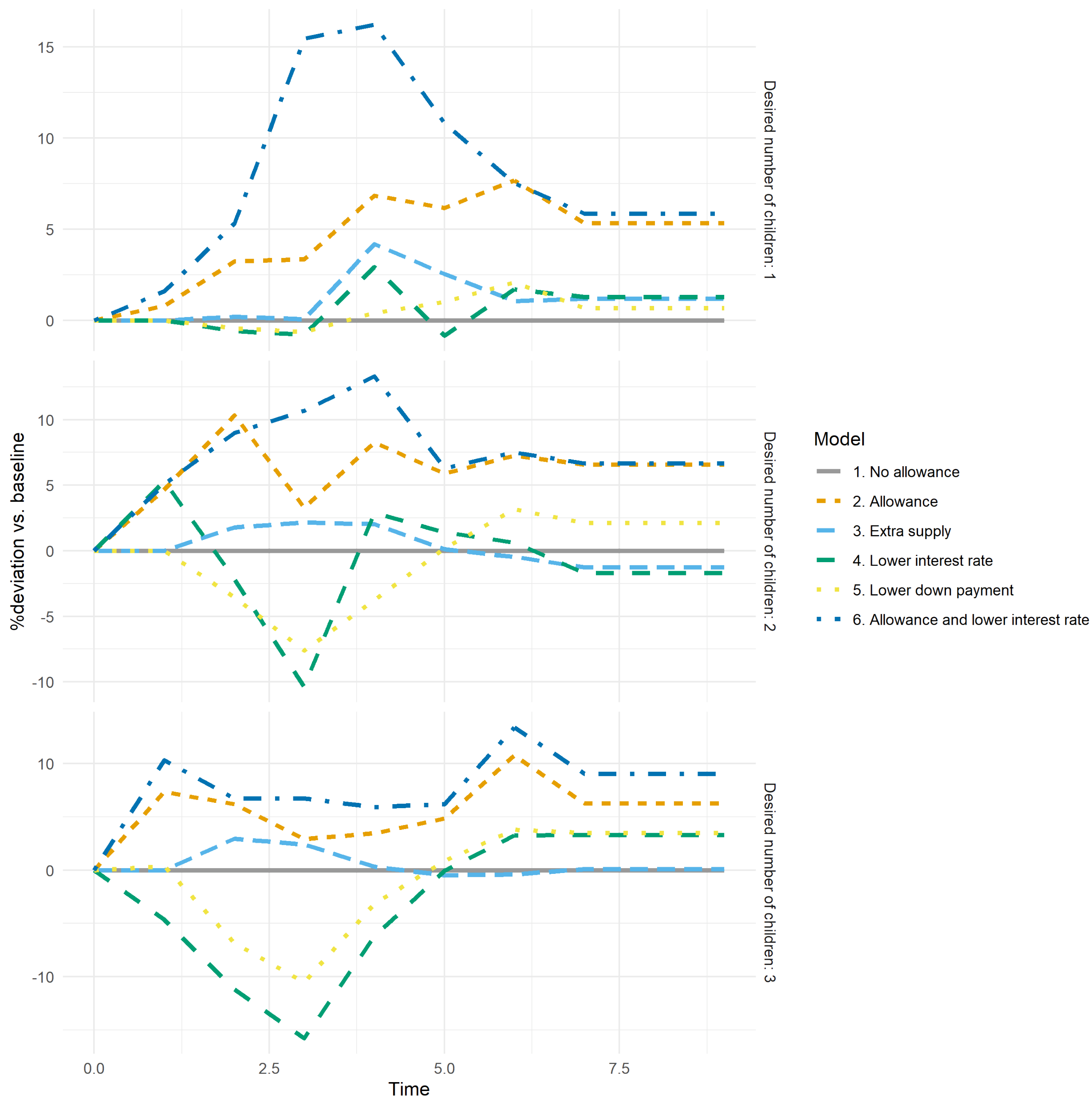


Note: the author's calculations based on 3 simulations of 150 Cohort-0, Cohort-1 and 'old' households, to allow for visibility of the results.

5.2 Effect on fertility and welfare

One point of focus in this analysis is to evaluate the Family Housing Allowance policy's potential long-run effects on the completed fertility outcomes of households, vs. timing effects. Figure 16 shows the %-deviation compared to the baseline scenario, in the number of children for Cohort-0's life-cycle, by the number of desired children. We can see that in this model setting, the Allowance results in an around 5-10% increase in completed fertility by the end of the life-cycle. The policy also alters timing, as we can see that children are born earlier in the life-cycle under the Allowance policy (hence the larger difference at the earlier periods). Other policies also affect fertility but to a lower degree. Lower down payment seems to cause an around 2-5% increase in completed fertility. Lower interest rate increases fertility for households who want one or three children, but decreases fertility for those that want two children, extra supply results in a small increase for households that want one child, but results in small decrease for those that want two children, and no change for those that desire three.

Figure 16: Evolution of the total number of children compared to the baseline scenario, by desired number of children



