Composite Simpsons Rule for Numerical Integration — Study Guide (Basic)

Study Guide on Composite Numerical Integration

Key Concepts:

- Composite Numerical Integration
- Simpson's Rule
- Subintervals
- Composite Trapezoidal Rule
- Composite Midpoint Rule
- Approximation Error

Composite Numerical Integration

- Uses a piecewise approach to numerical integration.
- Breaks the interval into smaller subintervals for more accurate approximations.

Simpson's Rule

- Approximates the integral using quadratic polynomials on each subinterval.

Composite Simpson's Rule

- Involves applying Simpson's rule on consecutive pairs of subintervals.
- Formula:

\int_{a}^{b} f(x) dx & = h \left[f(a) + 2 \sum_{j=1}^{n/2} f(x_{2j}) + 4 \sum_{j=1}^{n/2} f(x_{2j-1}) + f(b) \right] / 180h^4 f^{(4)}(\mu)

\end{aligned}∫ab f(x)dx = h

H403z M403 1759 V0 H319 V1759 v0 v1759 h84z"/>f(a) + 2 j=1∑n/

2 f(x2j) + 4 j=1∑n/2 f(x2j−1) + f(b)

M347 1759 V0 H263 V1759 v0 v1759 h84z"/> /180h4f(4)(μ)

where $h = \frac{b-a}{n}$ and $\mu \in [a, b]$.

Composite Trapezoidal Rule

- Formula:

\int_{a}^{b} f(x) dx & = h \left[f(a) + 2 \sum_{j=1}^{n-1} f(x_j) + f(b) \right] / 12h^2 f''(\mu)

\end{aligned}∫ab f(x)dx = h [f(a) + 2 j=1∑n−1 f(xj) + f(b)] /12h2f′′(μ)

where $h = \frac{b-a}{n}$ and $\mu \in [a, b]$.

Composite Midpoint Rule

- Formula:

\int_{a}^{b} f(x) dx & = 2h \sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu)

\end{aligned}∫ab f(x)dx = 2h j=0∑n/2 f(x2j) + 6b − ah2f′′(μ)

where $h = \frac{b-a}{n+2}$ and $\mu \in [a, b]$.

Examples:

- **Example 24:**
- Use Simpson's rule to approximate $\int_0^4 e^x dx$ and compare it to the approximation obtained by adding the Simpson's rule approximations for $\int_0^2 e^x dx$ and $\int_2^4 e^x dx$.
- **Example 25:**
- Use composite Simpson's rule with $n = 4$ to approximate $\int_1^x \ln(x) dx$.
- **Example 26:**
- Determine values of n that will ensure the approximation error is less than 2×10^{-5} when approximating $\int_0^\pi \sin(x) dx$ using:

  a) Composite Trapezoidal Rule

  b) Composite Simpson's Rule.