

Design and Analysis of Algorithms: Problem 3-1

Ben Chaplin

Contents

1 Problem Statement	1
2 VEB Variants	1
2.1 Groups of $u^{2/3}$	1

1 Problem Statement

For each of the following variants on the van Emde Boas data structures, carefully describe what changes are needed to the pseudocode and analyze the costs of the vEB operations INSERT, DELETE and SUCCESSOR, comparing them with the costs of the same operations for the original vEB structure.

- a) Instead of dividing the structure into $u^{1/2}$ groups of $u^{1/2}$ elements each, use $u^{1/3}$ groups of $u^{2/3}$ numbers each.

2 VEB Variants

2.1 Groups of $u^{2/3}$

The algorithms for INSERT, DELETE and SUCCESSOR do not change. However, the recurrence changes. Our original recurrence:

$$\begin{aligned}T(n) &= T(\sqrt{n}) + C \\ &= O(\log_2 \log_2 n)\end{aligned}$$

The new recurrence:

$$\begin{aligned}T(n) &= T(n^{2/3}) + C' \\ &= O(\log_{3/2} \log_2 n) \\ &= O\left(\frac{\log_2 n}{\log_2 3/2} \log_2 n\right) \\ &= O(\log_2 \log_2 n)\end{aligned}$$

The runtime is asymptotically equivalent. However, we should expect $C' > C$, as groups are larger and therefore operating on them will require more time.