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

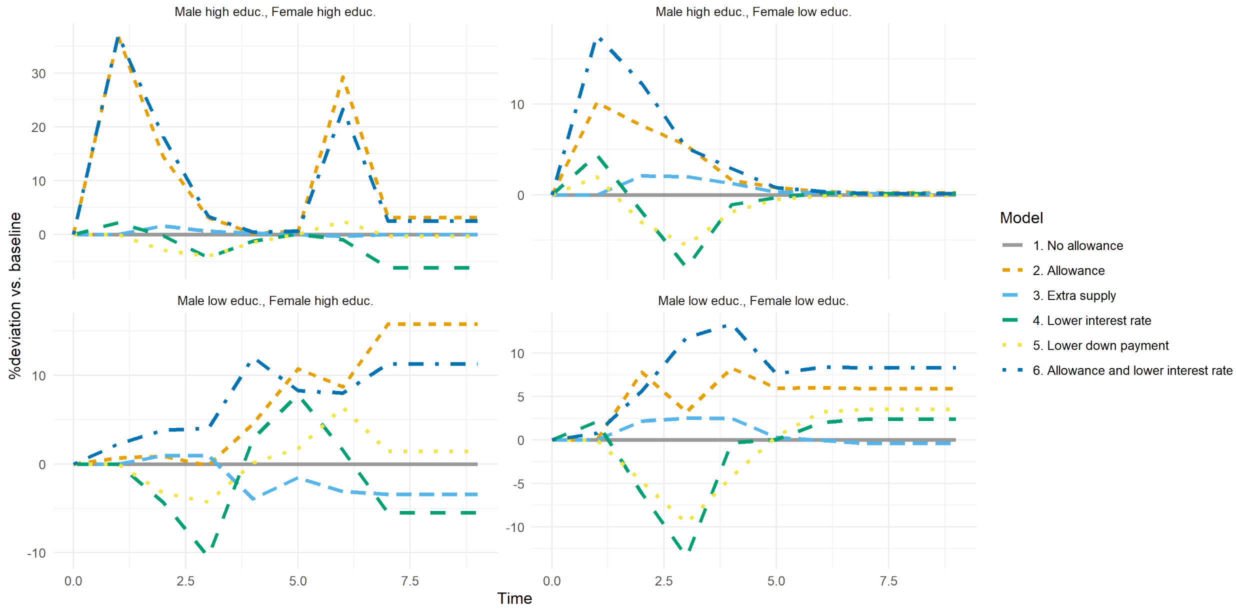
We can also examine which education groups change their fertility as consequence to different policies, Table 7 and Figure 17 display these results. Households with high education increase their completed fertility by 0.05 (around 2.5%) child per family as a response to either allowance scenario, while their fertility decreases substantially (-0.12 child per family) under the lower interest rate scenario. Households with lower education react somewhat differently: they increase their completed fertility more as a reaction to the allowance policies (0.11 and 0.15 child per family, or around 6-8% respectively), however, their fertility is unchanged or slightly changed under all other policies. Changes in timing of births depend on household education as well. Households with high male education bring their births forward in time due to the allowance, but as we saw, it does not imply much in completed fertility. In comparison, lower education households increase their completed fertility more with the allowance, while their reaction in terms of timing is lower.

Table 7: Average completed fertility by education

Education group	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Male high educ., Female high educ.	1.9282	1.9886	1.9279	1.8090	1.9215	1.9768
	(0.0126)	(0.0123)	(0.0126)	(0.0137)	(0.0128)	(0.0126)
Male high educ., Female low educ.	2.0596	2.0636	2.0620	2.0620	2.0580	2.0620
	(0.0172)	(0.0173)	(0.0173)	(0.0172)	(0.0173)	(0.0174)
Male low educ., Female high educ.	1.5697	1.8176	1.5160	1.4834	1.5920	1.7471
	(0.0154)	(0.0153)	(0.0155)	(0.0155)	(0.0154)	(0.0156)
Male low educ., Female low educ.	1.8219	1.9301	1.8146	1.8663	1.8860	1.9739
	(0.0061)	(0.0055)	(0.0062)	(0.0061)	(0.0060)	(0.0054)

Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

Figure 17: Evolution of the total number of children compared to the baseline scenario, by education



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

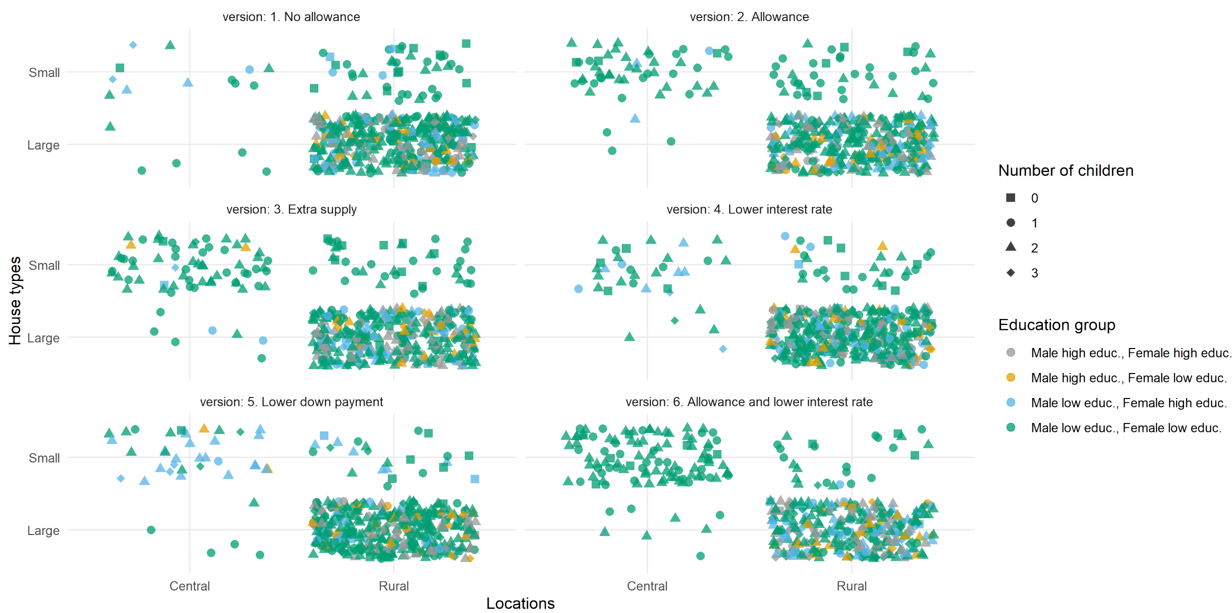
Finally we can look at how families sort into housing conditions under different scenarios, based on their fertility outcomes: Table 8 shows the distribution within the final number of children across house sizes and locations, while Figure 18 illustrates this with information on the households' education as well. (The evolution of the variables can be found in the Appendix, Figure A6) We can see that policies do not affect much where families without children, or with three children live: while families without children settle in large or small rural houses, families with three live in large rural houses almost exclusively. The policies however change where families with one or two children end up living. Compared to the baseline, in the scenarios with the Family Housing Allowance more of them reside in small central location houses (30% point increase), and most of those would otherwise have lived in large rural houses in the baseline scenario. This suggests, that according to our model those families who do not end up with three children are actually somewhat worse off regarding their housing conditions. The figure also suggests, that these families that end up in the central location are households with low education.

