RC Circuits and Electronic Filters

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Capacitors and resistors can be connected within a circuit to create high pass, low pass, and band pass filters. These filters attenuate signals depending on their frequency. Low pass filters attenuate signals with high frequencies and high pass filters attenuate signals with low frequencies. A band pass filter can be constructed by connecting a low pass filter and a high pass filter together within a circuit. A band pass filter attenuates frequencies that fall outside of a specific range. The frequency values that are attenuated in all three filters are determined by the equivalent values of the resistors and capacitors that are used in their construction. These filters also induce a phase shift between the input and output signals. This phase shift is dependent on the frequency of the input signal . Electronic filters are often used in audio equipment to eliminate unwanted noise from speakers or headphones.

The purpose of this experiment is to construct a low pass, high pass, and band pass filter, and to test their attenuation and phase shift behavior for a wide range of frequencies. The observed experimental behavior will be compared to predictions based on mathematical theory. By constructing these filters and analyzing their behavior for different input signals, a better understanding of their application and function can be established.

I. INTRODUCTION

A. Capacitors

A capacitor is a passive electrical device that stores electrostatic energy in the form of an electric field. A capacitor can be made with two pieces of conducting material separated by a dielectric material. When electrons reach one of the metal plates in the form of a current, their potential energy is stored as an electric field within the dielectric material. The diagram below illustrates a basic representation of a capacitor:

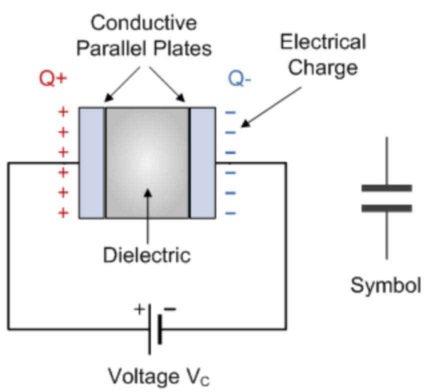

FIG. 1. [A parallel plate capacitor]. (n.d.). Retrieved from www.electronics-tutorials.ws/capacitor/cap_1.html

Capacitance is a quantifiable measure of a capacitor's ability to store electrostatic energy. The formula for capacitance is:

$$C = \frac{Q}{V} \tag{1}$$

where Q is the total charge on the capacitor and V is the voltage applied across it. The energy stored by a capacitor is given by the following formula:

$$E = \frac{QV}{2} \tag{2}$$

1. *Capacitors Connected in a Circuit*

Capacitors, like other passive electrical devices, can be connected in series within a circuit. In a series connection, the voltage drop across each capacitor is given by equation (1). The voltages across each capacitor must sum to the total input voltage of the circuit, which can be supplied by a battery. The current through each capacitor will be identical, and the charge on each capacitor will also be identical. The equivalent capacitance for capacitors connected in series is given by:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i} \tag{3}$$

Capacitors can also be connected in parallel. In a parallel connection, the voltage across each capacitor will be equivalent to the input voltage of the circuit. The current, however, will be divided between each capacitor. The equivalent capacitance for capacitors connected in parallel is given by:

$$C_{eq} = \sum_i C_i \tag{4}$$

Capacitors and their uses are discussed thoroughly in [1].

B. Transient Behavior

When a DC voltage source is applied to a capacitor, charge builds up on the conducting plates and the capacitor begins to store electrostatic energy. Equal and opposite charges will continue to build up on the plates until the voltage across the capacitor fully opposes the input voltage to the circuit. At this point, the current through the capacitor will be zero until the input voltage is reduced. Once the input voltage is lowered, the capacitor will begin to discharge and current will flow through the circuit again. [2] offers more in-depth analyses of RC circuits and extends to RL and RLC circuits as well.

The charging and discharging behavior of a capacitor can be expressed mathematically. Consider the following RC Circuit:

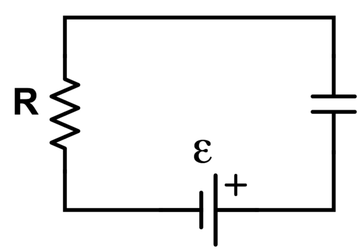

FIG. 2. [An RC Circuit with voltage source]. (n.d.). Retrieved from brilliant.org/wiki/rc-circuits-dc/

When power is applied to the circuit, Kirchoff's loop rule can be used to sum the voltage in series:

$$V_{in} = I(t)R + V_C(t) \tag{5}$$

Combining I(t) $= \frac{dQ(t)}{dt}$ and equation (1), equation (5) can be rewritten as:

$$V_{in} = \frac{RCdV_C}{dt} + V_C(t) \tag{6}$$

which can be expressed as a simple first order differential equation:

$$\frac{dt}{RC} = \frac{dV_C}{V_{in} - V_C} \tag{7}$$

At t = 0, there is no current flowing in the circuit and therefore no voltage across the capacitor. As such, the boundary condition of $V_C(0) = 0$ can be imposed. The differential equation can then be solved to find the voltage across the capacitor as a function of time:

$$V_C(t) = V_{in}[1 - e^{\frac{-t}{RC}}] \tag{8}$$

as well as the current in the circuit as a function of time. Since the resistor and capacitor are connected in series, the current through each device is identical.

$$I(t) = I_0 e^{\frac{-t}{RC}} \tag{9}$$

.2% of the electrostatic energy that it is able to. This exponential relationship can be utilized to determine how long it takes the capacitor to store various amounts of electrostatic energy, and how long it takes the current to decrease to various levels. We can also see that, due to the exponential term in equations (8) and (9), the voltage across a capacitor will never fully reach the input voltage and the current will never be exactly zero.

