

Presentation Title

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Introduction

planetmath.org

A *statistical model* is usually parameterised by a function, called a *parametrisation*

$\Theta \rightarrow \mathcal{P}$, given by $\theta \mapsto P_\theta$, so that $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$,

where Θ is the *parameter space*. Θ is usually a subset of \mathbb{R}^n .

McCullagh, 2002

This should be defined using category theory.

Three-Way Contingency Tables

Let X , Y , and Z be random variables that have a , b , and c states, respectively. A *probability distribution* P for these random variables is an $(a \times b \times c)$ -table of non-negative numbers which sum to one.

The entries of the table P are the probabilities

$$P_{ijk} = \text{Prob}(X = i; Y = j; Z = k).$$

The set of all distributions is a simplex Δ of dimension $abc - 1$. A *statistical model* is a subset \mathcal{M} of Δ which can be described by polynomial equations and inequalities in the coordinates P_{ijk} .

Usually, the model \mathcal{M} is presented as the image of a polynomial map $P: \Theta \rightarrow \Delta$, where Θ is a polynomially described subset of Δ .

Independence

The distribution P is called *independent* if each probability is the product of the corresponding *marginal probabilities*:

$$P_{ijk} = P_{i++} \cdot P_{+j+} \cdot P_{++k}.$$

A marginal probability is the probability of an event irrespective of the outcomes of the other variables, that is:

$$P_{i++} = \text{Prob}(X = i) = \sum_{j=1}^b \sum_{k=1}^c P_{ijk}.$$

Independence Model

The *independence model* has the parametric representation:

$$\begin{aligned}\Theta &= \Delta_{a-1} \times \Delta_{b-c} \times \Delta_{c-1} \rightarrow \Delta = \Delta_{abc-1}, \\ (\alpha, \beta, \gamma) &\mapsto (P_{ijk}) = (\alpha_i \beta_j \gamma_k).\end{aligned}$$

The image is known as the *Segre variety* in algebraic geometry.

Projective Space

Playing field is *n-dimensional projective space*, \mathbb{P}^n :

$$\mathbb{P}^n := \{(z_0, \dots, z_n) \in \mathbb{C}^n\} / (\mathbf{x} \sim \lambda \cdot \mathbf{y}), \quad \lambda \neq 0,$$

that is, its elements consists of *lines through the origin* in \mathbb{C}^n .

TODO: FIGURE

Varieties are the geometric objects studied in algebraic¹ geometry, which are the *zero sets* of polynomials.

Example:

$$S^1 = \{x^2 + y^2 = 1\} = V(x^2 + y^2 - 1).$$

TODO: FIGURE

¹classical algebraic geometry

These come from the maps $\sigma : \mathbb{P}^n \times \mathbb{P}^m \rightarrow \mathbb{P}^{(n+1)(m+1)-1}$, sending the pair $([X], [Y])$ to the coordinates formed by pairwise products of the individual $[X]$ and $[Y]$:

$$\sigma : ([X_0, \dots, X_n], [Y_0, \dots, Y_m]) \mapsto [\dots, X_i Y_j, \dots],$$

with the image is formed overall pairwise products of X_i and Y_j .

Example

$$\sigma : \mathbb{P}^1 \times \mathbb{P}^1 \rightarrow \mathbb{P}^3, ([X_0, X_1], [Y_0, Y_1]) \mapsto [X_0 Y_0, X_0 Y_1, X_1 Y_0, X_1 Y_1].$$

If we set $[X_0 Y_0, X_0 Y_1, X_1 Y_0, X_1 Y_1] = [Z_0, Z_1, Z_2, Z_3]$, then notice that $Z_0 Z_3 - Z_1 Z_2 = 0$.

Summary

Questions?