Undergrad Complexity: Lecture 1

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1 Overview

1.1 Course details

Course Title: Undergrad Complexity
Teacher: Professor Ryan O'Donnell
School: Carnegie Melon University
Lectures: https://www.youtube.com/playlist?list=PLm3J0oaFux3YL5vLXpzOyJiLtqLp6dCW2
Textbook: *Introduction to the Theory of Computation* by Michael Sipser

2 Introductory Definitions

2.1 Alphabets and Strings

Definition. Computational tasks are processes which, given an input, should produce a certain kind of output.

In general, we encode both inputs and outputs using a given set of characters.

Definition. An **alphabet** Σ is a non-empty finite set of symbols.

Example. $\Sigma = \{0, 1\}$.

Definition. Σ^n is the set of all **strings** of length exactly n made up of symbols in the alphabet Σ.

Example. Let $\Sigma = \{0, 1\}$, then $\Sigma^2 = \{00, 01, 10, 11\}$.

Note that $n = 0$ is allowed, the empty string is denoted as ϵ.

Definition. $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup ...$ is the set of all finite length strings made up of symbols in the alphabet Σ.

2.2 Encoding Mathematical Objects

This is an annoying topic, but necessary for a complete analysis of complexity.

Definition. If X is a mathematical object and Σ is an alphabet, then $\langle X \rangle_\Sigma \in \Sigma^*$ is an **encoding of** X (a unique representation of x using the alphabet σ).

In general, we are going to avoid rigorously describe encodings. It will be enough to imagine a theoretical "most sensible" one.

2.3 Computational Problems

There are three categories of computational problems:

- **Decision problems:** $f : \Sigma^* \to \{0, 1\}$. Problems for which the answer is either "yes" or "no".

  Example. Is a number prime? Does there exist a path in a given graph?

- **Function problems:** $f : \Sigma^* \to \Sigma'^*$. Problems for which the answer is another string (not necessarily of the same alphabet).

  Example. Convert a decimal number to binary ($\{0, 1, ..., 9\} \to \{0, 1\}$). Factor a prime.

- **Search problems:** $f : \Sigma^* \to \{x : x \in \Sigma'^*\}$. Problems for which there may be more than one answer, or no answer at all.

  Example. What are the paths in a given graph?

We primarily work with decision problems. In most cases, search problems and function problems can be easily reduced to decision problems, without added complexity.