QUESTION R.E. S^1-ACTION VIA A SUBTORUS

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1. Example

Take $(\mathbb{C}^2, \omega_{std})$ with T^2 acting on $\mathbb{C}^2$ as

$$(t_1, t_2) \cdot (z_1, z_2) = (t_1 z_1, t_2 z_2).$$

This action is Hamiltonian with moment map $\mu : \mathbb{C}^2 \to \mathbb{R}^2$ given by

$$\mu(z_1, z_2) = \frac{1}{2}\left(|z_1|^2, |z_2|^2\right).$$

For the symplectic cut, relabel $M := \mathbb{C}^2$ then consider $M \times \mathbb{C}$, along with the following S^1-action:

$$\tau \cdot (z_1, z_2, \xi) = (\tau z_1, \tau z_2, \tau \xi),$$

so S^1 can be thought of acting on the first factor, M, via the inclusion $S^1 \hookrightarrow T^2$ as $\tau \mapsto (\tau, \tau)$, and then via the diagonal product action on $M \times \mathbb{C}$. This action is also Hamiltonian, with moment map $\Phi : M \times \mathbb{C} \to \mathbb{R}$, given by

$$\Phi(z_1, z_2, \xi) = \frac{1}{2}\left(|z_1|^2 + |z_2|^2 + |\xi|^2\right).$$

Consider the preimage of $k \in \mathbb{Z}$ under Φ to get

$$\Phi^{-1}(k) = \left\{ (z_1, z_2, 0) \in M \times \mathbb{C} \;\middle|\; \|z\|^2 = 2k \right\} \bigsqcup \left\{ (z_1, z_2, \xi) \in M \times \mathbb{C} \;\middle|\; \|z\|^2 < 2k \right\}$$

References

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