10/23/21 # HW 5 Benjamin Ruddy

1. WMMY 8.37

For a chi-squared distribution, find:

a) $\chi^2_{0.025}$ when $\nu = 15$ $\overset{A.5}{\Longrightarrow}$ $\chi^2_{0.025} = \boxed{27.488}$

$\Rightarrow$ R: qchisq(0.025, 15, lower.tail = FALSE)

= $\boxed{27.48839}$

b) $\chi^2_{0.01}$ when $\nu = 7$ $\overset{A.5}{\Longrightarrow}$ = $\boxed{18.475}$

$\overset{R}{\Longrightarrow}$ qchisq(0.01, 7, lower.tail = FALSE)

= $\boxed{18.47531}$

c) $\chi^2_{0.05}$ when $\nu = 24$ $\overset{A.5}{\Longrightarrow}$ = $\boxed{36.415}$

$\overset{R}{\Longrightarrow}$ qchisq(0.025, 24, lower.tail = FALSE) = $\boxed{36.41503}$

2. WMMY 8.40

a) $P(X^2 > \chi^2_\alpha) = 0.01$ when $\nu = 21$

By definition $\alpha = P(X > \chi^2_\alpha)$
$= 0.01$

$\overset{A.5}{\Rightarrow} \chi^2_{0.01}$ with $\nu = 21$ $= \boxed{38.932}$
$\overset{R}{\Rightarrow}$ qchisq(0.01, 21, lower.tail = FALSE) $= \boxed{38.93217}$

b) $P(X < \chi^2_\alpha) = 0.95$ when $\nu = 6$

$\rightarrow \therefore P(X \geq \chi^2_\alpha) = 1 - 0.95 = 0.05 \Rightarrow \alpha = 0.05$

$\overset{A.5}{\Rightarrow} \chi^2_{0.05}$ w/ $\nu = 6$ $= \boxed{12.592}$
$\overset{R}{\Rightarrow}$ qchisq(0.05, 6, lower.tail = FALSE) $= \boxed{12.59159}$

c) $P(\chi^2_\alpha < X^2 < 23.209) = 0.015$ when $\nu = 10$

$\therefore \underbrace{P(X^2 > \chi^2_\alpha)}_{=\alpha} - \underbrace{P(X^2 > 23.209)} = 0.015$

$\chi^2_\beta = 23.209, \nu = 10$ implies $\beta = 0.01$ by table A.5

$\Rightarrow \alpha - 0.01 = 0.015 \Rightarrow \alpha = 0.025$

$\overset{A.5}{\Rightarrow} \chi^2_{0.025}, \nu = 10 = 20.483$
$\overset{R}{\Rightarrow}$ qchisq(0.025, 10, lower.tail = FALSE) $= \boxed{20.48318}$

3. WMMY 8.41

Assume sample variances to be continuous measurements. Find the probability that a random sample of 25 observations from a normal population with $\sigma^2 = 6$ will have a sample variance S^2

a) greater than 9.1: $\Rightarrow S^2 > 9.1$; $\underbrace{\chi^2 = \frac{(25-1)S^2}{6} = 4S^2}_{\frac{1}{4}\chi^2 > 9.1}$

$P(\frac{1}{4}\chi^2 > 9.1) = P(\chi^2 > 36.4)$

$\Rightarrow$ with $v = n-1 = 24$, $P(\chi^2 > 36.4) \overset{A.5}{=} \boxed{0.05}$

b) between 3.462 and 10.745

$P(3.462 < \frac{1}{4}\chi^2 < 10.745)$ with $v = n-1 = 24$

$= P(\chi^2 > 13.848) - P(\chi^2 > 42.98)$

$\overset{A.5}{=} 0.95 - 0.01 = \boxed{0.94}$

4. WMMY 8.47

Given a random sample of $\overbrace{\text{size } 24}^{\Rightarrow \nu = 23}$ from a normal dist, find k s.t.

