

random sample X_1, X_2, \dots, X_n $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

ST 2334

$$\bar{X} = \frac{1}{n} \sum X_i$$

$$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$$

No. :

Continuity Correction (CLT) or approx

Date :

$$S^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$$

Chi-square (χ^2)

$$t\text{-dist} \quad \frac{Z}{\sqrt{U/n}} \quad Z \sim N(0,1); U \sim \chi^2(n)$$

$$Y \sim \chi^2(n) \rightarrow E(Y) = n; V(Y) = 2n$$

$$\lim_{n \rightarrow \infty} t_{n;\alpha} = z_\alpha$$

$$\sum_{i=1}^k Y_i \sim \chi^2(\sum n_i)$$

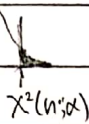
$$T \sim t(n) \rightarrow E(T) = 0, V(T) = \frac{n}{n-2}$$

$$Z \sim N(0,1) \rightarrow Z^2 \sim \chi^2(1)$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \rightarrow \text{take } Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}, U = \frac{(n-1)S^2}{\sigma^2}$$

$$P(Y \geq \chi^2(n; \alpha)) = \alpha$$

$$= P(Y \leq \chi^2(n; 1-\alpha))$$



$$F\text{-dist} \quad F = \frac{U/n_1}{V/n_2} \rightarrow U \sim \chi^2(n_1), V \sim \chi^2(n_2)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$E(X) = \frac{n_2}{n_2-2}; V(X) = \frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$$

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$$

$$F(n, m) = 1/F(m, n) \rightarrow F(n_1, n_2; 1-\alpha) = \frac{1}{F(n_2, n_1; \alpha)}$$

$$Pr(F > F(n_1, n_2; \alpha)) = \alpha$$

CI

① MEAN: single (μ_1)

③ MEAN: Paired data. Use $D_i = X_i - Y_i$

$$i) \text{ known } \sigma, \text{ normal, } n \geq 30 \rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

④ Variance single

$$2) \text{ unknown } \sigma; \text{ normal, } n < 30 \rightarrow \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

$$i) \text{ known } \mu, \text{ normal} \rightarrow \frac{\sum (X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$3) \text{ ———— } n \geq 30 \rightarrow \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim N(0,1)$$

$$ii) \text{ unknown } \mu, \text{ normal} \rightarrow \frac{\sum (X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\rightarrow \frac{(n-1)S^2}{\sigma^2}$$

② MEAN difference ($\mu_1 - \mu_2$)

⑤ Variance

$$i) \text{ known variance, } \sigma_1^2 \neq \sigma_2^2, \text{ approx. normal}$$

$$i) \text{ unknown } \mu_1, \mu_2; \text{ both normal}$$

$$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$$

$$\hookrightarrow \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$$

$$ii) \text{ unknown variance, in sufficiently large } (\sigma \rightarrow S)$$

$$iii) \text{ ———— }, \sigma_1 = \sigma_2, \text{ normal, small}$$

$$(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)$$

$$\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})}$$

$$\sim t_{n_1+n_2-2}$$

change to z if big

$$\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$\text{Type I: } P(\text{reject } H_0 | H_0)$$

$$\text{Type II: } P(\text{don't reject } H_0 | H_1)$$

$$\text{Power: } P(\text{reject } H_0 | H_1)$$

$$n \rightarrow \infty \quad p \rightarrow 0 \quad \text{Poisson}$$

$$p \rightarrow 1/2 \quad \text{Normal} \quad np > 5, n_1 > 5$$