C. AC Circuits

In an AC circuit, the voltage will oscillate as a function of time as follows:

$$V_{in} = V_0 \sin \omega t \tag{10}$$

where ω is the angular frequency of the incoming signal: 2πf. Since the incoming voltage is oscillating over time, it follows that the current is oscillating as well:

$$I_{in} = I_0 \sin(\omega t + \phi) \tag{11}$$

The value of either the voltage or current is best expressed as the RMS value, which is calculated by squaring the voltage at every point along its cycle and taking the average value.

1. AC Circuit with Capacitors

When a capacitor is connected to a DC voltage supply, charge will build up on the plates and the capacitor will store electrostatic energy until the input voltage is reduced. If a capacitor is connected to an AC voltage supply, however, the capacitor will charge and discharge at a rate determined by the frequency of the input signal. Capacitors in an AC circuit have a form of current resistance known as Capacitive Reactance. This reactance is a quantitative measure of how the capacitor resists current flow as a function of signal frequency. It is given by:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \tag{12}$$

where X_C is the reactance, f is the frequency and ω is the angular frequency of the input signal. From equation (12), the capacitive reactance is inversely proportional to the signal frequency. At a very large frequency, the capacitor acts as a near perfect conductor, and at very low frequencies the capacitor acts as a very large resistor.

D. Phasors

A phasor is a graphical method of expressing a sinusoidal function as a complex number with an invariant initial phase and peak amplitude. In an AC circuit, the voltage and current are sinusoidal functions of time. If an AC circuit is constructed with only a resistor, then the current and voltage will be in phase with one another.

If the input voltage has a phase shift such that:

$$V(t) = V_0 sin(\omega t + \phi) \qquad I(t) = I_0 sin(\omega t) \tag{13}$$

Then the voltage will lead the current by ϕ, as is illustrated in the phasor diagram below:

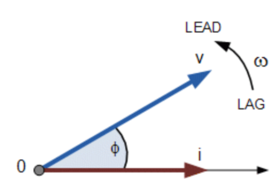

FIG. 3. [Phasor Diagram leading voltage]. (n.d.). Retrieved from www.electronics-tutorials.ws/accircuits/phasors.html

This phase shift can be generated by adding a capacitor to an AC circuit with a resistor. The voltage will lead the current by 2π. This phase shift can be graphically represented as follows:

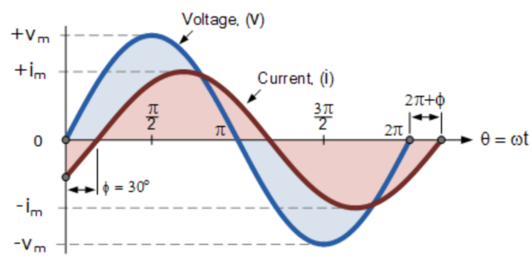

FIG. 4. [Graph of leading voltage]. (n.d.). Retrieved from www.electronics-tutorials.ws/accircuits/phasors.html

Phasors and phasor relationships for different electrical devices and circuit elements are examined in greater detail in [2].

E. Impedance

In an AC circuit, impedance is the electrical resistance of the components connected in the circuit and represented by the letter Z. Impedance differs from resistance and reactance, such that impedance must be used when discussing AC circuits and can be a complex number. The impedance expressions for a resistor and capacitor are given below:

$$Z_R = R \tag{14}$$

$$Z_C = \frac{1}{j\omega C} \tag{15}$$

where R is the resistance, j is the imaginary unit and C is the capacitance. This might look similar to equation (12), however, note that the capacitive impedance is imaginary.

F. Low Pass Filter

A low-pass filter attenuates high-frequency signals and allows low-frequency signals to pass through. A low-pass filter can be constructed as follows:

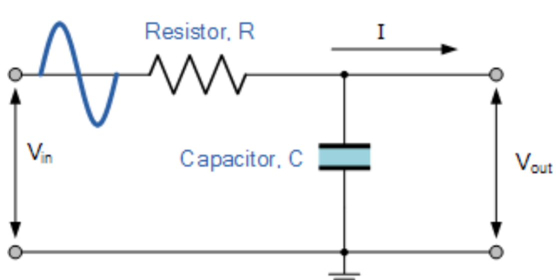

FIG. 5. [A basic low pass filter]. (n.d.). Retrieved from www.electronics-tutorials.ws/filter/filter_2.html

This circuit has a resistor and capacitor connected in series, with the output voltage measured across the capacitor. When the signal frequency is very high, the capacitive reactance will be very low. While the capacitive reactance is much lower than the resistance of the resistor, the voltage across the capacitor will be very small with respect to the voltage across the resistor. Conversely, when the signal frequency is very low, the capacitive reactance will be very low. When the capacitive reactance is much higher than the resistor's resistance, the voltage across the capacitor will be very high with respect to the voltage across the resistor. As such, the output voltage will be high for low frequency signals and low for high frequency signals.