Table 8: Household sorting into house sizes and location, by number of children

Size	Location	Children	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Large	Central	0	0.02	0.01	0.02	0.01	0.02	0.01
Large	Rural		0.70	0.65	0.73	0.73	0.70	0.69
Small	Central		0.05	0.05	0.05	0.04	0.04	0.04
Small	Rural		0.23	0.29	0.20	0.22	0.25	0.26
Large	Central	1	0.02	0.02	0.02	0.01	0.03	0.03
Large	Rural		0.72	0.47	0.66	0.80	0.83	0.40
Small	Central		0.09	0.29	0.14	0.05	0.06	0.38
Small	Rural		0.17	0.22	0.18	0.14	0.08	0.18
Large	Central	2	0.01	0.00	0.00	0.01	0.01	0.01
Large	Rural		0.89	0.85	0.86	0.89	0.92	0.64
Small	Central		0.06	0.10	0.08	0.06	0.05	0.32
Small	Rural		0.05	0.04	0.05	0.05	0.02	0.04
Large	Central	3	0.00	0.00	0.00	0.01	0.00	0.00
Large	Rural		0.91	0.99	0.90	0.90	0.90	0.97
Small	Central		0.08	0.00	0.08	0.08	0.08	0.02
Small	Rural		0.01	0.00	0.01	0.02	0.02	0.01

Note: the author's calculation based on 150 simulations of the model to allow for visibility of the results.

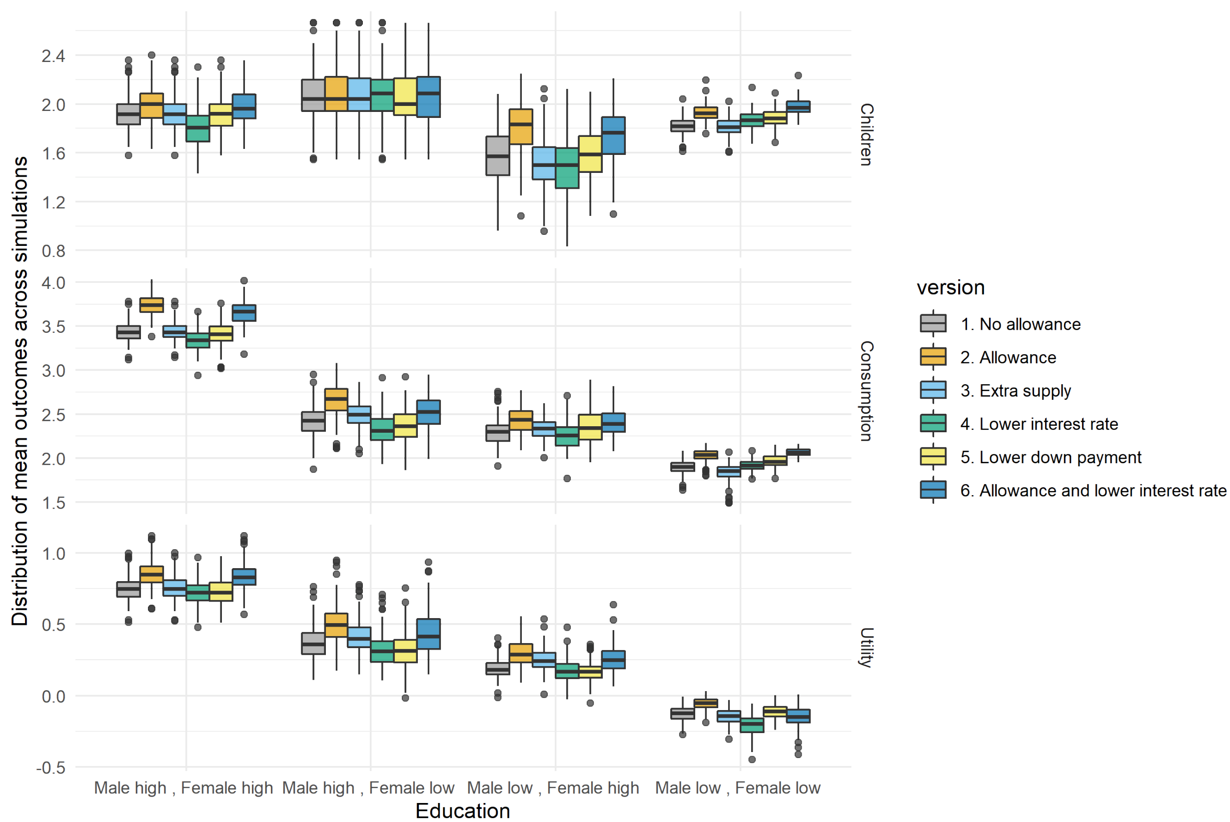
Figure 18: Household sorting into house size, location by number of children and education, under 3 scenarios



Note: the author's calculations based on 3 simulations of 150 Cohort-0, Cohort-1 and 'old' households, to allow for visibility of the results.

Finally we can also compare the scenarios directly by calculating the average discounted utilities realized over the life-cycles of the households. Figure 19 shows these utilities, along with showing average life-time consumption, and the again the completed fertility, now as the distribution of averages over simulations. Concerning utility, we can see that for households with high education for at least one parent, the Allowance policies increased average utilities, while all other policies leave it more or less unaffected. However, for households where both male and female have lower education, lower interest rates seems to decrease utility substantially, so much so that even with the Allowance the net effect is not positive (while Allowance without lower interest rates seems to have a positive utility effect). I have already discussed that housing conditions of such low education households worsen under lower interest rates, which is then compensated by increased consumption and number of children, to have a net neutral effect in the end. Our figure shows, that for all groups, non-surprisingly the available government subsidy will result in higher consumption, while the other policies leave it unaffected.

Figure 19: Effect of scenarios on non-housing outcomes



Note: the author's calculations based on 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results for Cohort-0.

6 Discussion and conclusion

In this paper I constructed a life-cycle model of household behavior focusing on fertility, location and housing choices, with unobserved heterogeneity in the preferences for the desired number of children, with endogenously evolving house prices. Using the period of 2004-2014, I estimated the parameters characterizing the preferences of the households, and with the estimates I analyzed the effects of different policy scenarios on the housing conditions and completed fertility of households, concentrating on the potential effects of the Family Housing Allowance program of the Hungarian Government, running from 2015.

I find that without additional changes, the Allowance program itself cannot explain the house price increases that were observed in Hungary after its introduction. However, combining it with lower interest rates (as these policies indeed coexisted in the relevant time frame) does induce a house price evolution relative to the baseline scenario that is comparable to the one observed after 2015 in Hungary. According to this model, the combination of the fiscal and monetary policy jointly could be responsible for the house price increases, possibly negating some of the welfare effects of the policy.

