

4.14

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1 \\ 0, & \text{else} \end{cases} \rightarrow \int_0^1 x \cdot \frac{2x+4}{5} dx = \int_0^1 \frac{2x^2}{5} + \frac{4x}{5} dx$$

$$= \left[ \frac{2x^3}{15} + \frac{4x^2}{10} \right]_0^1 = \frac{2}{15} + \frac{4}{10} = \frac{2}{15} + \frac{6}{15} = \boxed{\frac{8}{15}}$$

4.38]  $\int_0^1 (x - \frac{8}{15})^2 \cdot \frac{2x+4}{5} dx = \frac{37}{450} = \boxed{0.082}$

4.40]  $E(3x^2 + 4) = \mu_{g(x)} = 3E(x^2) + 4 = 5.1$

$$E(x^2) = \int_0^1 x^2 \cdot \frac{2x+4}{5} dx = 0.36$$

$$E((3x^2 + 4)^2) = \int_0^1 (3x^2 + 4)^2 \cdot \frac{2x+4}{5} dx = 26.84 - 5.1^2 = \boxed{0.83} = \sigma_{g(x)}^2$$

4.62] X & Y independent,  $\sigma_x^2 = 5$      $\sigma_y^2 = 3$

$$Z = -2X - 4Y - 3$$

$$\sigma_z^2 = \sigma_{-2X-4Y-3}^2 = \sigma_{-2X-4Y}^2 = 4\sigma_x^2 + 16\sigma_y^2 = 20 + 48 = \boxed{68}$$

4.63] X & Y not independent,  $\sigma_{xy} = 1$

$$\sigma_z^2 = \sigma_{-2X-4Y-3}^2 = \sigma_{-2X-4Y}^2 = 4\sigma_x^2 + 16\sigma_y^2 - 2 \cdot 2 \cdot 4 \sigma_{xy} \\ = 4 \cdot 5 + 16 \cdot 3 - 16 \cdot 1 = \boxed{52}$$

$$\text{correlation} = \frac{1}{\sqrt{5 \cdot 3}} = \frac{1}{\sqrt{15}} = \boxed{0.258}$$

$$\sigma_{xy} = E(XY) - E(X)E(Y)$$

4.83]  $f(x,y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, x+y \leq 1 \\ 0, & \text{else} \end{cases}$



$$E(XY) = \int_0^1 \int_0^{1-x} xy \cdot 24xy \, dy \, dx$$

$$= 24 \int_0^1 \int_0^{1-x} x^2 y^2 \, dy \, dx = 0.13$$

$$g(x) = \int_0^{1-x} 24xy \, dy$$

$$= 12x(1-x)^2 \mathbb{1}(0 < x < 1)$$

$$h(y) = \int_0^{1-y} 24xy \, dx$$

$$= 12y(1-y)^2 \mathbb{1}(0 < y < 1)$$

$$\text{Correlation} = \frac{-0.027}{\sqrt{0.04 \cdot 0.04}} = \boxed{-0.668}$$

$$E(X) = \int_0^1 12x^2(1-x)^2 \, dx = 0.4$$

$$E(Y) = \int_0^1 12y^2(1-y)^2 \, dy = 0.4$$

$$\sigma_{xy} = 0.13 - 0.4 \cdot 0.4 = \boxed{-0.027}$$

$$E(X^2) = \int_0^1 12x^3(1-x)^2 \, dx = 0.2$$

$$E(Y^2) = \int_0^1 12y^3(1-y)^2 \, dy = 0.2$$

$$\sigma_x^2 = 0.2 - 0.4^2 = 0.04$$

$$\sigma_y^2 = 0.2 - 0.4^2 = 0.04$$

4.84]  $E(X+Y) = \int_0^1 \int_0^{1-x} (x+y) \cdot 24xy \, dy \, dx = \boxed{0.8}$