1. *Solving the Low-Pass Filter*

The ratio between the output and input voltage in a low-pass filter can be solved for mathematically:

Starting from Kirchoff's loop rule for figure (5), we can sum the voltages:

$$V_{in} = IR + IZ_C \tag{16}$$

Since V_{out} is simply the voltage drop across the capacitor, this can be written as:

$$V_{out} = IZ_C \tag{17}$$

and take the ratio, substituting in equation (15):

$$\frac{V_{out}}{V_{in}} = \frac{IZ_C}{IR + IZ_C} = \frac{1}{\sqrt{(\omega RC)^2 + 1}} \tag{18}$$

This ratio can be examined for three different cases:

$$\omega RC << 1 \quad => \quad \frac{V_{out}}{V_{in}} \approx 1 \tag{19}$$

$$\omega RC = 1 \quad => \quad \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} \tag{20}$$

$$\omega RC >> 1 \quad => \quad \frac{V_{out}}{V_{in}} \approx 0 \tag{21}$$

We can also solve for the phase angle between the input and output voltage for the low-pass filter:

$$\phi = tan^{-1}(\omega RC) \tag{22}$$

The phase angle can also be examined for three different cases:

$$\omega RC << 1 \quad => \quad \phi \approx 0 \tag{23}$$

$$\omega RC = 1 \quad => \quad \phi = \frac{\pi}{4} \tag{24}$$

$$\omega RC >> 1 \quad => \quad \phi \approx \frac{\pi}{2} \tag{25}$$

G. High Pass Filter

A high-pass filter attenuates low-frequency signals and allows high-frequency signals to pass through. A high-pass filter can be constructed as follows:

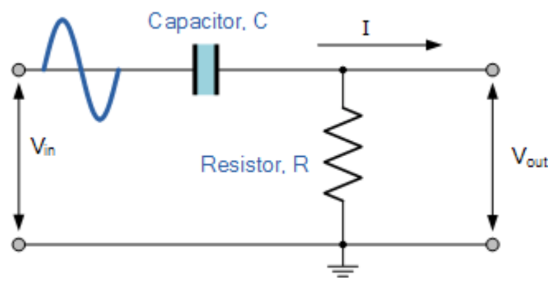

FIG. 6. [A basic high pass filter.] (n.d.). Retrieved from www.electronics-tutorials.ws/filter/filter_3.html

The circuit is similar to the low-pass filter, however, the circuit and resistor have swapped places and the output voltage is measured across the resistor. When the signal frequency is very high, the voltage across the resistor is very large, and when the signal frequency is very low, the voltage across the resistor is very small.

1. *Solving the High Pass Filter*

Kirchoff's loop rule can be used to solve for the input and output voltage ratios, as well as the phase shift between them:

$$V_{in} = IZ_C + IR \tag{26}$$

Since V_{out} is simply the voltage drop across the resistor, this is written as:

$$V_{out} = IR \tag{27}$$

Following similar algebraic steps as for the low pass filter solution, the voltage ratio is found to be:

$$\frac{V_{out}}{V_{in}} = \frac{IR}{IR + IZ_C} = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} \tag{28}$$

different cases:

$$\omega RC << 1 \quad => \quad \frac{V_{out}}{V_{in}} \approx 0 \tag{29}$$

$$\omega RC = 1 \quad => \quad \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} \tag{30}$$

$$\omega RC >> 1 \quad => \quad \frac{V_{out}}{V_{in}} \approx 1 \tag{31}$$

The phase angle between the input and output voltage can also be solved for:

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right) \tag{32}$$

The phase angle can be examined for three different cases:

$$\omega RC << 1 \quad => \quad \phi \approx \frac{\pi}{2} \tag{33}$$

$$\omega RC = 1 \quad => \quad \phi = \frac{\pi}{4} \tag{34}$$

$$\omega RC >> 1 \quad => \quad \phi \approx 0 \tag{35}$$

2. *The -3dB point*

Decibels can be used to express the magnitude of an output voltage with respect to the input voltage and can be described by the following formula:

$$dB = 20 \log \frac{V_{out}}{V_{in}} \tag{36}$$

The -3 dB point is particularly interesting because the ratio between the input and output voltage is approximately .7071. From equations (24) and (34), the conditions for achieving this ratio is $\omega RC = 1$ for both the high pass and low pass filter. If the RC constant for any two filters are the same, then the voltage ratio should be approximately .7071 at the same frequency for both filters. This condition will be satisfied when the following relationship between the RC constant and input signal frequency is met:

$$RC = \frac{1}{2\pi f} \tag{37}$$

H. Band Pass Filter

A low-pass filter and high-pass filter can be connected to create a band pass filter. A band pass filter attenuates signals that fall outside of a certain range. This is achieved by the the low-pass and high-pass filters acting on the input signal simultaneously. For filters with the same RC constant, signals with high and low frequencies will be attenuated while a band of signals with mid-range frequencies are allowed to pass through. A sample band pass filter is illustrated below:

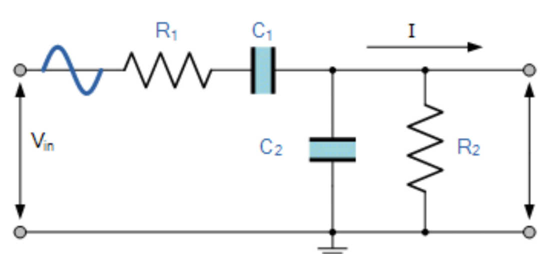

FIG. 7. [A basic band pass filter]. (n.d.). Retrieved from www.electronics-tutorials.ws/filter/filter_4.html

Graphical comparisons between the behaviors of each type of filter can be found in [3].

