

STA 3032 Exam 1 (Practice)

(100 points)

Print your registered name and UFID clearly.

Name: _____ UFID: _____

Instructions:

- Write all your answers on the separate answer sheet provided and submit only the answer sheet at the end of the exam.
- All problems are short-answer questions. Write only your final answer.
- Round all your answers to three decimal places.
- All notations are consistent with those given in class unless otherwise specified.

1. Consider the following data and answer the questions below. Use the definitions given in class.

2 3 5 10 10 11 12 13 14 15 15 22 23 25

- (a) (2 points) Find Q_1 .
 - (b) (2 points) Find Q_2 .
 - (c) (2 points) Find Q_3 .
 - (d) (3 points) Suppose you construct a horizontal boxplot using the data above. What is the horizontal length of the box?
 - (e) (4 points) How many observations will be shown separately in the boxplot?
 - (f) (2 points) It is calculated that $\sum_{i=1}^n x_i = 180$, where x_i is the i th observation of the data and n is the sample size. Find the sample mean of the data.
 - (g) (4 points) It is calculated that $\sum_{i=1}^n x_i^2 = 2956$. Find the sample variance of the data.
2. Suppose you roll a fair die once. Let A be the event that you roll an odd number. Let B be the event that you roll a number less than or equal to 3.
 - (a) (3 points) Find $P(A \cap B)$.
 - (b) (3 points) Find $P(A \cup B)$.
 - (c) (3 points) Find $P(A|B)$.
 - (d) (2 points) Are A and B mutually exclusive?
Yes No
 - (e) (2 points) Are A and B independent?
Yes No
 3. Sociologists are studying divorce rates in a community. For the population of married or once-married adults in that community, the number of divorces (X) and their respective probabilities are as follows.

$X = x$	0	1	2	3
$P(X = x)$	0.90	0.06	0.03	0.01

- (a) (4 points) Among those who have been divorced, find the probability they have been divorced more than once.
- (b) (3 points) Find $E(X)$.
- (c) (4 points) Find $\text{Var}(X)$.

4. Let X and Y be two random variables with the pdfs

$$g(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

and

$$h(y) = \begin{cases} \frac{1}{\pi}y^{-\frac{1}{2}}(1-y)^{-\frac{1}{2}}, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

respectively. Also, X and Y have the joint pdf

$$f(x, y) = \begin{cases} \frac{1}{2\pi}e^{-\frac{x}{2}}y^{-\frac{1}{2}}(1-y)^{-\frac{1}{2}}, & x > 0, 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) (3 points) It can be shown that $E(X) = 2$ and $E(X^2) = 8$. Find $\text{Var}(X)$.
 (b) (4 points) It can be shown that

$$\int_0^1 y^{\frac{1}{2}}(1-y)^{-\frac{1}{2}}dy = \frac{\pi}{2} \text{ and that } \int_0^1 \left(y - \frac{1}{2}\right)^2 y^{-\frac{1}{2}}(1-y)^{-\frac{1}{2}}dy = \frac{\pi}{8}.$$

Find $E(Y^2)$.

- (c) (3 points) Find $E(XY)$.
 (d) (2 points) Find $\text{Cov}(X, Y)$.
 (e) (3 points) Find $\text{Var}(X + Y)$.

5. Let X and Y be two random variables with the joint pdf

$$f(x, y) = \begin{cases} k, & 0 < x < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where k is a constant. Find the following.

- (a) (4 points) k .
 (b) (3 points) $E(X)$.
 (c) (3 points) $E(X^2)$.
 (d) (2 points) $\text{Var}(X)$.
 (e) (3 points) $E(Y)$.
 (f) (3 points) $E(Y^2)$.
 (g) (2 points) $\text{Var}(Y)$.
 (h) (4 points) $E(XY)$.
 (i) (3 points) $\text{Cov}(X, Y)$.
 (j) (3 points) $\text{Corr}(X, Y)$.
 (k) (4 points) $\text{Var}(X - Y)$.
 (l) (4 points) $P(1/4 < X < 1/2 | Y = 3/4)$.
 (m) (4 points) $P(1/3 < Y < 1/2 | X = 1/6)$.