Looking at that scenario more closely, the model suggests that poorer households (represented by both parents having attained non-tertiary education level) were affected ambivalently by the policy, mainly driven by the counteracting negative impact of higher house prices. While consumption and their number of children seemed to respond positively to the policy incentives, their housing conditions worsen in the long run, as many of them have to move to the less desirable central location, and rent small apartments, instead of living in self-owned, larger rural houses. In comparison, households with higher education seem to benefit from the policy, seemingly without significant drawbacks. These findings does not seem to contradict the recent brief assessment of HNB (2021), which suggests that the policy helped families with three children, but did not raise availability of housing for other family types.

As future improvements and extensions of this model, I am going to attempt to resolve the problems of model fit regarding rural and central location across households with different education background. Furthermore, a drawback of the approach I chose is that when allowed to fluctuate endogenously, house prices even in the baseline scenario are far from stable, which corrected would further strengthen the conclusions of this study.

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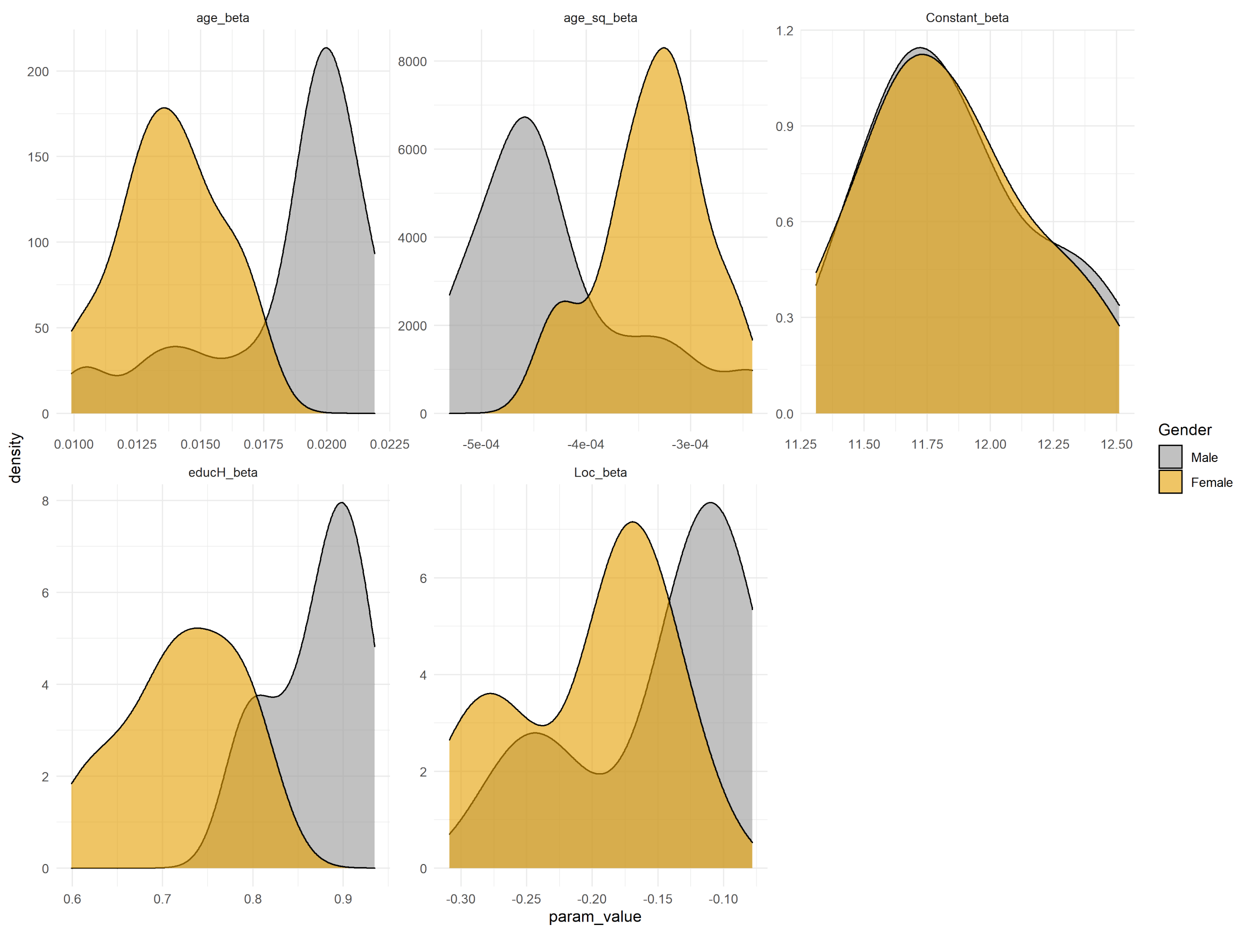
A Appendix