I. RC Integrators

A low-pass filter can be used to output the integral of an input signal. This can be shown mathematically by starting with the input voltage as expressed in equation (16) and substituting in equation (12):

$$V_{in} = I\sqrt{R^2 + (\frac{1}{\omega C})^2} \tag{38}$$

For the RC integrator, only signals with frequencies such that $\omega >> 1$ are considered. Assuming a large enough ω, the $\frac{1}{\omega C}$ term can be approximated to 0:

$$V_{in} = IR \tag{39}$$

Since the output voltage is measured as the voltage across the capacitor in figure (5), the voltage of the output signal is:

$$V_{out} = \frac{Q}{C} = \frac{1}{C}\int Idt = \frac{1}{RC}\int V_{in}dt \tag{40}$$

When the input frequency is large enough with respect to RC, the output signal will be an integral of the input signal multiplied by a constant.

J. RC Differentiators

A high-pass filter can be used to output the derivative of an input signal. The input voltage is given by equation (26), however, assuming a very small ω makes the R^2 term negligible. This results in the following expression for the input voltage to a high pass filter at very low frequencies:

$$V_{in} = \frac{1}{\omega C} \tag{41}$$

The voltage is measured across the resistor, so the output voltage is:

$$V_{out} = IR = RC\frac{d}{dt}V_{in} \tag{42}$$

For a small enough signal frequency, the output signal will be the derivative of the input multiplied by a constant. Additional insight into differentiators and integrators can be found in [1].

K. Better Filters

A filter is supposed to attenuate signals with either low or high frequencies. In practical uses, a single filter will likely not suffice for adequate attenuation of unwanted signals. It is possible to chain together multiple filters to improve their attenuation and better reduce noise outside of a desired frequency range. As the number of filters connected in a circuit increases, the precision of frequency attenuation increases significantly. [3] offers a graphical illustration of the benefits of chaining together more than one filter within a circuit.

II. METHODS

The resistance and capacitance measurements noted in this experiment were recorded using a digital multimeter. The circuits were all built on a breadboard using wires, alligator clips, and other materials supplied by the laboratory. The input signals to the circuits were supplied using either an AC power source or a waveform generator. Voltage values were measured using digital multi-meters connected to the breadboard. Phase shift values were measured using the dual-channel feature of the oscilloscope and recording the time difference between when the input and output waveforms crossed the axis. Errors on recorded values are a result of either instrument limitation or display fluctuation. Errors on theoretical resistance and capacitance values are derived from the labels on each device. There are no errors associated with the frequency values because they were digitally produced.

III. RESULTS AND ANALYSIS

A. -3 dB Frequency

We chose specific resistance, capacitance, and frequency values to achieve a voltage division of approximately .7071 in both the high-pass and low-pass filter that we constructed. We specified the - 3 dB frequency as 1 kHz, forcing us to find resistors and capacitors that satisfied the following equation:

$$RC = \frac{1}{2000\pi} = .00016 \;\; k\Omega * nF \tag{43}$$

Ideally we would have connected multiple resistors and capacitors together to obtain extremely accurate resistance and capacitance values. It might also have been possible to improve accuracy by using a variable resistor. Due to time and resource constraints, however, we simply found a single resistor and capacitor that roughly satisfied equation (43). The recorded values for each of our devices and the corresponding RC constant for both of the high and low pass filters are below:

TABLE I. RC Values

Theoretical R	3.90 ± 0.20 kΩ
Measured R	4.10 ± 0.05 kΩ
Theoretical C	47.00 ± 4.70 nF
Measured C	43.40 ± 0.05 nF
Theoretical RC	0.0001833 ± 0.0000202 kΩ*nF
Measured RC	0.0001779 ± 0.0000002 kΩ*nF

The theoretical values in table (1) were found using the markings printed on the devices. The measured values were determined by directly measuring each device.

The theoretical RC value falls within one standard deviation of the measured RC value, which indicates that the values derived from the device markings are consistent with the values recorded from measurement. Moving forward, we will be using the measured RC value for calculations.

The difference between the measured RC value and the desired value specified in equation (43) is statistically significant because they do not agree within 3 standard deviations. Moving forward, our calculations and analyses will use the adjusted frequency, instead of the originally predicted 1 kHz, which is calculated using the measured RC value in table (1):

$$f = 2000 * \pi * RC = 1.117 \pm .038 \text{ kHz} \tag{44}$$

B. Voltage Ratio

Both the high pass and low pass filter should attenuate a signal with the frequency specified in equation (44) by roughly .7071. After constructing the low and high pass filters, we can input a signal with a frequency of 1.117 kHz. We expect that the voltage ratio for both filters will be roughly .7071, since the RC constants are the same for each filter. Our experimental results for this test are below:

TABLE II. Voltage Ratios at -3 dB frequency

High Pass Filter		Low Pass Filter	
In	3.04 ± .005 V	In	3.04 ± .005 V
Out	2.16 ± .005 V	Out	2.04 ± .005 V
Ratio	.711 ± .004	Ratio	.671 ± .004

The voltage ratio for the high pass filter falls within two standard deviations of the expected value of .7071. The voltage ratio for the low pass filter, however, does not fall within 3 standard deviations of the expected value. The difference between these values, however, is roughly 8.6%. Given the nature of the equipment that we used to build the circuit and record measurements, it is reasonable that this discrepancy is a result of imperfections in the circuit connections. The statistical significance is still worth noting.

