Geometric Quantisation

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Women in STEM Society Talk

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Some Physics

Classical Mechanics

- In classical mechanics, the space of all possible states of a system is given by *phase space*, M.
- "*State*" describes the *position* and the *momentum*.

Quantum Mechanics

- In quantum mechanics, still have phase space M but states are replaced by *wavefunctions*.
- In my research, wavefunctions are just *homogeneous polynomials*, for example:

$$\phi(\mathbf{z}) = z_1^{k_1} z_2^{k_2} z_3^{k_3}, \qquad \text{where } k_1 + k_2 + k_3 = k.$$

Lattice Points

- Certain types of spaces M are special[1]: each has an associated polytope.
- Each lattice point inside correspond to a polynomial.

[1] *"Symplectic toric manifolds"*.

Quantisation Dimension

- Degree k of the polynomial analogous to quantised "energy" of the system.
- Quantisation dimension M equals the lattice point count (how many wavefunctions there are).

- Useful to realise that a polytope equals the intersection of half-spaces.

Hyperplane Arrangements

- What if we included both sides of the hyperplane?
- Get something unbounded[2].

[2] Corresponding to *"hypertoric manifolds"*.

Quantisation?

- But now, # lattice points $= \infty$!
- Compactify the arrangement into a *"polyptych"* [3].

[3] Coined by J. Martens.

A-Levels & Undergraduate

A-Levels

- During my A-Levels, I originally wanted to study chemistry.
- Decided that Physics and Further Maths would be beneficial for this.

Undergraduate Studies

- Eventually studied integrated Masters in Mathematics & Physics at The University of Warwick.
- In my 2nd Year, became interested in geometry because of its deep relationship with physics.

Postgraduate Studies

- Unsuccessful in my first round of PhD applications 😭.
- Stayed at Warwick for a MASt in Mathematics, to strengthen my mathematics.
- Received an offer for Edinburgh 😇.
- Met lots of lovely people.