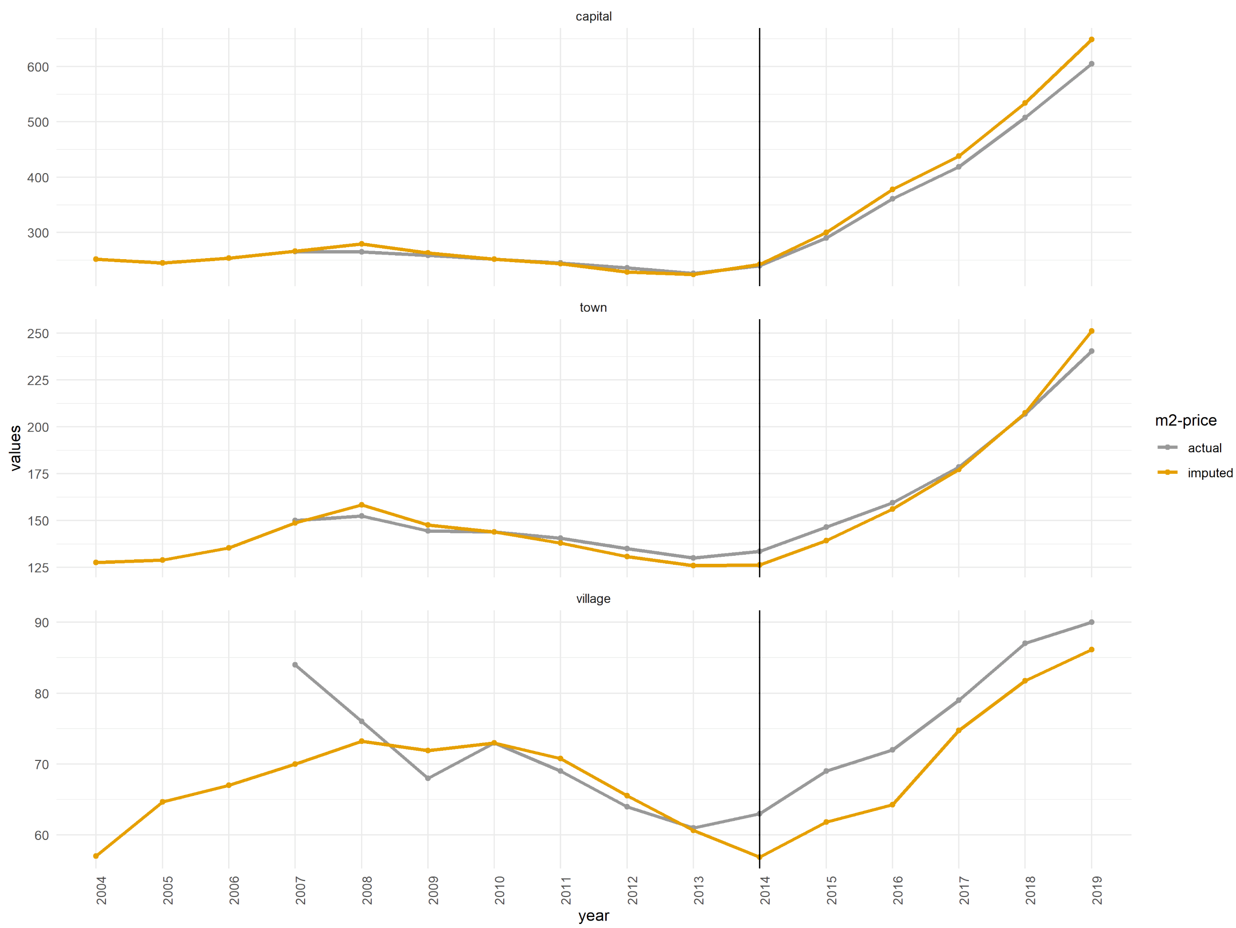
All additional figures and programs that belong to this paper can be found attached to this document.

Figure A1: Estimated densities of the estimated wage regression parameters, 2004-2018



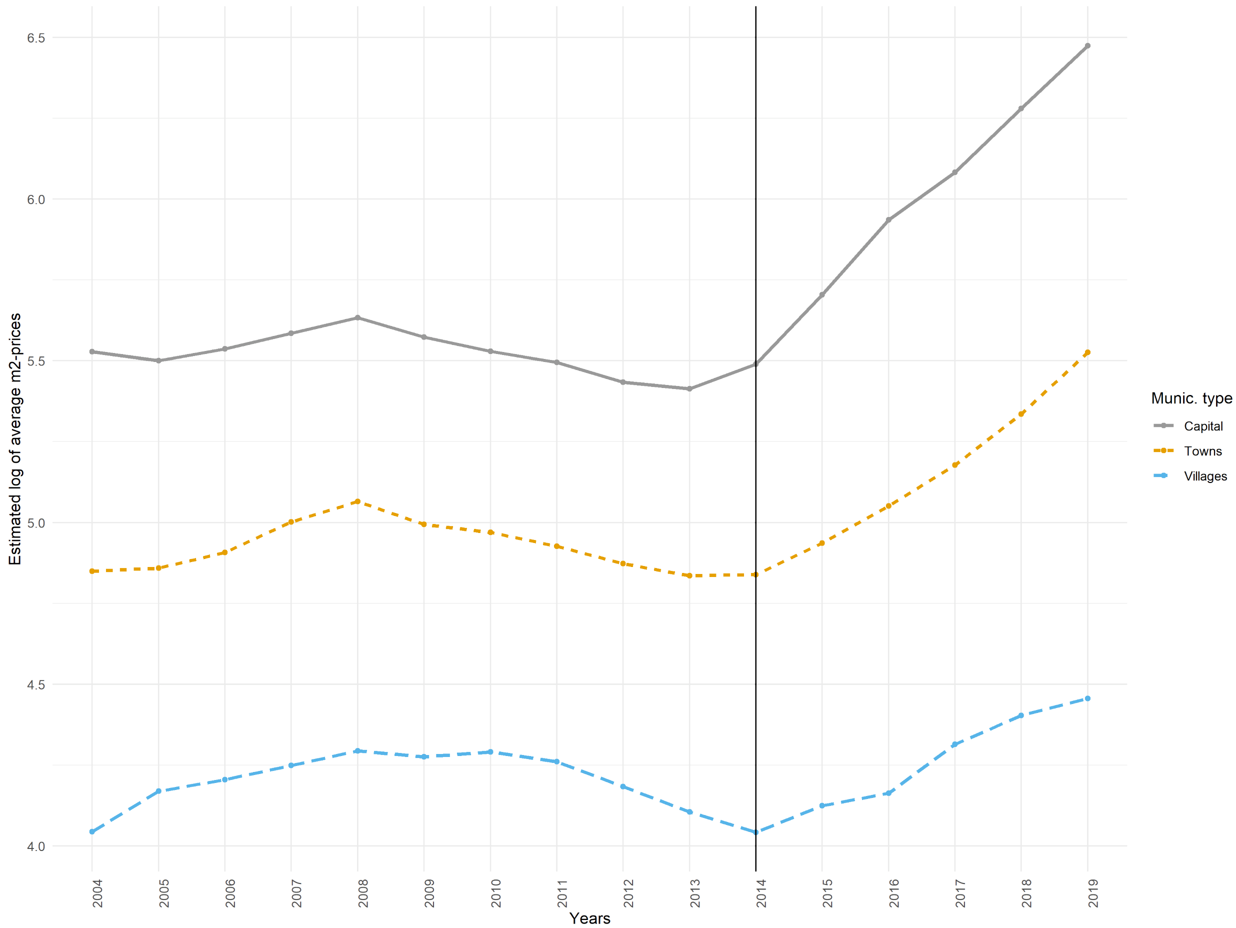
Note: the author's calculations based on the regression estimates from the Wage Survey data of Hungary, 2004-2018.