It would have been ideal to take repeated measurements for this test, as well as record the frequency that gave a voltage ratio of exactly .7071 for each filter. This was not done in the lab, and should be considered for any future experiments.

1. *Low Pass Filter*

We can expand on the previous section and record the voltage ratio of the low pass filter for a wide range of frequencies. A table highlighting our results is below:

TABLE III. Low-Pass Filter Voltage Ratios

f (Hz)	Voltage In	Voltage Out	Ratio
100	3.04 ± .05 V	2.96 ± .05 V	.974 ± .022
500	3.04 ± .05 V	2.64 ± .05 V	.868 ± .020
1000	3.04 ± .05 V	2.08 ± .05 V	.684 ± .019
5000	3.04 ± .05 V	.52 ± .05 V	.172 ± .017
10000	3.04 ± .05 V	.28 ± .05 V	.092 ± .017
50000	3.04 ± .05 V	.042 ± .005 V	.013 ± .002
100000	3.04 ± .05 V	.014 ± .005 V	.005 ± .002

A plot of the data shown in table (3) with the theoretical curve given by equation (18) is below:

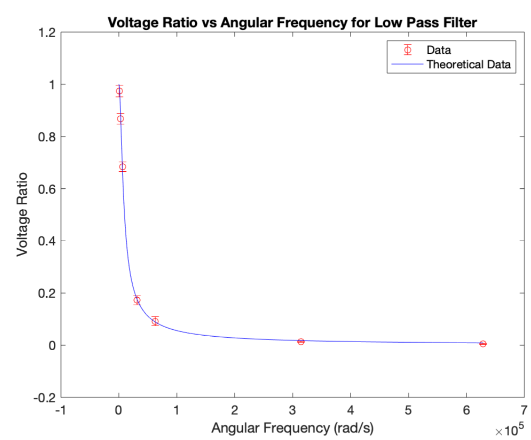

FIG. 8. Voltage ratio vs angular frequency for low pass filter

The voltage ratio decreases non-linearly as the angular frequency increases. This behavior that is expected from equation (18). As the angular frequency approaches 0, the voltage ratio approaches 1. This agrees with our expectations, because for very low frequencies the low pass filter should not attenuate any signals. As the angular frequency approaches infinity, the voltage ratio tends towards 0. This also agrees with our expectations, because for very high frequencies the low pass filter should attenuate all input signals.

To better analyze the behavior of the low pass filter as a function of angular frequency, the data in table (3) can be plotted on a logarithmic scale with the theoretical curve given by equation (18):

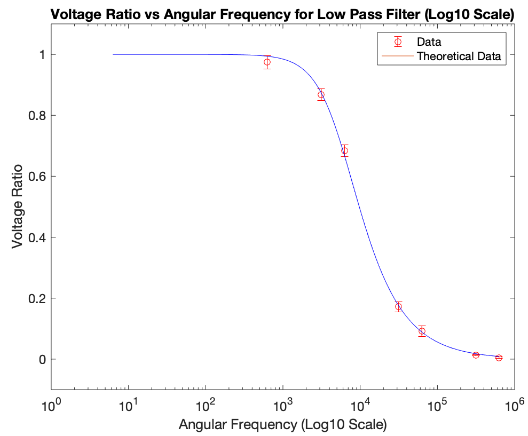

FIG. 9. Voltage ratio vs angular frequency on log scale for low pass filter

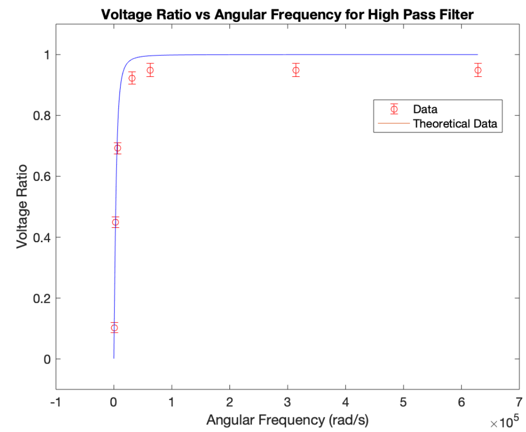

FIG. 10. Voltage ratio vs angular frequency for high pass filter

The low pass filter does not begin to filter input voltages until the angular frequency exceeds an order of magnitude of roughly 10^3. As the angular frequency exceeds an order of magnitude of 10^5, the voltage ratio tends towards 0. For angular frequencies of orders of magnitude between roughly 10^3 and 10^5, a fraction of the input signal is attenuated.

2. *High Pass Filter*

We can repeat the voltage ratio measurements over a range of frequencies for the high-pass filter. Our results are shown in the table below:

TABLE IV. High-Pass Filter Voltage Ratios

f (Hz)	Voltage In	Voltage Out	Ratio
100	3.12 ± .05 V	0.32 ± .05 V	.103 ± .016
500	3.12 ± .05 V	1.40 ± .05 V	.449 ± .018
1000	3.12 ± .05 V	2.16 ± .05 V	.692 ± .019
5000	3.12 ± .05 V	2.88 ± .05 V	.923 ± .020
10000	3.12 ± .05 V	2.96 ± .05 V	.949 ± .022
50000	3.12 ± .05 V	2.96 ± .05 V	.949 ± .022
100000	3.12 ± .05 V	2.96 ± .05 V	.949 ± .022

A plot of the data shown in table (4) with the theoretical curve given by equation (28) is below:

The voltage ratio increases non-linearly as the angular frequency increases. For low angular frequencies, the voltage ratio tends towards 0 as all of these signals are attenuated. As the angular frequency increases, the voltage ratio tends towards 1 as all of these signals are allowed to pass through. This agrees with the expected behavior seen in equation (28).

