Design and Analysis of Algorithms: Lecture 2

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Contents

1 Divide & Conquer

1.1 Paradigm

Given a problem of size n, the general strategy of divide & conquer can be summarized:

1. Divide the problem into a ($a \geq 1$) subproblems of size $\frac{n}{b}$ ($b > 1$).
2. Recursively solve subproblems.
3. Combine solutions of the subproblems into one overall solution.

2 The Convex Hull problem

2.1 Definitions

Definition. Given a set of points S on the real plane, **convex hull of S** ($CH(S)$) is the smallest convex polygon containing all of the points in S.

We'll represent a convex hull as a sequence of points in clockwise order.

2.2 Brute-force algorithm

Algorithm 1 Brute-force algorithm for the Convex Hull problem

Input

S a set of points in the real plane, where:

1. no two points have the same x-coordinate,
2. no two points have the same y-coordinate,
3. and no three points are colinear

Output

$CH(S)$ the convex hull of S (as a sequence of points in clockwise order)

1: $C \leftarrow \{\}$
2: **for** each pair of points $p, q \in S$ **do**
3: Draw the line which holds both points
4: **if** all remaining points lie on one side of the line **then**
5: Add the points p, q to C
6: **end if**
7: **end for**
8: **return** the elements of C, clockwise-ordered

2.3 Brute-force runtime

The brute-force algorithm:

1. iterates $O(n^2)$ times (line 2)
2. during each, iterates $O(n)$ times (line 4)

The sorting of the elements before returning runs in linear time and occurs only once, therefore, **Algorithm 1** runs in $O(n^3)$ time.

2.4 Divide & conquer algorithm

Algorithm 2 Divide & conquer algorithm for the Convex Hull problem

Input

S a set of points in the real plane, where:

1. no two points have the same x-coordinate,
2. no two points have the same y-coordinate,
3. and no three points are colinear

Output

$CH(S)$ the convex hull of S (as a sequence of points in clockwise order)

1: b