

# Geometric Quantisation

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Women in STEM Society Talk



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### *Classical Mechanics*

- ▶ In classical mechanics, the space of all possible states of a system is given by *phase space*,  $M$ .
- ▶ “*State*” describes the *position* and the *momentum*.

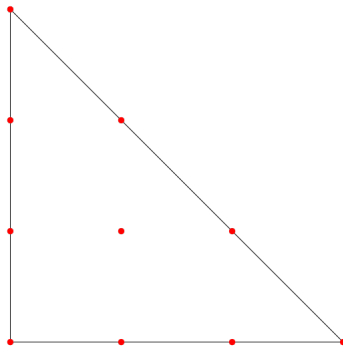
### *Quantum Mechanics*

- ▶ In quantum mechanics, still have phase space  $M$  but states are replaced by *wavefunctions*.
- ▶ In my research, wavefunctions are just *homogeneous polynomials*, for example:

$$\phi(\mathbf{z}) = z_1^{k_1} z_2^{k_2} z_3^{k_3}, \quad \text{where } k_1 + k_2 + k_3 = k.$$

## Lattice Points

- ▶ Certain types of spaces  $M$  are special<sup>1</sup>: each has an associated polytope.
- ▶ Each lattice point inside corresponds to a polynomial.

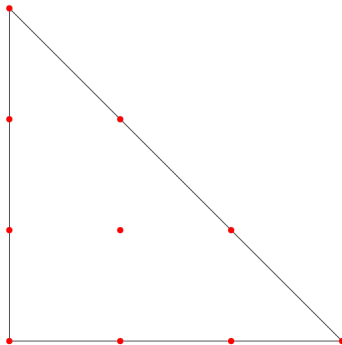


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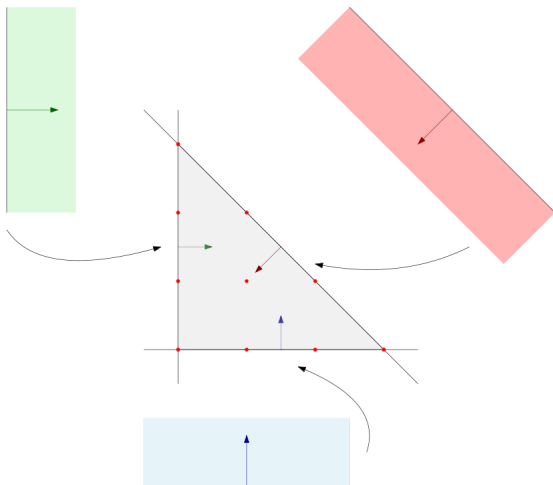
<sup>1</sup> “Symplectic toric manifolds”.

## Quantisation Dimension

- ▶ Degree  $k$  of the polynomial analogous to quantised “energy” of the system.
- ▶ Quantisation dimension  $M$  equals the lattice point count (how many wavefunctions there are).

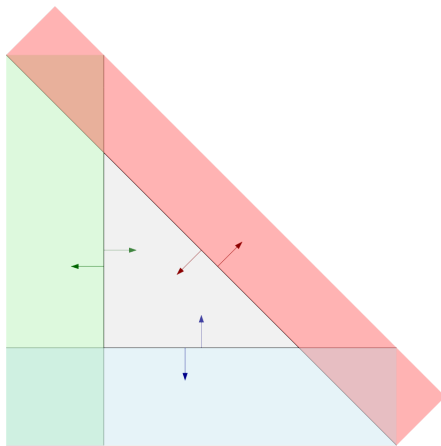


- Useful to view the polytope as an intersection of half-spaces.



## Hyperplane Arrangements

- ▶ What if we included both sides of each hyperplane?
- ▶ Get something unbounded<sup>2</sup>.

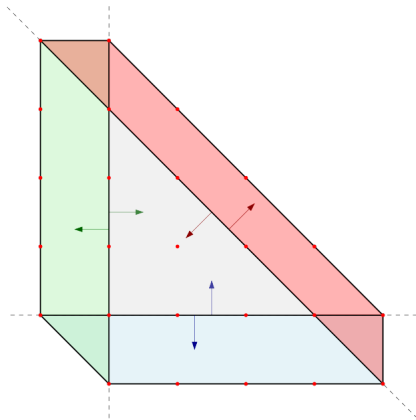


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<sup>2</sup>Corresponding to “hypertoric manifolds”.

## Quantisation?

- ▶ But now, # lattice points =  $\infty$ !
- ▶ Compactify the arrangement into a “polyptych”<sup>3</sup>.



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<sup>3</sup>Coined by J. Martens.

### A-Levels & Undergraduate

#### *A-Levels*

- ▶ During my A-Levels, I originally wanted to study chemistry.
- ▶ Decided that Physics and Further Maths would be beneficial for this.

#### *Undergraduate Studies*

- ▶ Eventually studied integrated Masters in Mathematics & Physics at The University of Warwick.
- ▶ In my 2nd Year, became interested in geometry because of its deep relationship with physics.



#### *Postgraduate Studies*

- ▶ Unsuccessful in my first round of PhD applications 😞.
- ▶ Stayed at Warwick for a MAST in Mathematics, to strengthen my mathematics.
- ▶ Received an offer for Edinburgh 😊.
- ▶ Met lots of lovely people.