Geometry and Spin Transport in Skyrmion Magnets

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Abstract

We investigate the emergent electrodynamic fields that arise from potentials when a free electron travels through a skyrmion magnetic texture. In the adiabatic limit we project these potentials onto the spin axis, allowing us to investigate the nature of the emergent fields. We consider the cases where the magnetic texture is time-independent and time-dependent, and find that a spin-motive force makes itself manifest.

Introduction

The equilibrium magnetisation field in chiral magnets often possess a non-trivial topology. An example of such a configuration is the magnetic skyrmion, which is a vortex-like whirl found under certain conditions in transition metal compounds such as MnSi [1].

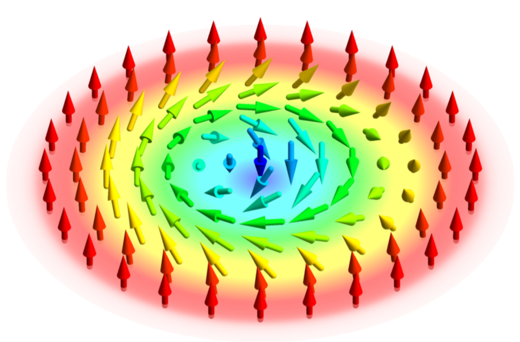

Figure 1: A vortex-like magnetic whirl, *i.e.* a skyrmion [2].

An interesting consequence of the skyrmion texture is that it endows to the wavefunction of a free election travelling through the texture a geometric phase, which can be interpreted as a Lorentz force due to emergent electromagnetic fields [2]. The aim of this project is to describe the dynamics of these electrons due to these fields.

Conclusion

We have described the emergent electromagnetic fields that arise when a free electron travels though a skyrmion magnetic texture. When a magnetic field is applied orthogonally to the basal plane, an emergent electric field arises which acts to deflect free electrons radially from the skyrmion core. To take this project further, it would be interesting to investigate:

1. How different forms of the LLG equation determine the emergent electric field.
2. The effect that non-adiabicity has on the potentials.

References

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Method

For the skyrmion illustrated in Fig. 1, the magnetic order parameter function $\hat{\boldsymbol{M}}(\boldsymbol{r})$ which minimises the energy of the skyrmion system is of the form:

$$\hat{\boldsymbol{M}}(\boldsymbol{r}) = \left(\frac{2\lambda r}{r^2+\lambda^2}\cos\Phi(\phi), \frac{2\lambda r}{r^2+\lambda^2}\sin\Phi(\phi), \frac{r^2-\lambda^2}{r^2+\lambda^2}\right),$$

where $r=\sqrt{x^2+y^2}$, λ is the size of the skyrmion, and $\Phi(\phi)=\mathcal{W}\phi+\phi_0$ (ϕ_0 is a constant) describes the winding of $\hat{\boldsymbol{M}}$ of winding number $\mathcal{W}$ around the 2-sphere [3]. This winding configuration is what gives rise to the non-trivial topology and stability of the magnetic texture [2].

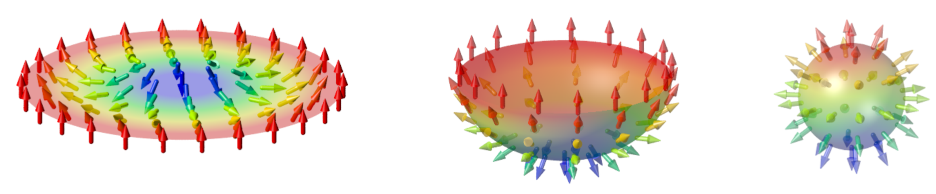

Figure 2: The wrapping of an anti-skyrmion configuration around the 2-sphere with $\mathcal{W}=-1$ [2].

A free electron travelling through a temporally or spatially inhomogeneous magnetic texture $\hat{\boldsymbol{M}}(\boldsymbol{r},t)$ is described by the Schrödinger equation with Hamiltonian $H(\boldsymbol{r},t)$:

$$i\hbar\partial_t\left|\chi\right\rangle = \left[\frac{\boldsymbol{p}^2}{2m}\mathbb{1} + J\frac{g\mu_B}{2}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{M}}(\boldsymbol{r},t)\right]\left|\chi\right\rangle = H(\boldsymbol{r},t)\left|\chi\right\rangle$$

for the two component spinor $\left|\chi\right\rangle=(\chi_\uparrow,\chi_\downarrow)^T$, with rows corresponding to majority and minority spin electrons respectively [2].