Figure A2: Imputed m^2-prices compared to available estimates



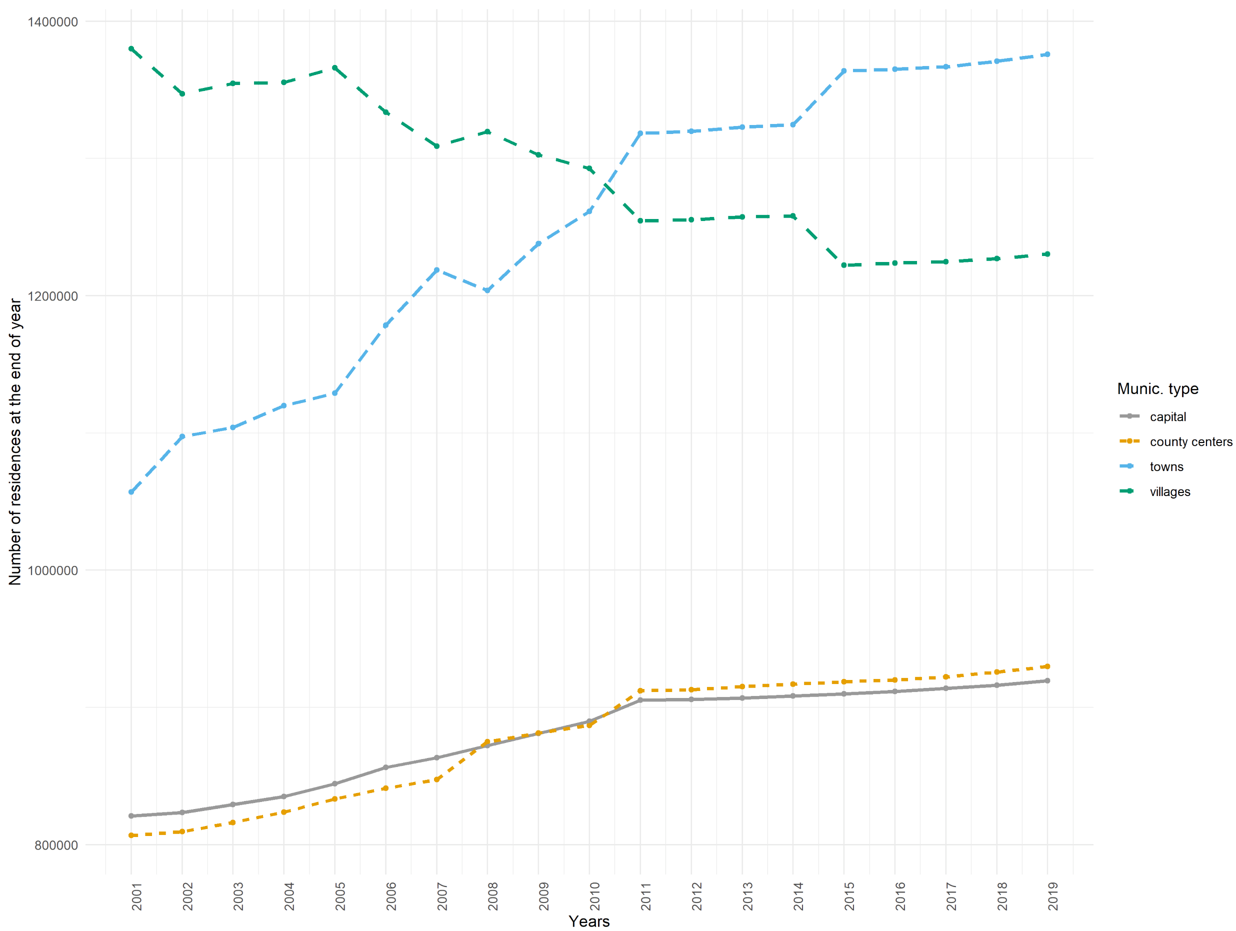
Note: the author's calculations based on the publicly available data of the Hungarian National Bank (HNB) and the Hungarian Central Statistical Office (HCSO). The earliest average m^2-price estimates of the HCSO are only available from 2007, while the HNB publishes nominal and real house price indices for earlier years as well. I use the two sources of information to estimate the pre-2007 prices, using 2010 as the baseline.

Figure A3: Logarithm of yearly average m^2-prices in 1000s of HUF, by type of municipality, 2004-2019



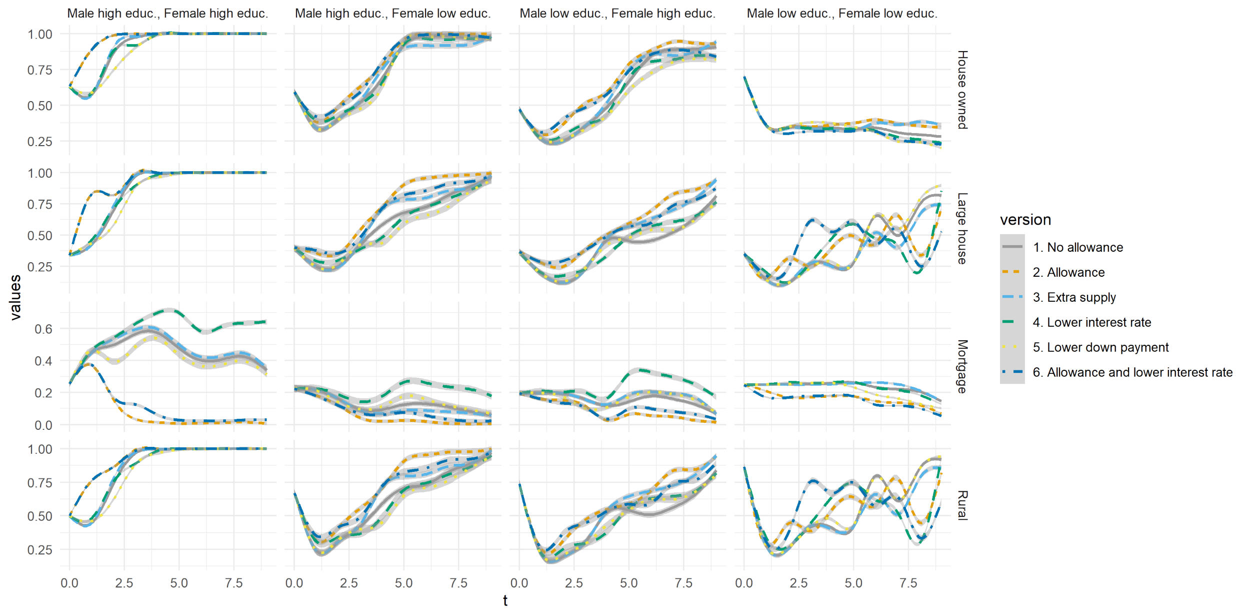
Note: the author's calculations based on the nominal house price index of the Hungarian National Bank, and house price estimates of the Hungarian Central Statistical Office.

