Presentation Title

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Introduction

planetmath.org

A *statistical model* is usually parameterised by a function, called a *parametrisation*

$$\Theta \to \mathcal{P}, \quad \text{given by} \quad \theta \mapsto P_\theta, \quad \text{so that} \quad \mathcal{P} = \{P_\theta : \theta \in \Theta\},$$

where Θ is the *parameter space.* Θ is usually a subset of $\mathbb{R}^n$.

McCullagh, 2002

This should be defined using category theory.

Let X, Y, and Z be random variables that have a, b, and c states, respectively. A *probability distribution* P for these random variables is an $(a \times b \times c)$-table of non-negative numbers which sum to one.
The entries of the table P are the probabilities

$$P_{ijk} = \mathrm{Prob}(X = i; Y = j; Z = k).$$

The set of all distributions is a simplex Δ of dimension $abc - 1$.
A *statistical model* is a subset $\mathcal{M}$ of Δ which can be described by polynomial equations and inequalities in the coordinates P_{ijk}.
Usually, the model $\mathcal{M}$ is presented as the image of a polynomial map $P : \Theta \to \Delta$, where Θ is a polynomially described subset of Δ.

The distribution P is called *independent* is each probability is the product of the corresponding *marginal probabilities*:

$$P_{ijk} = P_{i++} \cdot P_{+j+} \cdot P_{++k}.$$

A marginal probability is the probability of an event irrespective of the outcomes of the other variables, that is:

$$P_{i++} = \mathrm{Prob}(X = i) = \sum_{j=1}^{b} \sum_{k=1}^{c} P_{ijk}.$$

The *independence model* has the parametric representation:

$$\Theta = \Delta_{a-1} \times \Delta_{b-c} \times \Delta_{c-1} \to \Delta = \Delta_{abc-1},$$
$$(\alpha, \beta, \gamma) \mapsto (P_{ijk}) = (\alpha_i \beta_j \gamma_k).$$

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Summary

Questions?