In contrast to the data illustrated in figure (9), there is a discrepancy between the theoretical curve and the data after the voltage ratio begins to level off. In table (4), the voltage ratio appears to have an upper limit of roughly .949, rather than 1 as is expected and shown in equation (29). This might be explained by an imperfection within the circuit that allowed for a fraction of the signal to be lost independently of filter attenuation.

To better analyze the behavior of the high pass filter, our data can be plotted on a logarithmic scale with the theoretical curve given by equation (28):

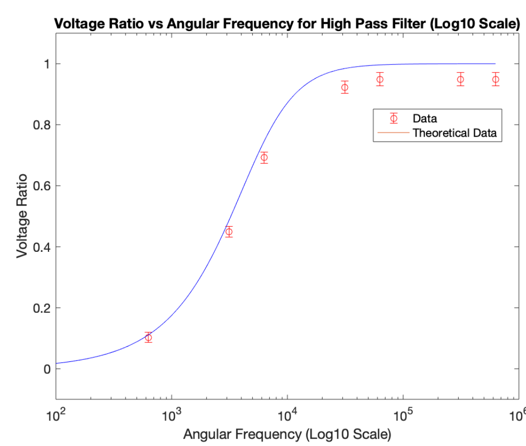

FIG. 11. Voltage ratio as a function of angular frequency on log scale for high pass filter

The high pass filter attenuates nearly all of the input signal until the angular frequency reaches a magnitude of roughly 10^3. As the angular frequency exceeds a magnitude of roughly 10^5, the theoretical voltage ratio tends towards 1. For angular frequencies of magnitude between roughly 10^3 and 10^5, a fraction of the input signal is attenuated.

C. Phase Shift

We can also measure the phase shift between the input and output voltage signals as a function of frequency.

1. *Low Pass Filter*

Data collected for low pass filter phase shift measurements is below:

TABLE V. Low pass filter phase shift vs frequency

f (kHz)	Period (s)	Phase Shift (μs)	Phase Shift (rad)
100	.01	200 $\pm$ 25	.126 $\pm$ .016
500	.002	150 $\pm$ 25	.471 $\pm$ .079
1000	.001	125 $\pm$ 25	.785 $\pm$ .157
5000	.0002	50 $\pm$ 5	1.571 $\pm$ .157
10000	.0001	25 $\pm$ 5	1.571 $\pm$ .314
50000	.00002	5.25 $\pm$ .25	1.649 $\pm$ .079

The phase shift values have relatively large errors because the oscilloscope had lots of fluctuation and there was a large degree of uncertainty when measuring the phase shift value along the time axis. A plot of the data shown in table (5) with the theoretical data given by equation (22) is below:

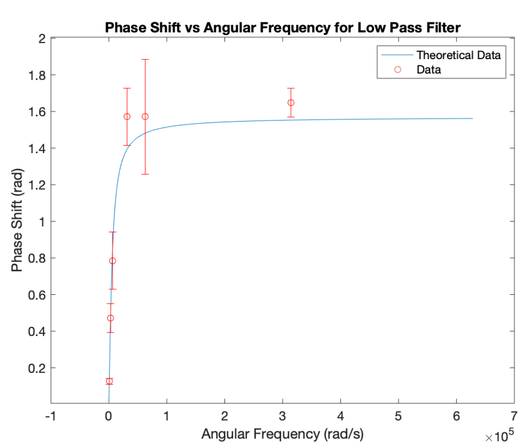

FIG. 12. Phase shift vs angular frequency for low pass filter

As the angular frequency tends towards 0, the phase shift also tends towards 0. As the angular frequency increases to very large values, the phase shift levels off around roughly 1.6 radians. Based on the theoretical data given by equation (25), we expected that for very large frequencies the phase shift will approach approximately 1.57 radians = $\frac{\pi}{2}$. Similar behavior can be observed for the data in table (5) as well as in figure (12). All of the phase shift values for 5000 Hz, 10000 Hz, and 50000 Hz, fall within one standard deviation of 1.57 radians.

In figure (12), it is clear that the experimental data begins to level off at a lower frequency than the theoretical data. There are relatively large error bars on the phase shift measurements, however, this might also be explained by considering that there was significant fluctuation within our equipment for frequencies higher than 1000 Hz.

To better analyze the phase shift behavior for the low pass filter, the data and theoretical data from equation (22) can be plotted on a logarithmic scale as follows:

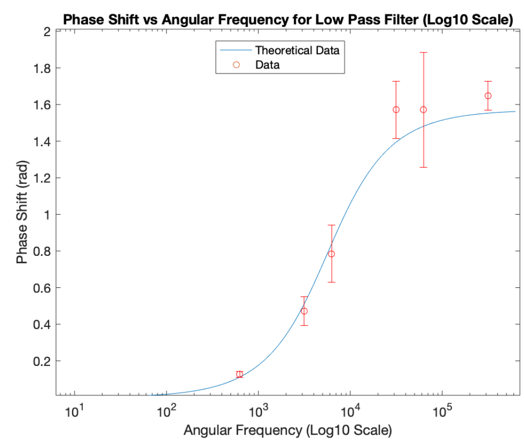

FIG. 13. Phase shift as a function of frequency for the low-pass filter on log scale

Before the angular frequency reaches a magnitude of roughly 10^3, there is negligible phase shift. As the angular frequency exceeds a magnitude of roughly 10^5, the phase shift levels off around $\frac{\pi}{2}$.