Figure A4: Number of houses at the end of the year, by type of municipality, 2001-2019



Note: based on the publicly available data of the Hungarian Central Statistical Office.

Figure A5: Evolution of housing variables under different scenarios

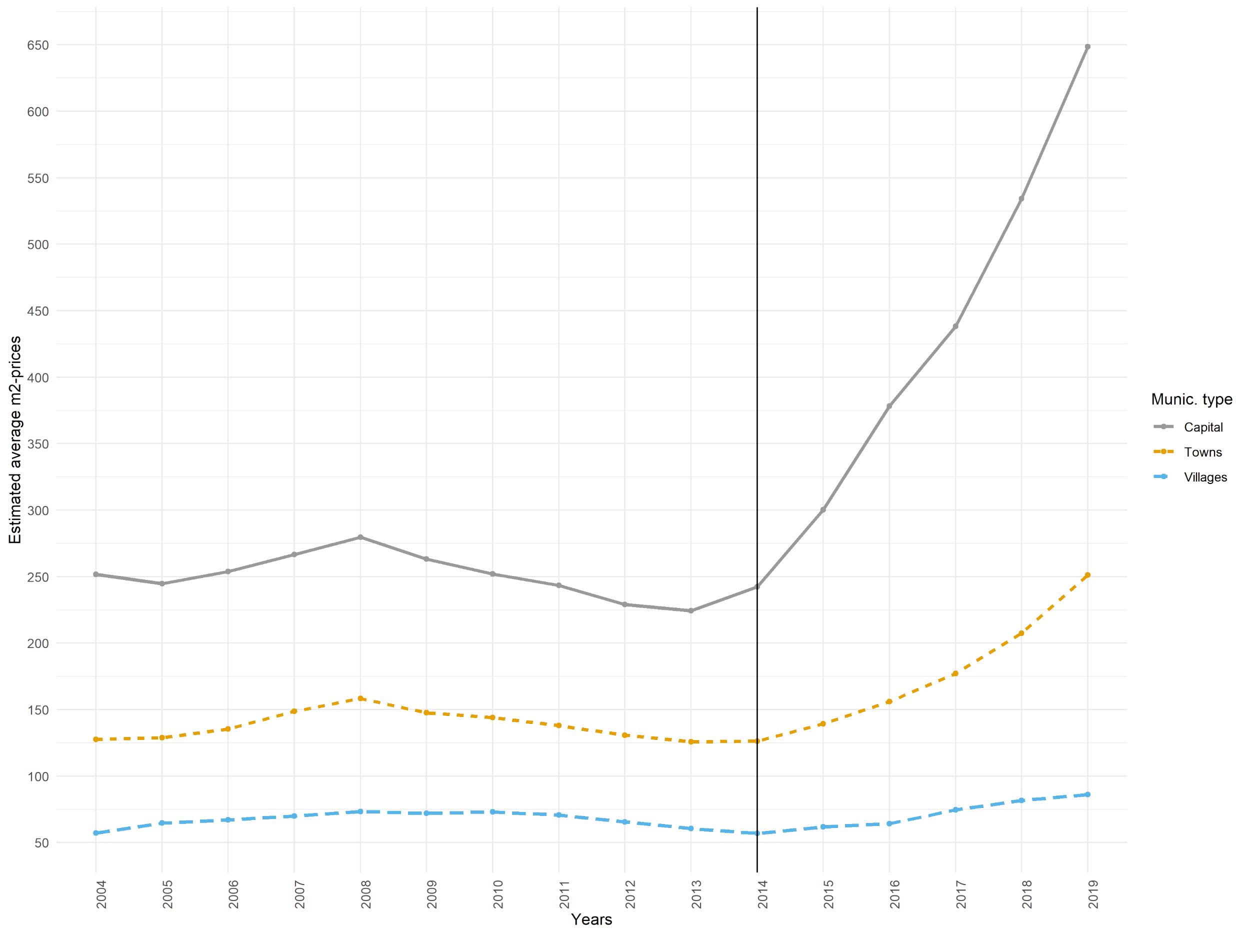


Note: the author's calculations based 150 simulations of the model. One period on the horizontal axis equals two years.

Figure A6: Evolution of non-housing variables under different scenarios

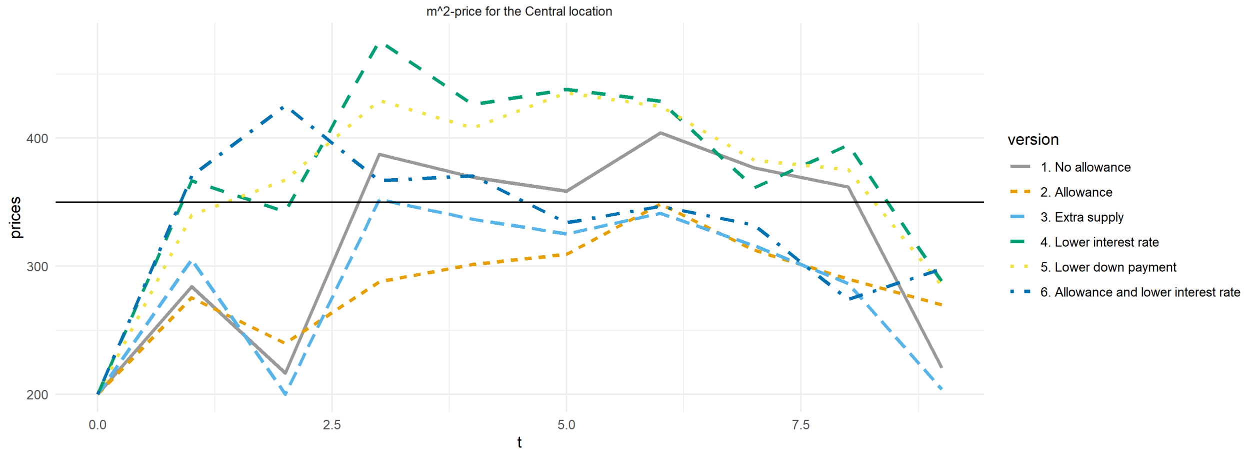
Note: the author's calculations based 150 simulations of the model. One period on the horizontal axis equals two years.