a) $P(-2.069 < T < k) = 0.965$

$= P(T > -2.069) - P(T > k) = 0.965$

$= \underbrace{P(T > -t_{0.025})}_{-t_x = t_{1-x}} - P(T > k) = 0.965$

$\Rightarrow P(T > k) = 1 - 0.025 - 0.965$

$= 0.01$

$\therefore k = t_{0.01} \overset{A.4}{=} \boxed{2.500}$ w/ $\nu = 23$

$\overset{R}{\Rightarrow}$ qt(0.01, 23, lower.tail = FALSE) $= \boxed{2.499867}$

b) $P(k < T < 2.807) = 0.095$

$= P(T > k) - P(T > 2.807) = 0.095$

$= P(T > k) - P(T > t_{0.005}) = 0.095$

$\Rightarrow P(T > k) = 0.095 + 0.005$

$= 0.100$

$\therefore k = t_{0.100}$ w/ $\nu = 23$ $\overset{A.4}{=} \boxed{1.319}$; $\overset{R}{=}$ qt(0.1, 23, lower.tail = FALSE) $= \boxed{1.31946}$

c) $P(-k < T < k) = 0.90$

$= \underbrace{P(T > -k) - P(T > k)}_{\text{the middle 90\% of values}} = 0.90$

the middle 90% of values $\Rightarrow$ 5% to the right of k, 5% to left of $-k$

$\Rightarrow k = t_{0.05} = \boxed{1.714}$ w/ $\nu = 23$

$\overset{R}{\Rightarrow}$ qt(0.05, 23, lower.tail = FALSE) $= \boxed{1.713872}$

5. WMMY 8.51

a) $f_{0.05}$ with $v_1 = 7$, $v_2 = 15$:

$\overset{A.6}{=} \boxed{2.71}$; $\overset{R}{=}$ qf(0.05, 7, 15, lower.tail = FALSE) = $\boxed{2.706627}$

b) $f_{0.05}$, $v_1 = 15$, $v_2 = 7$

$\overset{A.6}{=} \boxed{3.51}$; $\overset{R}{=}$ qf(0.05, 15, 7, lower.tail = FALSE) = $\boxed{3.51074}$

c) $f_{0.01}$; $v_1 = 24$, $v_2 = 19$

$\overset{A.6}{=} \boxed{2.92}$ $\overset{R}{=}$ qf(0.01, 24, 19) = $\boxed{2.924866}$

d) $f_{0.95}$, $v_1 = 19$, $v_2 = 24$

using $f_{1-\alpha}^{(v_1,v_2)} = \frac{1}{f_\alpha^{(v_2,v_1)}}$ $\quad f_{0.95}^{(v_1,v_2)} = \frac{1}{f_{0.05}^{(v_2,v_1)}} \overset{A.6}{=} \frac{1}{2.11} = \boxed{0.474}$

order of (v_1, v_2) swaps

$\overset{R}{=}$ qf(0.95, 19, 24, lower.tail = FALSE) = $\boxed{0.4730049}$

e) $f_{0.99}$, $v_1 = 28$, $v_2 = 12$

$f_{0.99}(28, 12) = f_{0.01}(12, 28) \overset{A.6}{=} \frac{1}{2.12} = \boxed{0.345}$

$\overset{R}{=}$ qf(0.99, 12, 28) = $\boxed{0.3453181}$

8. WMMY 9.6

$$n = \left(\frac{z_{\alpha/2}\sigma}{e}\right)^2$$

$$= \left(\frac{(2.05)(40)}{10}\right)^2$$

$$= 67.24 \rightarrow \boxed{68}$$

9. WMMY 9.12

$n=10,\ \bar{x}=230,\ s=15,\ \alpha = 1-.99 = 0.01$

for 99% confidence:

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$= 230 - t_{0.005}\frac{15}{\sqrt{10}} < \mu < 230 + t_{0.005}\frac{15}{\sqrt{10}}$$

A.5 $= 3.25$

$$\Rightarrow \boxed{214.6 < \mu < 245.4}$$