Results

In order to determine the motion of a free electron as a result of this Hamiltonian, we rotate the spin axis $\left|\sigma\right\rangle$ to align with $\hat{\boldsymbol{M}}(\boldsymbol{r},t)$, which may be achieved by the rotation matrix $U\propto\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{M}}+\hat{\boldsymbol{e}}_z)$ [3]. The transformed Hamiltonian is then:

$$i\hbar\partial_t\left|\chi\right\rangle = H\left|\chi\right\rangle \xrightarrow{U(\boldsymbol{r},t)} H'\left|\psi\right\rangle = \left[q^eV^e + \frac{(\boldsymbol{p}\mathbb{1}-q^e\boldsymbol{A}^e)^2}{2m} + J'\sigma_z\right]\left|\psi\right\rangle, \qquad \left|\psi\right\rangle = U(\mathbf{r},t)\left|\chi\right\rangle$$

where we have introduced the emergent scalar and vector potentials with an emergent charge q^e:

$$V^e = -(i\hbar/q^e)U^\dagger\partial_t U, \qquad \boldsymbol{A}^e = (i\hbar/q^e)U^\dagger\boldsymbol{\nabla}U.$$

In the adiabatic limit, we project these potentials onto the spin axis $\left|\sigma\right\rangle$ to determine that:

$$\mathcal{V}^e_\sigma = \left\langle\sigma\right|V^e\left|\sigma\right\rangle = -(i\hbar/q^e)\left\langle\Psi_\sigma\right|\partial_t\left|\Psi_\sigma\right\rangle, \qquad \boldsymbol{\mathcal{A}}^e_\sigma = \left\langle\sigma\right|\boldsymbol{A}^e\left|\sigma\right\rangle = (i\hbar/q^e)\left\langle\Psi_\sigma\right|\boldsymbol{\nabla}\left|\Psi_\sigma\right\rangle,$$

for $\left|\Psi_\sigma\right\rangle = U(\boldsymbol{r},t)\left|\sigma\right\rangle$. These potentials can be identified with the famous Berry connection [4], with the following Berry curvatures for each spin band σ as emergent electric and magnetic fields:

$$\boldsymbol{E}^e_\sigma = -\boldsymbol{\nabla}\mathcal{V}^e_\sigma - \partial_t\boldsymbol{\mathcal{A}}^e_\sigma, \qquad \boldsymbol{B}^e_\sigma = \boldsymbol{\nabla}\times\boldsymbol{\mathcal{A}}^e_\sigma.$$

If $\hat{\boldsymbol{M}}(\boldsymbol{r},t)\equiv\hat{\boldsymbol{M}}(\boldsymbol{r})$, then it can be easily seen that $\boldsymbol{E}^e=0$ and that:

$$\boldsymbol{B}^e_\sigma = \mp\frac{2\mathcal{W}\hbar\lambda^2}{q^e(r^2+\lambda^2)^2}\hat{\boldsymbol{e}}_z \equiv \pm\frac{\mathcal{W}\hbar}{2q^e}\hat{\boldsymbol{M}}\cdot(\partial_x\hat{\boldsymbol{M}}\times\partial_y\hat{\boldsymbol{M}})\hat{\boldsymbol{e}}_z,$$

where $\boldsymbol{B}^e_\sigma$ is negative (resp. positive) for majority (resp. minority) spin electrons. It follows immediately that the surface integral over the skyrmion texture of $|\boldsymbol{B}^e_\sigma|$ is topologically quantised:

$$\frac{1}{4\pi}\iint_{\text{skyrmion}}\boldsymbol{B}^e_\sigma\cdot d\boldsymbol{S} = \frac{-\mathcal{W}\hbar\lambda^2}{q^e}\int_0^\lambda\frac{r\,dr}{(r^2+\lambda^2)^2} = -\mathcal{W}\hbar/2q^e.$$

Now if we apply a magnetic field $\boldsymbol{B}=(0,0,-B)$ over the skyrmion, $\hat{\boldsymbol{M}}$ develops a time-dependence described by the dissipationless Landau-Lifshitz-Gilbert (LLG) equation [3]:

$$\partial_t\hat{\boldsymbol{M}} = \alpha\boldsymbol{B}\times\hat{\boldsymbol{M}} \quad\Longrightarrow\quad \hat{\boldsymbol{M}}(\boldsymbol{r},t) = \left(\frac{2\lambda r}{r^2+\lambda^2}\cos\tilde{\Phi}(\phi,t), \frac{2\lambda r}{r^2+\lambda^2}\sin\tilde{\Phi}(\phi,t), \frac{r^2-\lambda^2}{r^2+\lambda^2}\right),$$

where now $\tilde{\Phi}(\phi,t)=\mathcal{W}\phi-\alpha Bt+\phi_0$. The matrix $U(\boldsymbol{r},t)$ thus inherits this time-dependence, which lets us determine the new emergent electromagnetic fields; $\boldsymbol{B}^e_\sigma$ remains the same as before in the adiabatic limit, but now we have a nonzero $\boldsymbol{E}^e_\sigma$:

$$\boldsymbol{E}^e_\sigma = \pm\frac{2\alpha B\hbar\lambda^2 r}{q^e(r^2+\lambda^2)^2}\hat{\boldsymbol{e}}_r,$$

which acts in the positive (resp. negative) radial direction for majority (resp. minority) spin electrons. Hence we have found that the skyrmion texture gives rise to a spin-motive force that acts on free electrons, *i.e.* the trajectory of the electron depends upon its spin state.