Undergrad Complexity

Ben Chaplin

Contents

1 Primitive Recursive Functions

1.1 Various functions and operators

Definition. The **constant function** C_n^k is defined for all $n, k \in \mathbb{N}$ as $C_n^k(x_1, \ldots, x_k) = n$.

Definition. The **successor function** S is defined for all $x\ inN$ as $S(x) = x + 1$.

Definition. The **projection function** P_i^k is defined for all $i, k \in \mathbb{N}$, $1 \leq i \leq k$ as $P_i^k(x_1, \ldots, x_k) = x_i$.

Definition. Given an m-ary function h and m k-ary functions $g_1, \ldots, g_m$, the **composition operation** $\circ$ is defined:

$$f = h \circ (g_1, \ldots, g_m)$$
$$f(x_1, \ldots, x_k) = h(g_1(x_1, \ldots, x_k), \ldots, g_m(x_1, \ldots, x_k))$$

Definition. Given a k-ary function g and a $(k + 2)$-ary function h, the **primitive recursion operator** ρ is defined:

$$f = \rho(g, h)$$
$$f(0, x_1, \ldots, x_k) = g(x_1, \ldots, x_k)$$
$$f(S(y), x_1, \ldots, x_k) = h(y, f(y, x_1, \ldots, x_k), x_1, \ldots, x_k)$$

1.2 Definition and examples

Definition. The constant function, successor function and projection function are **primitive recursive functions**, as well as any finite number of composition or primitive recursive operations on those functions.

Example.