Presentation Title

Author Name
Author Email

Presentation Location

26th October 2020

Contents I

Introduction

planetmath.org

A *statistical model* is usually parameterised by a function, called a *parametrisation*

$$\Theta \to \mathcal{P}, \quad \text{given by} \quad \theta \mapsto P_\theta, \quad \text{so that} \quad \mathcal{P} = \{P_\theta : \theta \in \Theta\},$$

where Θ is the *parameter space.* Θ is usually a subset of $\mathbb{R}^n$.

McCullagh, 2002

This should be defined using category theory.

Let X, Y, and Z be random variables that have a, b, and c states, respectively. A *probability distribution* P for these random variables is an $(a \times b \times c)$-table of non-negative numbers which sum to one.
The entries of the table P are the probabilities

$$P_{ijk} = \mathrm{Prob}(X = i; Y = j; Z = k).$$

The set of all distributions is a simplex Δ of dimension $abc - 1$.
A *statistical model* is a subset $\mathcal{M}$ of Δ which can be described by polynomial equations and inequalities in the coordinates P_{ijk}.
Usually, the model $\mathcal{M}$ is presented as the image of a polynomial map $P : \Theta \to \Delta$, where Θ is a polynomially described subset of Δ.

The distribution P is called *independent* is each probability is the product of the corresponding *marginal probabilities*:

$$P_{ijk} = P_{i++} \cdot P_{+j+} \cdot P_{++k}.$$

A marginal probability is the probability of an event irrespective of the outcomes of the other variables, that is:

$$P_{i++} = \mathrm{Prob}(X = i) = \sum_{j=1}^{b} \sum_{k=1}^{c} P_{ijk}.$$

The *independence model* has the parametric representation:

$$\Theta = \Delta_{a-1} \times \Delta_{b-c} \times \Delta_{c-1} \to \Delta = \Delta_{abc-1},$$
$$(\alpha, \beta, \gamma) \mapsto (P_{ijk}) = (\alpha_i \beta_j \gamma_k).$$

The image is known as the *Segre variety* in algebraic geometry.

Projective Space

Playing field is *n-dimensional projective space*, $\mathbb{P}^n$:

$$\mathbb{P}^n := \{(z_0, \ldots, z_n) \in \mathbb{C}^n\}/(\mathbf{x} \sim \lambda \cdot \mathbf{y}), \quad \lambda \neq 0,$$

that is, its elements consists of *lines through the origin* in $\mathbb{C}^n$.

TODO: FIGURE

Varieties

Varieties are the geometric objects studied in algebraic[1] geometry, which are the *zero sets* of polynomials.
Example:

$$S^1 = \{x^2 + y^2 = 1\} = V(x^2 + y^2 - 1).$$

TODO: FIGURE

[1]classical algebraic geometry

Segre Varieties

These come from the maps $\sigma : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^{(n+1)(m+1)-1}$, sending the pair $([X],[Y])$ to the coordinates formed by pairwise products of the individual $[X]$ and $[Y]$:

$$\sigma : ([X_0, \ldots, X_n], [Y_0, \ldots, Y_m]) \mapsto [\ldots, X_i Y_j, \ldots],$$

with the image is formed overall pairwise products of X_i and Y_j.

Example

$$\sigma : \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^3,\ ([X_0, X_1], [Y_0, Y_1]) \mapsto [X_0 Y_0, X_0 Y_1, X_1 Y_0, X_1 Y_1].$$

If we set $[X_0 Y_0, X_0 Y_1, X_1 Y_0, X_1 Y_1] = [Z_0, Z_1, Z_2, Z_3]$, then notice that $Z_0 Z_3 - Z_1 Z_2 = 0$.

Summary

Questions?