

Quantisation & Lattice Points in Polytopes

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EUWiSTEM & MathSoc Talk



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Classical Mechanics

- ▶ In classical mechanics, the space of all possible states of a system is given by *phase space*, M .
- ▶ “*State*” describes the *position* and the *momentum*.

Quantum Mechanics

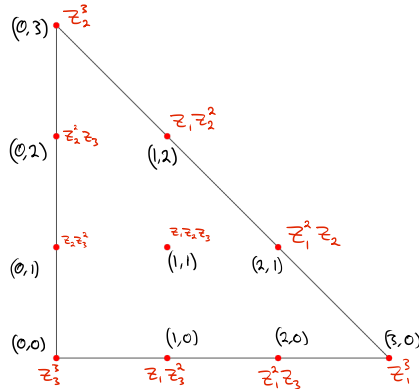
- ▶ In quantum mechanics, still have M but states are replaced by *wavefunctions*.
- ▶ In my research, wavefunctions are just *homogeneous polynomials*, for example:

$$\phi(\mathbf{z}) = z_1^{k_1} z_2^{k_2} z_3^{k_3}, \quad \text{where } k_1 + k_2 + k_3 = k.$$

Lattice Points

- ▶ Certain types of spaces M are special¹: each has an associated polytope.
- ▶ Each lattice point inside corresponds to a polynomial.

$k=3$.



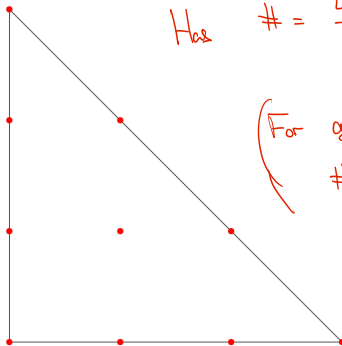
$$(k_1, k_2) \leftrightarrow z_1^{k_1} z_2^{k_2} z_3^{3-k_1-k_2}$$

¹Called “Symplectic toric manifolds”.

Quantisation Dimension

- ▶ Degree k of the polynomial analogous to quantised “energy” of the system.
- ▶ Quantisation dimension M equals the lattice point count (how many wavefunctions there are).

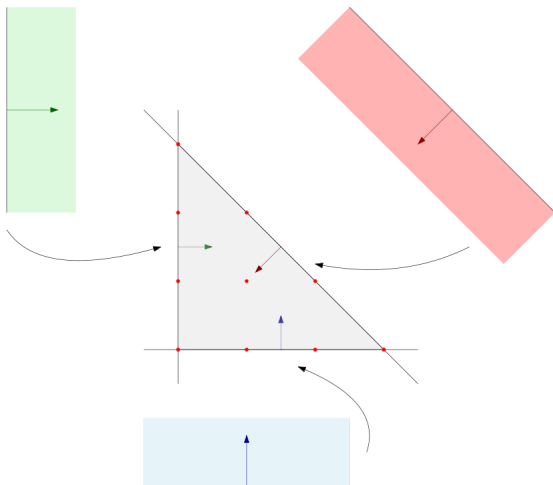
$$k = 3$$



$$\text{Has } \# = \frac{4 \cdot 5}{2} = 10 \text{ points.}$$

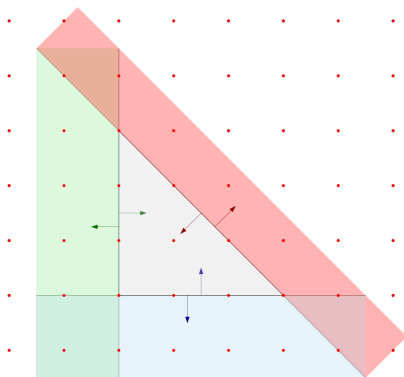
$$\left(\text{For general } k, \right. \\ \left. \# = \frac{(k+1)(k+2)}{2} \right)$$

- Useful to view the polytope as an intersection of half-spaces.



Hyperplane Arrangements

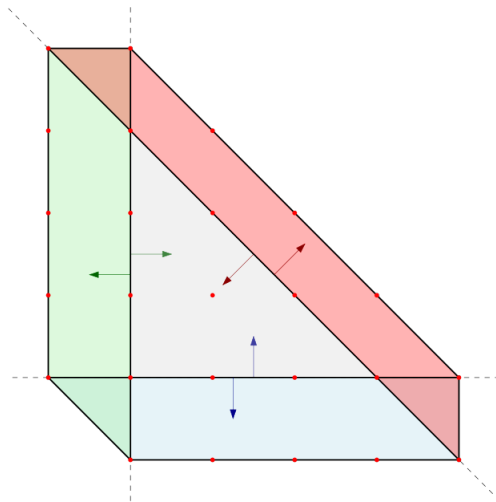
- ▶ What if we include both sides of each hyperplane?
- ▶ Get something unbounded² \implies # lattice points = ∞ .



²Corresponding to “hypertoric manifolds”.

Quantisation?

- Compactify the arrangement into a “polyptych”³.



³Coined by J. Martens.

A-Levels & Undergraduate

A-Levels

- ▶ During my A-Levels, I originally wanted to study chemistry.
- ▶ Decided that Physics and Further Maths would be beneficial for this.

Undergraduate Studies

- ▶ Eventually studied integrated Masters in Mathematics & Physics at The University of Warwick.
- ▶ In my 2nd Year, became interested in geometry because of its deep relationship with physics.

Postgraduate Studies

- ▶ Unsuccessful in my first round of PhD applications 😞.
- ▶ Stayed at Warwick for a MAST in Mathematics, to strengthen my mathematics.
- ▶ Received an offer for Edinburgh 😊.
- ▶ Met lots of lovely people.