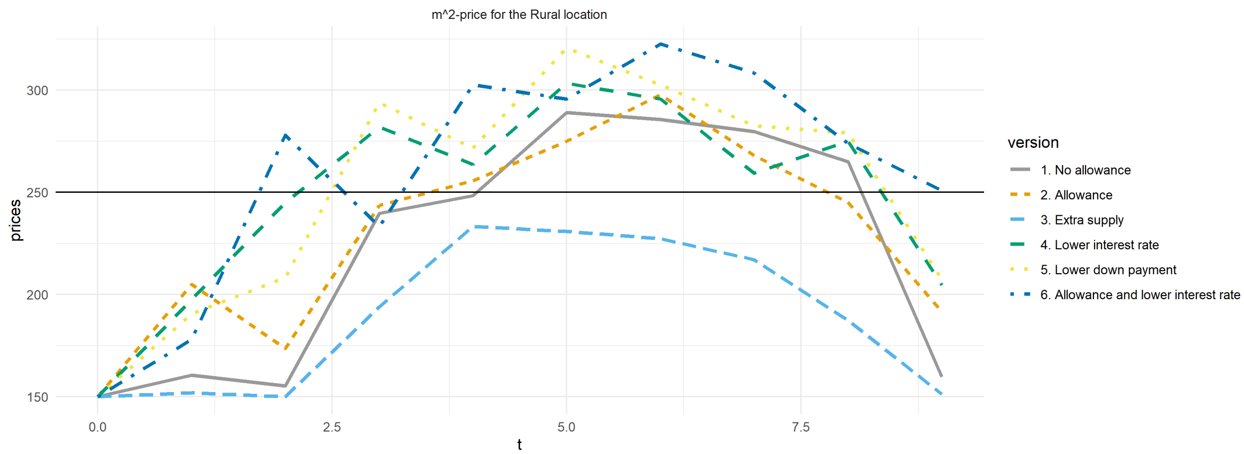
Figure A7: Average m^2-prices in 1000s of HUF, by type of municipality, 2004-2019



Note: the author's calculations based on the nominal house price index of the Hungarian National Bank, and house price estimates of the Hungarian Central Statistical Office.

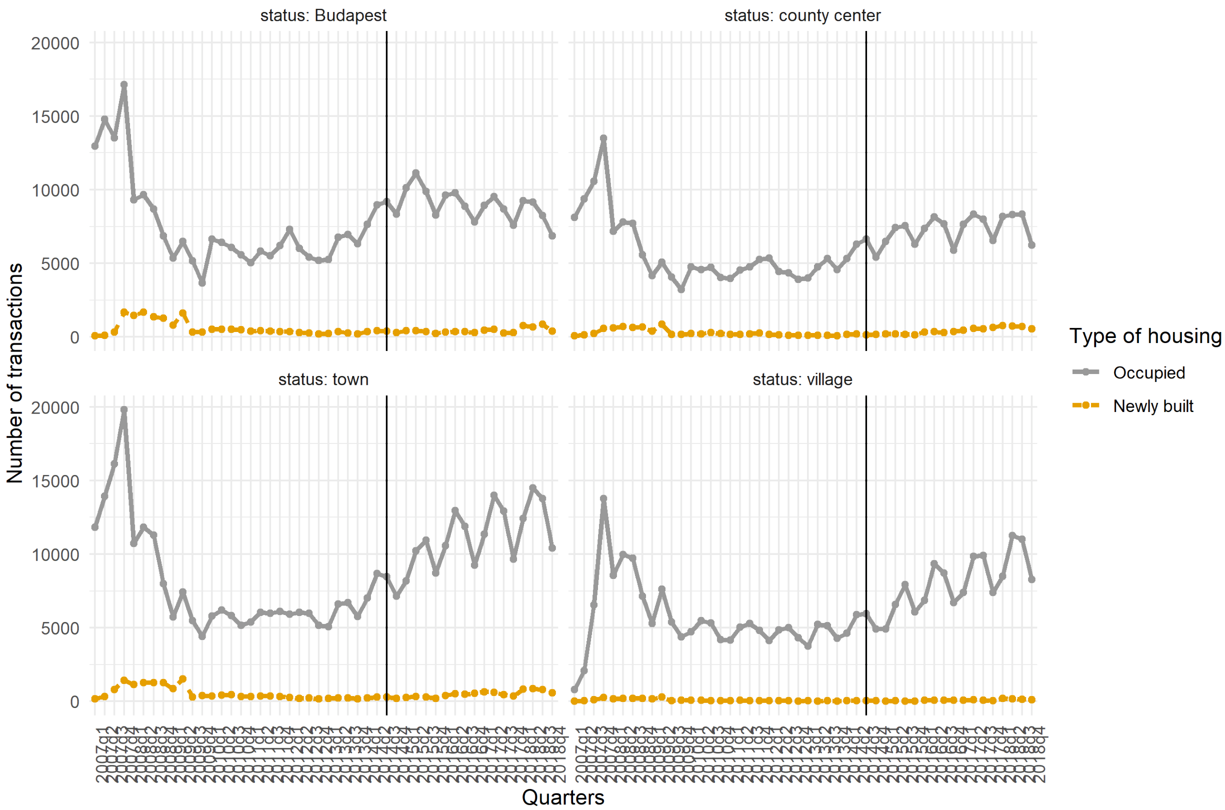
Figure A8: Average m^2-prices for the scenarios





Note: the author's calculations based 150 simulations of the model. One period on the horizontal axis equals two years. Targets of the prices are indicated with the horizontal lines at 350 for the central, and 250 for the rural area.

Figure A9: Number of housing transactions by type of municipality and house, 2007q1-2018q4



Note: based on the publicly available data of the Hungarian Central Statistical Office.