2. *High Pass Filter*

Data collected for high pass filter phase shift measurements is below:

TABLE VI. High-Pass Filter Phase-Shift

f (Hz)	Period (s)	Phase Shift (μs)	Phase Shift (rad)
100	.01	2250 $\pm$ 250	1.413 $\pm$ .157
500	.002	390 $\pm$ 25	1.225 $\pm$ .079
1000	.001	120 $\pm$ 20	.754 $\pm$ .126
5000	.0002	8 $\pm$ 2	.251 $\pm$ .063
10000	.0001	2 $\pm$ 1	.126 $\pm$ .063
50000	.00002	0 $\pm$ 1	0 $\pm$ .314

Some of the phase shift values have relatively large errors because the oscilloscope display fluctuated significantly. A plot of the data shown in table (6) with the theoretical data given by equation (32) is below:

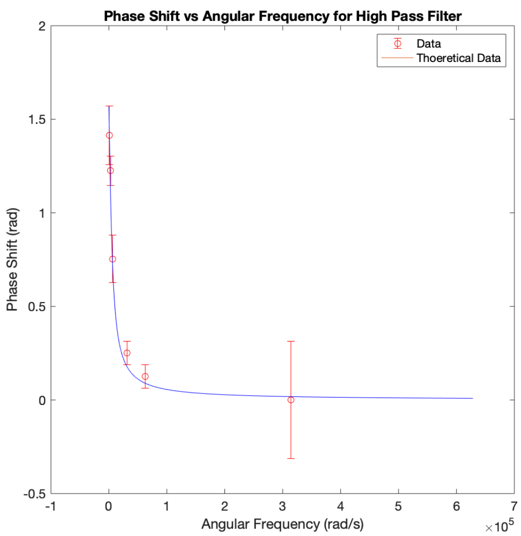

FIG. 14. Phase shift as a function of frequency for the high pass filter

The phase shift tends towards 0 as the angular frequency increases to very larger values, which agrees with equation (35). As the angular frequency approaches 0, the phase shift approaches roughly 1.57 radians, which agrees with the expected behavior given by equation (33). The phase shift at 100 Hz falls within one standard deviation of 1.57 radians, and the phase shift at 50000 Hz falls within one standard deviation of 0 radians.

The data from table (6) can be plotted on a logarithmic scale with the theoretical data given by equation (32) to better observe the phase shift behavior:

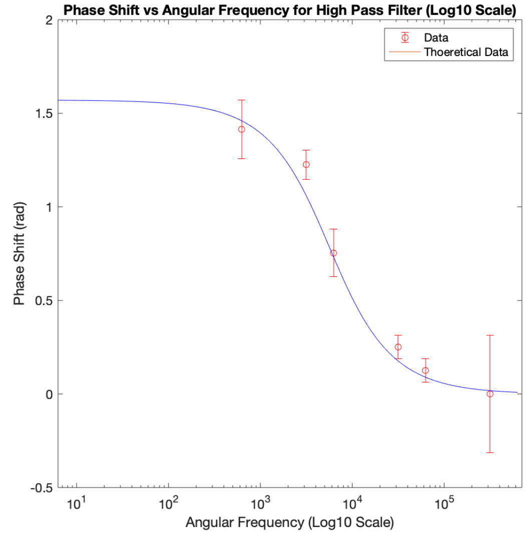

FIG. 15. Phase shift vs angular frequency for the high pass filter on log scale

For angular frequencies below an order of magnitude of roughly 10^3, the phase shift does not significantly decrease. Beyond angular frequencies above an order of magnitude of roughly 10^5, the phase shift tends towards 0.

D. Band Pass Filter

We connected our low and high pass filter to create a band pass filter. Our voltage ratio data for the band pass filter is below:

TABLE VII. Voltage ratio data for band pass filter

f (Hz)	Voltage In	Voltage Out	Ratio
100	3.04 $\pm$ .05 V	.296 $\pm$ .005 V	.097 $\pm$ .002
300	3.04 $\pm$ .05 V	.720 $\pm$ .005 V	.237 $\pm$ .004
500	3.04 $\pm$ .05 V	.880 $\pm$ .005 V	.289 $\pm$ .005
1000	3.12 $\pm$ .05 V	.960 $\pm$ .005 V	.307 $\pm$ .005
2000	3.12 $\pm$ .05 V	.880 $\pm$ .005 V	.282 $\pm$ .005
2500	3.12 $\pm$ .05 V	.800 $\pm$ .005 V	.256 $\pm$ .004
5000	3.12 $\pm$ .05 V	.480 $\pm$ .005 V	.154 $\pm$ .003
10000	3.12 $\pm$ .05 V	.240 $\pm$ .005 V	.077 $\pm$ .002

A plot showing the results from table (7) is below:

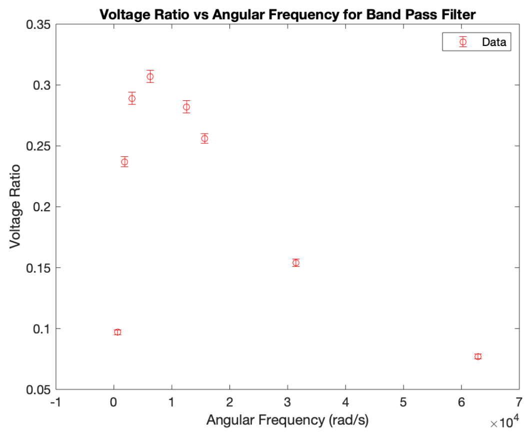

FIG. 16. Voltage Ratio vs angular frequency for band-pass filter

To better analyze the behavior for the band pass filter, the data in table (7) can be plotted on a logarithmic scale:

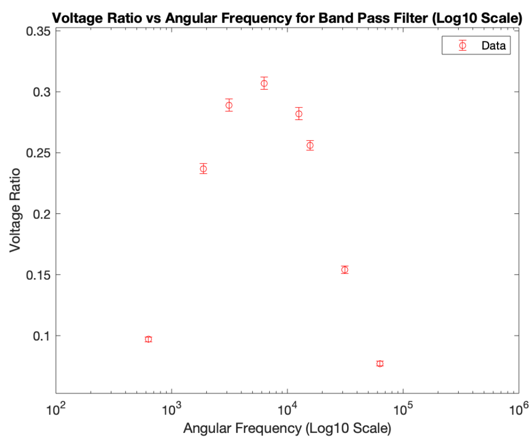

FIG. 17. Voltage Ratio vs angular frequency for band pass filter on log scale

Within the range of roughly 10^3 and 10^5 angular frequency, the band pass filter does not fully attenuate the signals. Data was not collected for frequencies outside of this range, but we expect that corresponding voltage ratios would follow the trend seen in figure (17).

The behavior seen in figure (17) is consistent with our data for the high and low pass filters. The low pass filter attenuated signals beyond roughly 10^5, while the high pass filters did not attenuate these signals. The high pass filter attenuated signals smaller than roughly 10^3, while the low pass filter did not. Therefore, we expect the band pass filter to fully attenuate signals in both of these regions and not to fully attenuate signals for angular frequencies in the middle. The band pass filter did not fully attenuate signals with angular frequency with an order of magnitude between roughly 10^3 and 10^5. This agrees with the data for the high and low pass filters because between these angular frequency ranges we do not observe full attenuation either.

The voltage ratio for the band pass filter should reach a maximum at the -3 dB frequency. We did not record data for the band pass filter at the adjusted -3 dB frequency in equation (44). The data in table (7) shows that the voltage ratio is at a statistically significant maximum at a frequency of 1000 Hz. If we had taken more data around this peak, we would have been able to test our expected - 3 dB frequency for the band pass filter.

E. RC Differentiator

We built an RC differentiator but did not record data for the results. We simply made qualitative observations of its function at varying frequencies. The frequency range for which the high-pass filter worked well as a differentiator was between roughly 50 Hz and 450 Hz. This agrees with equation (42), as the differentiator is only functional at very low frequencies. We noticed that as we increased the frequency, the differentiation become worse. At 50 Hz, the differentiation was rather clear and easy to observe. Once we reached 450 Hz, the differentiated waveform became distorted and was not clear to observe. This behavior agrees with what was expected based on equation (42).

IV. DISCUSSION

The purpose of this experiment was to construct a high, low, and band pass filter, record voltage ratio and phase shift data as a function of angular frequency, and compare mathematical and behavioral results to theoretical expectations.

The first part of the experiment involved constructing low and high pass filters and measuring their voltage ratios at the expected - 3 dB frequency. We found that the high pass voltage ratio agreed with our expectation, but the low pass ratio did not. This discrepancy might be attributed to imperfections in the circuit or low quality of the equipment that we used. To improve this measurement, we could have recorded voltage ratio data for frequencies around the expected value. This would have allowed us to determine the -3 dB frequency experimentally.

The next task was measuring the voltage ratio for the low pass filter across a wide range of frequencies. We found that the experimental behavior agreed with theoretical expectations for both the linear and logarithmic plots. We repeated this same experiment for the high pass filter, and observed behavior that agreed with our theoretical expectations. The only exception was that the voltage ratio leveled off before it reached

1, which can be attributed to circuit imperfections or the quality of our equipment. This could have been improved by recording more data points, which would have allowed us to fit a curve to the data and predict the -3 dB frequency point as well as other interesting information. Only having 7 data points for each filter significantly limited our analysis.

We also measured the phase shift for each filter as a function of frequency. Overall, the experimental behavior for each filter seemed to agree with theoretical expectations. We could have improved this data by taking more precise measurements. The error on our phase shift values were relatively large, which decreases the usefulness of our results. We also could have recorded more data, allowing us to perform deeper analyses of the results.

We recorded voltage ratio data for a band pass filter. The observed behavior agreed with theoretical expectations as well as the experimental data for the high and low pass filters in this experiment. We could have recorded more data for this test as well, which would have allowed us to properly estimate the -3 dB frequency. Phase shift data for the band pass filter was not recorded, which would have been useful for further analysis.

The final part of this experiment involved building an RC differentiator, with which we were able to roughly identify the frequency range over which it worked. This test could have been improved by recording images at various frequencies. We did not build an RC integrator, which would have been another interesting circuit to observe.

We were successful in obtaining voltage ratio and phase shift data for the low, high, and band pass filters. We were also successful in building and observing an RC differentiator. Our experiment could have been improved by collecting more data and in some cases more precise data, however, we were able to observe general behavior and compare it to theoretical expectations.

V. REFERENCES

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