random sample $X_1, X_2, \dots, X_n$ $\quad \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ ST2334

$\bar{X} = \frac{1}{n}\sum X_i$ $\quad \bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$

$\boxed{S^2 = \frac{1}{n-1}\sum (X_i - \bar{X})^2}$

No. : Continuity Correction (CLT) or approx

Date :

Chi-square (χ^2)

- $Y \sim \chi^2(n) \rightarrow E(Y) = n$; $V(Y) = 2n$
- $\sum^k Y_i \sim \chi^2(\sum^k n_i)$
- $Z \sim N(0,1) \rightarrow Z^2 \sim \chi^2(1)$
- $P(Y \geq \chi^2(n;\alpha)) = \alpha$

  $= P(Y \leq \chi^2(n; 1-\alpha))$. $\quad \chi^2(n;\alpha)$

$$\boxed{\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)}$$

t-dist

$\frac{Z}{\sqrt{U/n}}$ $\quad Z \sim N(0,1)$; $U \sim \chi^2(n)$.

- $\lim_{n\to\infty} t_{n;\alpha} = z_\alpha$.
- $T \sim t(n) \rightarrow E(T) = 0$, $V(T) = \frac{n}{n-2}$
- $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$ $\rightarrow$ take $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$, $U = \frac{(n-1)S^2}{\sigma^2}$

F-dist

$F = \frac{U/n_1}{V/n_2} \rightarrow U \sim \chi^2(n_1)$, $V \sim \chi^2(n_2)$

- $E(X) = \frac{n_2}{n_2 - 2}$; $V(X) = \frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$
- $\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1)$
- $F(n,m) = 1/F(m,n)$ $\longrightarrow F(n_1, n_2; 1-\alpha) = \frac{1}{F(n_2, n_1; \alpha)}$
- $Pr(F > F(n_1, n_2; \alpha)) = \alpha$

CI.

① MEAN. single (μ_1).

i) known σ, normal, $n \geq 30$. $\rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

2) unknown σ; normal, $n < 30 \rightarrow \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

3) — " — $n \geq 30$. $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N(0,1)$

② MEAN difference ($\mu_1 - \mu_2$)

i) known variance, $\sigma_1^2 \neq \sigma_2^2$, n_1, n_2 approx. normal

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$

ii) unknown variance, n sufficiently large ($\sigma \to s$)

iii) — " —, $\sigma_1 = \sigma_2$, normal, small

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t_{n_1+n_2-2}$$

change to Z if big

$\hookrightarrow S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

③ MEAN. Paired data. Use $D_i = X_i - Y_i$

④ Variance single

i) known μ, normal $\rightarrow \frac{\sum (X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$

ii) unknown μ, normal $\rightarrow \frac{\sum (X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$

$\hookrightarrow = \frac{(n-1)S^2}{\sigma^2}$

⑤ Variance

i) unknown μ_1, μ_2; both normal

$\hookrightarrow \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$

Type I : P(reject H_0 | H_0)

Type II : P(don't reject H_0 | H_1)

Power : P(reject H_0 | H_1)

$n \to \infty$ $p \to 0$. Poisson

$p \to \frac{1}{2}$ Normal $np > 5$, $nq > 5$.